CALCULUS
GRADE 12

THE EWING PUBLIC SCHOOLS
2099 Pennington Road
Ewing, NJ 08618

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In accordance with The Ewing Public Schools' Policy 2230, Course Guides, this curriculum has been reviewed and found to be in compliance with all policies and all affirmative action criteria.

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Course Description and Rationale

Calculus was invented by Isaac Newton, because in his day, mathematics fell short. For Newton it left unknowns and approximations. He invented calculus to apply a higher level of mathematics to answer things about the world that the math of his time could not.

In Calculus, students will learn how to apply math to a higher level to discover what Newton found.

The Ewing Public Schools' Math Vision

The Ewing Public Schools will deliver an instructional program in mathematics where students are actively engaged in the discovery of math concepts and are applying these concepts in ways that they find meaningful and relevant.

Ewing students will be mathematical thinkers who can reason, communicate and solve problems.

Ultimately, Ewing students will master and will be able to utilize these math concepts and skills throughout their lives.

21st Century Skills - During this course, students will work on developing, to an age appropriate level, the following 21st century skills:

Career Readiness Pathways:

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP12. Work productively in teams while using cultural global competence.

Learning and Innovation Skills

Creativity and Innovation

Think Creatively

- Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts

Work Creatively with Others

- View failure as an opportunity to learn; understand that creativity and innovation is a long-term, cyclical process of small successes and frequent mistakes

CRITICAL THINKING AND PROBLEM SOLVING

Reason Effectively

- Use various types of reasoning (inductive, deductive, etc.) as appropriate to the situation

Use Systems Thinking

- Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems

Make Judgments and Decisions

- Effectively analyze and evaluate evidence, arguments, claims and beliefs
- Synthesize and make connections between information and arguments
- Interpret information and draw conclusions based on the best analysis

Solve Problems

- Identify and ask significant questions that clarify various points of view and lead to better solutions

COMMUNICATION AND COLLABORATION

Communicate Clearly

- Articulate thoughts and ideas effectively using oral, written and nonverbal communication skills in a variety of forms and contexts
- Listen effectively to decipher meaning, including knowledge, values, attitudes and intentions
- Use communication for a range of purposes (e.g. to inform, instruct, motivate and persuade)
- Utilize multiple media and technologies, and know how to judge their effectiveness a priori as well as assess their impact
- Communicate effectively in diverse environments (including multi-lingual)

Collaborate with Others

- Assume shared responsibility for collaborative work, and value the individual contributions made by each team member

Information, Media, and Technology Skills

Informational Literacy

Access and Evaluate Information

- Evaluate information critically and competently

Use and Manage Information

- Use information accurately and creatively for the issue or problem at hand

Life and Career Skills

Social and Cross-Cultural Skills

Interact Effectively with Others

- Know when it is appropriate to listen and when to speak

Work Effectively in Diverse Teams

- Respond open-mindedly to different ideas and values

Be Responsible to Others

- Act responsibly with the interests of the larger community in mind

Technology Integration

8.1 Educational Technology
All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.

ELA Integration:

SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

Unit 1: Functions, Graphs and Concepts Leading to Limits (7 Days)

Why Is This Unit Important?

In this unit, previous learning on functions and their graphs will be extended upon to lay the foundation for the concepts of limits.

Enduring Understandings:

- The concept of absolute value applies to radicals and determining the distance between points on a coordinate line.
- Derive the formula for the distance between two points in the coordinate plane.
- Use derived distance formulas to study equations of circles.

Essential Questions:

- How is absolute value related to radicals and the distance between two points on a line?
- How is the distance formula derived from the Pythagorean Theorem?
- Standard form equations of a circle - how does this relate to the distance formula?

