Big Ideas Math: Algebra 1

Chapter Summary

Chapter 3: Graphing Linear Functions

Learning Goals

Determine whether relations are functions.

Find the domain and range of a function.

Identify the independent and dependent variables of functions.

Identify linear functions using graphs, tables, and equations.

Graph linear functions using discrete and continuous data.

Write real-life problems to fit data.

Use function notation to evaluate and interpret functions.

Use function notation to solve and graph functions.

Solve real-life problems using function notation.

Graph equations of horizontal and vertical lines.

Graph linear equations in standard form using intercepts.

Use linear equations in standard form to solve real-life problems.

Find the slope of a line.

Use the slope-intercept form of a linear equation.

Use slopes and *y*-intercepts to solve real-life problems.

Translate and reflect graphs of linear functions.

Stretch and shrink graphs of linear functions.

Combine transformations of graphs of linear functions.

Translate graphs of absolute value functions.

Stretch, shrink, and reflect graphs of absolute value functions.

Combine transformations of graphs of absolute value functions.

Games

- How Are We Related?
- Transform Me

These are available online in the *Game Closet* at www.bigideasmath.com.

Core Vocabulary

A ***relation*** pairs inputs with outputs.

A relation that pairs each input with exactly one output is a ***function***.

The ***domain*** of a function is the set of all possible input values.

The ***range*** of a function is the set of all possible output values.

The variable that represents the input values of a function is the ***independent variable***.

The variable that represents the output values of a function is the ***dependent variable***.

A ***linear equation in two variables***, x and y, is an equation that can be written in the form $y = mx + b$, where m and b are constants.

A ***linear function*** is a function whose graph is a nonvertical line.

A function that does not have a constant rate of change and whose graph is not a line is a ***nonlinear function.***

A ***solution of a linear equation in two variables*** is an ordered pair (x, y) that makes the equation true.

A ***discrete domain*** is a set of input values that consists of only certain numbers in an interval.

A ***continuous domain*** is a set of input values that consists of all numbers in an interval.

Function notation is another name for y denoted as $f(x)$ and read as “the value of f at x”.

The ***x-intercept*** of a graph is the x-coordinate of a point where the graph crosses the x-axis.

The ***y-intercept*** of a graph is the y-coordinate of a point where the graph crosses the y-axis.

The ***standard form*** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

Slope is the rate of change between any two points on a line.

The change in y of the slope of a nonvertical line is the ***rise.***

The change in x of the slope of a nonvertical line is the ***run***.

A linear equation written in the form $y = mx + b$ is in ***slope-intercept form***.

A linear equation written in the form $y = 0x + b$, or $y = b$, is a ***constant function***.

A ***family of functions*** is a group of functions with similar characteristics.

The most basic function in a family of functions is the ***parent function***.

A ***transformation*** changes the size, shape, position, or orientation of a graph.

A ***translation*** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

A ***reflection*** is a transformation that flips a graph over a line called the line of reflection.

A ***horizontal shrink*** is a transformation that causes the graph of a function to shrink toward the y-axis when all the x-coordinates are multiplied by a factor a, where $a > 1$.

A ***horizontal stretch*** is a transformation that causes the graph of a function to stretch away from the y-axis when all the x-coordinates are multiplied by a factor a, where $0 < a < 1$.

A ***vertical stretch*** is a transformation that causes the graph of a function to stretch away from the x-axis when all the y-coordinates are multiplied by a factor a, where $a > 1$.

A ***vertical shrink*** is a transformation that causes the graph of a function to shrink toward the x-axis when all the y-coordinates are multiplied by a factor a, where $0 < a < 1$.

An ***absolute value function*** is a function that contains an absolute value expression.

The ***vertex*** is the point where a graph changes direction.

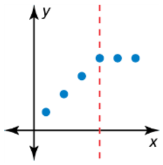
An absolute value function written in the form $g(x) = a\,|\,x - h\,|\, + k$, where $a \neq 0$, is in ***vertex form***.

Core Concept

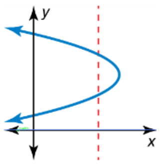
Vertical Line Test

A graph represents a function when no vertical line passes through more than one point on the graph.

Function



Not a function



The Domain and Range of a Function

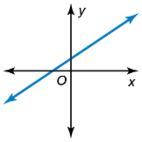
- The domain of a function is the set of all possible input values.
- The range of a function is the set of all possible output values.

Discrete and Continuous Domains

- A discrete domain is a set of input values that consists of only certain numbers in an interval.
- A continuous domain is a set of input values that consists of all numbers in an interval.

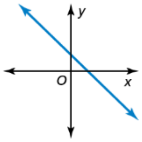
Slope

Positive slope



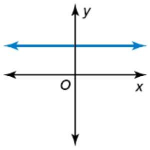
The line rises from left to right.

Negative slope



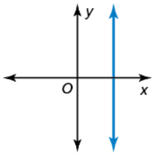
The line falls from left to right.

Slope of 0



The line is horizontal.

