College Preparatory Integrated Mathematics Course I Notebook

This notebook is based on

Marecek, L., & Honeycutt Mathis, A. (2020). *Intermediate Algebra* (2 ed.). Houston, TX: OpenStax. Retrieved from https://openstax.org/details/books/intermediate-algebra-2e

Marecek, L., Anthony-Smith, M., & Honeycutt Mathis, A. (2020). *Prealgebra* (2 ed.). Houston, TX: OpenStax. Retrieved from https://openstax.org/details/books/prealgebra-2e

Notebook developed by

Edith Aguirre, El Paso Community College
Fan Chen, El Paso Community College
Ivette Chuca, El Paso Community College
Sandra Cuevas, El Paso Community College
Shahrbanoo Daneshtalab, El Paso Community College
Lorena Gonzalez, El Paso Community College
Jose Ibarra, Ysleta Independent School District

College Preparatory Integrated Mathematics
Course I Notebook

TABLE OF CONTENTS

UNIT I

I. Identify and apply properties of real numbers and perform accurate arithmetic operations with numbers in various formats and number systems. Apply basic geometric theorems and formulas

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome I

Learning Objective I.1: Add, subtract, multiply and divide, using order of operations, real numbers and manipulate certain expressions including exponential operations.

Learning Objective I.1.1: Order of operations
Read Section 1.1 on pages 12-14 in the textbook and answer the questions below.

Definitions

1. In the expression 2^3, the 2 is called the ______________ and the 3 is called the ________________.
2. The symbols ( ), [ ], and { } are examples of _________________ symbols.
3. ____________________notation may be used to write $2 \cdot 2 \cdot 2$ as 2^3.
4. **Order of Operations**: Simplify expressions using the order below.
   1. If grouping symbols such as ______________ are present, simplify expressions within those first, starting with the innermost set.
   2. Evaluate ___________________expressions.
   3. Perform ___________________or _____________________in order from left to right.
   4. Perform__________________ or______________________in order from left to right.

Example 1: Simplify each expression.

a) $18 \div 6 + 4(5 - 2)$

b) $30 \div 5 + 10(3 - 2)$

c) $9 + 5^3 - [4(9 + 3)]$

d) $5 + 2^3 + 3[6 - 3(4 - 2)]$

Example 2: Simplify each expression.

a) $\left(\frac{3}{5}\right)^2 \cdot |-5|$

b) $\frac{2(15-6)}{|-3|}$

c) $\frac{4^2-6}{1+|3-2|\cdot 4}$

d) $3[20 - 2(5 - 3)]$

College Preparatory Integrated Mathematics Course I

Learning Objective I.1.2: Evaluating Algebraic Expressions

Read section 1.1 on pages 10 and 15 in the textbook and answer the questions below.

Definitions

1. A symbol that is used to represent a number is called a ___________________.
2. A number whose value always remains the same is called a ___________________.
3. An ____________________expression is a collection of numbers, variables, operation symbols, and grouping symbols.
4. If we give a specific value to a variable, we can _______________ an algebraic expression.

Example 3: Evaluate each expression if $x = 3$.

a) x^2

b) 4^x

c) $3x^2 + 4x + 1$

Example 4: Evaluate each expression if $x = 3$ and $y = 5$.

a) $2x + y$

b) $\frac{4x}{3y}$

c) $\frac{3}{x} + \frac{y}{5}$

d) $x^3 + y^2$

College Preparatory Integrated Mathematics Course I

Learning Objective I.1.3: Determining Whether a Number is a Solution of an Equation

Definitions

1. An *equation* is a mathematical statement that two expressions have equal value. The equal symbol "=" is used to equate the two expressions.
2. A *solution* of an equation is a value of a variable that makes a true statement when substituted into the equation.

Example 5: Decide whether 3 is a solution of $5x - 9 = 2x$

Learning Objective I.1.4: Translating English Phrases to an Algebraic Expression

Read section 1.1 on page 18 in the textbook to fill the table below.

Keywords

Addition (+)	Subtraction (-)	Multiplication (·)	Division (÷)

Example 6: Translate the English phrase into an algebraic expression.

a. The difference of $14x$ and 9

b. The quotient of $8y^2$ and 3

c. Twelve more than y

d. Seven less than $49x^2$

e. The difference of two times x and 8

f. Two times the difference of x and 8

Example 7: Write an algebraic expression that represents each phrase. Let the variable x represent the unknown number.

a. Five times a number

b. The product of a number and 8

c. The sum of 9 and a number

d. A number decreased by 4

e. Two times a number, plus 7

Example 8: The width of a rectangle is 6 less than the length. Let L represent the length of the rectangle. Write an expression for the width of the rectangle**.**

Example 9: Write each sentence as an equation or inequality. Let x represent the unknown number.

a) A number is increased by 4 is equal to 17.

b) Two less than a number is 15.

c) Double a number, added to 5, is not equal to 40.

d) Five times 8 is greater than or equal to an unknown number.

Learning Objective I.1.5: Adding Real Numbers

Definitions

1. **Adding Two Numbers with the Same Sign**
   Add their absolute values. Use their common signs as the sign of the sum.

2. **Adding Two Numbers with Different Signs**
   Subtract the *smallest* absolute value from the *largest* absolute value. Use the sign of the number whose absolute value is *larger* as the sign of the sum.

Example 10: Add.

a) $(-3) + (-7)$

b) $3 + (-7)$

c) $-3 + 7$

d) $(-0.8) + 0.3$

Example 11: Add.

a) $-\frac{1}{4} + \left(-\frac{1}{2}\right)$

b) $(-3) + (-2) + (-9)$

c) $19 - |11 - 4(3 - 1)|$

Example 12: If the temperature was -10° Fahrenheit at 4 a.m., and it rose 8 degrees by 7 a.m and then rose another 5 degrees in the hour from 7 a.m. to 8 a.m., what was the temperature at 8 a.m.?

Learning Objective I.1.6: Finding the Opposite of a Number

Definitions

1. Two numbers that are the same distance from 0 but lie on opposite sides of 0 are called *opposite* or *additive inverses* of each other.
3. If a is a number, then $-(-a) = a$.
4. The *opposite* of a number a and its opposite $-a$ is 0. $a + (-a) = 0$

Example 13: Find the opposite or additive inverse of each number.

a) $-\frac{8}{12}$ b) 4 c) -2.7
d) 6

Example 14: Simplify each expression.

a) $-(-4)$ b) $-|-2|$ c) $-(-2x)$ d) $-(-\frac{2}{5})$

Learning Objective I.1.7: Subtracting Real Numbers
Read Section 1.2 on page 31 in the textbook and answer the questions below.

Definitions

If a and b are real numbers, then $a - b =$ _____________.

Example 15: Subtract.

a) $4 - 6$ b) $-6 - (-4)$ c) $-6 - 4$ d) $6 - (-4)$

Example 16: Subtract.

a) $-\frac{3}{7}-\left(-\frac{4}{7}\right)$ b) $8-(-3-1)-9$ c) $-2.6+5-(-3.7)$

Example 17: Subtract 7 from -5.

Example 18: Simplify each expression.

a) $-11+[(-4-7)-3^2]$ b) $|-15|-(-5)+[2-(-6)]$

Example 19: Find the value of each expression when $x=-2\ and\ y=5$.

a) $\frac{3-x}{y+x}$ b) x^2-y

Example 20: The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

Learning Objective I.1.8: Multiplying Real Numbers
Read Section 1.2 on page 33 in the textbook and answer the questions below.

Definitions

1. The product of two numbers with the ___________sign is a positive number.
2. The product of two numbers with ________________signs is a negative number.
3. If a is a real number, then $a \cdot 0 = 0$

Example 21: Subtract.

a) $4(-5)$

b) $(-7)(-2)$

c) $(-3)(9)$

Example 22: Subtract.

a) $\left(-\frac{6}{7}\right) \cdot \left(-\frac{2}{9}\right)$

b) $\left(-\frac{3}{8}\right)(-24)$

c) $(-8)(-3) - (-5)(2)$

Example 23: Evaluate.

a) $(-5)^2$

b) -5^2

c) $(-2)^3$

d) -2^3

College Preparatory Integrated Mathematics Course I

Learning Objective I.1.9: Dividing Real Numbers

Read Section 1.2 on page 33 in the textbook and answer the questions below.

Definitions

1. Two numbers whose product is 1 are called *reciprocals* or multiplicative inverses of each other.
2. If *a* and *b* are real numbers and *b* is not 0, then $a \div b = \frac{a}{b}$ ($\frac{a}{0}$ is *undefined*)
3. The quotient of zero and any real number except 0 is 0. ($\frac{0}{b} = 0$)
4. The product or quotient of two numbers with the same sign is a ______________________number.
5. The product or quotient of two numbers with different signs is a ______________________ number.

Example 24: Divide.

a) $\frac{-18}{-9}$ b) $-\frac{39}{3}$ c) $\frac{8}{3} \div \left(-\frac{2}{9}\right)$ d) $-\frac{3}{16} \div 6$

Example 25: Simplify each expression.

a) $\frac{8(-2)^2+4(-3)}{-5(2)+3}$ b) $\frac{(-6)(-11)-1}{-9-(-4)}$

Example 26: A card player had a score of -13 for each of the four games. Find the total score.

College Preparatory Integrated Mathematics Course I

Learning Objective I.1.10: Using Commutative, Associative, and Distributive Properties
Read Section 1.5 on page 74 in the textbook and answer the questions below.

Definitions

If a and b are real numbers, then:

1. **Commutative Properties**
   Addition: $a + b = b + a$
   Multiplication: $a \cdot b = b \cdot a$
2. **Associative Properties**
   Addition: $(a + b) + c = a + (b + c)$
   Multiplicative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
3. **Distributive Property**
   $a(b + c) = ab + ac$
4. **Identity Property**
   Addition: $a + 0 = 0 + a = a$
   Multiplicative: $a \cdot 1 = 1 \cdot a = a$

Example 27: Simplify each expression.

a) $9 - (2 + x)$ b) $5(-3x) + 2$ c) $-3(x - y) + 5x - y$

Example 28: Simplify each expression.

a) $-2(3x - 7y + z)$ b) $\frac{1}{2}(6x - 2) + 5x$ c) $37m + 21n + 4m - 15n$

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date:_____________________

Learning Objective I.1

To check your understanding of the section, work out the following exercises.

1. Simplify each expression.

a) $2-5[-3(1-7)-(5-2)]$

b) $63 \div (-9)+(-36) \div (-4)$

c) $\frac{-(-2)^2-4(2-3)}{-5(1-3)-3^2}$

d) $\frac{(-6)(-1)-3(-1)^3}{-7-(-4)}$

e) $12 \cdot \frac{3}{4}\,(-8+1)-11$

f) $\left(\frac{1}{5}+\frac{8}{15}\right)-2(\frac{4}{15}-\frac{2}{5})$

2. Simplify each expression.

a) $-5(-2x+3)-6x$

b) $-2(3b-a)-4(a-b)$

c) $-(x-y)+x-y$

d) $(t-2y)-5(t-y)$

e) $-2(t+2x-3)+5t-x$

f) $(3x+1)-5(x-y+2)$

3. Find the value of each expression when $x=-3$, $y=2, and\ z=-1$.

a) $-2x-(y-5z)$

b) $x^2-x.y+z^3$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome I

Learning Objective I.2: Find square roots of perfect square numbers
Read Section 1.4 on page 66 in the textbook and answer the questions below.

Definitions

1. The numbers such as 1, 4, 9, and 25 are called ______________squares.
2. The opposite of squaring a number is taking the __________________ of a number.
3. The notation $\sqrt{a}$ is used to denote the _____________, or principal, square root of a nonnegative number *a.*

Example 1: Find the square roots.

a) $\sqrt{36}$ b) $\sqrt{169}$ c) $-\sqrt{225}$ d) $\sqrt{121}$

Example 2: Find the square roots.

a) $\sqrt{100}$ b) $\sqrt{\frac{1}{25}}$ c) $-\sqrt{64}$ d) $\sqrt{-64}$

Example 3: Simplify each expression.

a) $10 \div (\sqrt{144} - 8 - 3)$ b) $\frac{\sqrt{81}}{50 \div 10 - 2}$

Example 4: Simplify. Assume that all variable represent positive numbers.

a) $\sqrt{x^8}$ b) $\sqrt{9b^4}$ c) $-2\sqrt{a^4}$ d) $5b\sqrt{4b^6}$

Example 5: Use a calculator to approximate$\sqrt{57}$. Round the approximation to three decimal places and check to see that your approximation is reasonable.

College Preparatory Integrated Mathematics Course I

Name: ________________________________Date:_____________________

Learning Objective I.2

To check your understanding of the section, work out the following exercises.

1. Simplify each expression.

a) $\sqrt{121}$

b) $-2\sqrt{81}$

c) $-\frac{1}{5}\sqrt{225}$

d) $3-2\sqrt{121}$

e) $\frac{\sqrt{25}}{\sqrt{16}}$

f) $\sqrt{\frac{1}{36}}$

2. Simplify each expression.

a) $2\sqrt{25} \div (\sqrt{64}-\sqrt{9}-3)$

b) $\frac{-2\sqrt{4}}{\sqrt{100}\div 10-2}$

3. Simplify. Assume that all variable represent positive numbers.

a) $-5\sqrt{4a^6}$

b) $2\sqrt{121t^4}$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome I

Learning Objective I.3: Solve problems involving calculations with Percentage and interpret the result.
Read Section 2.2 on page 116 and write down the seven General Strategies for Problem Solving.

Definitions

General Strategy for Problem Solving

1.
2.
3.
4.
5.
6.
7.

Example 1: The number 35 is what percent of 56?

Example 2: The number 198 is 55% of what number?

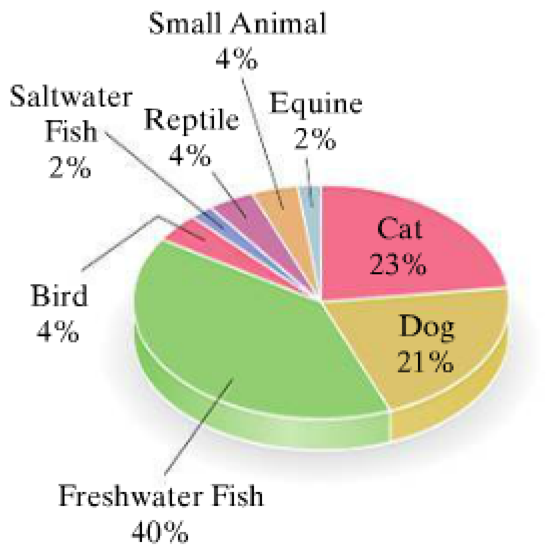
Example 3: 7.5% of what amount is $1.95?

Example 4: One serving of wheat square cereal has 7 grams of fiber, which is 28% of the recommended daily amount. What is the total recommended daily amount of fiber?

Example 5: Mitzi received some gourmet brownies as a gift. The wrapper said each 28% brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat? Round the answer to the nearest whole percent.

Example 6: **Use the circle graph to answer each question.**

Pets Owned in the United States



Data from American Pet Products Association's Industry Statistics and Trends results

a) What percent of pets owned in the United States are freshwater fish or saltwater fish?

b) What percent of pets owned in the United States are not Reptile?

c) Currently, 377.41 million pets are owned in the United States. How many of these would be cats? (Round to the nearest tenth of a million.)

College Preparatory Integrated Mathematics Course I

Name: ________________________________Date:_____________________

Learning Objective I.3

To check your understanding of the section, work out the following exercises.

1. The number 110 is what percent of 88?

2. 8.5 % of what number is $3.06 ?

3. What number is 45% of 80 ?

4. One serving of rice has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

5. The mix Ricardo plans to use to make brownies says that each brownie will be 190 calories, and 76 calories are from fat. What percent of the total calories are from fat? Round the answer to the nearest whole percent.

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome I

(Section is from Prealgebra 2e (Openstax))

Learning Objective I.4: Use estimation skills, and know why, and when to estimate results.

Learning Objective I.4.1: Solve Sales Tax and total cost applications
Read Section 6.3 on page 546 Book Prealgebra 2e (Openstax)

Example 1: Alexandra bought a television set for \$724 in El Paso, where the sales tax rate was 8.25% of the purchase price.

a) Estimate the sales tax.
b) Calculate the sales tax.

Example 2: Kim bought a winter coat for \$250 in St. Luis, where the tax rate is 8.2% of the purchase price.

a) Estimate the sales tax and the total cost.
b) Calculate the sale tax and the total cost.

Example 3: Diego bought a new car for \$26,525. He was surprised that the dealer then added \$2,387.25.

a) Estimate the sales tax rate for this purchase.
b) Calculate the sales tax rate for this purchase.

Learning Objective I.4.2: Solve Discount and Mark-Up applications
Read Section 6.3 on page 551 Book Prealgebra 2e (Openstax)

Example 4: Marta bought a dishwasher that was on sale for 25% off. The original price of the dishwasher was \$525.

a) Estimate the amount of discount and the sale price before tax.
b) Calculate the amount of discount and the sale price before tax.

Example 5: Lena bought a kitchen table at the sale price of \$375.20. The original price of the table was \$560.

a) Estimate the amount of discount.
b) Calculate the amount of discount.

Example 6: A used treadmill, originally purchased for \$480, was sold at a garage sale at a discount of 85% of the original price.

a) Estimate the amount of discount and the new price.
b) Calculate the amount of discount and the new price.

Name: _______________________________Date:____________________

Learning Objective I.4

To check your understanding of the section, work out the following exercises.

1) John bought a smartphone set for $540 in Boston, where the sales tax rate was 6.25% of the purchase price.
    a) Estimate the sales tax.
    b) Calculate the sales tax.

2) Lee bought a TV coat for $1350 in TX, where the tax rate is 8.25% of the purchase price.

    a) Estimate the sales tax and the total cost.
    b) Calculate the sale tax and the total cost.

3) Jose purchased a piano for $7,594 with $569.55 of sales tax added to it.
    a) Estimate the sales tax rate for this purchase.
    b) Calculate the sales tax rate for this purchase.

4) Mike bought a computer that was on sale for 35% off. The original price of the computer was $1,999.
    a) Estimate the amount of discount and the sale price before tax.
    b) Calculate the amount of discount and the sale price before tax.

5) Mia bought a dress at the sale price of $125.99. The original price of the table was $499.99
    a) Estimate the amount of discount.
    b) Calculate the amount of discount.

6) A used car, originally purchased for $32,500, was sold at a discount of 68% of the original price.
    a) Estimate the amount of discount and the new price.
    b) Calculate the amount of discount and the new price.

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome I

Learning Objective I.5: Find the perimeter and area of rectangles, squares, parallelograms, triangles, trapezoids and circles; volume and surface area, relations between angle measures, congruent and similar triangles, and properties of parallelograms. PreAlgebra e2 Pages 747 – 839 textbook available in OpenStax.

Angles and Similar Triangles

Learning Objective I.5.1: Relations between angle measures, congruent and similar triangles

Read Textbook(PreAlgebra e2) Section 9.3 on page 747 and answer the questions below.

BEPREPARED: Before you get started, try:

1. Solve $x + 3 + 6 = 11$
2. Solve $\frac{a}{45} = \frac{4}{3}$
3. Simplify $\sqrt{36 + 64}$

Example 1: An angle measure 25°. Find its

(a) supplement

(b) complement

Example 2: An angle measure 77°. Find its

(a) supplement

(b) complement

Example 3: Two angles are supplementary. The larger angle is 100° more than the smaller angle. Find the measures of both angles.

Example 4: Two angles are supplementary. The larger angle is 40° more than the smaller angle. Find the measures of both angles.

Example 5: The measures of two angles of a triangle are 31° and 128°. Find the measure of the third angle.

Example 6: The measures of two angles of a triangle are 49° and 75°. Find the measure of the third angle.

Example 7: One angle of a right triangle measures 56°. What is the measure of the other angle?

Example 8: One angle of a right triangle measures 45°. What is the measure of the other angle?

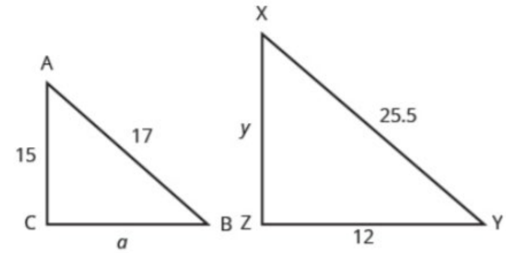
Example 9: One angle of a right triangle measures 56°. What is the measure of the other angle?

Example 10: One angle of a right triangle measures 45°. What is the measure of the other angle?

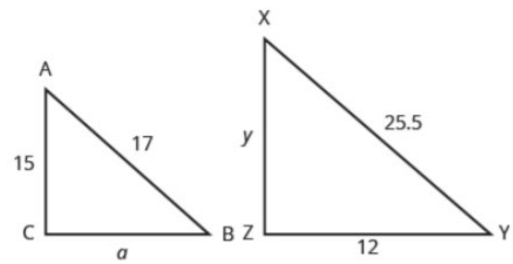
Example 11: The measure of one angle of a right triangle is 50° more than the measure of the smallest angle. Find the measures of all three angles.

Example 12: The measure of one angle of a right triangle is 30° more than the measure of the smallest angle. Find the measures of all three angles.

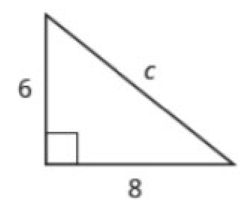
Example 13: ΔABC is similar to ΔXYZ. Find a.



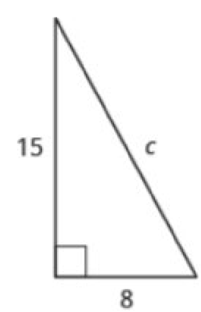
Example 14: ΔABC is similar to ΔXYZ. Find y.



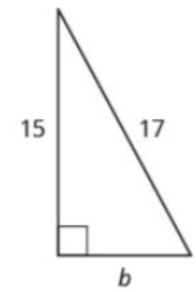
Example 15: Use the Pythagorean Theorem to find the length of the hypotenuse.



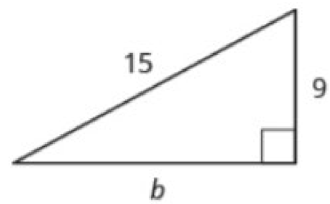
Example 16: Use the Pythagorean Theorem to find the length of the hypotenuse.



