

# Chapter Summary

## Chapter 9: Solving Quadratic Equations

### Core Vocabulary

A **counterexample** is an example that proves that a general statement is *not* true.

A **radical expression** is an expression that contains a radical.

A radical that has no radicands with perfect  $n$ th powers as factors other than 1, no radicands that contain fractions, and no radicals that appear in the denominator of a fraction is in **simplest form**.

The process of eliminating a radical from the denominator of a fraction by multiplying by an appropriate form of 1 is called **rationalizing the denominator**.

The binomials  $a\sqrt{b} + c\sqrt{d}$  and  $a\sqrt{b} - c\sqrt{d}$ , where  $a$ ,  $b$ ,  $c$ , and  $d$  are rational numbers, are called **conjugates**.

Radicals with the same index and radicand are called **like radicals**.

A **quadratic equation** is a nonlinear equation that can be written in the standard form  $ax^2 + bx + c = 0$ , where  $a \neq 0$ .

The process of adding a constant  $c$  to an expression of the form  $x^2 + bx$  so that  $x^2 + bx + c$  is a perfect square trinomial is called **completing the square**.

The **Quadratic Formula** states that the real solutions of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0 \text{ and } b^2 - 4ac \geq 0.$$

The expression  $b^2 - 4ac$  in the Quadratic Formula is called the **discriminant**.

A **system of nonlinear equations** is a system in which at least one of the equations is nonlinear.

### Games

- Quadratic Quandary
- Equation Tic-Tac-Toe

These are available online in the *Game Closet* at [www.bigideasmath.com](http://www.bigideasmath.com).

### Essential Questions

How can you multiply and divide square roots?

How can you use a graph to solve a quadratic equation in one variable?

How can you determine the number of solutions of a quadratic equation of the form  $ax^2 + c = 0$ ?

How can you use “completing the square” to solve a quadratic equation?

How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?

How can you solve a system of two equations when one is linear and the other is quadratic?

### Core Concept

#### Product Property of Square Roots

- The square root of a product equals the product of the square roots of the factors.
- $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ , where  $a, b \geq 0$

#### Quotient Property of Square Roots

- The square root of a quotient equals the quotient of the square roots of the numerator and denominator.
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ , where  $a \geq 0$  and  $b > 0$

### Learning Goals

Use properties of radicals to simplify expressions.

Simplify expressions by rationalizing the denominator.

Perform operations with radicals.

Solve quadratic equations by graphing.

Use graphs to find and approximate the zeros of functions.

Solve real-life problems using graphs of quadratic functions.

Solve quadratic equations using square roots.

Approximate the solutions of quadratic equations.

Complete the square for expressions of the form  $x^2 + bx$ .

Solve quadratic equations by completing the square.

Find and use maximum and minimum values.

Solve real-life problems by completing the square.

Solve quadratic equations using the Quadratic Formula.

Interpret the discriminant.

Choose efficient methods for solving quadratic equations.

Solve systems of nonlinear equations by graphing.

Solve systems of nonlinear equations algebraically.

Approximate solutions of nonlinear systems and equations.

## Core Concept

### Solving Quadratic Equations by Graphing

**Step 1** Write the equation in standard form,  $ax^2 + bx + c = 0$ .

**Step 2** Graph the related function  $y = ax^2 + bx + c$ .

**Step 3** Find the x-intercepts, if any.

The solutions, or roots, of  $ax^2 + bx + c = 0$  are the x-intercepts of the graph.

### Number of Solutions of a Quadratic Equation

A quadratic equation has:

- two real solutions when the graph of its related function has two x-intercepts.
- one real solution when the graph of its related function has one x-intercept.
- no real solutions when the graph of its related function has no x-intercepts.

### Solutions of $x^2 = d$

- When  $d > 0$ ,  $x^2 = d$  has two real solutions,  $x = \pm\sqrt{d}$ .
- When  $d = 0$ ,  $x^2 = d$  has one real solution,  $x = 0$ .
- When  $d < 0$ ,  $x^2 = d$  has no real solutions.

### Completing the Square

To complete the square for an expression of the form  $x^2 + bx$ , follow these steps.

**Step 1** Find one-half of  $b$ , the coefficient of  $x$ .

**Step 2** Square the result from Step 1.

**Step 3** Add the result from Step 2 to  $x^2 + bx$ .

Factor the resulting expression as the square of a binomial.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

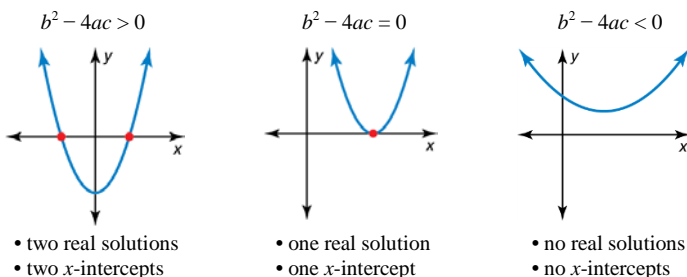
### Quadratic Formula

The real solutions of the quadratic equation  $ax^2 + bx + c = 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

where  $a \neq 0$  and  $b^2 - 4ac \geq 0$ .

### Interpreting the Discriminant



### Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring	<ul style="list-style-type: none"> <li>• Straightforward when the equation can be factored easily</li> </ul>	<ul style="list-style-type: none"> <li>• Some equations are not factorable.</li> </ul>
Graphing	<ul style="list-style-type: none"> <li>• Can easily see the number of solutions</li> <li>• Use when approximate solutions are sufficient.</li> <li>• Can use a graphing calculator</li> </ul>	<ul style="list-style-type: none"> <li>• May not give exact solutions</li> </ul>
Using Square Roots	<ul style="list-style-type: none"> <li>• Use to solve equations of the form <math>x^2 = d</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• Can only be used for certain equations</li> </ul>
Completing the Square	<ul style="list-style-type: none"> <li>• Best used when <math>a = 1</math> and <math>b</math> is even</li> </ul>	<ul style="list-style-type: none"> <li>• May involve difficult calculations</li> </ul>
Quadratic Formula	<ul style="list-style-type: none"> <li>• Can be used for any quadratic equation</li> <li>• Gives exact solutions</li> </ul>	<ul style="list-style-type: none"> <li>• Takes time to do calculations</li> </ul>

### What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 9: Golden Rectangle STEM Video is available online at [www.bigideasmath.com](http://www.bigideasmath.com).

### Additional Review

- Rationalizing the Denominator, p. 482
- Performing Operations with Radicals, p. 484
- Finding Zeros of Functions, p. 492
- Approximating Solutions of Quadratic Equations, p. 499
- Solving Systems of Nonlinear Equations, p. 526