Undefined slope



The line is vertical.

Transformations of Graphs

The graph of $y = a \cdot f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1 Translate the graph of $y = f(x)$ horizontally h units.

Step 2 Use a to stretch or shrink the resulting graph from Step 1.

Step 3 Reflect the resulting graph from Step 2 when $a < 0$.

Step 4 Translate the resulting graph from Step 3 vertically k units.

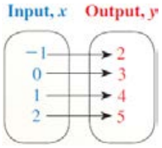
Representations of Functions

- An output is 3 more than the input.
- Equation: $y = x + 3$

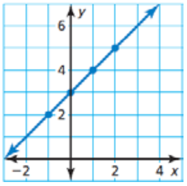
Input-Output Table

Input, x	Output, y
−1	2
0	3
1	4
2	5

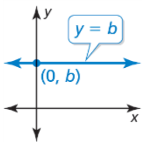
Mapping Diagram



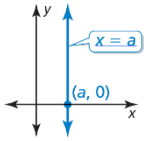
Graph



Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.

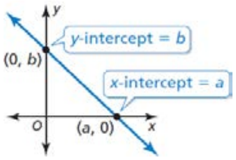


The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

Using Intercepts to Graph Equations

The x-intercept of a graph is the x-coordinate of a point where the graph crosses the x-axis. It occurs when $y = 0$.

The y-intercept of a graph is the y-coordinate of a point where the graph crosses the y-axis. It occurs when $x = 0$.



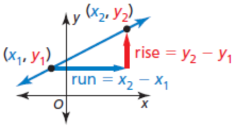
To graph the linear equation $Ax + By = C$, find the intercepts and draw the line that passes through the two intercepts.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.

Slope

The slope m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the ratio of the rise (change in y) to the run (change in x).

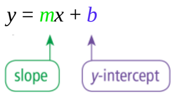
$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

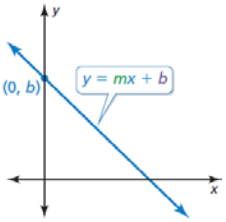


When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

Slope-Intercept Form

- A linear equation written in the form $y = mx + b$ is in slope-intercept form.
- The slope of the line is m, and the y-intercept of the line is b.





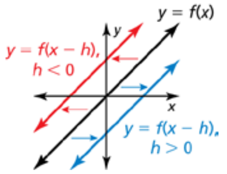
Core Concept

Translations

A translation is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

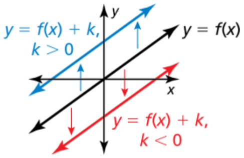
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



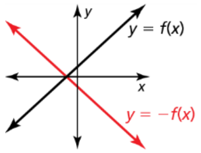
Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

Reflections

A reflection is a transformation that flips a graph over a line called the line of reflection.

Reflections in the x-axis

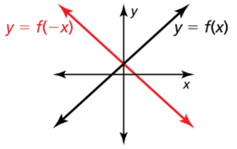
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by −1 changes their signs.

Reflections in the y-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.

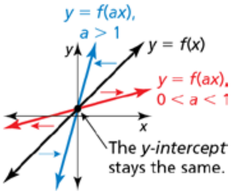


Multiplying the inputs by −1 changes their signs.

Stretches and Shrinks

Horizontal Stretches and Shrinks

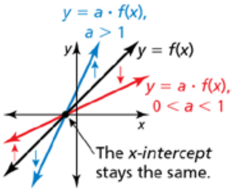
The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



The y-intercept stays the same.

Vertical Stretches and Shrinks

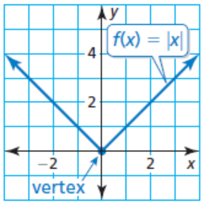
The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



The x-intercept stays the same.

Absolute Value Function

- An absolute value function is a function that contains an absolute value expression.
- The parent absolute value function is $f(x) = | x |$.
- The graph of $f(x) = | x |$ is V-shaped and symmetric about the y-axis.
- The vertex is the point where the graph changes direction. The vertex of the graph of $f(x) = | x |$ is (0, 0).
- The domain of $f(x) = | x |$ is all real numbers.
- The range is $y \geq 0$.



Vertex Form of an Absolute Value Function

- An absolute value function written in the form $g(x) = a | x - h | + k$, where $a \neq 0$, is in vertex form.
- The vertex of the graph of g is (h, k).
- Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

Essential Questions

What is a function?

How can you determine whether a function is linear or nonlinear?

How can you use function notation to represent a function?

How can you describe the graph of the equation $Ax + By = C$?

How can you describe the graph of the equation $y = mx + b$?

How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

How do the values of a, h, and k affect the graph of the absolute value function $g(x) = a | x - h | + k$?

What's the Point?

The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 3: Speed of Light STEM Video is available online at www.bigideasmath.com.

Additional Review

- Determining Whether Relations Are Functions, *p. 104*
- Independent and Dependent Variables, *p. 107*
- Linear and Nonlinear Functions, *p. 112*
- Using Function Notation, *p. 122*