Example 17: Use the Pythagorean Theorem to find the length of the leg.

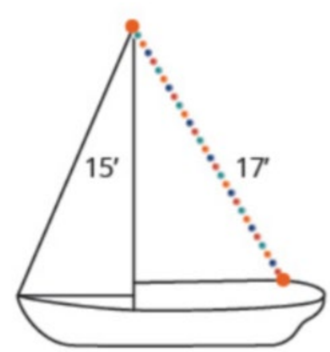


Example 18: Use the Pythagorean Theorem to find the length of the leg.



Example 19: John puts the base of a 13-ft ladder 5 feet from the wall of his house. How far up the wall does the ladder reach?

Example 20: Randy wants to attach a 17-ft string of lights to the top of the 15-ft mast of his sailboat. How far from the base of the mast should he attach the end of the light string?



College Preparatory Integrated Mathematics Course I

Name: ____________________________ Date:__________________

Learning Objective I.5.1

To check your understanding of the section, work out the following exercises.

1. Find the supplement and the complement of the given angles.
   a) 81° b)53° c) 16° d) 29°

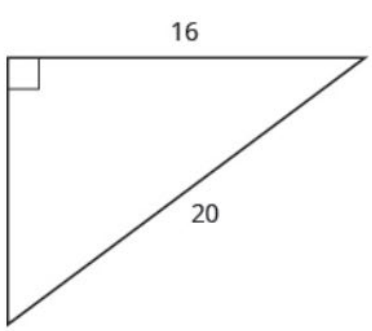
2. Two angles are supplementary. The larger angle is 56° more than the smaller angle. Find the measures of both angles.

3. The measures of two angles of a triangle are 26° and 98°. Find the measure of the third angle.

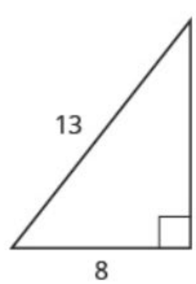
4. One angle of a right triangle measures 33°. What is the measure of the other angle?

5. Use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

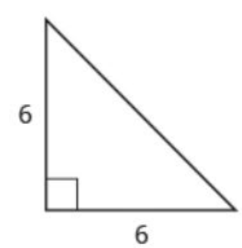
a)



b)



c)



College Preparatory Integrated Mathematics Course I

Rectangles, Triangles, and Trapezoids

Learning Objective I.5.2: Use properties of rectangles, triangles, and trapezoids

Read Textbook (PreAlgebra e2) Section 9.4 on page 774 and answer the questions below.

BEPREPARED: Before you get started, try:

1. The length of a rectangle is 3 less than the width. Let w represent the width. Write an expression for the length of the rectangle
2. Simplify: $\frac{1}{2}(6h)$
3. Simplify $\frac{5}{2}(10.3 - 7.9)$

Example 1: Determine whether you would use linear, square, or cubic measure for each item.

ⓐ amount of paint in a can

ⓑ height of a tree

ⓒ floor of your bedroom

ⓓ diameter of bike wheel

ⓔ size of a piece of sod

ⓕ amount of water in a swimming pool

Example 2: Determine whether you would use linear, square, or cubic measure for each item.

ⓐ volume of a packing box

ⓑ size of patio

ⓒ amount of medicine in a syringe

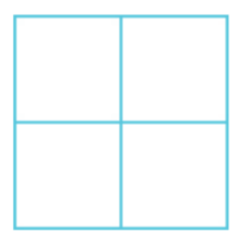
ⓓ length of a piece of yarn

ⓔ size of housing lot

ⓕ height of a flagpole.

Example 3: Each box in the figure below is 1 square inch. Find the ⓐ perimeter and ⓑ area of the figure:

Example 4: Each box in the figure below is 1 square inch. Find the ⓐ perimeter and ⓑ area of the figure:



Example 5: The length of a rectangle is 120 yards and the width is 50 yards. Find ⓐ the perimeter and ⓑ the area.

Example 6: The length of a rectangle is 62 feet and the width is 48 feet. Find ⓐ the perimeter and ⓑ the area.

Example 7: Find the length of a rectangle with a perimeter of 80 inches and width of 25 inches.

Example 8: Find the length of a rectangle with a perimeter of 30 yards and width of 6 yards.

Example 9: The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Example 10: The width of a rectangle is eight fee more than the length. The perimeter is 60 feet. Find the length and width.

Example 11: The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Example 12: The width of a rectangle is 21 meters. The area is 609 square meters. What is the length?

Example 13: Find the area of a triangle with base 13 inches and height 2 inches.

Example 14: Find the area of a triangle with base 14 inches and height 7 inches.

Example 15: The perimeter of a triangular garden is 48 feet. The lengths of two sides are 18 feet and 22 feet. How long is the third side?

Example 16: The lengths of two sides of a triangular window are 7 feet and 5 feet. The perimeter is 18 feet. How long is the third side?

Example 17: The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Example 18: The area of a triangular painting is 15 square feet. The height is 5 feet. What is the height?

Example 19: Find the length of each side of an equilateral triangle with perimeter 39 inches.

Example 20: Find the length of each side of an equilateral triangle with perimeter 51 centimeter.

Example 21: A backyard deck is in the shape of an isosceles triangle with a base of 20 feet. The perimeter of the deck is 48 feet. How long is each of the equal sides of the deck?

Example 22: A boat's sail is an isosceles triangle with base of 8 meters. The perimeter is 22 meters. How long is each of the equal sides of the sail?

Example 23: The height of a trapezoid is 14 yards and the bases are 7 and 16 yards. What is the area?

Example 24: The height of a trapezoid is 18 centimeters and the bases are 17 and 8 centimeters. What is the area?

Example 25: The height of a trapezoid is 7 centimeters and the bases are 4.6 and 7.4 centimeters. What is the area?

Example 26: The height of a trapezoid is 9 meters and the bases are 6.2 and 7.8 meters. What is the area?

Example 27: Lin wants to sod his lawn, which is shaped like a trapezoid. The bases are 10.8 yards and 6.7 yards, and the height is 4.6 yards. How many square yards of sod does he need?

Example 28: Kira wants cover his patio with concrete pavers. If the patio is shaped like a trapezoid whose bases are 18 feet and 14 feet and whose height is 15 feet, how many square feet of pavers will he need?

College Preparatory Integrated Mathematics Course I

Name: _______________________________ Date:____________________

Learning Objective I.5.2

To check your understanding of the section, work out the following exercises.

1. Determine whether you would measure each item using linear, square, or cubic units.

   a) amount of water in a fish tank

   b) length of dental floss

   c) living area of an apartment

   d) height of a doorway

2. The length of a rectangle is 85 feet and the width is 45 feet. Find the perimeter and the area of the rectangle.

3. Find the length of a rectangle with perimeter 124 inches and width 38 inches.

4. The perimeter of a rectangular painting is 306 centimeters. The length is 17 centimeters more than the width. Find the length and the width.

5. Find the area of a triangle with base 12 inches and height 5 inches.

6. The perimeter of an isosceles triangle is 42 feet. The length of the shortest side is 12 feet. Find the length of the other two sides

College Preparatory Integrated Mathematics Course I

Circles

Learning Objective I.5.3: Use properties of circles

Read Textbook(PreAlgebra e2) Section 9.5 on page 803 and answer the questions below.

BEPREPARED: Before you get started, try:

1. Evaluate x^2 when $x = 5$
2. Using 3.14 for π , approximate the (a) circumference and (b) the area of a circle with radius 8 inches.
3. Simplify $\frac{22}{7}(0.25)^2$ and round to the nearest thousandth.

Example 1: A circular mirror has radius of 5 inches. Find the ⓐ circumference and ⓑ area of the mirror.

Example 2: A circular spa has radius of 4.5 feet. Find the ⓐ circumference and ⓑ area of the spa.

Example 3: Find the circumference of a circular fire pit whose diameter is 5.5 feet.

Example 4: If the diameter of a circular trampoline is 12 feet, what is its circumference?

Example 5: Find the diameter of a circle with circumference of 94.2 centimeters.

Example 6: Find the diameter of a circle with circumference of 345.4 feet.

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date:_____________________

Learning Objective I.5.3

1. An extra-large pizza is a circle with radius 8 inches. Find the ⓐ circumference and ⓑ area of the pizza.

2. A round coin has a diameter of 3 centimeters. What is the circumference of the coin?

3. A circle has a circumference of 59.66 feet. Find the diameter.

4. A circle has a circumference of 80.07 centimeters. Find the diameter.

5. A circle has a circumference of 251.2 centimeters.

College Preparatory Integrated Mathematics Course I

Volume and Surface Area

Learning Objective I.5.4: Find volumes and surface areas of rectangular solids.

Read Textbook (PreAlgebra e2) Section 9.6 on page 815 and answer the questions below.

BEPREPARED: Before you get started, try:

1. Evaluate x^3 when $x = 5$
2. Evaluate 2^x when $x = 5$
3. Find the area of a circle with radius $\frac{7}{2}$.

Example 1: Find the ⓐ volume and ⓑ surface area of rectangular solid with the: length 8 feet, width 9 feet, and height 11 feet.

Example 2: Find the ⓐ volume and ⓑ surface area of rectangular solid with the: length 15 feet, width 12 feet, and height 8 feet.

Example 3: A rectangular box has length 9 feet, width 4 feet, and height 6 feet. Find its ⓐ volume and ⓑ surface area.

Example 4: A rectangular box has length 22 inches, width 14 inches, and height 9 inches. Find its ⓐ volume and ⓑ surface area.

Example 5: For a cube with side 4.5 meters, find the ⓐ volume and ⓑ surface area of the cube.

Example 6: For a cube with side 7.3 yards, find the ⓐ volume and ⓑ surface area of the cube.

Example 7: A packing box is a cube measuring 4 feet on each side. Find its ⓐ volume and ⓑ surface area.

Example 8: A wall is made up of cube-shaped bricks. Each cube is 16 inches on each side. Find the ⓐ volume and ⓑ surface area of each cube.

Example 9: Find the ⓐ volume and ⓑ surface area of a sphere with radius 3 centimeters.

Example 10: Find the ⓐ volume and ⓑ surface area of each sphere with a radius of 1 foot.

Example 11: A beach ball is in the shape of a sphere with radius of 9 inches. Find its ⓐ volume and ⓑ surface area.

Example 12: A Roman statue depicts Atlas holding a globe with radius of 1.5 feet. Find the ⓐ volume and ⓑ surface area of the globe.

Example 13: Find the ⓐ volume and ⓑ surface area of the cylinder with radius 4 cm and height 7cm.

Example 14: Find the ⓐ volume and ⓑ surface area of the cylinder with given radius 2 ft and height 8 ft.

Example 15: Find the ⓐ volume and ⓑ surface area of a can of paint with radius 8 centimeters and height 19 centimeters. Assume the can is shaped exactly like a cylinder.

Example 16: Find the ⓐ volume and ⓑ surface area of a can of paint with radius 2.7 feet and height 4 feet. Assume the can is shaped exactly like a cylinder.

Example 17: Find the volume of a cone with height 7 inches and radius 3 inches.

Example 18: Find the volume of a cone with height 9 centimeters and radius 5 centimeter.

Example 19: How many cubic inches of candy will fit in a cone-shaped piñata that is 18 inches long and 12 inches across its base? Round the answer to the nearest hundredth.

Example 20: What is the volume of a cone-shaped party hat that is 10 inches tall and 7 inches across at the base? Round the answer to the nearest hundredth.

1. Find the volume and the surface area of the indicated solid with the given dimensions. Round answers to the nearest hundredth

a) Rectangular solid: given length 5 feet, width 8 feet, height 2.5 feet.

b) Cube: given side lenth12.5 meters.

c) Sphere: given radius 2.1 yards.

d) Cylinder: given radius 5 centimeters, height 15 centimeters.

e) Cone: given height 9 feet and radius 2 feet

2. Gift box A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its volume and surface area.

3. Shipping container A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its volume and surface area.

4. Barber shop pole A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its volume and surface area.

5. Popcorn cup What is the volume of a cone-shaped popcorn cup that is 8 inches tall and 6 inches across at the base?

6. Ice cream cones A regular ice cream cone is 4 inches tall and has a diameter of 2.5 inches. A waffle cone is 7 inches tall and has a diameter of 3.25 inches. To the nearest hundredth,

   a) Find the volume of the regular ice cream cone.
   b) Find the volume of the waffle cone.
   c) How much more ice cream fits in the waffle cone compared to the regular cone?

UNIT II

II. Demonstrate the ability to graph and solve linear equations and inequalities.

Framework Student Learning Outcome II.1

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities.

Read Textbook Section 2.5 on page 179 and fill in the following.

Definition: Graph Inequalities on the Number Line

What number would make the inequality x > 3 true? Are you thinking, "*x* could be four"? That's correct, but *x* could be 6, too, or 37, or even 3.001. Any number greater than three is a solution to the inequality x > 3.

We show all the solutions to the inequality x > 3 on the number line by shading in all the numbers to the right of three, to show that all numbers greater than three are solutions. Because the number three itself is not a solution, we put an open parenthesis at three.

We can also represent inequalities using **interval notation**. There is no upper end to the solution to this inequality. In interval notation, we express x > 3 as (3 , ∞). The symbol ∞ is read as "**infinity**." It is not an actual number.

Figure 2.2 shows both the number line and the interval notation.

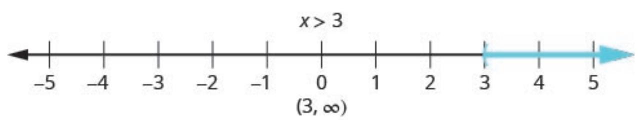
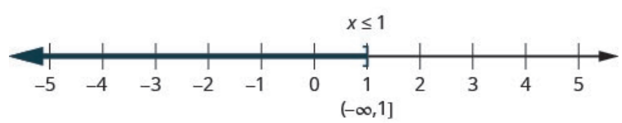
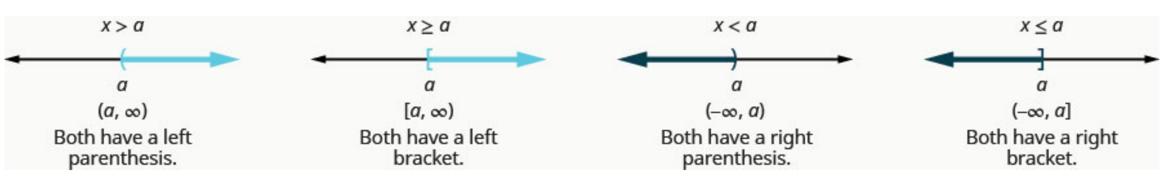


Figure 2.3 The inequality x ≤ 1 is graphed on this number line and written in interval notation.



Inequalities, Number Lines and Interval Notation:



Graph each inequality on the number line and write in interval notation.

Example 1: $x \geq -3$

Example 2: $x < 2.5$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II.1

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities **Read Textbook Section 2.5 on page 181 and fill in the following.**

What numbers are greater than two but less than five? Are you thinking say, 2.5,3,3.23,4,4.99? We can represent all the numbers between two and five with the inequality $2 < x < 5$. We can show $2 < x < 5$ on the number line by shading all the numbers between two and five. Again, we use the parentheses to show the numbers two and five are not included.

The interval notation would be (2, 5).

Graph each inequality on the number line and write in interval notation.

Example 3: $-3 < x < 4$

Example 4: $0 \leq x \leq 2.5$

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities **Read Textbook Section 2.5 on page 181 and fill in the following.**

Definition: Linear Inequalities

A linear inequality is much like a linear equation—but the equal sign is replaced with an inequality sign. A **linear inequality** is an inequality in one variable that can be written in one of the forms, $ax + b < c$, $ax + b \leq c$, $ax + b > c$, or $ax + b \geq c$

When we solve linear equations we are able to use the properties of equality to add, subtract, multiply, or divide both sides and still keep the equality. Similar properties hold true for inequalities. In addition to the same properties that we use for linear equations however for inequalities we need to be aware of the following property:

Definition: Multiplication and Division Property

For any number a, b and c

multiply or divide by a positive

if $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

multiply or divide by a negative

if $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II.1

Solve each inequality. Graph the solution on a number line and write solution in interval notation.

Example 5: $x - \frac{3}{8} \leq \frac{3}{4}$

Example 6: $9y < 54$

Example 7: $-15 < \frac{3}{5}x$

Example 8: $-8q > 32$

Example 9: $\frac{k}{12} \leq 15$

Example 10: $6y \leq 11y + 17$

Example 11: $9y + 2(y + 6) > 5y - 24$

Example 12: $-5(2x + 6) \leq 4x - 28$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II.1

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities. Read Textbook Section 2.6 on page 198 and fill in the following.

Definitions

Inequalities containing one inequality symbol are called ____________inequalities, while inequalities containing two inequality symbols are called ____________inequalities.

Solve the compound inequality. Graph the solution and write in interval notation:

Example 13: $-5 \leq 4x - 1 < 7$

Example 14: $-3 < 2x - 5 \leq 1$

Example 15: $1 - 2x \leq -3 \; or \; 7 + 3x \leq 4$

Example 16: $2 - 5x \leq -3 \; or \; 5 + 2x \leq 3$

Framework Student Learning Outcome II.1

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities. Read Textbook Section 2.7 on page 209 and fill in the following.

Definitions If a is a positive number, then $|X| = a$ is equivalent to $X = a$ or $X = -a$.

Steps to solving absolute value equations

1. Isolate the absolute value expression.
2. Write the equivalent equations.
3. Solve each equation.
4. Check each solution.

Solve the following.

Example 17: $|x| = 2$

Example 18: $|y| = -4$

Example 19: $|z| = 0$

Example 20: $|3x - 5| - 1 = 6$

Example 21: $|4x - 3| - 5 = 2$

Framework Student Learning Outcome II.1

Learning Objective II.1: Solve problems using equations and inequalities, absolute value inequalities

Definitions Absolute Value Inequalities with < or ≤

For any algebraic expression, *u*, and any positive real number, *a*

if $|u| < a$, then $-a < u < a$

if $|u| \leq a$, then $-a \leq u \leq a$

After solving an inequality, it is often helpful to check some points to see if the solution makes sense. The graph of the solution divides the number line into three sections. Choose a value in each section and substitute it in the original inequality to see if it makes the inequality true or not. While this is not a complete check, it often helps verify the solution.

Steps to solving absolute value inequalities with < or ≤

1. Isolate the absolute value expression
2. Write the equivalent compound inequality
3. Solve the compound inequality
4. Graph the solution
5. Write the solution in interval notation

Solve and graph the following solutions and write solutions in interval notation.

Example 22: $|x| < 9$

Example 23: $|x| < 1$

Example 24: $|4x - 3| \geq 5$

Example 25: $|3x - 4| \geq 2$

Example 26: $\left|3x + \frac{5}{8}\right| < -4$

Example 27: $\left|\frac{3(x-2)}{5}\right| \leq 0$

College Preparatory Integrated Mathematics Course I

Name: ______________________________ Date:___________________

Learning Objective II.1

To check your understanding of the section, work out the following exercises.

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

1. $a + 34 \geq 710$

2. $-6y < 48$

3. $4v \geq 9v - 40$

4. $5u \leq 8u - 21$

Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

5. $9p > 14p - 18$

6. $12x + 3(x + 7) > 10x - 24$

Graph the solution on the number line, and write the solution in interval notation

7. $-3 < 2x - 5 \leq 1$

8. $5 < 4x + 1 < 9$

9. $-1 < 3x + 2 < 8$

10. $-8 < 5x + 2 \leq -3$

11. $4 - 7x \geq -3 \text{ or } 5(x - 3) + 8 > 3$

12. $12x - 5 \leq 3 \text{ or } 14(x - 8) \geq -3$

Graph the solution and write the solution in interval notation.

13. $|3x - 4| + 5 = 7$

14. $|4x + 7| + 2 = 5$

Graph the solution and write the solution in interval notation.

15. $\left|\frac{1}{2}x + 5\right| + 4 = 1$

16. $\left|\frac{3}{5}x - 2\right| + 4 = 2$

17. $|x| < 5$

18. $|x| \leq 8$

19. $|2x + 3| + 5 < 4$

20. $|x| > 3$

21. $|3x - 2| > 4$

22. $|2x - 1| > 5$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II

Learning Objective II.2 Solving linear equations.
Read Textbook Section 2.1 on page 107 and fill in the following.

Definition

Solution of an Equation:

A solution of an equation is a value of a variable that makes a ______________________________ when substituted into the equation.

How to Determine Whether a Number is a Solution to an Equation:

Step 1:

Step 2:

Step 3:

Example 1: Determine whether the values are solutions to the equation: $9y + 2 = 6y + 3$.

a. $y = \frac{4}{3}$

b. $y = \frac{1}{3}$

Example 2: Determine whether the values are solutions to the equation: $4x - 2 = 2x + 1$.

a. $x = \frac{3}{2}$

b. $x = -\frac{1}{2}$

Learning Objective II.2 Solving linear equations.
Read Textbook Section 2.1 on page 109 and fill in the following.

Definition

Linear Equation:

A linear equation is an equation in one variable that can be written, where a and b are real numbers and $a \neq 0$, as ____________________.

How to Solve A Linear Equation Using a General Strategy:

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Example 3: Solve: $2(m-4)+3=-1$.

Example 4: Solve: $5(a-3)+5=-10$.

Example 5: Solve: $\frac{1}{3}(6u+3)=7-u$.

Example 6: Solve: $\frac{2}{3}(9x-12)=8+2x$.

Note: Collecting the variable terms on the side with the larger coefficient helps prevent potential errors due to negative signs.

Example 7: Solve: $6(p-3)-7=5(4p+3)-12$.

Example 8: Solve: $8(q+1)-5=3(2q-4)-1$.

Example 9: Solve: $6[4-2(7y-1)]=8(13-8y)$.

Example 10: Solve: $12[1-5(4z-1)]=3(24+11z)$.

Learning Objective II.2 Solving linear equations.
Read Textbook Section 2.1 on page 112 and fill in the following.

Classify Equations

Conditional Equation:

An equation that is ________ for one or more values of the variable and _________ for all other values of the variable is a conditional equation.

Note: All equations so far have been conditional equations. That will not always be the case.

