Big Ideas Math: Algebra 1

Chapter Summary

Chapter 8: Graphing Quadratic Functions

Learning Goals

Identify characteristics of quadratic functions.

Graph and use quadratic functions of the form of $f(x) = ax^2$.

Graph quadratic functions of the form $f(x) = ax^2 + c$.

Solve real-life problems involving functions of the form $f(x) = ax^2 + c$.

Graph quadratic functions of the form $f(x) = ax^2 + bx + c$.

Find maximum and minimum values of quadratic functions.

Identify even and odd functions.

Graph quadratic functions of the form $f(x) = a(x - h)^2$.

Graph quadratic functions of the form $f(x) = a(x - h)^2 + k$.

Model real-life problems using $f(x) = a(x - h)^2 + k$.

Graph quadratic functions of the form $f(x) = a(x - p)(x - q)$.

Use intercept form to find zeros of functions.

Use characteristics to graph and write quadratic equations.

Use characteristics to graph and write cubic functions.

Choose functions to model data.

Write functions to model data.

Compare functions using average rates of change.

Solve real-life problems involving different function types.

Core Vocabulary

A ***quadratic function*** is a nonlinear function that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.

The U-shaped graph of a quadratic function is called a ***parabola***.

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the ***vertex***.

The vertical line that divides the parabola into two symmetric parts is the ***axis of symmetry***.

A ***zero of a function*** f is an x-value for which $f(x) = 0$.

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$, where $a < 0$, is the ***maximum value*** of the function.

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$, where $a > 0$, is the ***minimum value*** of the function.

A function $y = f(x)$ is ***even*** when $f(-x) = f(x)$ for each x in the domain of f.

A function $y = f(x)$ is ***odd*** when $f(-x) = -f(x)$ for each x in the domain of f.

The ***vertex form*** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

The ***intercept form*** of a quadratic function is $f(x) = a(x - p)(x - q)$, where $a \neq 0$.

The ***average rate of change*** of a function $y = f(x)$ between $x = a$ and $x = b$ is the slope of the line through $(a, f(a))$ and $(b, f(b))$.

Essential Questions

What are some of the characteristics of the graph of a quadratic function of the form $f(x) = ax^2$?

How does the value of c affect the graph of $f(x) = ax^2 + c$?

How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?

How can you describe the graph of $f(x) = a(x - h)^2$?

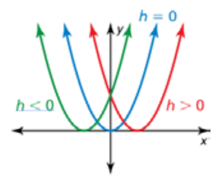
What are some of the characteristics of the graph of $f(x) = a(x - p)(x - q)$?

How can you compare the growth rates of linear, exponential, and quadratic functions?

Core Concept

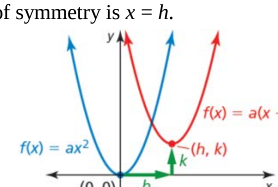
Graphing $f(x) = a(x - h)^2$

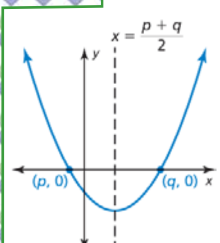
- When $h > 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation h units right of the graph of $f(x) = ax^2$.
- When $h < 0$, the graph of $f(x) = a(x - h)^2$ is a horizontal translation $|h|$ units left of the graph of $f(x) = ax^2$.
- The vertex of the graph of $f(x) = a(x - h)^2$ is $(h, 0)$, and the axis of symmetry is $x = h$.



Graphing $f(x) = a(x - h)^2 + k$

- The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$.
- The graph of $f(x) = a(x - h)^2 + k$ is a translation h units horizontally and k units vertically of the graph of $f(x) = ax^2$.
- The vertex of the graph of $f(x) = a(x - h)^2 + k$ is (h, k), and the axis of symmetry is $x = h$.





Graphing $f(x) = a(x - p)(x - q)$

- The x-intercepts are p and q.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p+q}{2}$.
- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.

Games

- Transform Me
- Polynomial Tic-Tac-Toe

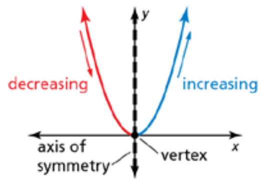
These are available online in the *Game Closet* at www.bigideasmath.com.

Core Concept

Characteristics of Quadratic Functions

The *parent quadratic function* is $f(x) = x^2$. The graphs of all other quadratic functions are *transformations* of the graph of the parent quadratic function.

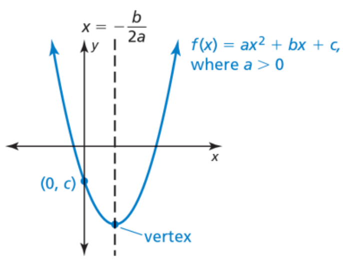
The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the vertex. The vertex of the graph of $f(x) = x^2$ is (0, 0).



