Big Ideas Math: Algebra 1

Chapter Summary

Chapter 6: Exponential Functions and Sequences

Learning Goals

Use zero and negative exponents.

Use the properties of exponents.

Solve real-life problems involving exponents.

Find *n*th roots.

Evaluate expressions with rational exponents.

Solve real-life problems involving rational exponents.

Identify and evaluate exponential functions.

Graph exponential functions.

Solve real-life problems involving exponential functions.

Use and identify exponential growth and decay functions.

Interpret and rewrite exponential growth and decay functions.

Solve real-life problems involving exponential growth and decay.

Solve exponential equations with the same base.

Solve exponential equations with unlike bases.

Solve exponential equations by graphing.

Identify geometric sequences.

Extend and graph geometric sequences.

Write geometric sequences as functions.

Write terms of recursively defined sequences.

Write recursive rules for sequences.

Translate between recursive rules and explicit rules.

Write recursive rules for special sequences.

Core Vocabulary

For an integer n greater than 1, if $b^n = a$, then b is an ***nth root of a***.

An expression of the form $\sqrt[n]{a}$ is called a ***radical.***

The value of n in the radical $\sqrt[n]{a}$ is the ***index*** of the radical.

An ***exponential function*** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

Exponential growth occurs when a quantity increases by the same factor over equal intervals of time.

Exponential decay occurs when a quantity decreases by the same factor over equal intervals of time.

Exponential equations are equations in which variable expressions occur as exponents.

A ***geometric sequence*** is an ordered list of numbers in which the ratio between each pair of consecutive terms is the same.

The constant ratio r between consecutive terms of a geometric sequence is called the ***common ratio***.

An ***explicit rule*** gives a_n as a function of the term's position number n in the sequence.

A ***recursive rule*** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms.

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$ or $y = ab^x$, where $a > 0$ and $b > 1$ is an ***exponential growth function***.

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $0 < r < 1$ or $y = ab^x$, where $a > 0$ and $0 < b < 1$ is an ***exponential decay function***.

The interest earned on the principal and on previously earned interest is ***compound interest***.

Essential Questions

How can you write general rules involving properties of exponents?

How can you write and evaluate an *n*th root of a number?

What are some of the characteristics of the graph of an exponential function?

What are some of the characteristics of exponential growth and exponential decay functions?

How can you solve an exponential equation graphically?

How can you use a geometric sequence to describe a pattern?

How can you define a sequence recursively?

Core Concept

Zero Exponent
For any nonzero number a, $a^0 = 1$. The power 0^0 is undefined.
- $a^0 = 1$, where $a \neq 0$

Negative Exponents
For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.
- $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

Product of Powers Property
Let a be a real number, and let m and n be integers.
- To multiply powers with the same base, add their exponents.
- $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property
Let a be a nonzero real number, and let m and n be integers.
- To divide powers with the same base, subtract their exponents.
- $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property
Let a be a real number, and let m and n be integers.
- To find a power of a power, multiply the exponents.
- $(a^m)^n = a^{mn}$

Power of a Product Property
Let a and b be real numbers, and let m be an integer.
- To find a power of a product, find the power of each factor and multiply.
- $(ab)^m = a^m b^m$

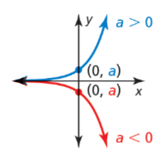
Rational Exponents
Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.
- $a^{m/n} = (a^{1/n})^m = \left(\sqrt[n]{a}\right)^m$

Property of Equality for Exponential Equations
- Two powers with the *same positive base b*, where $b \neq 1$, are equal if and only if their exponents are equal.
- If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

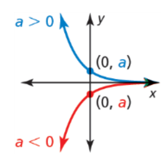
Power of a Quotient Property
Let a and b be real numbers with $b \neq 0$, and let m be an integer.
- To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Real *n*th Roots of *a*
Let n be an integer greater than 1, and let a be a real number.
- If n is odd, then a has one real nth root: $\sqrt[n]{a} = a^{1/n}$
- If n is even and $a > 0$, then a has two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If n is even and $a = 0$, then a has one real nth root: $\sqrt[n]{0} = 0$
- If n is even and $a < 0$, then a has no real nth roots.

Graphing $y = ab^x$ When $b > 1$



Graphing $y = ab^x$ When $0 < b < 1$



Compound Interest
Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

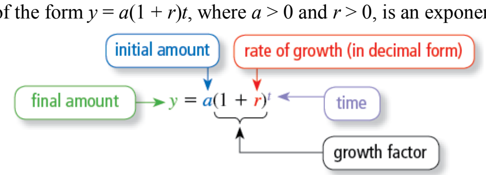
Geometric Sequence
- In a geometric sequence, the ratio between each pair of consecutive terms is the same. This ratio is called the common ratio.
- Each term is found by multiplying the previous term by the common ratio.

Equation for a Geometric Sequence
Let a_n be the nth term of a geometric sequence with first term a_1 and common ratio r. The nth term is given by
$a_n = a_1 r^{n-1}$.

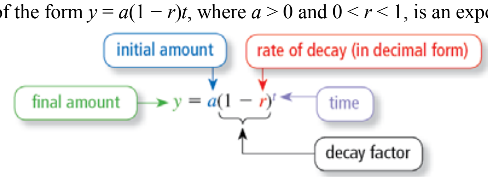
Recursive Equation for an Arithmetic Sequence
$a_n = a_{n-1} + d$, where d is the common difference.

Recursive Equation for a Geometric Sequence
$a_n = r \cdot a_{n-1}$, where r is the common ratio.

Exponential Growth Functions
A function of the form $y = a(1 + r)t$, where $a > 0$ and $r > 0$, is an exponential growth function.



Exponential Decay Functions
A function of the form $y = a(1 - r)t$, where $a > 0$ and $0 < r < 1$, is an exponential decay function.



Game

- Equation Tic-Tac-Toe

These are available online in the *Game Closet* at www.bigideasmath.com.

Additional Review

- Solving Exponential Equations by Graphing, *p. 328*

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 6: Mathematical Recursion STEM Video is available online at www.bigideasmath.com.