

WGSD Curriculum -- Algebra 2  
DRAFT  
Trigonometry

**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

CCSS.Math.Content.HSS-CP.A.2 Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

CCSS.Math.Content.HSS-CP.A.3 Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .

CCSS.Math.Content.HSS-CP.B.6 Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.

CCSS.Math.Content.HSS-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

CCSS.Math.Content.HSS-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

**Learning Goal**

Students will be able to use probability to compute the

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

<p>likelihood an event will occur.</p>	<p>3: The student demonstrates mastery of the learning goal by:</p> <ul style="list-style-type: none"> <li>● using the Fundamental Counting Principle, combinations, and permutations to compute probabilities of compound events and solve realistic problems in both familiar and unfamiliar contexts.</li> <li>● calculating expected values and using them to solve problems.</li> <li>● finding conditional probability in various ways, including the use of the Multiplication Rule, Addition Rule, and probability trees.</li> <li>● interpreting Venn diagrams and using unions, complement, and intersections to find probabilities.</li> <li>● calculating and interpreting the measures of central tendency, variance, and standard deviation of a distribution of data.</li> </ul> <p>2: The student demonstrates he/she is nearing proficiency by:</p> <ul style="list-style-type: none"> <li>● recognizing or recalling specific vocabulary, such as: <i>sample, variance, mean, median, mode, permutation, combination, success, failure, sample space, random variable, expected value, experiment, population, complement, union and intersection.</i></li> <li>● performing specific processes, such as <ul style="list-style-type: none"> <li>○ solving basic probability in real world situations.</li> <li>○ determining whether an event is the <i>union</i> or <i>intersection</i> of two other events.</li> </ul> </li> </ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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### Learning Targets

<p><b><u>Learning Design</u></b></p>

**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

CCSS.Math.Content.HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

CCSS.Math.Content.HSF-BF.B.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CCSS.Math.Content.HSF-BF.B.4 Find inverse functions.

**Learning Goal**

Students will be able to use functional notation and perform operations on functions including compositions.

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- adding, subtracting, multiplying, and dividing functions and verifying the properties of the outcome with respect to domain and range.
- generating all compositions of functions without error.
- finding the inverse of a function and determining whether two functions or relations are inverses of each other.

2: The student demonstrates he/she is nearing proficiency by:

- recognizing or recalling specific vocabulary, such as sum, product, quotient, difference,

substitution, composition, evaluate, domain, range, element, input, output, function, and relation.

- performing specific processes, such as:
  - Adding, subtracting, multiplying, and dividing functions.
  - Finding compositions with numerical inputs.
  - Finding the inverse of a function or relation.
  - Finding compositions with no major errors regarding the simpler functions, but some errors or omissions regarding the more complex functions.

1: The student demonstrates limited understanding or skill with the learning goal.

### **Learning Targets**

### **Learning Design**

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### Trigonometry

#### High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)

CCSS.Math.Content.HSN-VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

CCSS.Math.Content.HSN-VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

CCSS.Math.Content.HSN-VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

CCSS.Math.Content.HSN-VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

CCSS.Math.Content.HSN-VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

CCSS.Math.Content.HSA-REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

CCSS.Math.Content.HSA-REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

#### Learning Goal

Students will be able to perform the basic operations on matrices.

#### Proficiency Scales

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- analyzing, organizing, and representing data using matrices.
- adding, subtracting, and multiplying (scalar and matrix) matrices in complex problems by hand as well as with a graphing calculator.
- solving systems of linear equations by applying the properties of inverse matrices using a graphing calculator.

2: The student demonstrates he/she is nearing proficiency by:

- recognizing or recalling specific vocabulary, such as matrix, element, dimensions, row, column, square matrix, zero matrix, identity matrix, scalar, coefficient matrix, variable matrix, inverse matrix, and determinant.
- performing specific processes, such as:
  - adding, subtracting, and multiplying matrices by a scalar.
  - recognizing matrix dimensions that are necessary to perform operations.
  - using the properties of matrix multiplication with no major errors.
  - recognizing that systems of equations can be solved using inverse matrices.