Identity:

An equation that is true for ______________________ of the variable is called an identity. The solution of an identity is ______________________________.

Contradiction:

An equation that is ___________ for all values of the variable is called a contradiction. A contradiction has _________________________.

Example 11: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4+9(3x-7)=-42x-13+23(3x-2)$.

Example 12: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$.

Example 13: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$.

Example 14: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$.

Example 15: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $12c+5(5+3c)=3(9c-4)$.

Example 16: Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $4(7d+18)=13(3d-2)-11d$.

Learning Objective II.2 Solving linear equations.
Read Textbook Section 2.1 on page 115 and fill in the following.

Solve Equations with Fraction or Decimal Coefficients

How to Solve A Linear Equation Using a General Strategy:

Step 1:

Step 2:

Step 3:

Note: When you multiply both sides of an equation by the LCD of the fractions, make sure you multiply each term by the LCD – even if it does not contain a fraction.

Example 17: Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

Example 18: Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Example 19: Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Example 20: Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Example 21: Solve: $\frac{1}{5}(n+3) = \frac{1}{4}(n+2)$.

Example 22: Solve: $\frac{1}{2}(m-3) = \frac{1}{4}(m-7)$.

Example 23: Solve: $\frac{3r+5}{6} + 1 = \frac{4r+3}{3}$.

Example 24: Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$.

Solving Equations with Decimal Coefficients:
Decimals can also be expressed as fractions. So, we can use the same method we used to clear fractions – multiply both sides of the equation by the least ____________________________________.

Example 25: Solve: $0.25n+0.05(n+5)=2.95$.

Example 26: Solve: $0.10d+0.05(d-5)=2.15$.

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date: ____________________

Learning Objective II.2

To check your understanding of the section, work out the following exercises.

1. Solve: $3(10 - 2x) + 54 = 0.$

2. Solve: $-15 + 4(2 - 5y) = -7(y - 4) + 4.$

3. Solve: $10[5(n + 1) + 4(n - 1)] = 11[7(5 + n) - (25 - 3n)].$

4. Solve: $18u - 51 = 9(4u + 5) - 6(3u - 10).$

5. Solve: $11(8c+5)-8c=2(40c+25)+5$.

6. Solve: $\frac{1}{3}x+\frac{2}{5}=\frac{1}{5}x-\frac{2}{5}$.

7. Solve: $\frac{3p+6}{3}=\frac{p}{2}$.

8. Solve: $\frac{3y-6}{2}+5=\frac{11y-4}{5}$.

9. Solve: $1.2x-0.91=0.8x+2.29$.

10. Solve: $0.10d+0.25(d+7)=5.25$.

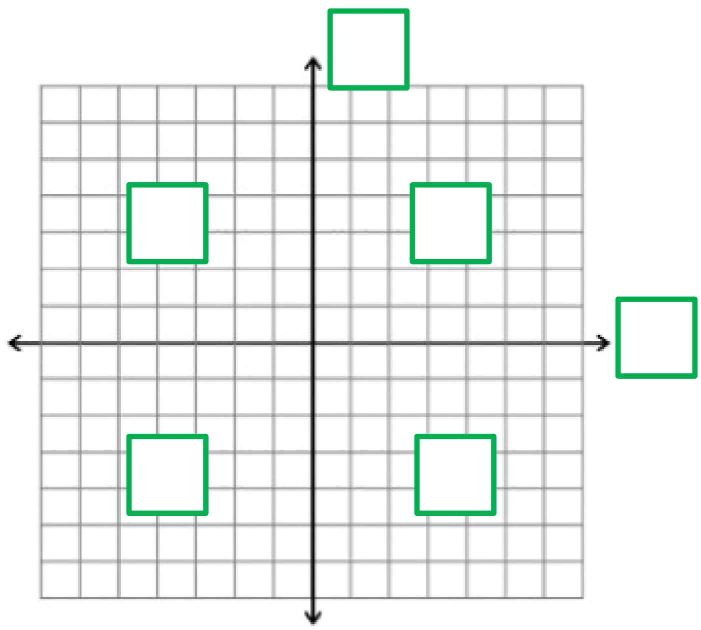
College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 235 and fill in the following.

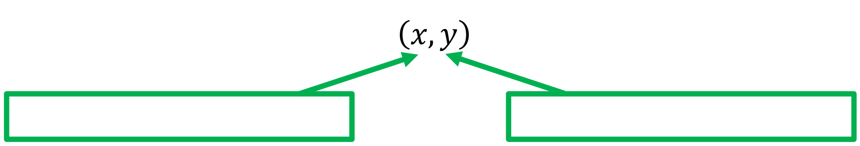
Plot Points on a Rectangular Coordinate System

We use a grid system in algebra to show a relationship between two variables in a rectangular coordinate system. The rectangular coordinate system is also called the ______________________ or the ______________________________________. The rectangular coordinate system is formed by two intersecting number lines, one ____________________ and one _____________________. The horizontal number line is called the _____________. The vertical number line is called the _____________. These axes divide a plane into __________ regions, called quadrants. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise.



Ordered Pair

An ordered pair, _________ gives the coordinates of a point in a rectangular coordinate system. The first number is the __________________________. The second number is the __________________________.



The Origin

The point __________ is called the origin. It is the point where the x – axis and y – axis _______________.

Points on the Axes

Points with a y – coordinate equal to 0 are on the ______________, and have coordinates ___________.

Points with a x – coordinate equal to 0 are on the ______________, and have coordinates ___________.

College Preparatory Integrated Mathematics Course I

Example 1: Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

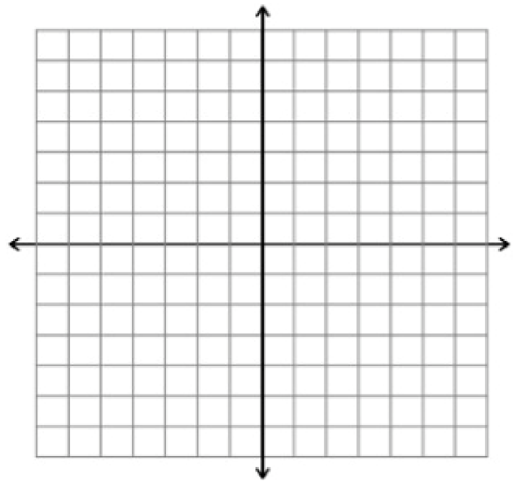
a. $(-2, 1)$

b. $(-3, -1)$

c. $(4, -4)$

d. $(-4, 4)$

e. $\left(-4, \frac{3}{2}\right)$



Example 2: Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

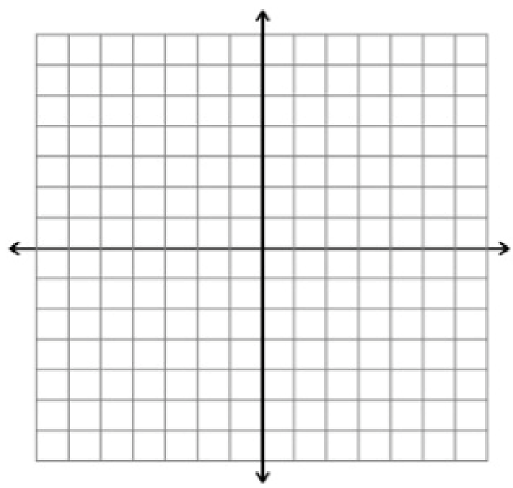
a. $(-4, 1)$

b. $(-2, 3)$

c. $(2, -5)$

d. $(-2, 5)$

e. $\left(-3, \frac{5}{2}\right)$



Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 238 and fill in the following.

Quadrants

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
________	________	________	________
________	________	________	________

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 239 and fill in the following.

Linear Equation
An equation of the form ______________________, where A and B are not both zero, is called a linear equation in two variables.

Standard Form of Linear Equation
A linear equation is in standard form when it is written ______________________.

Note: All linear equations can be written in standard form.

Example 3: Determine whether each equation is a linear equation in two variables.

a. $3x + 2.7y = -5.3$

b. $x^2 + y = 8$

c. $y = 12$

d. $5x = -3y$

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 239 and fill in the following.

Solution of a Linear Equation in Two Variables
An ordered pair (x, y) is a solution of the linear equation $Ax + By = C$, if the equation is a __________ statement when the x – and y – values of the ordered pair are substituted into the equation.

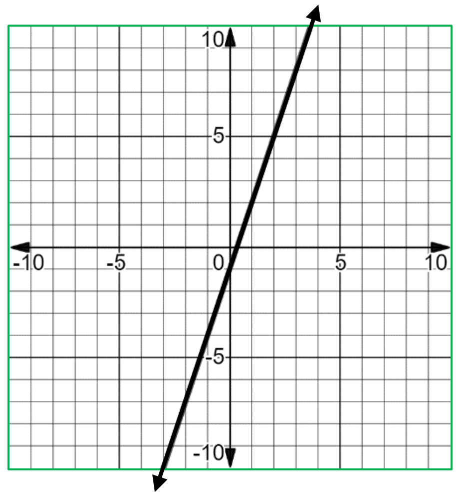
Linear equations have ______________________________ solutions. For every number that is substituted for _____ there is a corresponding _____ value.

Graph of a Linear Equation
The graph of a linear equation $Ax + By = C$ is a ______________________________.

- Every point on the line is a _________________ of the equation.
- Every solution of this is equation is a _________________ on this line.

Example 4: Use the graph of $y = 3x - 1$. For each ordered pair, decide:

- Is the ordered pair a solution to the equation?
- Is the point on the line?



a. $(0, -1)$ b. $(2, 5)$ c. $(3, -1)$ d. $(-1, -4)$

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 242 and fill in the following.

Graph a Linear Equation by Plotting Points

How to Graph a Linear Equation by Plotting Points:

Step 1:

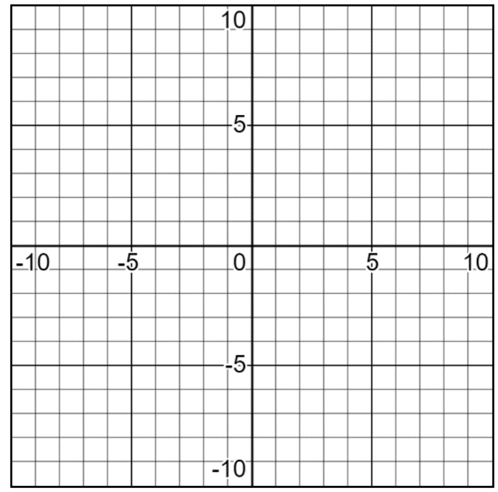
Step 2:

Step 3:

Note: Chose x values that will make the arithmetic and plotting easiest.

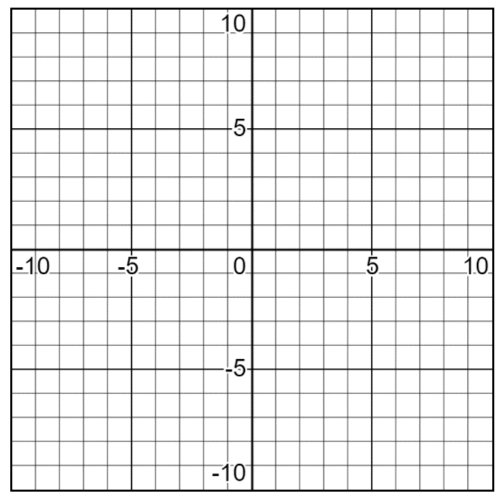
Example 5: Graph the equation by plotting points: $y = 2x - 3$.

x	y	Ordered Pair



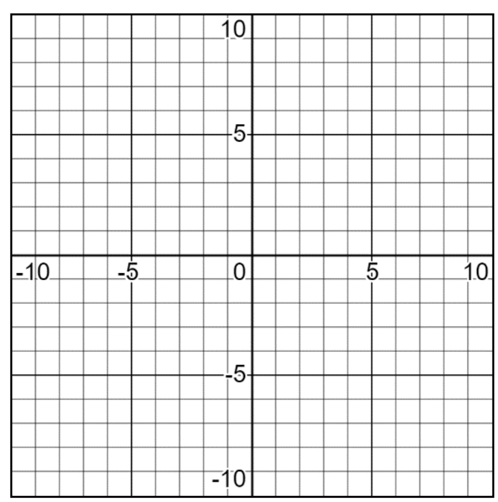
Example 6: Graph the equation by plotting points: $y = -2x + 4$.

x	y	Ordered Pair



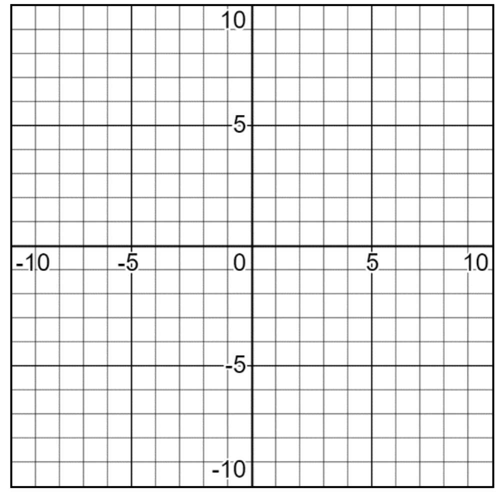
Example 7: Graph the equation: $y = \frac{1}{3}x - 1$.

x	y	Ordered Pair



Example 8: Graph the equation: $y = \frac{1}{4}x + 2$.

x	y	Ordered Pair



Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.1 on page 245 and fill in the following.

Graph Vertical and Horizontal Lines

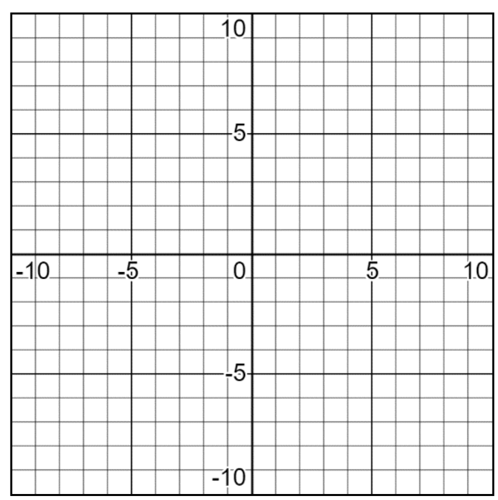
A vertical line is the graph of an equation of the form __________.
The line passes through the x – axis at _________.

A horizontal line is the graph of an equation of the form __________.
The line passes through the y – axis at _________.

Example 9: Graph the equations:

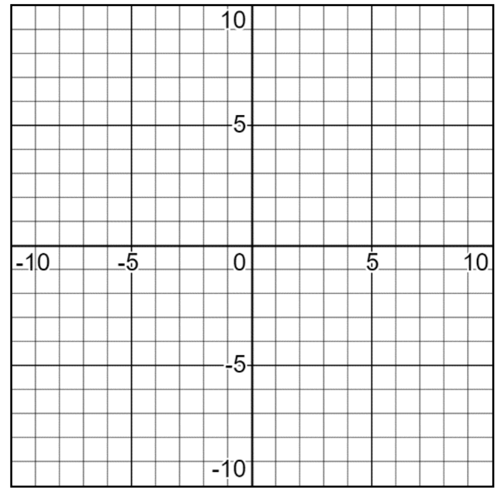
a. $x = 5$

x	y	Ordered Pair



b. $y = -4$

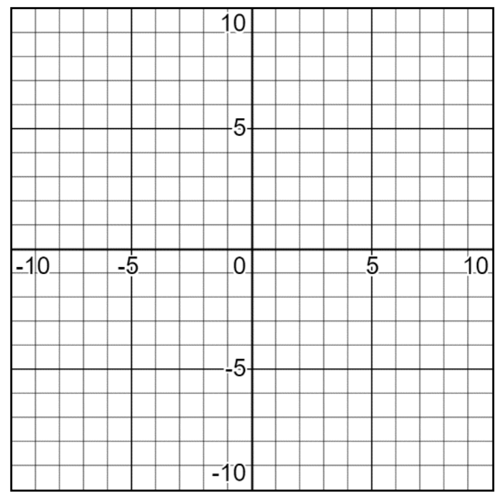
x	y	Ordered Pair



Example 10: Graph the equations:

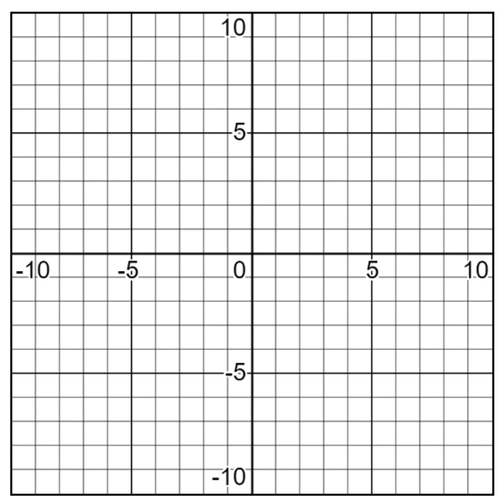
a. $x = -2$

x	y	Ordered Pair



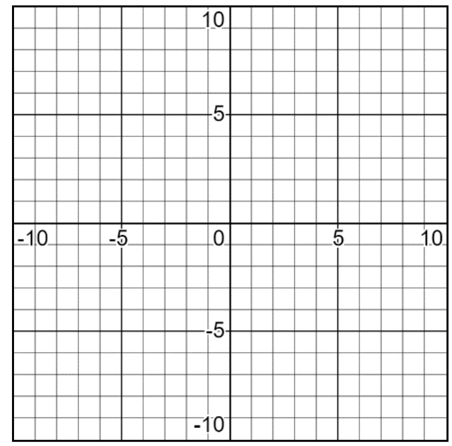
b. $y = 3$

x	y	Ordered Pair



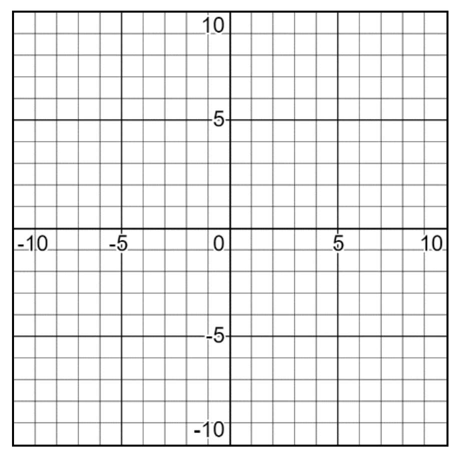
Example 11: Graph the equations in the same rectangular coordinate system:

$y = -4x$ $\qquad$ $y = -4$



Example 12: Graph the equations in the same rectangular coordinate system:

$y = 3$ $\qquad$ $y = 3x$



College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date: ____________________

Learning Objective II.3

To check your understanding of the section, work out the following exercises.

1. Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

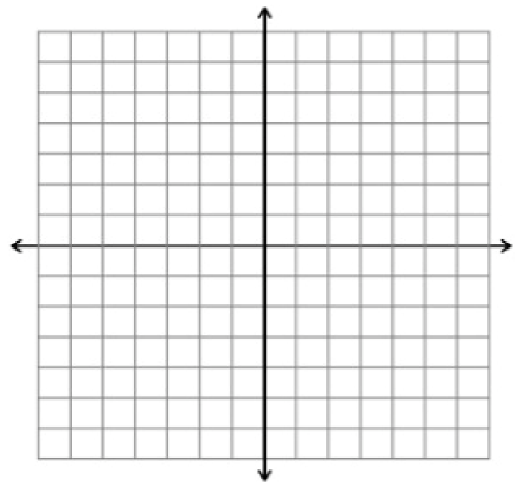
   a. $(-2, -3)$

   b. $(3, -3)$

   c. $(-4, 1)$

   d. $(4, -1)$

   e. $\left(\frac{3}{2}, 1\right)$



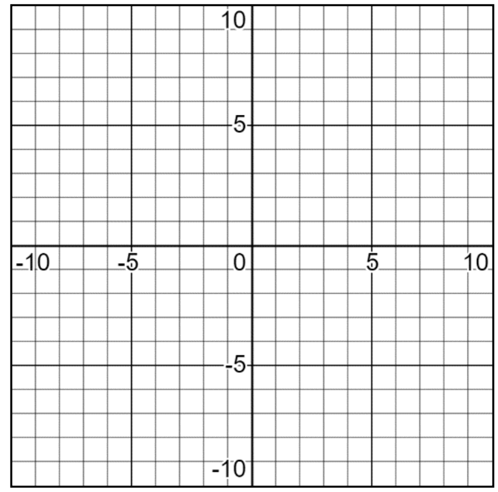
2. Determine if each ordered pair is a solution to the equation: $y = \frac{1}{3}x + 2$.

   a. $(0, 2)$ b. $(3, 3)$

   c. $(-3, 2)$ d. $(-6, 0)$

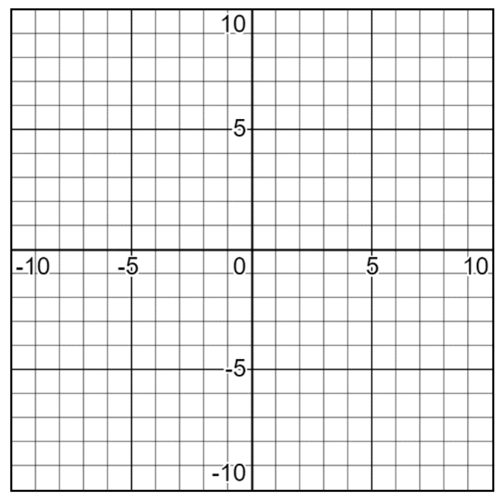
3. Graph by plotting points: $y = -x - 2$.

x	y	Ordered Pair

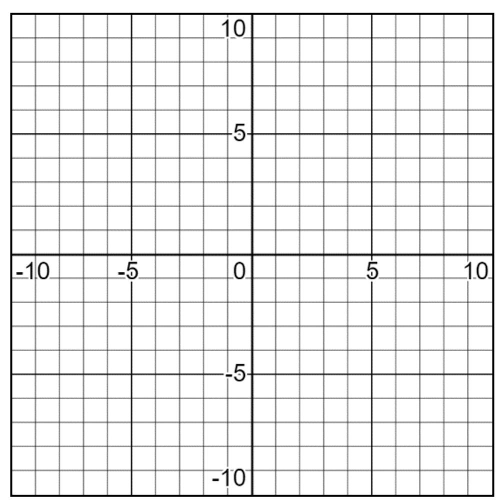


4. Graph by plotting points: $y = -\frac{5}{3}x + 4$.

x	y	Ordered Pair



5. Graph each pair of equations in the same rectangular coordinate system: $y = 5x$ and $y = 5$



College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II

Learning Objective II.4: Graph linear equations & linear inequalities in two variables.
Read Textbook Section 3.4 on page 306 and fill in the following.