Acquired Knowledge: Students will understand the following:

- Absolute values
- Distance formula
- Pythagorean Theorem
- Midpoint Formula
- Standard form equations of a circle
- General equation of a circle

Acquired Skills:

- Write all ranges in set and interval notation
- Visualize algebraic equations as geometric curves
- Represent geometric curves by algebraic equations
- Find and work with slope of a line
- Recognize equations whose graphs are straight lines
- Find equations for lines specified geometrically

Differentiation:

Enrichment:

- Find the standard equation of the circle satisfying the given conditions.

Supplement:

- Define integer, rational or irrational number
- Write inequalities in interval notation
- Find absolute values

Assessments:

Formative Assessments:

- Assessment Checklist for Distance
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Functions, Graphs, and Concepts Leading to Limits Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

NJSLS.MP.1-8

Suggested Activities:

- Solve inequalities involving absolute values.
- Find x and y intercepts and determine whether a graph is symmetric about the x-axis, the y-axis or the origin.
- Sketch graph portions and using symmetry to complete the rest of the graph.
- Find the slope of the line through the two points.
- Find slope and its relation to angle of inclination – $m = \tan \theta$
- Determine motion along a line
- Graph equations
- Classify the lines as parallel, perpendicular or neither.
- How is the distance formula derived from the Pythagorean Theorem?
- Midpoint Formula
- Standard form equations of a circle – how does this relate to the distance formula?
- General equation of a circle
- Find the distance between A and B, and the midpoint of the line segment joining A and B.

Unit 2: Rates of Change and Limits (12 Days)

Why Is This Unit Important?

Newton looked at the world, trying to explain and quantify what he observed. Unfortunately, he often ran into difficulty because mathematics was only able to aid in his descriptions to certain points. Often, approximation processes had to be utilized to find critical values. This led him to push the boundaries of mathematics and invent new approaches which led to an area of mathematics called *calculus*. In this unit, rates of change will be explored, discovering limits and the need to push math beyond what has been previously developed.

Enduring Understandings:

- Find the average and instantaneous rate of change of a function
- Find tangents and normal to a curve

Essential Questions:

- There is an appealing geometrical interpretation of the average rate of change. Can you figure it out?
- How do you evaluate a limit?
- How does the slope of the secant compare to the slope of the tangent as Q approaches P?

Acquired Knowledge: Students will understand the following:

- Continuity of polynomial functions
- Properties of continuous functions
- Continuity of rational functions
- Continuity of composite functions
- Types of discontinuity
- The Intermediate Value Theorem
- Normal to a Curve

Acquired Skills:

- Calculate instantaneous rate of change and average rate of change
- Compare and contrast instantaneous rate of change versus average rate of change
- Evaluate limits as x goes to infinity, using graphical and algebraic techniques
- Identify vertical and horizontal asymptotes
- Find and verify end behavior models for various functions
- Know and understand the concept of continuity and its properties
- Classify continuity as removable, jump, infinite, or oscillating
- Use the definition of continuity to find points of continuity and discontinuity
- Remove a discontinuity

Differentiation:

Enrichment:

- Use a given graph to compute the average rate of change and successive approximations of the slope of the secant line PQ

Supplement:

- Analyze graphs that describe the result of some scientific observation (the measurement of the value of the variable y at different times t)

Assessments:

Formative Assessments:

- Assessment Checklist for Average Versus Instantaneous
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Rates of Change and Limits Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

NJSLS.MP.1-8

Suggested Activities:

- The Tangent Line Problem
- The Instantaneous Velocity Problem
- Investigate the behavior of the function f (x) near a given value of x
- Functions that do not reach their intended height
- Limits that cannot be verified algebraically
- Right-Hand Limit and Left-Hand Limit
- Existence of a Limit
- Properties of Limits
- Limits of Rational Functions:
- How do you evaluate a limit?
  - Substitution
  - Factoring
  - Conjugate
- The Sandwich Theorem
- Examine the behavior of f(x) = 1/x and determine limits as x goes to + or minus infinity
- Find End Behavior Models
- Identify points of discontinuity
- Write a conjecture that explain the causes the discontinuity
- Explain Slope of a Curve at a Point

Unit 3: Derivative of a Function (25 Days)

Why Is This Unit Important?