1: The student demonstrates limited understanding or skill with the learning goal.

### Learning Targets

### Learning Design

**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSN-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

CCSS.Math.Content.HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.Math.Content.HSA-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

CCSS.Math.Content.HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CCSS.Math.Content.HSA-REI.B.4 Solve quadratic equations in one variable.

CCSS.Math.Content.HSA-REI.B.4a Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

CCSS.Math.Content.HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

**Learning Goal**

Students will be able to solve quadratic equations using all algebraic methods.

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- explaining that each step in solving an equation is equivalent to the previous step because of the properties of real numbers.
- developing equivalent forms of an expression to solve by factoring.
- developing equivalent forms of an equation and solving by completing the square.
- deriving and applying the quadratic formula to solve quadratic equations.
- recognizing that the solutions to a quadratic equation are the x-intercepts (zeros) of a quadratic function and finding them using various algebraic methods.
- justifying the most appropriate method to solve a given quadratic equation.
- finding the maximum and minimum of a quadratic function by analyzing the vertex form of a



	<p>quadratic function.</p> <p>2: The student demonstrates he/she is nearing proficiency by:</p> <ul style="list-style-type: none"><li>● recognizing or recalling specific vocabulary, such as quadratic function in standard form, zeros, factors, maximum, minimum, vertex form, discriminant, and completing the square.</li><li>● performing specific processes, such as:<ul style="list-style-type: none"><li>○ writing a quadratic function in standard form.</li><li>○ solving quadratic equations by completing the square.</li><li>○ solving quadratic equations by factoring and finding the zeros.</li><li>○ solving quadratic equations by using the quadratic formula.</li><li>○ recognizing the vertex form of a quadratic function.</li><li>○ finding the value of the discriminant of a quadratic equation.</li><li>○ writing a quadratic function in standard form.</li></ul></li></ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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CCSS.Math.Content.HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

CCSS.Math.Content.HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

CCSS.Math.Content.HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

CCSS.Math.Content.HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .

CCSS.Math.Content.HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

CCSS.Math.Content.HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Learning Goal

Students will be able to solve systems of equations and inequalities.

#### Proficiency Scales

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- modeling real world situations using systems.
- evaluating the most efficient method (elimination, substitution, graphing, and inverse

	<p>matrices) for solving a given system in two variables.</p> <ul style="list-style-type: none"> <li>• solving a system using all aforementioned methods with no errors.</li> <li>• solving a system with three variables using elimination with no errors.</li> </ul> <p>2: The student demonstrates he/she is nearing proficiency by:</p> <ul style="list-style-type: none"> <li>• recognizing or recalling specific vocabulary, such as: inconsistent, consistent system, elimination, substitution, graphing, inverse matrices, and point of intersection</li> <li>• performing specific processes, such as: <ul style="list-style-type: none"> <li>○ solving a system algebraically using substitution, elimination, and inverse matrices but with some errors on the more complex systems.</li> <li>○ recognizing that the point of intersection of the graphs is the solution to a system.</li> </ul> </li> </ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSF-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.  
CCSS.Math.Content.HSF-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  
CCSS.Math.Content.HSF-LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.  
CCSS.Math.Content.HSF-LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

**Learning Goal**

Students will be able to model  
real world problems using  
functions.

**Proficiency Scales**

- 4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.
- 3: The student demonstrates mastery of the learning goal by:
- developing a function (linear, quadratic, exponential) to model a relationship between two quantities, and using these models to solve problems in both familiar and unfamiliar contexts.
  - interpreting regression equations for a given set of data and making predictions of future events.
- 2: The student demonstrates he/she is nearing proficiency by:
- recognizing or recalling specific vocabulary, such as linear regression, best fit line, positive and negative correlation, scatter plot, rate of change, piecewise function, compound function, maximum and minimum.
  - performing specific processes, such as
    - creating a function to model a relation between two variables.
    - solving simple real world problems and some complex problems in a familiar context.
    - graphing linear scatter plots and finding regression equations by hand.

	1: The student demonstrates limited understanding or skill with the learning goal.
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**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSG-SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

CCSS.Math.Content.HSG-SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.