Verify Solutions to an Inequality in Two Variables

Linear Inequality

A linear inequality is an inequality that can be written in one of the following forms:

__________________ __________________ __________________ __________________

Where A and B are not both zero.

Solution to a Linear Inequality

An ordered pair (x, y) is a solution to a linear inequality if the inequality is __________ when we substitute the values of x and y.

Example 1: Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

a. $(0, 0)$ b. $(4, 9)$ c. $(-2, 1)$

d. $(-5, -3)$ e. $(5, 1)$

Example 2: Determine whether each ordered pair is a solution to the inequality $y < x + 1$:

a. $(0, 0)$ b. $(8, 6)$ c. $(-2, -1)$

d. $(3, 4)$ e. $(-1, -4)$

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.4 on page 308 and fill in the following.

Recognize the Relation Between the Solutions of an Inequality and its Graph

Boundary Line

The line with equation $Ax + By = C$ is the boundary line that ___________________ the region where $Ax + By > C$ from the region where $Ax + By < C$.

$Ax + By < C$
$Ax + By > C$
Boundary line is $Ax + By = C$
Boundary line is _________________ in solution.
Boundary line is _____________.

$Ax + By \leq C$
$Ax + By \geq C$
Boundary line is $Ax + By = C$
Boundary line is _________________ in solution.
Boundary line is _____________.

Learning Objective II.3: Plot ordered pairs on a rectangular coordinate system and graph linear equations.
Read Textbook Section 3.4 on page 315 and fill in the following.

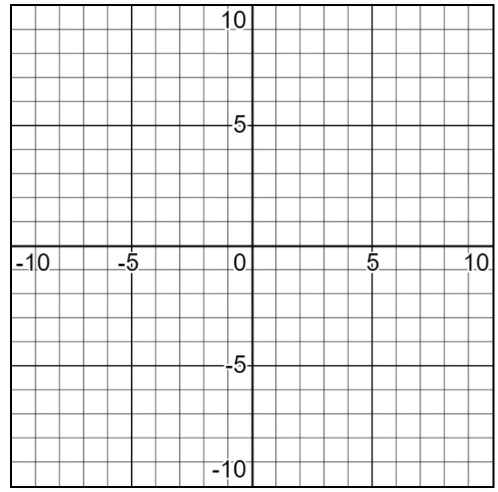
How to Graph a Linear Inequality in Two Variables

Step 1:

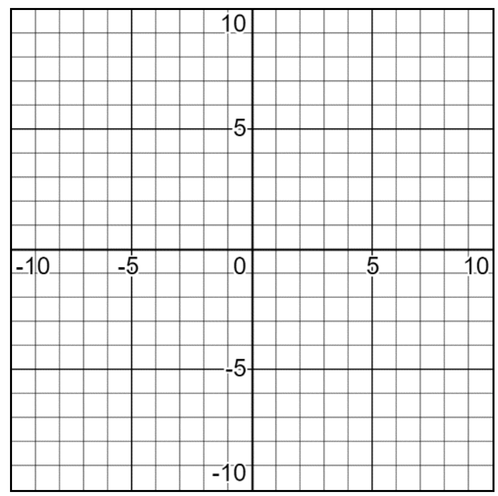
Step 2:

Step 3:

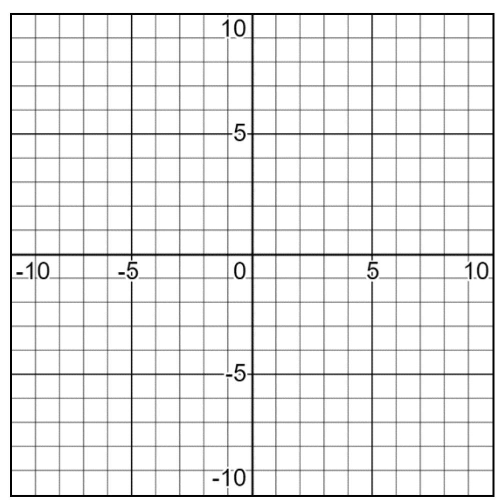
Example 3: Graph the linear inequality: $y \geq \frac{5}{2}x - 4$.



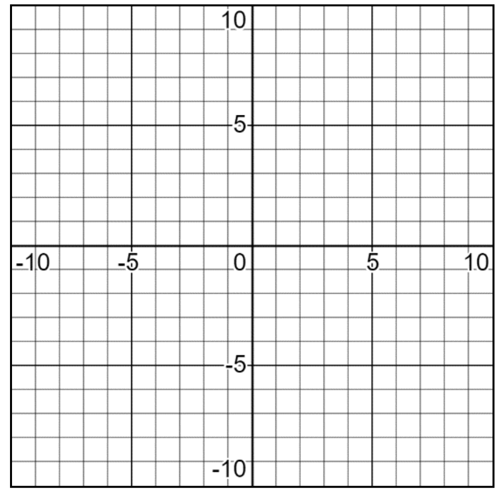
Example 4: Graph the linear inequality: $y \leq \frac{2}{3}x - 5$.



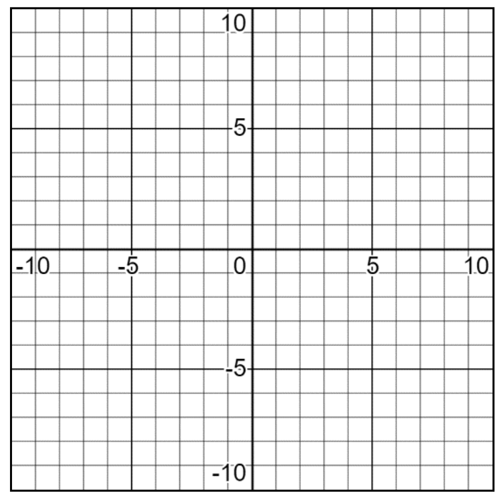
Example 5: Graph the linear inequality: $2x - 3y < 6$.



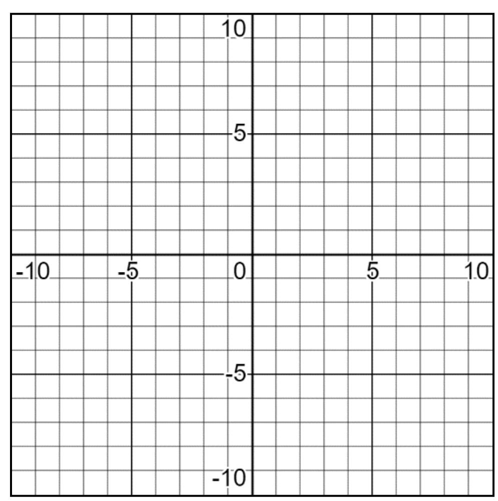
Example 6: Graph the linear inequality: $2x - y > 3$.



Example 7: Graph the linear inequality: $y < 5$.



Example 8: Graph the linear inequality: $y \leq -1$.



College Preparatory Integrated Mathematics Course I

Name: ______________________________ Date: ____________________

Learning Objective II.4

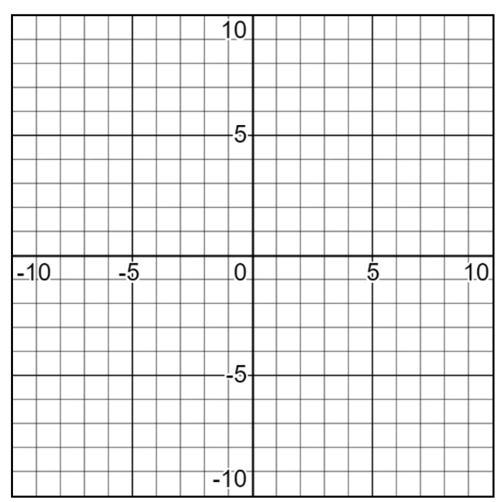
To check your understanding of the section, work out the following exercises.

1. Determine whether each ordered pair is a solution to the inequality $2x + 3y > 2$.

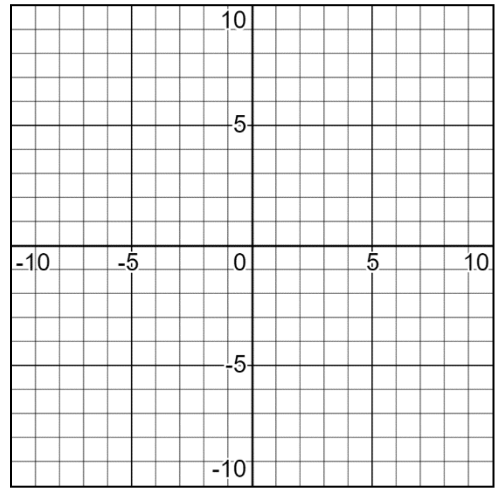
   a. $(1, 1)$ b. $(4, -3)$ c. $(0, 0)$

   d. $(-8, 12)$ e. $(3, 0)$

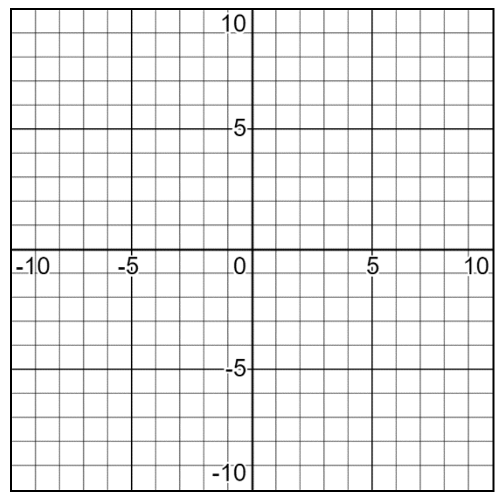
2. Graph the linear inequality: $y \geq -\frac{1}{3}x - 2$.



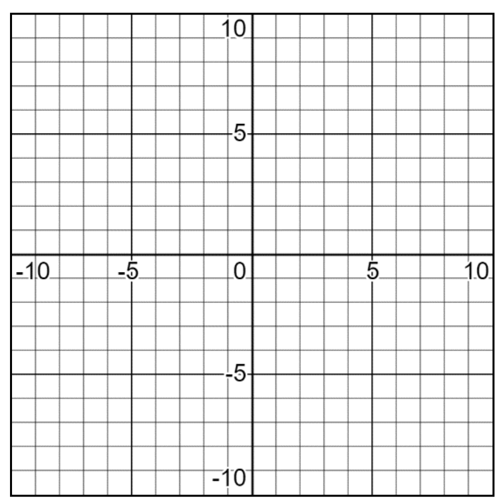
3. Graph the linear inequality: $y < -3x - 4$.



4. Graph the linear inequality: $2x - 5y > 10$.



5. Graph the linear inequality: $x \leq 5$.



Learning Objective II.5: Finding intercepts graphically and algebraically.
Read Textbook Section 3.1 on page 250 and fill in the following.

Find x- and y- intercepts graphically.

In the previous lesson we graphed lines by plotting points. In those lines we used three ordered pairs to graph the line. The three points you select might be different than the points your friend selected and the graphs might appear to be different. However, the lines will be the same if the work was done correctly. The two lines will eventually cross the x-axis and the y-axis. The points were the lines cross these axis are called ____________________ of a line.

Identify the x- and y- intercepts of the line and fill in the table below the graphs.

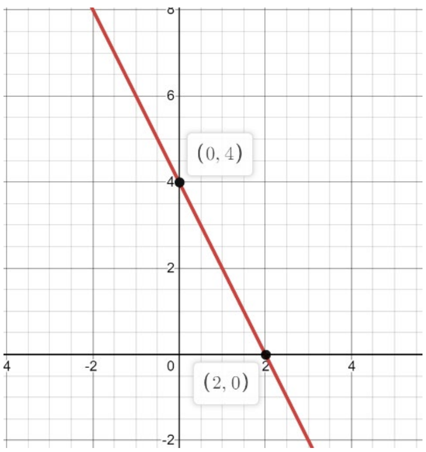


Figure 1

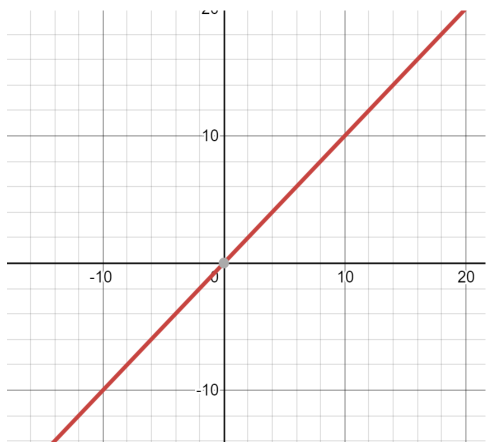


Figure 2

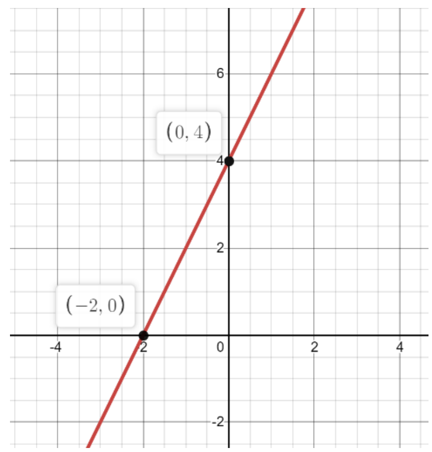


Figure 3

Figure	Line Crosses x-axis at:	Order pair for this point	Line Crosses at y-axis at:	Order pair for this point
1				
2				
3				
For any graph				

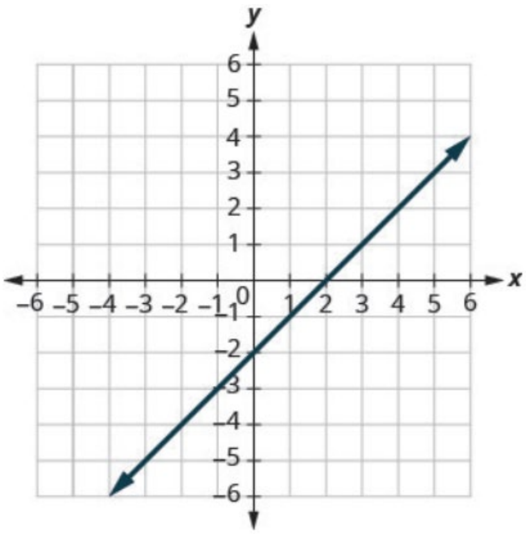
Notice the pattern in the table. The value of y is always zero when the line crosses the x-axis and the value of x is always zero when the line crosses the y-axis.

The ________________________is the point $(a, 0)$ where the line crosses the x-axis.

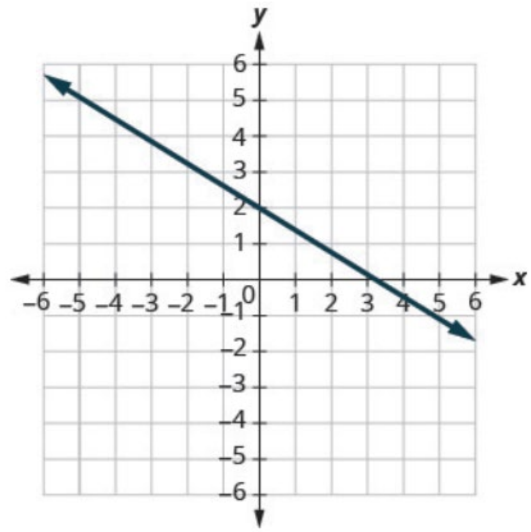
The ________________________is the point $(0, b)$ where the line crosses the y-axis.

Example 1: Find the x- and y- intercepts on the graphs.

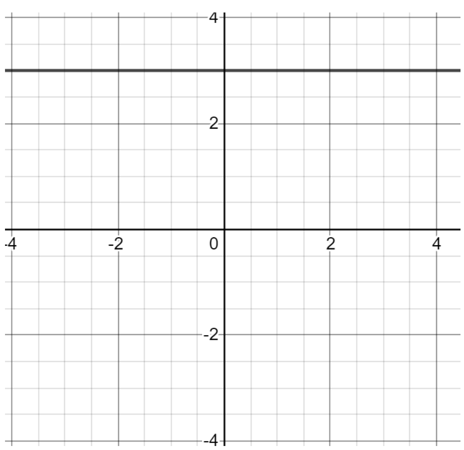
a)



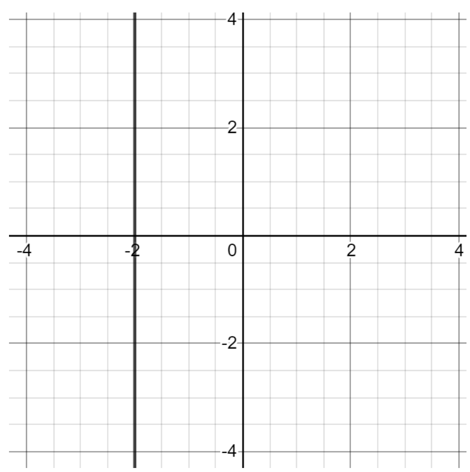
b)



c)



d)



Steps to find the intercepts algebraically.

To find the x- and y- intercepts algebraically follow the steps below.

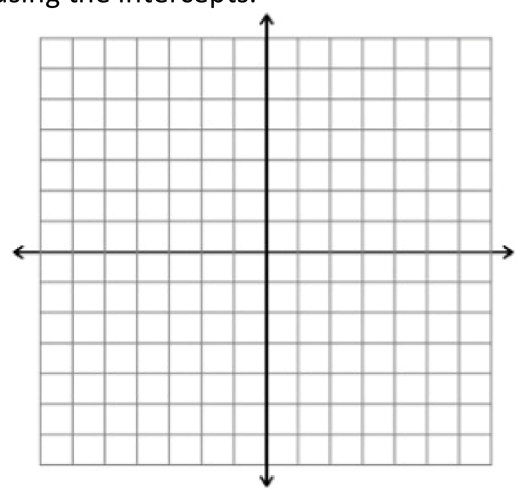
1. To find the x-intercepts, let $y = 0$ and solve for x. The results will be an ordered pair $(a, 0)$, where a is the value of x after solving the equation for x.

2. To find the y-intercepts, let $x = 0$ and solve for y. The results will be an ordered pair $(0, b)$, where b is the value of y after solving the equation for y.

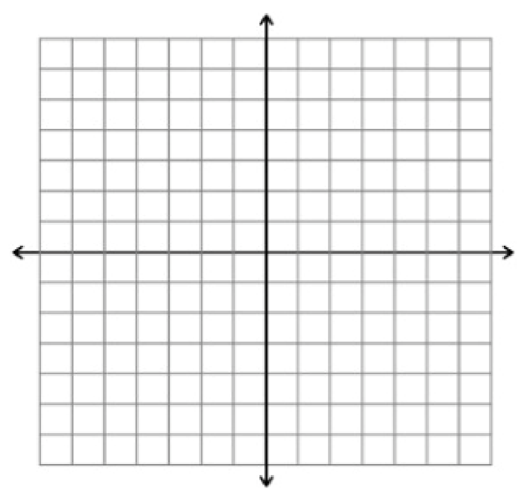
Steps to graph a linear equation using the intercepts.

1. Find the x-intercepts. Let $y = 0$ and solve for x.
2. Find the y-intercept. Let $x = 0$ and solve for y.
3. Find a third solution to the equation.
4. Plot the three points.
5. Draw the line.

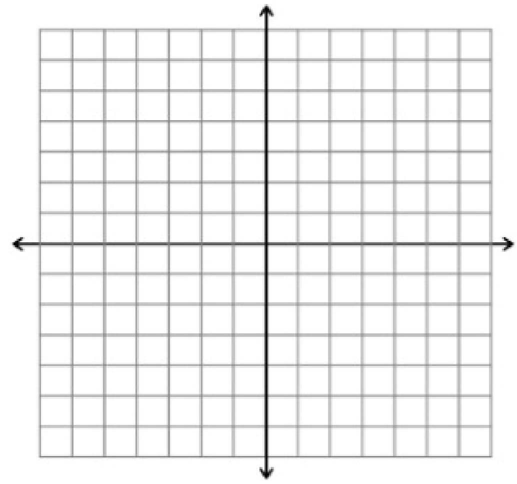
Example 2: Find the intercepts of 3x + y = 12 and graph the equation using the intercepts.



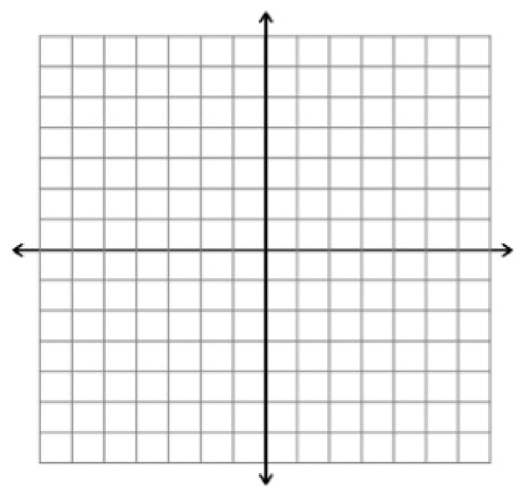
Example 3: Find the intercepts of $x + 4y = 8$ and graph the equation using the intercepts.



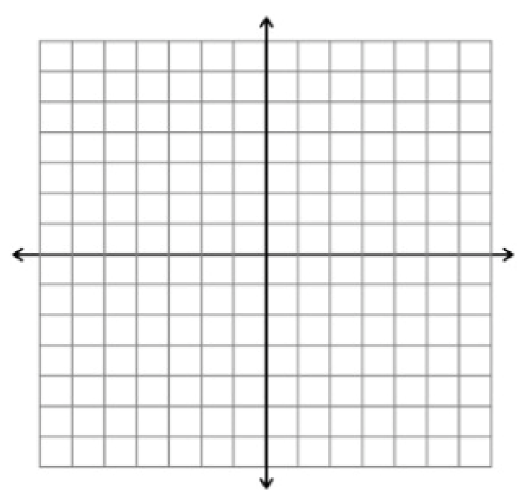
Example 4: Find the intercepts of $-x + 3y = 6$ and graph the equation using the intercepts.