The vertical line that divides the parabola into two symmetric parts is the axis of symmetry. The axis of symmetry passes through the vertex. For the graph of $f(x) = x^2$, the axis of symmetry is the y-axis, or $x = 0$.

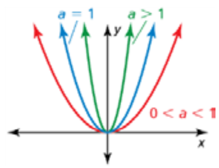
Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



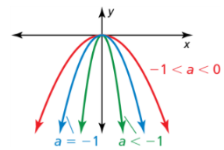
Graphing $f(x) = ax^2$ When $a > 0$

- When $0 < a < 1$, the graph of $f(x) = ax^2$ is a vertical shrink of the graph of $f(x) = x^2$.
- When $a > 1$, the graph of $f(x) = ax^2$ is
- a vertical stretch of the graph of $f(x) = x^2$.



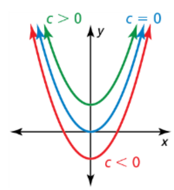
Graphing $f(x) = ax^2$ When $a < 0$

- When $-1 < a < 0$, the graph of $f(x) = ax^2$ is a vertical shrink with a reflection in the x-axis of the graph of $f(x) = x^2$.
- When $a < -1$, the graph of $f(x) = ax^2$ is a vertical stretch with a reflection in the x-axis of the graph of $f(x) = x^2$.



Graphing $f(x) = ax^2 + c$

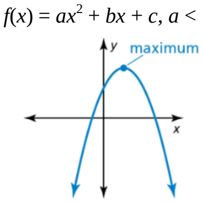
- When $c > 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation c units up of the graph of $f(x) = ax^2$.
- When $c < 0$, the graph of $f(x) = ax^2 + c$ is a vertical translation $|\,c\,|$ units down of the graph of $f(x) = ax^2$.
- The vertex of the graph of $f(x) = ax^2 + c$ is $(0, c)$, and the axis of symmetry is $x = 0$.



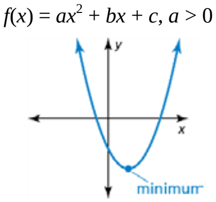
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the maximum value of the function when $a < 0$ or the minimum value of the function when $a > 0$.

$f(x) = ax^2 + bx + c,\ a < 0$



$f(x) = ax^2 + bx + c,\ a > 0$



Even and Odd Functions

- A function $y = f(x)$ is even when $f(-x) = f(x)$ for each x in the domain of f.
- The graph of an even function is symmetric about the y-axis.
- A function $y = f(x)$ is odd when $f(-x) = -f(x)$ for each x in the domain of f.
- The graph of an odd function is symmetric about the origin.
- A graph is *symmetric about the origin* when it looks the same after reflections in the x-axis and then in the y-axis.

Factors and Zeros

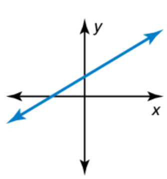
For any factor $x - n$ of a polynomial, n is a zero of the function defined by the polynomial.

Comparing Functions Using Average Rates of Change

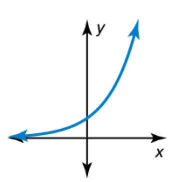
- Over the same interval, the average rate of change of a function increasing quadratically eventually exceeds the average rate of change of a function increasing linearly. So, the value of the quadratic function eventually exceeds the value of the linear function.
- Over the same interval, the average rate of change of a function increasing exponentially eventually exceeds the average rate of change of a function increasing linearly or quadratically. So, the value of the exponential function eventually exceeds the value of the linear or quadratic function.

Linear, Exponential, and Quadaratic Functions

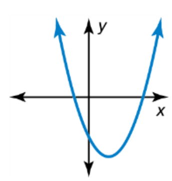
Linear Function
$y = mx + b$



Exponential Function
$y = ab^x$



Quadratic Function
$y = ax^2 + bx + c$



Differences and Ratios of Functions

You can use patterns between consecutive data pairs to determine which type of function models the data. The differences of consecutive y-values are called *first differences*. The differences of consecutive first differences are called *second differences*.

- **Linear Function** The first differences are constant.
- **Exponential Function** Consecutive y-values have a common *ratio*.
- **Quadratic Function** The second differences are constant.

In all cases, the differences of consecutive x-values need to be constant.

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 8: Comparing Growth Models STEM Video is available online at www.bigideasmath.com.

Additional Review

- Writing Quadratic Functions of the Form $f(x) = a(x - h)2 + k$, *p. 445*
- Using Characteristics to Graph and Write Quadratic Functions, *p. 452*
- Using Characteristics to Graph and Write Cubic Functions, *p. 453*
- Writing Functions to Model Data, *p. 462*