CCSS.Math.Content.HSG-SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\*

CCSS.Math.Content.HSG-SRT.D.9: Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

CCSS.Math.Content.HSG-SRT.D.10: Prove the Laws of Sines and Cosines and use them to solve problems.

CCSS.Math.Content.HSG-SRT.D.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

**Learning Goal**

Students will be able to use trigonometry to solve for sides and angles in right and non-right triangles.

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- developing trig ratios to solve real world problems.
- applying Law of Sines and Cosines to find missing angles and sides in real world situations.
- identifying and interpreting the ambiguous case for law of Sines.
- evaluating the effectiveness and efficiency of the formula used when solving triangles.
- deriving and using the formula to find the area of a triangle given two sides and the included angle.

2: The student demonstrates he/she is nearing proficiency by:

- recognizing or recalling specific vocabulary, such as: opposite side, adjacent side,

	<p>hypotenuse, inverse trig ratio, cross-multiplication, sine, cosine, tangent, and ambiguous case.</p> <ul style="list-style-type: none"> <li>performing specific processes, such as: <ul style="list-style-type: none"> <li>finding missing sides and angles in a right triangle.</li> <li>finding missing sides and angles in a non-right triangle using Law of Sines/Cosines.</li> </ul> </li> </ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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<u>Learning Targets</u>
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**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

CCSS.Math.Content.HSF-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.

CCSS.Math.Content.HSF-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

**Learning Goal**

Students will be able to evaluate trig ratios of angles (in radians and degrees) in the unit circle.

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- explaining the relationship between angles and radians.
- creating the unit circle with efficiency.
- interpreting patterns in the trig ratios of angles using the unit circle.
- understanding that sine and cosine represent periodic functions.

2: The student demonstrates he/she is nearing proficiency by:

- recognizing or recalling specific vocabulary, such as coterminal, reference angle, 30-60-90, 45-45-90, triangle relationships, terminal side, radians, and initial sides.



	<ul style="list-style-type: none"> <li>performing specific processes, such as:             <ul style="list-style-type: none"> <li>converting angles from degrees and radians, and from radians to degrees.</li> <li>creating the unit circle.</li> <li>recognizing trig ratios from the unit circle.</li> </ul> </li> </ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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<u>Learning Targets</u>
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**High Priority Standards (CCSS, State, National, TILS, CREDE, etc.)**

CCSS.Math.Content.HSF-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

CCSS.Math.Content.HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCSS.Math.Content.HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

CCSS.Math.Content.HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

CCSS.Math.Content.HSF-LE.A.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

CCSS.Math.Content.HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

**Learning Goal**

Students will be able to graph and solve exponential and logarithmic functions.

**Proficiency Scales**

4: Student demonstrates an in-depth inference or advanced application, or innovates with the learning goal.

3: The student demonstrates mastery of the learning goal by:

- explaining the inverse relationship between logarithmic functions and exponential functions.
- graphing logarithmic and exponential functions.
- interpreting real world logarithmic and exponential graphs.
- solving exponential and logarithmic equations.
- deriving equivalent logarithmic expressions using the laws of logarithms.

	<ul style="list-style-type: none"> <li>● creating exponential functions which model exponential growth and decay.</li> <li>● explaining how the laws of exponents are used to derive equivalent exponential functions.</li> </ul> <p>2: The student demonstrates he/she is nearing proficiency by:</p> <ul style="list-style-type: none"> <li>● recognizing or recalling specific vocabulary, such as: exponential growth, exponential decay, base, inverse function, common logarithm, laws of logarithms, laws of exponents, base, and exponent.</li> <li>● performing specific processes, such as: <ul style="list-style-type: none"> <li>○ graphing logarithmic and exponential graphs with a few errors or omissions on the more complex functions.</li> <li>○ recognizing when laws of logarithms are applied.</li> <li>○ solving exponential and logarithmic equations with a few errors on the more complex equations.</li> </ul> </li> </ul> <p>1: The student demonstrates limited understanding or skill with the learning goal.</p>
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<b><u>Learning Targets</u></b>
<b><u>Learning Design</u></b>