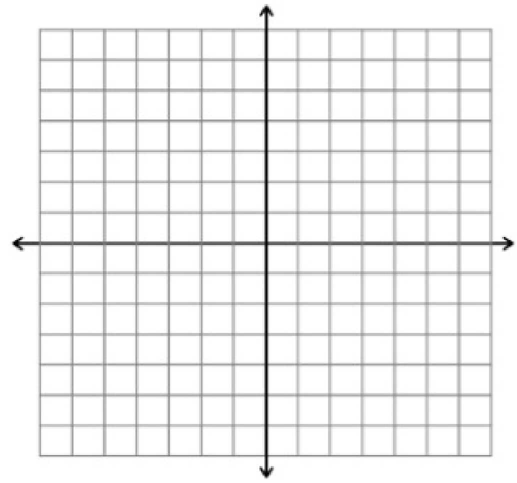
Example 5: Find the intercepts of $5x - 2y = 10$ and graph the equation using the intercepts.



Example 7: Find the intercepts of $3x - 4y = 12$ and graph the equation using the intercepts.



Example 9: Find the intercepts of $y = 4x$ and graph the equation using the intercepts.



College Preparatory Integrated Mathematics Course I

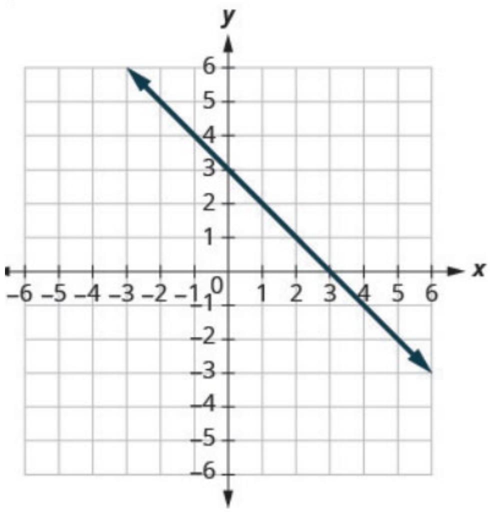
Name: ________________________________ Date: ____________________

Learning Objective II.5

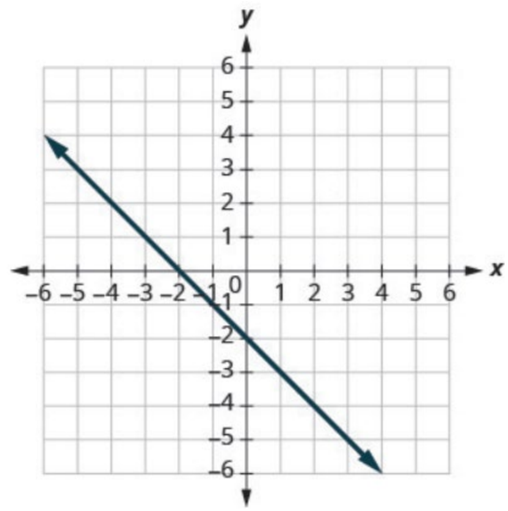
To check your understanding of the section, work out the following exercises.

1. Find the x- and y-intercepts on each graph.

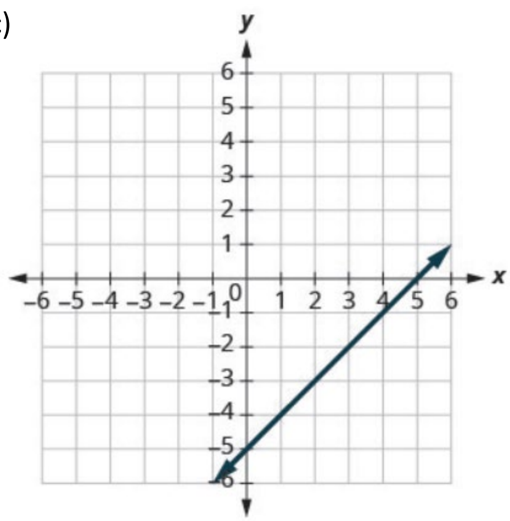
a)



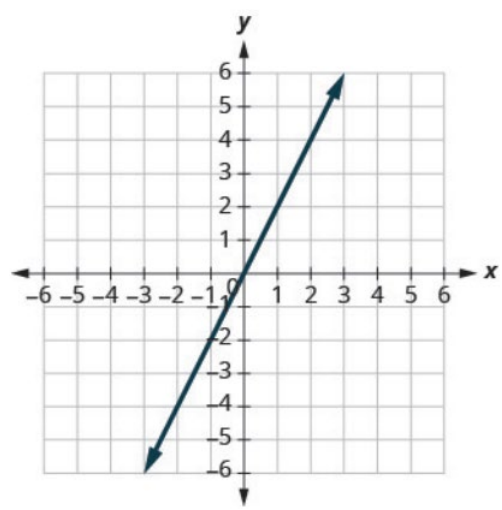
b)



c)



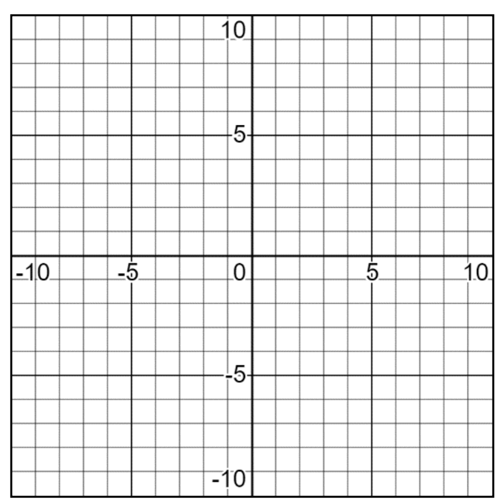
d)



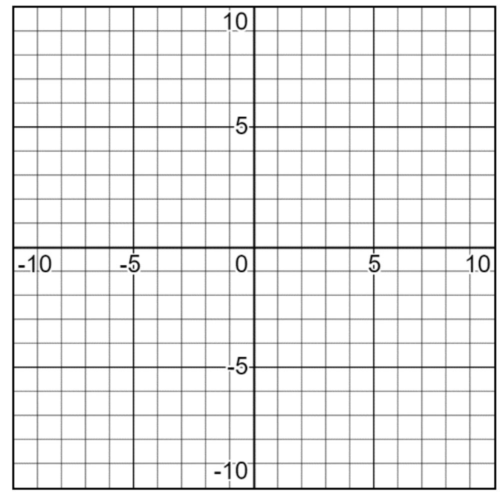
2. Find the intercepts for each equation:
   a. $x - y = -4$
   b. $3x - 2y = 12$

3. Find the intercepts for each equation:
   a. $5x - y = 5$
   b. $-x + 4y = 8$

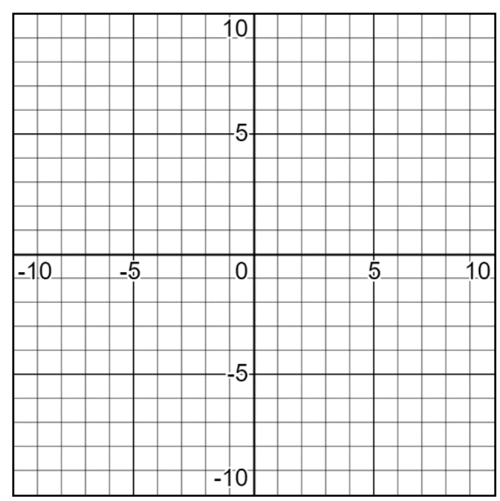
4. Graph using intercepts: $3x - y = -6$.



5. Graph using intercepts: $2x - 5y = -20$.



6. Graph the equation using any method: $y = \frac{1}{4}x - 2$



College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II

Learning Objective II.6A: Find the slope of a line.
Read Textbook Section 3.2 on page 264 and fill in the following.

Find the slope of a line.

In the previous lessons we graphed lines by plotting points and using the intercepts. As you graphed those lines you might have noticed that some lines are steeper than other lines. The slope of a line measures the steepness of a line and determines whether a line is increasing, decreasing, vertical, or horizontal.

Earlier we learned to graph lines by plotting points and using the x & y intercepts. Some lines are steeper than other lines. Slope measures the steepness of a line. In the examples below, we will determine the slope of a line and whether the line is increasing, decreasing, vertical, or horizontal.

The _____________measures the vertical change and the ____________measures the horizontal change.
The ______________of the line is $m = \frac{rise}{run}$.

Find the slope of a line from its graph using $m = \frac{rise}{run}$. (See page 257.)

1.

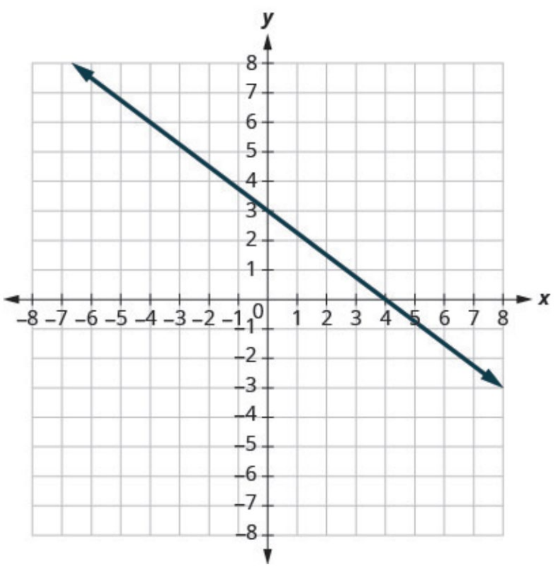
2.

3.

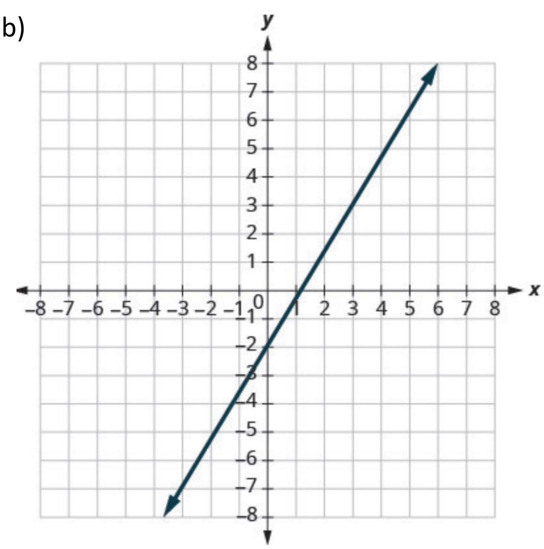
4.

__Example 1:__ Find the slope of the lines shown. Determine if the line is increasing, decreasing, vertical, or horizontal.

a)



b)



College Preparatory Integrated Mathematics Course I

Learning Objective II.6A: Slope of a Horizontal and Vertical Line.
Read Textbook Section 3.2 on page 267 and fill in the following.

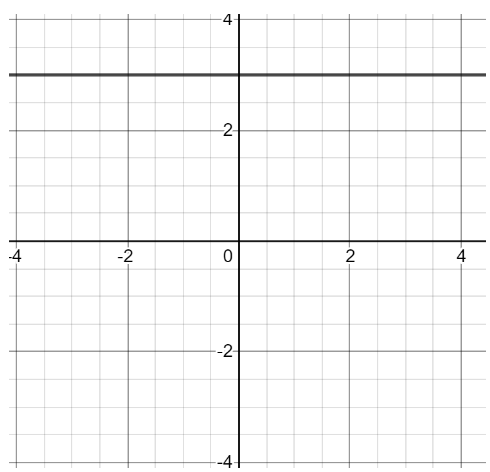
Slope of a Horizontal and Vertical Line

The slope of a ________________ line, $y = b, is\ 0$.

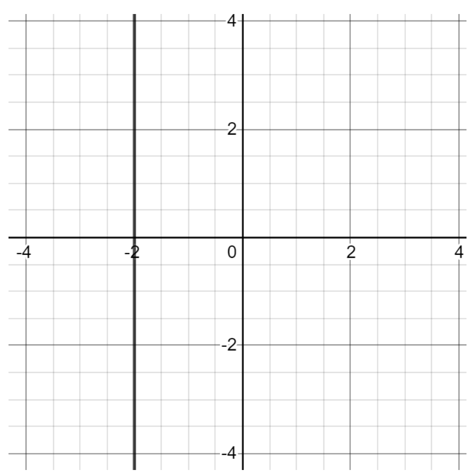
The slope of a ______________ line, $x = a$, is undefined.

Example 2: Find the slope of the lines shown. Determine if the line is increasing, decreasing, vertical, or horizontal.

a)



b)



Example 3: Find the slope of each line.

a) $x = -4$

b) $y = 7$

Learning Objective II.6A: Slope of a Line Between Two Points.
Read Textbook Section 3.2 on page 268 and fill in the following.

Slope of a Line Between Two Points

The ____________ of a line between two points $(x_1, y_1) and\ (x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example 4: Use the slope formula to find the slope of the line through the points $(-3, 4)$ and $(2, -1)$.

Example 5: Use the slope formula to find the slope of the line through the points $(-2,6)$ and $(-3,-4)$.

Learning Objective II.6A: Graph a line given a point and the slope.
Read Textbook Section 3.2 on page 270 and fill in the following.

HOW TO GRAPH A LINE GIVEN A POINT AND THE SLOPE

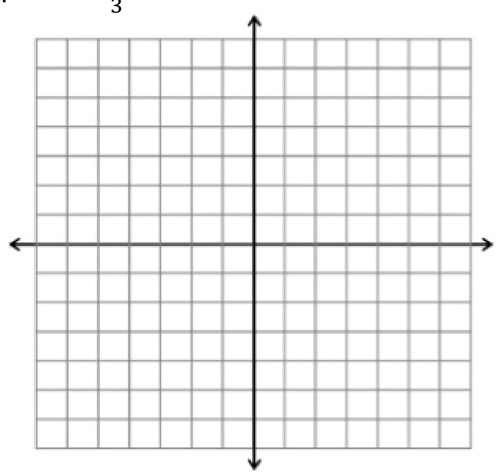
Step 1:

Step 2:

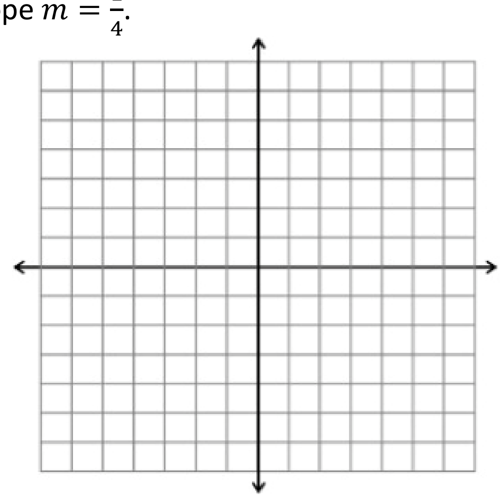
Step 3:

Step 4:

Example 6: Graph the line passing through the point $(2,-2)$ with the slope $m = \frac{4}{3}$.



Example 7: Graph the line passing through the point $(-2,3)$ with the slope $m = \frac{1}{4}$.



Learning Objective II.6A: Slope Intercept Form of an Equation of a Line
Read Textbook Section 3.2 on page 272 and fill in the following.

In the previous lessons you graphed equations using a variety of methods. If a linear equation is written in **slope-intercept form** then this will be an additional method that can be used to graph.

Slope Intercept Form of an Equation of a Line
The ________________ form of an equation of a line with slope m and $y - intercept$, $(0, b)$ is $y = mx + b$.

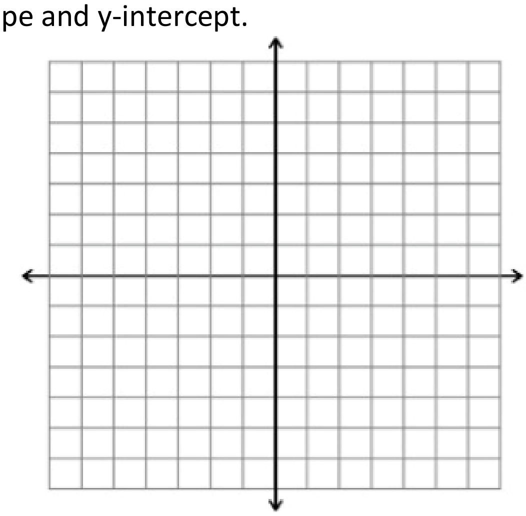
Example 8: Identify the slope and y-intercept from the equation of the line.

a) $y = \frac{2}{5}x - 1$ b) $x + 4y = 8$ c) $3x + 2y = 12$

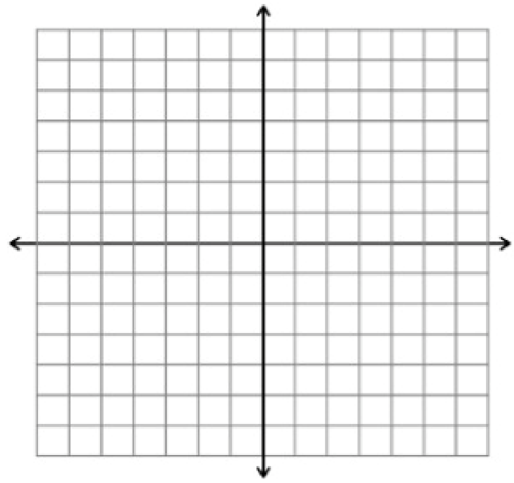
Steps to graph a linear equation using the Slope-Intercept Form of a Line.

1. Identify and Plot the y-intercept of the line.
2. Identify the slope of the line.
3. Use the slope to identify the rise over the run.
4. From the y-intercept count out the rise and run to find a second point.
5. Draw the line.

Example 9: Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.



Example 10: Graph the line of the equation $x + 4y = 8$ using slope and y-intercept.



To graph a line you can use any one of the following methods.

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
x , y	$y = mx + b$	x , y 0 , , 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Which method do you find easiest? Why?

On page 266, a Strategy for Choosing the Most Convenient Method to Graph a Line is given. Use the strategy to answer the next example.

Example 11: Determine the most convenient method to graph each line:

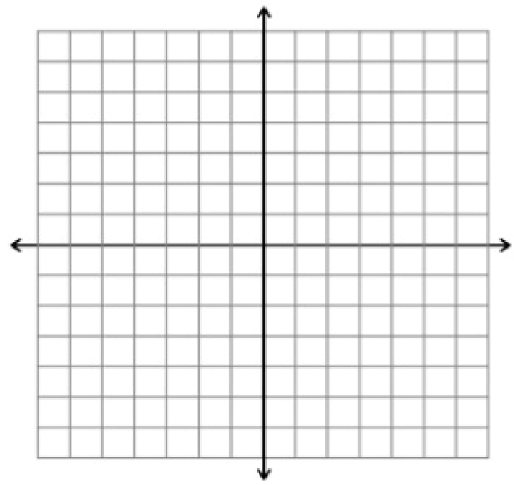
a) $3x + 2y = 12$ b) $y = 4$ c) $y = 15x - 4$ d) $x = -7$

College Preparatory Integrated Mathematics Course I

Graph and Interpret Applications of Slope–Intercept

Example 12: The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h, based on her shoe size, s.

a) Estimate the height of a child who wears women's shoe size 0.
b) Estimate the height of a woman with shoe size 8.
c) Interpret the slope and h −intercept of the equation.
d) Graph the equation.



Learning Objective II.6A: Use Slopes to Identify Parallel and Perpendicular Lines

Read Textbook Section 3.2 on page 279 and fill in the following.

Two lines that have the same slope are called __________________ lines.
Two lines that have the same ____________ and different y-intercepts are called parallel lines.

If m_1 and m_2 are the slopes of two __________________ lines, then:

- their slopes are negative reciprocals of each other, $m_1 = -\frac{1}{m_2}$.
- the product of their slopes is −1 , $m1 \cdot m2 = -1$.
- A vertical line and a horizontal line are always perpendicular to each other.

Example 13: Use slopes and y-intercepts to determine if the lines are parallel:

a) $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$

b) $y = -\frac{1}{2}x - 1$ and $x + 2y = -2$.

Example 14: Use slopes to determine if the lines are perpendicular:

a) $y = -3x + 2$ and $x - 3y = 4$

b) $5x + 4y = 1$ and $4x + 5y = 3$.

College Preparatory Integrated Mathematics Course I

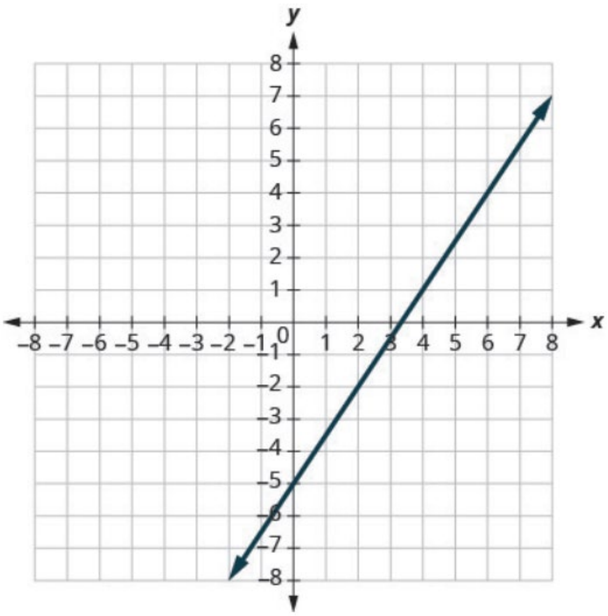
Name: ________________________________ Date: _____________________

Learning Objective II.6A

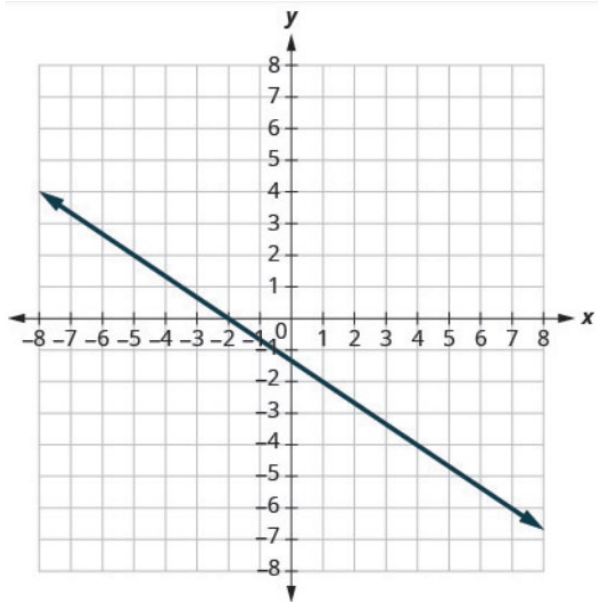
To check your understanding of the section, work out the following exercises.