Newton knew that the slope of a tangent on a position-time graph would yield the object's instantaneous velocity at that point in time, a value that can never truly be directly measured. Unfortunately, the mathematical processes available could only provide an approximate value (which ironically can be measured). Inventing the use of derivatives enabled Newton to find the exact values he desired. In this unit, the concept, calculation and use of derivatives will be learned.

Enduring Understandings:

- Find the derivative of a composite function by using the chain rule, repeated use of the chain rule and the power chain rule
- Use implicit differentiation to find the derivative of a function

Essential Questions:

- Does the limit of the difference quotient exist at the point(s) of discontinuity?
- Is it possible to draw a discontinuous function where this limit exists at the point of discontinuity?
- Is it possible to draw continuous functions that fail to have a tangent at a given point?

Acquired Knowledge: Students will understand the following:

- Power Rule
- The Constant Multiple Rule
- The Sum and Difference Rule
- The Product Rule
- The Quotient Rule
- Power Rule for Negative Powers
- Chain Rule
- Outside-Inside Rule
- Implicit Differentiation Process

Acquired Skills:

- Understand the definition of the derivative of a function
- Find the derivative of a function by using the definition
- Compare the graph of f and f' and determine how they relate
- Recognize when the derivative might fail to exist
- Interpret differentiability as local linearity
- Know how differentiability relates to continuity
- Estimate derivatives using a graphing calculator
- Use the constant, power, sum, difference, product and power rules to find the derivative of a function
- Use instantaneous rate of change to model problem situations
- Find the derivative of trigonometric functions
- Apply this concept to solve real life problems
- Recognize functions that are defined implicitly

Differentiation:

Enrichment:

- Use the change of base formula to find an expression for the derivative of $y = \log_a x$

Supplement:

- Definition of Derivative (Difference quotient)

Assessments:

Formative Assessments:

- Assessment Checklist for Chain Rule
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Derivative of a Function Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

NJSLS.MP.1-8

Suggested Activities:

- Alternate Definition of Derivative at a Point
- Relationship between the Graphs of f and f’
- Go Off on a Tangent
- The linear nature of graphs
- Derivative of the Sine and Cosine Functions
- Derivative of the other Trigonometric Functions
- Find the derivative of f(x) by applying function composition
- Diagram that illustrates the derivative of a composition of functions
- Implicitly Defined Functions
- Find a rule for the derivative of the function f(x) = ex

Unit 4: Extreme Values of a Function to Related Rates (25 Days)

Why Is This Unit Important?

In this unit, the use of derivatives will be applied to delve into analyzing graphical descriptions of phenomena.

Enduring Understandings:

- Find an equation that relates the corresponding rates of change of any equation involving two or more variables
- Design and apply a solution strategy to related rates problems

Essential Questions:

- Can endpoints give relative extrema?
- What happens when the hypotheses of the theorem are not met?

Acquired Knowledge: Students will understand the following:

- The Extreme Value Theorem
- Definition of Critical Point
- Relative Maximum
- Relative Minimum
- Rolle's Theorem
- The Mean Value Theorem
- Point of Inflection
- Second Derivative Test for Local Extrema
- Maximum Profit Theorem
- Minimum Cost Theorem

Acquired Skills:

- Understand the difference between global extrema and relative extrema
- Find the extreme values of a function by using the first derivative test
- Understand and apply Rolle's theorem
- Understand and use the Mean Value Theorem
- Determine similarities and differences between both theorems
- Determine intervals of increase and decrease by using the first and second derivative test
- Investigate the concavity of a function by using the first and second derivative test
- Use the first and second derivative to graph polynomial and rational function
- Use optimum dimensions when designing a geometric figure by using the first derivative to determine the critical dimensions
- Given the profit function, to determine the production level at which the maximum profit occurs
- Given the cost function, to determine the production level at which the minimum cost occurs

