

# Chapter Summary

## Chapter 7: Polynomial Equations and Factoring

### Learning Goals

Find the degrees of monomials.

Classify polynomials.

Add and subtract polynomials.

Solve real-life problems.

Multiply binomials.

Use the FOIL method.

Multiply binomials and trinomials.

Use the square of a binomial pattern.

Use the sum and difference pattern.

Use special product patterns to solve real-life problems.

Use the Zero-Product Property.

Factor polynomials using the GCF.

Use the Zero-Product Property to solve real-life problems.

Factor  $x^2 + bx + c$ .

Factor  $ax^2 + bx + c$ .

Factor the difference of two squares.

Factor perfect square trinomials.

Factor polynomials by grouping.

Factor polynomials completely.

Use factoring to solve real-life problems.

### Essential Questions

How can you add and subtract polynomials?

How can you multiply two polynomials?

What are the patterns in the special products  $(a + b)(a - b)$ ,  $(a + b)^2$ , and  $(a - b)^2$ ?

How can you solve a polynomial equation?

How can you use algebra tiles to factor the trinomial  $x^2 + bx + c$  into the product of two binomials?

How can you use algebra tiles to factor the trinomial  $ax^2 + bx + c$  into the product of two binomials?

How can you recognize and factor special products?

How can you factor a polynomial completely?

### Games

- Make My Team
- Equation Tic-Tac-Toe

These are available online in the *Game Closet* at [www.bigideasmath.com](http://www.bigideasmath.com).

### Core Vocabulary

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial.

A **polynomial** is a monomial or a sum of monomials.

A polynomial with two terms is a **binomial**.

A polynomial with three terms is a **trinomial**.

The **degree of a polynomial** is the greatest degree of its terms.

A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right.

The coefficient of the first term of a polynomial written in standard form is the **leading coefficient**.

A set of numbers is **closed** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set.

A shortcut for multiplying two binomials by finding the sum of the products of the first terms, outer terms, inner terms, and last terms is called the **FOIL Method**.

A polynomial is in **factored form** when it is written as a product of factors.

The **Zero-Product Property** states that if the product of two real numbers is 0, then at least one of the numbers is 0.

The solutions of a polynomial equation are called **roots**.

Two or more roots of an equation that are the same number are called **repeated roots**.

The process of using the Distributive Property to factor a polynomial with four terms is called **factoring by grouping**.

A polynomial that is written as a product of unfactorable polynomials with integer coefficients is **factored completely**.

## Core Concept

### Polynomials

- A polynomial is a monomial or a sum of monomials.
- Each monomial is called a *term* of the polynomial.
- A polynomial with two terms is a binomial.
- A polynomial with three terms is a trinomial.
- The degree of a polynomial is the greatest degree of its terms.
- A polynomial in one variable is in standard form when the exponents of the terms decrease from left to right.
- When you write a polynomial in standard form, the coefficient of the first term is the leading coefficient.

### Square of a Binomial Pattern

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

### Sum and Difference Pattern

$$(a + b)(a - b) = a^2 - b^2$$

### Difference of Two Squares Pattern

$$a^2 - b^2 = (a + b)(a - b)$$

### Zero-Product Property

- If the product of two real numbers is 0, then at least one of the numbers is 0.
- If  $a$  and  $b$  are real numbers and  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

### Factoring $x^2 + bx + c$ When $c$ Is Positive

- $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .
- When  $c$  is positive,  $p$  and  $q$  have the same sign as  $b$ .

### Factoring $x^2 + bx + c$ When $c$ Is Negative

- $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .
- When  $c$  is negative,  $p$  and  $q$  have different signs.

### Guidelines for Factoring Polynomials Completely

To factor a polynomial completely, you should try each of these steps.

1. Factor out the greatest common monomial factor.
2. Look for a difference of two squares or a perfect square trinomial.
3. Factor a trinomial of the form  $ax^2 + bx + c$  into a product of binomial factors.
4. Factor a polynomial with four terms by grouping.

### FOIL Method

To multiply two binomials using the FOIL Method, find the sum of the products of the

**F**irst terms,  $(x + 1)(x + 2) \rightarrow x(x) = x^2$

**O**uter terms,  $(x + 1)(x + 2) \rightarrow x(2) = 2x$

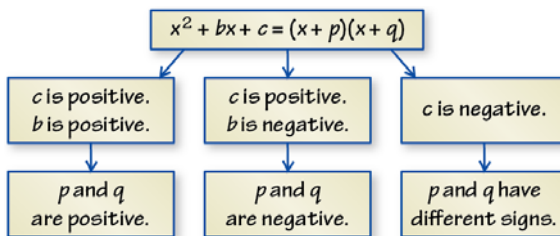
**I**nnner terms, and  $(x + 1)(x + 2) \rightarrow 1(x) = x$

**L**ast terms.  $(x + 1)(x + 2) \rightarrow 1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

### Factoring $x^2 + bx + c$ as $(x + p)(x + q)$

The diagram shows the relationships between the signs of  $b$  and  $c$  and the signs of  $p$  and  $q$ .



### Perfect Square Trinomial Pattern

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

### Factoring by Grouping

- To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called factoring by grouping.

Example:

$$3x^2 + 6x = 3x(x + 2)$$

$$x^2 + 4x + 4 = (x + 2)^2$$

$$3x^2 - 5x - 2 = (3x + 1)(x - 2)$$

$$x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$$

### What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations.

The Chapter 7: Birds Dropping Food STEM Video is available online at [www.bigideasmath.com](http://www.bigideasmath.com).

### Additional Review

- Adding Polynomials, p. 360
- Subtracting Polynomials, p. 360
- Multiplying Binomials, p. 366
- Multiplying Binomials and Trinomials, p. 368
- Factoring Polynomials Using the GCF, p. 379