1. Find the x- and y-intercepts on each graph.

a)



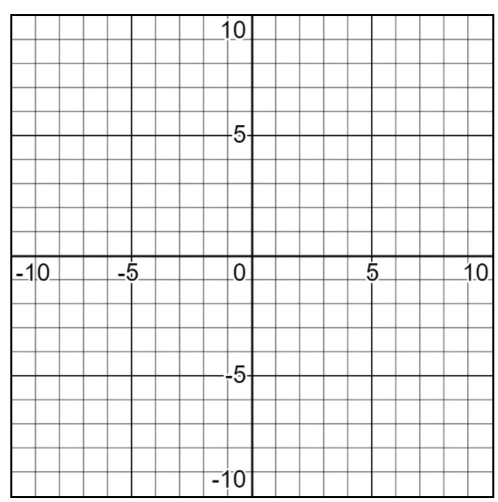
b)



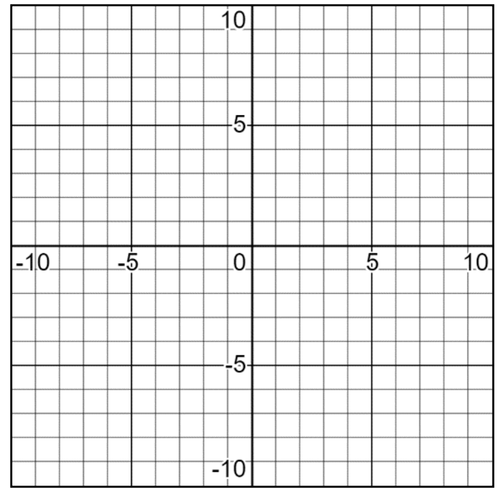
2. Find the slope of the line between the two pair points:

a. $(2, 5), (4, 0)$ b) (−2, −1), (6, 5) c) $(3, -6), (2, -2)$

3. Graph using the slope and y-intercept: $y = -7x + 3$.



4. Graph using intercepts: $3x - 4y = 8$.



5. Use slopes and y-intercepts to determine if the lines are parallel, perpendicular, or neither.

$a)\ y = \frac{3}{4}x - 3; 3x - 4y = -2$

b) $2x + 3y = 5;\ 3x - 2y = 7$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome II

Learning Objective II.6B: Find the equation of a line.
Read Textbook Section 3.3 on page 289 and fill in the following.

Find the equation of a line.

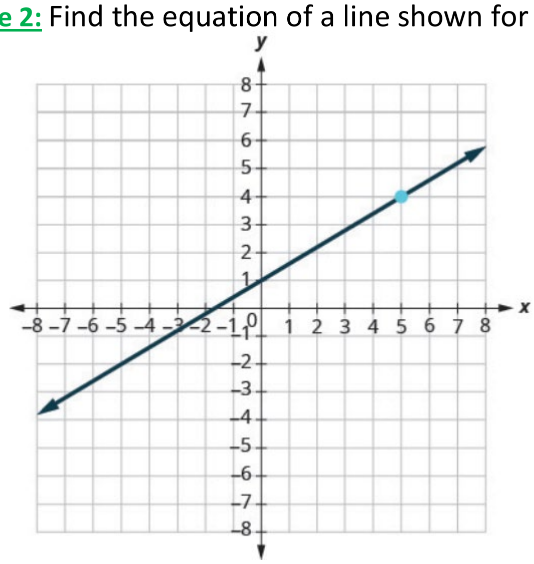
In this lesson we will find the equation of a line. Finding the equation of a line is important because it helps model real life events and the relationship between two variables.

In the previous lesson we identified the slope and y-intercept from the slope-intercept form of a line $y = mx + b$. Given the slope and y-intercept we can then find the equation.

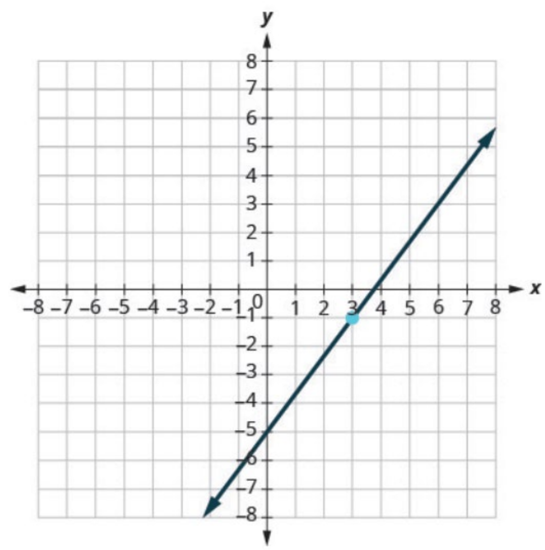
Example 1: Find the equation of a line with slope $\frac{2}{5}$ and y-intercept $(0,4)$.

Example 2: Find the equation of a line shown for each graph.

a)



b)



Learning Objective II.6B: Slope of a Line Between Two Points.
Read Textbook Section 3.3 on page 292 and fill in the following.

Point-slope Form of an Equation of a Line
The _______________ form of an equation of a line with slope m and containing the point (x_1, y_1) is:
$y - y_1 = m(x - x_1)$

The point-slope form of an equation can be used when given the slope and a point other than the y-intercept.

STEPS TO FIND AN EQUATION OF A LINE GIVEN THE SLOPE AND A POINT (See page 285).

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope-intercept form.

Example 3: Find the equation of a line with slope $m = -\frac{2}{5}$, and containing the point $(10, -5)$.

Example 4: Find the equation of a line with slope $m = -\frac{3}{4}$, and containing the point $(4, -7)$.

Example 5: Find the equation of a horizontal line containing the point $(-3, 8)$.

Learning Objective II.6B: Find an equation of a line given two points.
Read Textbook Section 3.3 on page 294 and fill in the following.

FIND AN EQUATION OF A LINE GIVEN TWO POINTS.

Step 1:

Step 2:

Step 3:

Step 4:

Example 6: Find the equation of a line containing the points $(5, 1)$ and $(5, -4)$.

Example 7: Find the equation of a line containing the points $(-4, 4)$ and $(-4, 3)$.

In the previous examples, we used different methods to write the equation of a line. See table below to help guide you in determining which method to use.

To Write an Equation of a Line		
If given:	**Use:**	**Form:**
Slope and *y*-intercept	slope-intercept	$y = mx + b$
Slope and a point	point-slope	$y - y_1 = m(x - x_1)$
Two points	point-slope	$y - y_1 = m(x - x_1)$

Learning Objective II.6B: Find an equation of a line parallel to a given line.
Read Textbook Section 3.3 on page 297 and fill in the following.

FIND AN EQUATION OF A LINE PARALLEL TO A GIVEN LINE.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Example 8: Find an equation of a line parallel to the line $y = 3x + 1$ that contains the point $(4,2)$. Write the equation in slope-intercept form.

Example 9: Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point $(6,4)$. Write the equation in slope-intercept form.

Learning Objective II.6B: FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE

Read Textbook Section 3.3 on page 299 and fill in the following.

FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Example 10: Find an equation of a line perpendicular to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope-intercept form.

Example 11: Find an equation of a line perpendicular to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date: ____________________

Learning Objective II.6B

To check your understanding of the section, work out the following exercises.

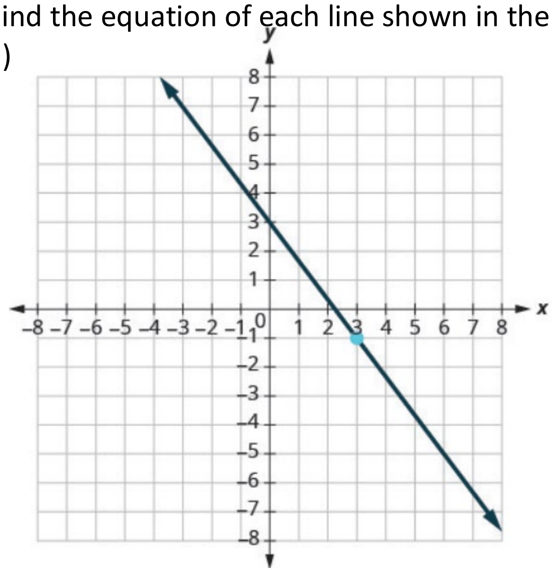
1. Find the equation of each line with given slope and y-intercept. Write the equation in slope-intercept form.

   a) slope 3 and y-intercept $(0, 5)$

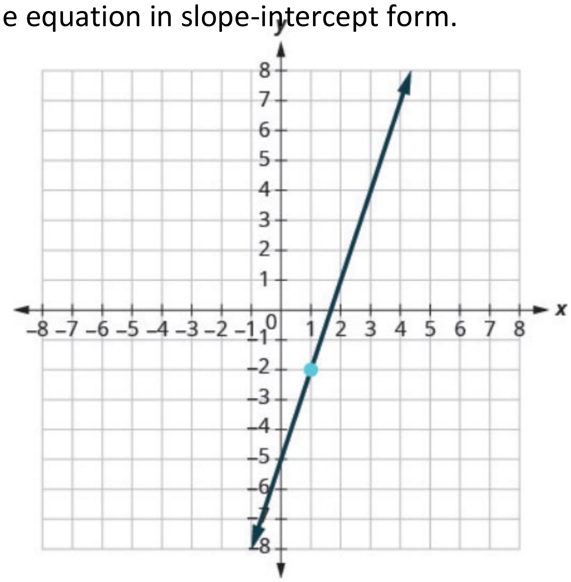
   b) slope $-\frac{3}{4}$ and y-intercept $(0, -2)$

2. Find the equation of each line shown in the graphs. Write the equation in slope-intercept form.

   a)



   b)



3. Find the equation of the lines with given slope containing the given point. Write equation in slope-intercept form.

   a) $m = \frac{5}{8}$, point $(8, 3)$

   b) Horizontal line containing $(4, -8)$

4. Find the equation of a line containing the given points. Write the equation in slope-intercept form.

   a) $(2, 6)$ and $(5, 3)$ b) $(0, -2)$ and $(-5, -3)$ c) $(7, 2)$ and $(7, -2)$

5. Find the equation of a line parallel to the line $y = 4x + 2$ and contains the point $(1, 2)$. Write the equation in slope-intercept form.

6. Find the equation of a line perpendicular to the line $4x - 3y = 5$ and contains the point $(-3, 2)$. Write the equation in slpe-intercept form.

UNIT III

III. Solve systems of equations using a variety of techniques.

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome III

Learning Objective III.1: Solve systems of linear equations in two variables by graphing.

Read Textbook Section 4.1 on page 380 and fill in the following.

Definitions

1. **When two or more linear equations are grouped together, they form a ____________________________________.**
2. **The ___________________of a system of equations are the values of the variables that make *all* equations true. A solution of a system of two linear equations is represented by an _______________________ $(\boldsymbol{x}, \boldsymbol{y})$.**

Example 1: Determine if each ordered pair is a solution to the system $\begin{cases} 3x + y = 0 \\ x + 2y = -5 \end{cases}$

a. $(1, -3)$

b. $(0,0)$

Example 2: Determine if each ordered pair is a solution to the system $\begin{cases} x - 3y = -8 \\ -3x - y = 4 \end{cases}$

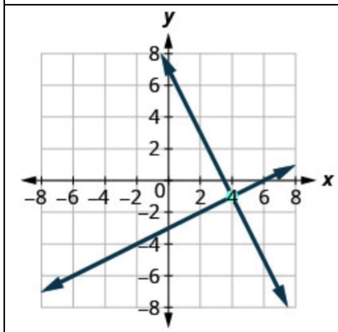
a. $(2, -2)$

b. $(-2,2)$

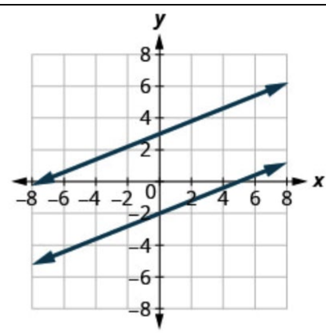
Learning Objective III.1: Solve systems of linear equations in two variables by graphing.

Read Textbook Section 4.1 on page 381 and fill in the following.

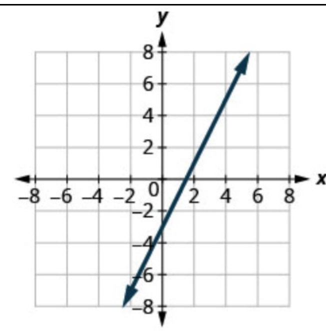
There are three possible cases for the graph of a system of two linear equations:



_________________ lines have one point in common. There is _______ solution to this system.

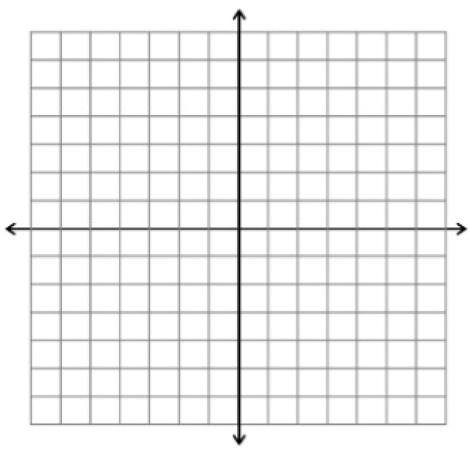


_________________ lines have no points in common. There is _______ solution to this system.

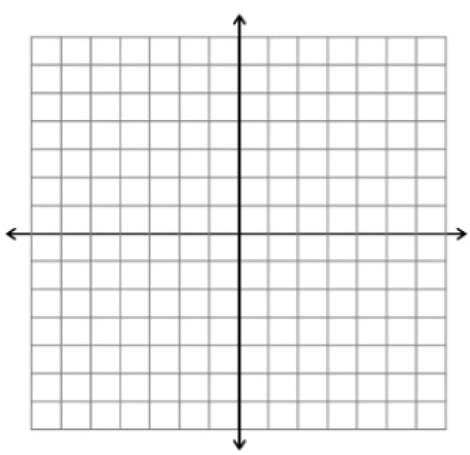


Both equations give the ________ _________. Because we have just one line, there are_____________ ________________ solutions.

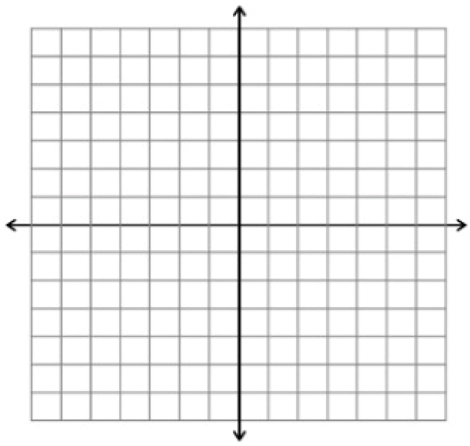
Example 3: Solve the system by graphing $\begin{cases} -x + y = 1 \\ 2x + y = 10 \end{cases}$



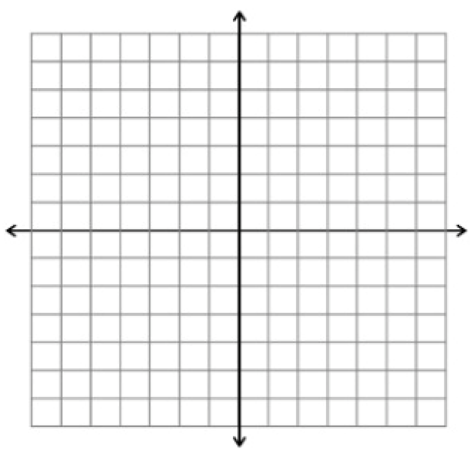
Example 4: Solve the system by graphing $\begin{cases} 2x + y = 6 \\ x + y = 1 \end{cases}$



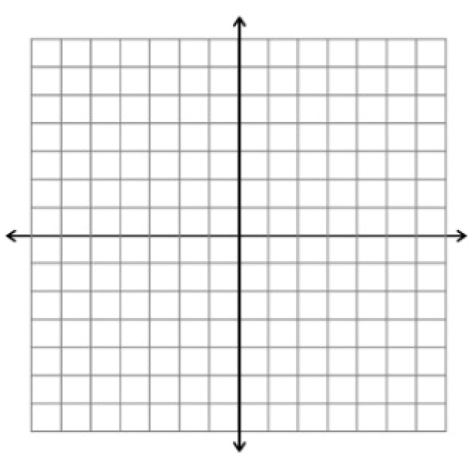
Example 5: Solve the system by graphing $\begin{cases} y = -\frac{1}{4}x + 2 \\ x + 4y = -8 \end{cases}$



Example 6: Solve the system by graphing $\begin{cases} y = 3x - 1 \\ 6x - 2y = 6 \end{cases}$



Example 7: Solve the system by graphing $\begin{cases} y = -\frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

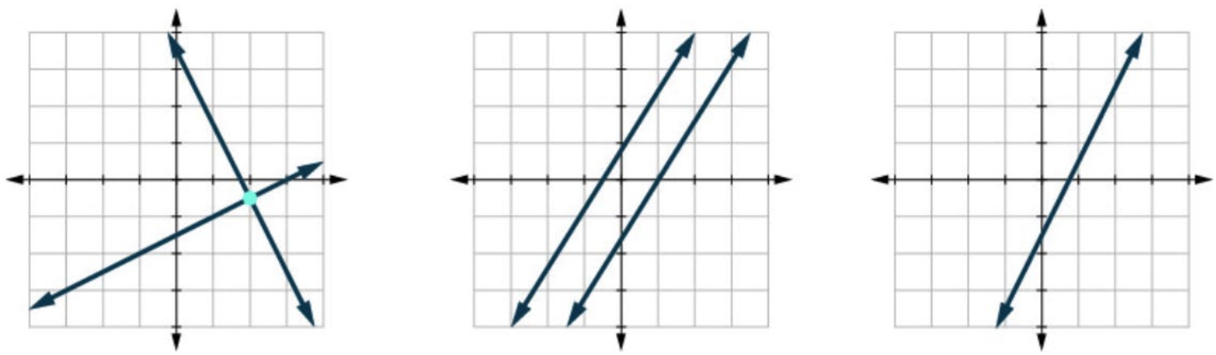


Learning Objective III.1: Solve systems of linear equations in two variables by graphing.

Read Textbook Section 4.1 on page 386 and fill in the following.

Definitions

1. **A ________ system of equations is a system of equations with at least one solution.**
2. **An _____________ system of equations is a system of equations with no solution.**
3. **If two equations are _____________, they each have their own set of solutions.**
4. **It two equations are _____________, all the solutions of one equation are also solutions of the other equation.**



Intersecting	Parallel	Coincident
Consistent System	**Inconsistent System**	**Consistent System**
Independent Equations	**Independent Equations**	**Dependent Equations**

Example 8: Without graphing, determine the number of solutions and then classify the system of equations.

a. $\begin{cases} y = -2x - 4 \\ 4x + 2y = 9 \end{cases}$

b. $\begin{cases} 3x + 2y = 2 \\ 2x + 2y = 1 \end{cases}$

Example 9: Without graphing, determine the number of solutions and then classify the system of equations.

a. $\begin{cases} y = \frac{1}{3}x - 5 \\ x - 3y = 6 \end{cases}$

b. $\begin{cases} x + 4y = 12 \\ -x + y = 3 \end{cases}$

College Preparatory Integrated Mathematics Course I

Name: ______________________________ Date: ____________________

Learning Objective III.1

To check your understanding of the section, work out the following exercises.