Differentiation:

Enrichment:

- Use the critical points of f along with the derivative tests to construct a sign chart for f’ and f

Supplement:

- Definition of Absolute Extreme Values

Assessments:

Formative Assessments:

- Assessment Checklist for Rates of Change
- Teacher’s observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students’ recording sheets

Summative Assessments:

- Teacher’s observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Extreme Values of a Function to Related Rates Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

NJSLS.MP.1-8

Suggested Activities:

- Find Extrema
- Find Extrema Using Graphical Methods
- Interpret the mean value theorem with a real life example
- Derivative test for increasing and decreasing function
- Use the Mean Value Theorem to proof the derivative test
- Use the Mean Value Theorem to show that functions with the same derivative differ by a constant
- Examine the change of the tangent line

Unit 5: Integration (17 Days)

Why Is This Unit Important?

Newton knew that the area under the curve on a velocity-time graph would yield the object's displacement over that range of time. Unfortunately, the mathematical processes available could only provide an approximate value. Inventing the use of integrals enabled Newton to find the exact values he desired. In this unit, the concept, calculation and use of integrals will be learned.

Enduring Understandings:

- Find an integral by using anti-derivatives
- Solve differential equations using separation of variables

Essential Questions:

- What variable determines the value of the integral?
- Can you see why Newton and Leibniz concluded that $F'(x) = f(x)$?

Acquired Knowledge: Students will understand the following:

- Express Limits as Integrals
- Properties of Definite Integrals
- Mean Value Theorem for Definite Integrals
- The Integral as the Anti-derivative of f
- The Average Value of a Function
- The Fundamental Theorem of Calculus

Acquired Skills:

- Know and apply the rules for Definite Integrals
- Understand and apply the Average Value of a Function
- Understand and apply the Mean Value Theorem for Definite Integrals
- Find an integral by using anti-derivatives
- Turn an unfamiliar integral into one that can be evaluated by using substitution
- Solve differential equations using separation of variables

Differentiation:

Enrichment:

- Find a rule for the derivative of the function $f(x) = e^x$
- Derivative of a^x.
- Derivative of ln x

Supplement:

- Exploration: Finding the Derivative of an Integral

Assessments:

Formative Assessments:

- Assessment Checklist for Anti-Derivatives
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Integration Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

NJSLS.MP.1-8

Suggested Activities:

- Find bounds for an integral: Applying the Fundamental Theorem
- Graph the Function: $\int f(x)\,dt$
- Explore The Effect of Changing the lower limit in $\int_a^x f(t)dt$
- Evaluation of Integrals Using Anti-Derivatives
- Find the area of the region between curves analytically
- Find the area of the region between curves using a graphing calculator
- Use Substitution
- Substitution in Indefinite Integrals
- Substitution in definite Integrals
- Solve Differential Equations by Separation of Variables and Substitution

Sample Standards Integration

21st Century Skills & Career Readiness Practices

CRP4. Communicate clearly and effectively and with reason.

For example, in Unit 1 students will justify their reasoning in their choice of solution pathways involving analysis of functions

CRP6. Demonstrate creativity and innovation.

For example, in Unit 4 students will apply graphical analysis tools to describe real world phenomena.

CRP7. Employ valid and reliable research strategies.

For example, in Unit 3 students learn how to use derivatives to determine instantaneous rates that defy direct measurement in the real world.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

For example, in Unit 5 students will work to solve data distributions and what they tell us about what is really occurring.

CRP12. Work productively in teams while using cultural global competence.

For example, in Unit 2 students will work in small teams to develop Analysis of rates of change and limits.

8.1 Educational Technology

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.

For example, in Unit 3 students will access, manage, evaluate, and synthesize information to develop models for calculating instantaneous rates.

Interdisciplinary Connections

SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

These standards are met throughout the course. For example, in Unit 6 students will discuss their solutions to a variety of real world phenomena justifying their descriptions.