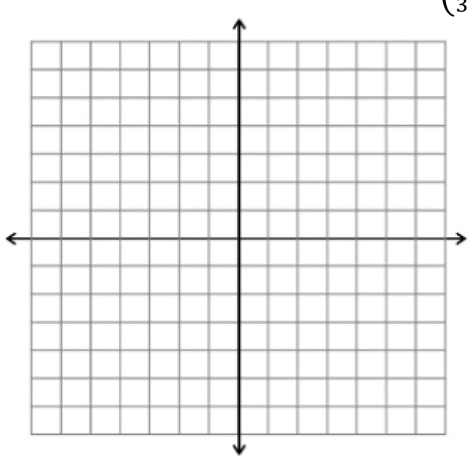
1. Determine if each ordered pair is a solution to the system $\begin{cases} -x + 3y = 9 \\ y = 2x - 2 \end{cases}$

   $(3, 4)$

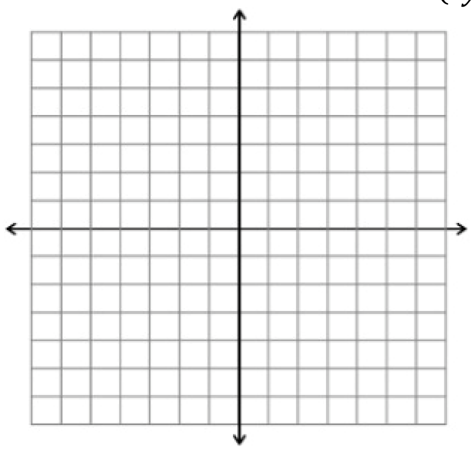
2. Determine if each ordered pair is a solution to the system $\begin{cases} y = -7x - 3 \\ y = 4 \end{cases}$

   $(2, 4)$

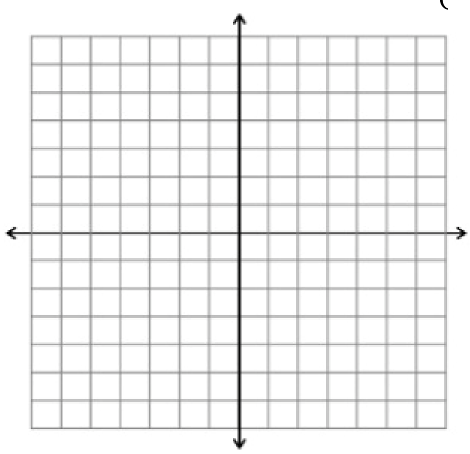
3. Solve the system by graphing $\begin{cases} y = \frac{1}{3}x - 4 \\ \frac{7}{3}x + y = 4 \end{cases}$



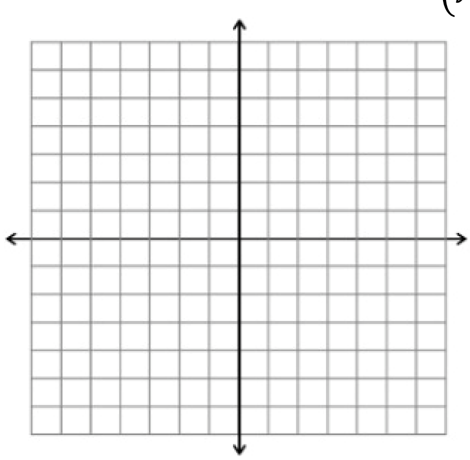
4. Solve the system by graphing $\begin{cases} -x + 3y = 9 \\ y = 2x - 2 \end{cases}$



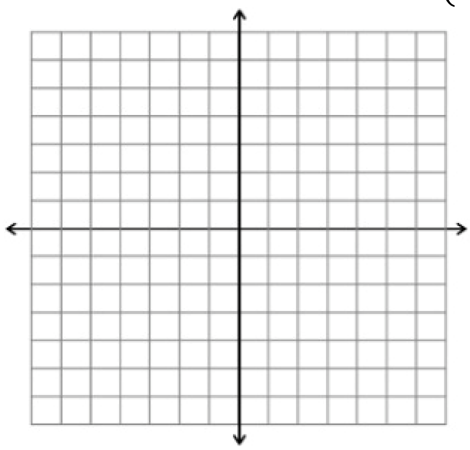
5. Solve the system by graphing $\begin{cases} y = -7x - 3 \\ y = 4 \end{cases}$



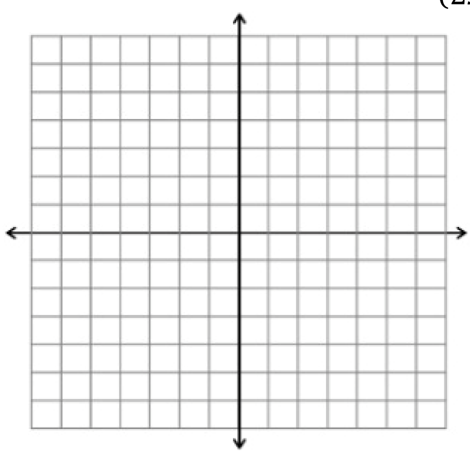
6. Solve the system by graphing $\begin{cases} y = -\frac{2}{3}x - 2 \\ y = -\frac{8}{3}x + 4 \end{cases}$



7. Solve the system by graphing $\begin{cases} y = -\frac{2}{3}x - 3 \\ y = -\frac{2}{3}x + 4 \end{cases}$



8. Solve the system by graphing $\begin{cases} y = -x + 3 \\ 2x + 2y = 6 \end{cases}$



9. Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = -6x - 3 \\ y = -x + 2 \end{cases}$$

10. Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} y = 2x + 5 \\ -x + \frac{1}{2}y = \frac{5}{2} \end{cases}$$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome III

Learning Objective III.2: Solve systems of linear equations in two variables by substitution.

Read Section 4.1 on page 380 and fill in the following.

Definitions

1. **Solving systems of linear equations by graphing is a good way to visualize the types of solutions that may result. A more accurate method for solving a system of equations is called the ______________method.**

How to solve a system of equations by substitution

Step 1.

Step 2.

Step 3.

Step 4.

Step 5.

Step 6.

Example 1: Solve the system by substitution: $\begin{cases} -2x + y = -11 \\ x + 3y = 9 \end{cases}$

Example 2: Solve the system by substitution: $\begin{cases} 2x + y = -1 \\ 4x + 3y = 3 \end{cases}$

Example 3: Solve the system by substitution: $\begin{cases} x - 4y = -4 \\ -3x + 4y = 0 \end{cases}$

Example 4: Solve the system by substitution: $\begin{cases} 4x - y = 0 \\ 2x - 3y = 5 \end{cases}$

Example 5: Solve the system by substitution: $\begin{cases} -2x + y = 5 \\ -2x + y = -1 \end{cases}$

Example 6: Solve the system by substitution: $\begin{cases} -\frac{1}{3}x + y = 5 \\ -x + 3y = 15 \end{cases}$

College Preparatory Integrated Mathematics Course I

Name: ______________________________ Date: ____________________

Learning Objective II.2

To check your understanding of the section, work out the following exercises.

1. Solve the system by substitution: $\begin{cases} y = 4x - 9 \\ y = x - 3 \end{cases}$

2. Solve the system by substitution: $\begin{cases} 4x + 2y = 10 \\ x - y = 13 \end{cases}$

3. Solve the system by substitution: $\begin{cases} y = -5 \\ 5x + 4y = -20 \end{cases}$

4. Solve the system by substitution: $\begin{cases} y = -2 \\ 4x - 3y = 18 \end{cases}$

5. Solve the system by substitution: $\begin{cases} -7x + 2y = 18 \\ 6x + 6y = 0 \end{cases}$

6. Solve the system by substitution: $\begin{cases} 4x - y = 20 \\ -2x - 2y = 10 \end{cases}$

7. Solve the system by substitution: $\begin{cases} y = 6x - 11 \\ -2x - 3y = -7 \end{cases}$

8. Solve the system by substitution: $\begin{cases} 2x - 3y = -1 \\ y = x - 1 \end{cases}$

9. Solve the system by substitution: $\begin{cases} -5x + y = -2 \\ -3x + 6y = -12 \end{cases}$

10. Solve the system by substitution: $\begin{cases} -5x + y = -3 \\ 3x - 8y = 24 \end{cases}$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome III

Learning Objective III.3: Solve systems of linear equations in two variables by addition (elimination).

Read Section 4.1 on page 391 and answer the questions below.

Definitions

1. **The third method of solving a system of linear equations accurately is the _______________method, also referred to as the addition method.**

How to solve a system of equations by elimination.

Step 1.

Step 2.

Step 3.

Step 4.

Step 5.

Step 6.

Step 7.

Example 1: Solve the system by elimination: $\begin{cases} 3x + y = 5 \\ 2x - 3y = 7 \end{cases}$

Example 2: Solve the system by elimination: $\begin{cases} 4x + y = -5 \\ -2x - 2y = -2 \end{cases}$

Example 3: Solve the system by elimination: $\begin{cases} 3x - 4y = -9 \\ 5x + 3y = 14 \end{cases}$

Example 4: Solve the system by elimination: $\begin{cases} 7x + 8y = 4 \\ 3x - 5y = 27 \end{cases}$

Example 5: Solve the system by elimination: $\begin{cases} \frac{1}{3}x - \frac{1}{2}y = 1 \\ \frac{3}{4}x - y = \frac{5}{2} \end{cases}$

Example 6: Solve the system by elimination: $\begin{cases} x + \frac{3}{5}y = -\frac{1}{5} \\ -\frac{1}{2}x - \frac{2}{3}y = \frac{5}{6} \end{cases}$

Example 7: The school that Lisa goes to is selling tickets to the annual talent show. On the first day of ticket sales the school sold 4 senior citizen tickets and 5 student tickets for a total of \$102. The school took in \$126 on the second day by selling 7 senior citizen tickets and 5 student tickets. What is the price of one senior citizen ticket and one student ticket?

Choose the Most Convenient Method to Solve a System of Linear Equations:

When you solve a system of linear equations in an application, you will not be told which method to use. You will need to make that decision yourself. So you'll want to choose the method that is easiest to do and minimizes your chance of making mistakes.

Graphing	Substitution	Addition (elimination)
Use when you need a ___________ of the situation.	Use when one equation is already ________ or be easily solved for one variable.	Use when the equations are in ____________ form.

Example7: For each system of linear equations decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

a. $\begin{cases} 4x - 5y = -32 \\ 3x + 2y = -1 \end{cases}$

b. $\begin{cases} x = 2y - 1 \\ 3x - 5y = -7 \end{cases}$

Example8: For each system of linear equations decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

a. $\begin{cases} y = 2x - 1 \\ 3x - 4y = -6 \end{cases}$

b. $\begin{cases} 6x - 2y = 12 \\ 3x + 7y = -13 \end{cases}$

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date: _____________________

Learning Objective III.3

To check your understanding of the section, work out the following exercises.

1. Solve the system by elimination: $\begin{cases} 5x + 2y = 2 \\ -3x - y = 0 \end{cases}$

2. Solve the system by elimination: $\begin{cases} 2x - 5y = 7 \\ 3x - y = 17 \end{cases}$

3. Solve the system by elimination: $\begin{cases} 3x - 5y = -9 \\ 5x + 2y = 16 \end{cases}$

4. Solve the system by elimination: $\begin{cases} 3x + 8y = -3 \\ 2x + 5y = -3 \end{cases}$

5. Solve the system by elimination: $\begin{cases} 3x + 8y = 67 \\ 5x + 3y = 60 \end{cases}$

6. Solve the system by elimination: $\begin{cases} \frac{1}{3}x - y = -3 \\ x + \frac{5}{2}y = 2 \end{cases}$

7. Solve the system by elimination: $\begin{cases} x + \frac{1}{3}y = -1 \\ \frac{1}{3}x + \frac{1}{2}y = 1 \end{cases}$

8. Lori and Missy are selling cookie dough for a school fundraiser. Customers can buy packages of chocolate chip cookie dough and packages of gingerbread cookie dough. Lori sold 8 packages of chocolate chip cookie dough and 12 packages of gingerbread cookie dough for a total of \$364. Missy sold 1 package of chocolate chip cookie dough and 4 packages of gingerbread dough for a total of \$93. Find the cost of each package of chocolate chip cookie dough and each package of gingerbread cookie dough.

9. Decide whether it would be more convenient to solve the system of equations by substitution or elimination. $\begin{cases} 8x - 15y = -32 \\ 6x + 3y = -5 \end{cases}$

10. Decide whether it would be more convenient to solve the system of equations by substitution or elimination. $\begin{cases} x = 4y - 3 \\ 4x - 2y = -6 \end{cases}$

UNIT IV

IV. Understand operations of polynomial functions and solve problems using scientific notation.

College Preparatory Integrated Mathematics Course I

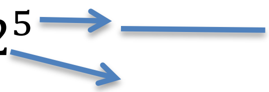
Framework Student Learning Outcome IV

Learning Objective IV.1: Exponents (Exponential Notation)
Read Textbook Section 5.2 on page 515 and answer the questions below.

Definition: (Review from Section 5.2, pg. 515)

1. Label the base and exponent for the expression below.

2^5



2. In exponential notation, 2^5 means multiply _______, five times
3. This expression 2^5 is read as _______ to the *5th* power.
4. In the expression 2^5 , the *exponent 2* tells us how many times we use the base ________ as a factor.

Example 1: Identify the base and exponent for each example below.

a) 4^3 b) 5^1 c) $(-9)^2$ d) -9^2

Example 2: Expand each expression to its Factors and Evaluate.

a) 4^3 b) 5^1 c) $(-9)^2$ d) -9^2

Example 3: Expand the exponential notation into a product of its factors.

a) x^3 b) y^2 c) $(-b)^3$ d) a^4

Learning Objective IV.1: Exponents (Simplify expressions using the Product Property for Exponents)

Read Textbook Section 5.2 on page 508 and answer the questions below.

Definitions

Product Property for Exponents

If *a* is a real number and *m* and *n* are integers, then

$$a^m \cdot a^n = a^{\_\_\_\_}$$

Add exponents.

Keep common base.

To multiply with like bases, add the exponents and keep the common base.

Example 4: Use the product property to simplify each expression.

a) $y^5 \cdot y^6$

b) $2^x \cdot 2^{3x}$

c) $2a^7 \cdot 3a$

d) $d^4 \cdot d^5 \cdot d^2$

Example 5: Use the product property to simplify each expression.

a) $b^9 \cdot b^8$

b) $4^{2x} \cdot 4^x$

c) $3p^5 \cdot 4p$

d) $x^6 \cdot x^4 \cdot x^8$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Quotient Property for Exponents)

Read Textbook Section 5.2 on page 517 and answer the questions below.

Definitions

Quotient Property for Exponents

1. If a is a real number , $a \neq 0$, and m and n are integers, then (IF m > n)

$$\frac{a^m}{a^n} = a^{\_\_\_\_\_}$$

Subtract exponents.

Keep common base.

To divide with like bases where m > n, subtract the exponents (m – n) and keep the common base.

2. If a is a real number , $a \neq 0$, and m and n are integers, then (IF n > m)

$$\frac{a^m}{a^n} = \frac{1}{a^{\_\_\_}}$$

Need a 1 in the numerator

Subtract larger exponent by smaller exponent

Keep common base.

To divide with like bases where m > n, subtract the exponents (m – n) and keep the common base, but place it denominator with a one as a numerator.

Example 6: Use Quotient Property to simplify each expression (Note: Check if m > n or m > n)

a) $\frac{x^9}{x^7}$

b) $\frac{3^{10}}{3^2}$

c) $\frac{b^8}{b^{12}}$

d) $\frac{7^3}{7^5}$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Zero Exponent Property for Exponents)

Read Textbook Section 5.2 on page 519 and answer the questions below.

Definitions

Zero Exponent Property

1. If a is a non-zero number, then a to the power of zero equals 1.

$$a^0 = \_\_\_\_\_$$

2. Any non-zero number raised to the zero power is 1.
3. See that $\frac{a^m}{a^m}$ simplifies to a— and to 1.

Example 7: Use Zero Exponent Property to simplify each expression

a) 9^0

b) n^0

c) $\frac{x^1}{x^1}$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Properties of Negative Exponents for Exponents)

Read Textbook Section 5.2 on page 519 and answer the questions below.

Definitions

Properties of Negative Exponents

If n is an integer and a ≠ 0, then

$$a^{-n} = \frac{1}{a} \qquad \text{or} \qquad \frac{1}{a^{-n}} = a$$

Example 8: Use Properties of Negative Exponents to simplify each expression

a) x^{-5}

b) 10^{-3}

c) $\frac{1}{y^{-4}}$

d) $\frac{1}{3^{-2}}$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Properties of Negative Exponents for Exponents)

Read Textbook Section 5.2 on page 522 and answer the questions below.

Definitions

Quotient to Negative Power Property

If *a* and *b* are real numbers, a ≠ 0, b ≠ 0, and *n* is an integer, then

$$\left(\frac{a}{b}\right)^{-n} = \left(-\right)^{n}$$

Take the reciprocal of the fraction and change the sign of the exponent.

Example 9: Use the Quotient to Negative Power Property to simplify each expression

a) $\left(\frac{5}{7}\right)^{-2}$

b) $\left(-\frac{x}{y}\right)^{-3}$

Example 10: Use the Product Property and the Properties of Negative Exponents to simplify each expression

a) $z^{-5} \cdot z^{-3}$

b) $(m^4 n^{-3})(m^{-5} n^{-2})$

c) $(2x^{-6} y^8)(-5x^5 y^{-3})$

College Preparatory Integrated Mathematics Course I

Learning Objective IV.1: Exponents (Simplifying Expressions using the Power Property of Exponents for Exponents)

Read Textbook Section 5.2 on page 524 and answer the questions below.

Definitions

Power Property for Exponents

If a is a real number and m and n are integers, then

$$(a^m)^n = a$$

Multiply exponents.

Keep common base.

To raise a power to a power, multiply the exponents.

Example 11: Use the Power Property of Exponents to simplify each expression

a) $(y^5)^9$ b) $(4^4)^7$ c) $(y^3)^6(y^5)^4$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Product to a Power Property of Exponents for Exponents)

Read Textbook Section 5.2 on page 525 and answer the questions below.

Definitions

Product to a Power Property for Exponents (p.525)

If a and b are real numbers and m is a whole number, then

$$(a \cdot b)^m = a \quad \cdot b$$

To raise a power to a power, multiply the exponents.

Example 12: Use the Product to a Power Property of Exponents to simplify each expression

a) $(-3mn)^3$ b) $(-4a^2b)^0$

c) $(6k^3)^{-2}$ d) $(5x^{-3})^2$

Learning Objective IV.1: Exponents (Simplifying Expressions using the Quotient to a Power Property of Exponents for Exponents)

Read Textbook Section 5.2 on page 527 and answer the questions below.

Definitions

Quotient to a Power Property for Exponents

If *a* and *b* are real numbers and b ≠ 0, and *m* is an integer, then

$$\left(\frac{a}{b}\right)^m = \frac{a}{b}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

Example 13: Use the Quotient to a Power Property of Exponents to simplify each expression

a) $\left(\frac{b}{3}\right)^4$

b) $\left(\frac{k}{j}\right)^{-3}$

c) $\left(\frac{2xy^2}{z}\right)^3$

d) $\left(\frac{4p^{-3}}{q^2}\right)^2$

Learning Objective IV.1: Exponents (Simplifying Expressions using all the Summary of Exponent Properties)

Read Textbook Section 5.2 on page 530 and answer the questions below.

Definitions

Summary of Exponent Properties

If *a* and *b* are real numbers, and *m* and *n* are integers, then

Property	Description
Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
Zero Exponent Property	$a^0 = 1, \quad a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad b \neq 0$
Properties of Negative Exponents	$a^{-n} = \frac{1}{a^n}$ *and* $\frac{1}{a^{-n}} = a^n$
Quotient to a Negative Exponent	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example 14: Simplify each expression by applying several properties. Write each results using positive exponents only.

a) $(3x^2y)^4(2xy^2)^3$

b) $\frac{(x^3)^4(x^{-2})^5}{(x^6)^5}$

c) $\left(\frac{2xy^2}{x^3y^{-2}}\right)^2\left(\frac{12xy^3}{x^3y^{-1}}\right)^{-1}$

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date:____________________

Learning Objective IV.1

To check your understanding of the section, work out the following exercises.

1. In the following exercises, simplify each expression using the properties for exponents

a) $2y \cdot 4y^3$ b) $\frac{u^{24}}{u^3}$ c) -27^0

2. In the following exercises, simplify each expression by applying several properties. Leave final answers with positive exponents only.

a) 10^{-3} b) $\frac{1}{t^{-9}}$ c) $\left(-\frac{1}{5}\right)^{-2}$ d) $(3 \cdot 4)^{-2}$

3. In the following exercises, simplify each expression by applying several properties. Leave final answers with positive exponents only.

a) $\left(\frac{p^{-1}q^4}{r^{-4}}\right)^2$ b) $(m^2n)^2(2mn^5)^4$

c) $\frac{(-2p^{-2})^4(3p^4)^2}{(-6p^3)^2}$

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome IV

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Determine the degree of polynomials)
Read Textbook Section 5.1 on page 502 and answer the questions below.

Definitions

1. A **monomial** is an algebraic expression with one term. A monomial in one variable is a term of the form ______, where *a* is a constant and *m* is a whole number

2. A monomial, or ______________ monomials combined by addition or subtraction, is a **polynomial**

3. Determine how many terms they the following polynomials have:
   monomial—A polynomial with exactly ______ term is called a monomial.
   binomial—A polynomial with exactly ______ terms is called a binomial.
   trinomial—A polynomial with exactly _________ terms is called a trinomial.

4. The **degree of a term** is the ________of the exponents of its variables.
   The **degree of a constant** is ______.
   The **degree of a polynomial** is the highest degree of ____ its terms.

Example 1: Count the number of terms, then determine the type (whether each polynomial is a monomial, binomial, trinomial, or other polynomial). Then, find the degree of each term and finally determine the degree of the polynomial. Complete the table below.

Polynomial	Number of Terms	Type	Degree of each term, separate by commas	Degree of Polynomial
$7y^2 - 5y + 3$				
$-2a^4b^2$				
$3x^5 - 4x^3 - 6x^2 + x - 8$				
$2y - 8xy^3$				
15				

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Add and Subtract polynomials)

Read Textbook Section 5.1 on page 504 and answer the questions below.

Definitions

1. Adding and subtracting monomials is the same as_______________ like terms.

2. If the monomials are like terms, we just combine them by ____________ or _____________ the coefficients

3. We can think of adding and subtracting polynomials as just adding and subtracting a series of _______________. Look for the like terms—those with the same variables and the same exponent. The __________________ Property allows us to rearrange the terms to put like terms together.

Example 2: Add or subtract.

a) $a^2 + 7b^2 - 6a^2$

b) $16pq^3 - (-7pq^3)$

c) $(7y^2 - 2y + 9) + (4y^2 - 8y - 7)$

d) $(9w^2 - 7w + 5) - (2w^2 - 4)$

e) $(a^3 - a^2b) - (ab^2 + b^3) + (a^2b + ab^2)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Evaluate a Polynomial Function)
Read Textbook Section 5.1 on page 507 and answer the questions below.

Definitions

A **polynomial function** is a function whose range is defined by a _________________.
For Example, $f(x) = x^2 + 5x + 6$ and $g(x) = 3x - 4$ are polynomial functions, because _______________________ and ____________________ are polynomials.

Example 3: For the function $f(x) = 5x^2 - 8x + 4$, find the following.

a) $f(4) =$

b) $f(-2) =$

c) $f(0) =$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Add and Subtract Polynomial Functions)
Read Textbook Section 5.1 on page 509 and answer the questions below.

Definitions

Addition and Subtraction of Polynomial Functions

For functions $f(x)$ and $g(x)$,

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$

Just as polynomials can be added and subtracted, polynomial _________________ can also be added and subtracted.

Example 4: For the functions $f(x) = 3x^2 - 5x + 7$ and $g(x) = x^2 - 4x - 3$ find:

a) $(f + g)(x) =$

b) $(f - g)(x) =$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply Monomials)

Read Textbook Section 5.3 on page 540 and answer the questions below.

Definitions

1. Since monomials are algebraic expressions, we can use the properties of ______________ to multiply monomials

Example 5: Multiply.

$a)$ $(3x^2)(-4x^3)$

b) $(\frac{5}{6}x^3y)(12xy^2)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply a Polynomial by a Monomial)

Read Textbook Section 5.3 on page 545 and answer the questions below.

Definitions

1. Multiplying a **polynomia**l by a **monomial** is really just applying the ____________________ Property

Example 6: Multiply.

a) $-2y(4y^2 + 3y - 5)$

b) $3x^2y(x^2 - 8xy + y^2)$

College Preparatory Integrated Mathematics Course I

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply a Binomial by a Binomial)

Read Textbook Section 5.3 on page 541 and answer the questions below.

Definitions

Just like there are different ways to represent multiplication of numbers, there are several methods that can be used to multiply a binomial times a binomial.

1. Multiplying a **binomia**l by a **binomial** is really just applying the ______________________ Property.

2. How to use **The FOIL method** to multiply two binomials:
   Step1: Multiply the F__________terms.
   Step2: Multiply the O__________terms.
   Step3: Multiply the I__________terms.
   Step4: Multiply the L__________terms.

3. To multiply binomials, use the:
   - Distributive Property
   - FOIL Method
   - Vertical Method

Example 7: Multiply by using the Distributive Property.

a) $(y+5)(y+8)$

b) $(4y+3)(2y-5)$

Example 8: Multiply by using The FOIL Method.

a) $(y-7)(y+4)$

b) $(4x+3)(2x-5)$

Example 9: Multiply by using the Vertical Method.

a) $(3y-1)(2y-6)$

b) $(b+3)(2b^2-5b+8)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply a Polynomial by a Polynomial)

Read Textbook Section 5.3 on page 545 and answer the questions below.

Definitions

Multiply a **polynomial** by a **polynomial**. Remember, FOIL will ____ work in this case, but we can use either the Distributive Property or the Vertical Method.

1. To multiply a **trinomial** by a **binomial,** use the
   ____________________ Property
   ____________________ Property

Example 11: Multiply by using the Distributive Property.

$(b+3)(2b^2-5b+8)$

Example 12: Multiply by using the Vertical Method

$(y-3)(y^2-5y+2)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply Special Products)
Read Textbook Section 5.3 on page 547 and answer the questions below.

Definitions

1. **Binomial Squares Pattern:** If a and b are real numbers,
   $(a+b)^2 = a^2 + \_\_\_\_\_\_\_\_ + b^2$
   $(a-b)^2 = a^2 - \_\_\_\_\_\_\_\_ + b^2$
   To square a binomial, square the first term, square the last term , double their product.
2. **Conjugate Pair:** A conjugate pair is two binomials of the form,
   $(a-b),\ \ (a+b)$
   The pair of binomials each have the same first term and the same last term, but one binomial is a ___________ and the other is a <u>difference</u>.
3. **Product of Conjugates Pattern:** If a and b are real numbers,
   $(a-b)(a+b) = \ a \qquad b$ This product is call the **difference of ____________.**
   To multiply conjugates, square the first term, square the last term, write it as a difference of squares.

Example 13: Multiply using the binomial squares pattern.

a) $(x+5)^2$

b) $(2x-3y)^2$

Example 14: Multiply using the product of conjugates pattern

a) $(2x+5)(2x-5)$

b) $(5m-9n)(5m+9n)$

Example 15: Special Products Choose the appropriate pattern and use it to find the product

a) $(2x-3)(2x+3)$

b) $(5x-8)^2$

c) $(6m+7)^2$

d) $(5x-6)(6x+5)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Multiply Polynomial Functions)

Read Textbook Section 5.3 on page 551 and answer the questions below.

Definitions

Just as polynomials can be multiplied, polynomial functions can also be multiplied

1. **Multiplication of Polynomial Functions**
   For functions $\boldsymbol{f(x)}$ and $\boldsymbol{g(x)}$,

$$\boldsymbol{(f \cdot g)(x) = f(x) \cdot g(x)}$$

Example 16: Multiply Polynomial Functions

a) For Functions $f(x) = x + 2$ and $(x) = x^2 - 3x - 4$, find

$$(f \cdot g)(x) =$$

b) For Functions $f(x) = x - 5$ and $(x) = x^2 - 2x + 3$, find

$$(f \cdot g)(x) =$$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing Monomials)
Read Textbook Section 5.4 on page 557 and answer the questions below.

Definitions

1. We are now familiar with all the properties of exponents and used them to multiply polynomials. Next, we'll use these properties to divide _______________ and polynomials.

Example 17: Find the Quotient

a) $54a^2b^3 \div (-6ab^5)$

b) $\frac{14x^7y^{12}}{21x^{11}y^6}$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing a Polynomial by a Monomial)
Read Textbook Section 5.4 on page 551 and answer the questions below.

Definitions

1. The method we'll use to divide a polynomial by a _______________ is based on the properties of fraction addition.
   The sum $\frac{y}{5} + \frac{2}{5}$ simplifies to $\frac{y+2}{5}$. Now we will do this in reverse to split a single fraction into separate fractions. For example $\frac{y+2}{5}$ can be written $\frac{y}{5} + \frac{2}{5}$.

2. This is the "reverse" of fraction addition and it states that if *a*, *b*, and *c* are numbers where $c \neq 0$, then $\frac{a+b}{c} = \frac{\ }{c} + \frac{\ }{c}$ =

3. **Division of a Polynomial by a Monomial**: To divide a polynomial by a monomial, divide each term of the polynomial by the _________________.

Example 18: Find the Quotient

a) $(18x^3y - 36xy^2) \div (-3xy)$

b) $(32a^2b - 16ab^2) \div (-8ab)$

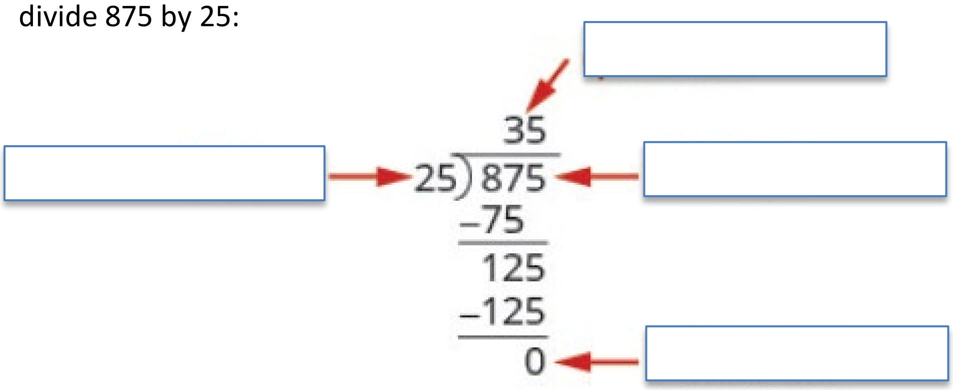
Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing Polynomials using Long Division)

Read Textbook Section 5.4 on page 559 and answer the questions below.

Definitions

1. Divide a **polynomial by a __________________**, we follow a procedure very similar to long division of numbers.

2. Identify the quotient, dividend, divisor, and remainder when you use long division to divide 875 by 25:



Now we will divide a **trinomial by a binomial**. As you read through the example, notice how similar the steps are to the numerical example above.

3. When we divided 875 by 25, we had no remainder. But sometimes division of numbers does leave a **remainder**. The same is true when we divide polynomials. We write the remainder as a fraction with the _________________ as the **denominator**.

4. The terms were written in **descending order of degrees**, and there were no missing degrees. For example, if the dividend is $x^4 - x^2 + 5x - 6$, and it's missing an x^3 term, then we add __________ as a placeholder

5. **Check your results:** If we did the division correctly, the product (quotient x divisor) should equal the ___________________.

Example 19: Find the Quotient. If needed, write the remainder as a fraction with the divisor as the denominator. Use a separate page to do all of the work, label each step carefully.

a) $(x^2 + 9x + 20) \div (x + 5)$

b) $(x^4 - x^2 + 5x - 6) \div (x + 2)$

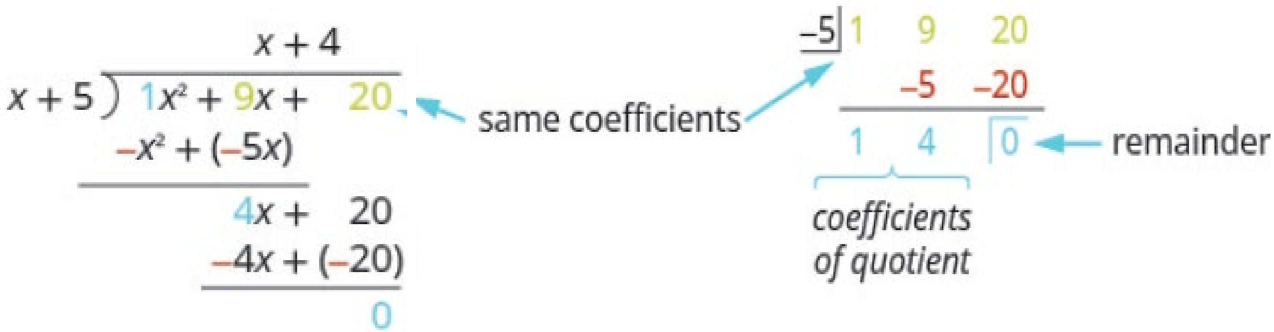
c) $(8a^3 + 27) \div (2a + 3)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing Polynomials using Synthetic Division)
Read Textbook Section 5.4 on page 562 and answer the questions below.

Definitions

1. Look at the pattern between Long Division and Synthetic Division.

Long Division **Synthetic Division**



   a) Synthetic division basically just removes unnecessary repeated variables and numbers.
   b) The first row of the synthetic division is the coefficients of the ______________.
   c) The second row of the synthetic division are the numbers shown in________ in the division problem.
   d) The third row of the synthetic division are the numbers shown in _________ in the division problem.
   e) Notice the quotient and _______________ are shown in the third row.
2. Synthetic Division **only works** when the **divisor is** of the form **x – c**
3. **Start** by writing the **dividend** with decreasing powers of **x.**
4. **End with…Check:** (quotient X divisor) + remainder = ________________

Example 20: (use your own paper to write down all steps and check your work)

Use Synthetic Division to find the Quotient and Remainder when

a) $2x^3 + 3x^2 + x + 8$ is divided by $x + 2$

b) $x^4 - 16x^2 + 3x + 12$ is divided by $x + 4$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing Polynomial Functions)

Read Textbook Section 5.4 on page 564 and answer the questions below.

Definitions

Just as polynomials can be divided, polynomial functions can also be divided.

1. Division of Polynomial Functions

   For $f(x)$ and $g(x)$, where $g(x) \neq 0$, $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example 21: For the functions $f(x) = x^2 - 5x - 14$ and $g(x) = x + 2$, find.

a) $\left(\frac{f}{g}\right)(x)$

b) $\left(\frac{f}{g}\right)(-4)$

Example 22: For the functions $f(x) = x^2 - 5x - 24$ and $g(x) = x + 3$, find.

b) $\left(\frac{f}{g}\right)(x)$

b) $\left(\frac{f}{g}\right)(-3)$

Learning Objective IV.2: Operations of polynomial functions to include addition, subtraction, multiplication, and division. (Dividing Polynomial Functions)

Read Textbook Section 5.4 on page 565 and answer the questions below.

Definitions

1. If function notation to get the dividend $f(x)$, we multiply the quotient, $q(x)$ times the divisor, $x - c$, and add the remainder, r.

(dividend) = (quotient X divisor) + (remainder)

$f(x) = q(x)(x-c) + r$

If we evaluate this at c we get: $f(c) = q(c)(c-c) + r$

$f(c) = q(c)(0) + r$

$f(c) =$ __________

2. **Remainder Theorem** If the polynomial function $f(x)$ is divided by $x - c$, then the remainder ____.

3. **Factor Theorem** For any polynomial function $f(x)$,
   - If $x - c$ is a factor of $f(x)$, then $f(c)$=__________
   - If $f(c) = 0$, then $x - c$ is a factor of _________

Example 23: Use the Remainder Theorem to find the Remainder when

a) $f(x) = x^3 + 3x + 19$ is divided by $x + 2$

b) $f(x) = x^3 - 7x + 12$ is divided by $x + 3$

Example 24: Use the Factor Theorem.

a) Use the Factor to determine if $x - 4$ is a factor of $f(x) = x^3 - 64$.

b) Use the Factor to determine if $x - 5$ is a factor of $f(x) = x^3 - 125$.

Name: ________________________________ Date:____________________

Learning Objective IV.2

To check your understanding of the section, work out the following exercises.

1. In the following exercises, add or subtract the monomials and polynomials.
a) $-12w + 18w + 7x^2y - (-12x^2y)$ b) $(x^3 - x^2y) - (4xy^2 - y^3) + (3x^2y - xy^2)$

2. **Add and Subtract Polynomial Functions.**
Given the following polynomial functions $f(x) = 2x^2 - 4x + 1$ and $g(x) = 5x^2 + 8x + 3$, find
   a) $(f + g)(x)$
   b) $(f + g)(2)$
   c) $(f - g)(x)$
   d) $(f - g)(-3)$

3. In the following exercises, **multiply** by any method.
a) Multiply two Monomials. $(-10x^5)(-3x^3)$

b) Multiply a Polynomial by a Monomial. $-5t(t^2 + 3t - 18)$

c) Multiply two Binomials. (y + 9)(y + 3)

d) Multiply two Binomials. $(2y - 3z)^2$

e) Multiply two Polynomials. $(x + 5)(x^2 + 4x + 3)$

4. **Multiply Polynomial Functions**
Given the following polynomial functions $f(x) = x^2 - 5x + 2$ and $g(x) = x^2 - 3x - 1$, find
   a) $(f \cdot g)(x)$
   b) $(f \cdot g)(-1)$

5. In the following exercises, **divide** by various method.
a) Divide two Monomials. $(20m^8n^4) \div (30m^5n^9)$

b) Divide a Polynomial by a Monomial. $(63m^4 - 42m^3) \div (-7m^2)$

c) Divide the Polynomials using Long Division. $(y^2 + 7y + 12) \div (y + 3)$

d) Divide the Polynomials using Synthetic Division. $(x^4 + x^2 + 6x - 10)$ is divided by $(x + 2)$

6. **Divide Polynomial Functions**
Given the following polynomial functions $f(x) = x^3 + x^2 - 7x + 2$ and $g(x) = x - 2$, find

a) $\left(\frac{f}{g}\right)(x)$

b) $(\frac{f}{g})(2)$

Framework Student Learning Outcome IV

Learning Objective IV.3: Solving Problems using scientific notation
Read Textbook Section 5.2 on page 532 and answer the questions below.

Definition:

1. A number is expressed in **scientific notation** when it is of the form.
   ____ x 10^n where $1 \leq a < 10$ and n is an integer.
   It is customary to use the x multiplication sign, even though we avoid using this sign elsewhere in algebra.

2. To **Convert a Decimal to Scientific Notation**.
   Step 1. Move the decimal point so that the first factor is greater than or equal to ____ but less than ______.
   Step 2. Count the number of decimal places, ____, that the decimal point was moved.

   Step 3. Write the number as a product with a power of _____. If the original number is
   - Greater than 1, the power of 10 will be __________
   - Between 0 and 1, the power of 10 will be __________

   Step 4. Check

3. To **Convert Scientific Notation to Decimal Form**.
   Step 1. Determine the exponent, *n*, on the factor _____.
   Step 2. Move the decimal ______ places, adding zeros if needed.
   - If the exponent is positive, move the decimal point *n* places to the __________.
   - If the exponent is negative, move the decimal point $|n|$ places to the __________.

   Step 3. Check

Example 1: Write each number in scientific notation.

a) 0.0052

b)37,000

Example 2: Convert each number to decimal form.

a) 6.2×10^3

b) -8.9×10^{-2}

Example 3: Multiply or Divide each scientific number and write final answers in decimal form

a) $(-4 \times 10^5)(2 \times 10^7)$

b) $\frac{9\times10^3}{3\times10^{-2}}$

College Preparatory Integrated Mathematics Course I

Name: ________________________________ Date:____________________

Learning Objective IV.3

To check your understanding of the section, work out the following exercises.

1. In the following exercises, write each number in scientific notation.

a) 57,000 b) 0.026 c) 8,750,000 d) 0.00000871

2. In the following exercises, convert each number to decimal form.

a) $5.2 \text{ X } 10^2$ b) $2.5 \text{ X } 10^{-2}$ c) $-8.3 \text{ X } 10^2$ d) $-4.13 \text{ X } 10^{-5}$

3. In the following exercises, multiply or divide as indicated. Write your answer in decimal form.

a) $(\mathbf{3 \times 10^{-5}})(\mathbf{3 \times 10^{9}})$ b) $(\mathbf{3.5 \times 10^{-4}})(\mathbf{1.6 \times 10^{-2}})$

c) $\dfrac{\mathbf{5 \times 10^{-2}}}{\mathbf{1 \times 10^{-10}}}$ d) $\dfrac{\mathbf{8 \times 10^{6}}}{\mathbf{4 \times 10^{-1}}}$

UNIT V

V. Understand, interpret, and make decisions based on financial information commonly presented to consumers.

College Preparatory Integrated Mathematics Course I

Framework Student Learning Outcome V.4

Learning Objective V.4 Factor Polynomials using the technologies of the greatest common factor (GCF) and grouping

Read Textbook Section 6.1 on page 584 and answer the questions below.

Definition: GREATEST COMMON FACTOR

The **greatest common factor** (GCF) of two or more expressions is the largest expression that is a factor of all the expressions.

Find the greatest common factor (GCF) of two expressions.

1. Step 1. Factor each coefficient into primes. Write all variables with exponents in expanded form.
2. Step 2. List all factors—matching common factors in a column. In each column, circle the common factors.
3. Step 3. Bring down the common factors that all expressions share.
4. Step 4. Multiply the factors.

Example 1: Find the greatest common factor of $21x^3$, $9x^2$, $15x$

Example 2: Find the greatest common factor of $25m^4$, $35m^3$, $20m^2$

Example 3: Find the greatest common factor of $14x^3$, $70x^2$, $105x$

Framework Student Learning Outcome V.4

Learning Objective V.4 Factor Polynomials using the technologies of the greatest common factor (GCF) and grouping

Read Textbook Section 6.1 on page 585 and answer the questions below.

Definition: Factor the Greatest Common Factor from a Polynomial

It is sometimes useful to represent a number as a product of factors, for example, **12** as **2·6** or **3·4**.

In algebra, it can also be useful to represent a polynomial in factored form. We will start with a product, such as $3x^2 + 15x$, and end with its factors, $3x(x + 5)$. To do this we apply the Distributive Property "in reverse."
We state the Distributive Property here just as you saw it in earlier chapters and "in reverse."

Definition: Distributive Property

If a, b, and c are real numbers, then

$$a(b + c) = ab + ac \text{ and } ab + ac = a(b + C)$$

The form on the left is used to multiply. To form on the right is used to factor.

Example 1: $8m^3 - 12m^2n + 20mn^2$

Example 2: $9xy^2 + 6x^2y^2 + 21y^3$

Example 3: $3p^3 - 6p^2q + 9pq^3$

Framework Student Learning Outcome V.4

Learning Objective V.4 Factor Polynomials using the technologies of the greatest common factor and grouping

Read Textbook Section 6.1 on page 586 and answer the questions below.

Steps: Factor the greatest common factor from a polynomial.

Step 1. Find the GCF of all the terms of the polynomial.
Step 2. Rewrite each term as a product using the GCF.
Step 3. Use the "reverse" Distributive Property to factor the expression.
Step 4. Check by multiplying the factors.

Example 1: $5x^3 - 25x^2$

Example 2: $6y^3 - 15y^2$

Example 3: $15x^3y - 3x^2y^2 + 6xy^3$

Example 4: $2x^3 + 12x^2$

Example 5: $8x^3y - 10x^2y^2 + 12xy^3$

Example 6: $8ab + 2a^2b^2 - 6ab^3$

When the leading coefficient is negative, we factor the negative out as part of the GCF.

Example 1: $-4a^3 + 36a^2 - 8a$

Example 2: $-4b^3 + 16b^2 - 8b$

So far our greatest common factors have been monomials. In the next example, the GCF is a binomial.

Example 1: $3y(y+7) - 4(y+7)$

Example 2: $4m(m+3)-7(m+3)$

Framework Student Learning Outcome V.4

Learning Objective V.4 Factor Polynomials using the technologies of the greatest common factor (GCF) and grouping.

Read Textbook Section 6.1 on page 588 and answer the questions below.

Definition: Factor by Grouping

Sometimes there is no common factor of all the terms of a polynomial. When there are four terms we separate the polynomial into two parts with two terms in each part. Then look for the GCF in each part. If the polynomial can be factored, you will find a common factor emerges from both parts. Not all polynomials can be factored. Just like some numbers are prime, some polynomials are prime.

Example 1: $xy + 3y + 2x + 6$

Example 2: $xy + 8y + 3x + 24$

Example 3: $ab + 7b + 8a + 56$

Learning Objective V.4 Factor Polynomials using the technologies of the greatest common factor and grouping

Steps: Factor by Grouping

Step 1. Group terms with common factors
Step 2. Factor out the common factor in each group.
Step 3. Factory the common factor from the expression
Step 4. Check by multiplying the factors

Example 1: $x^2 + 3x - 2x - 6$

Example 2: $6x^2 - 3x - 4x + 2$

Example 3: $x^2 + 2x - 5x - 10$

Example 4: $20x^2 - 16x - 15x + 12$

Example 5: $y^2 + 4y - 7y - 28$

Example 6: $42m^2 - 18m - 35m + 15$

Name: ________________________________ Date:_____________________

Learning Objective V.4

To check your understanding of the section, work out the following exercises.

Find the Greatest Common Factor of Two or More Expressions

1. $10p^3q$, $12pq^2$
2. $8a^2b^3$, $10ab^2$
3. $12m^2n^3$, $30m^5n^3$
4. $28x^2y^4$, $42x^4y^4$

Factor the greatest common factor from each polynomial.

5. $6m + 9$
6. $14p + 35$
7. $9n – 63$
8. $3x^2 + 6x – 9$

Factor by Grouping

9. $ab + 5a + 3b + 15$
10. $cd + 6c + 4d + 24$
11. $6y^2 + 7y + 24y + 28$
12. $x^2 – x + 4x – 4$