Family Support Materials

Area and Surface Area

Here are the video lesson summaries for Grade 6 Unit 1, Area and Surface Area. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 6, Unit 1: Area and Surface Area	Vimeo	YouTube
Video 1: Reasoning to Find Area (Lessons 1–3, 11)	Link	Link
Video 2: Parallelograms (Lessons 4–6)	Link	Link
Video 3: Triangles (Lessons 7–10)	Link	Link
Video 4: Surface Area (Lessons 12–15)	Link	Link
Video 5: Distinguishing between Surface Area and Volume (Lessons 16–18)	Link	Link

Video 1

Video 'VLS G6U1V1 Reasoning to Find Area (Lessons 1–3, 11)' available here: https://player.vimeo.com/video/443554693.

Video 2

Video 'VLS G6U2V2 Parallelograms (Lessons 4–6)' available here: https://player.vimeo.com/video/443559353.

Video 3

Video 'VLS G6U1V3 Triangles (Lessons 7–10)' available here: https://player.vimeo.com/video/443857237.

Video 4

Video 'VLS G6U1V4 Surface Area (Lessons 12–15)' available here: https://player.vimeo.com/video/443561431.

Video 5

Video 'VLS G6U1V5 Distinguishing between Surface Area and Volume (Lessons 16–18)' available here: https://player.vimeo.com/video/443563211.

Connecting to Other Units

- *Coming soon*

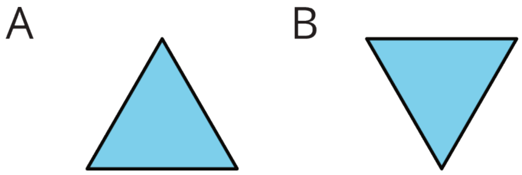
Reasoning to Find Area

Family Support Materials 1

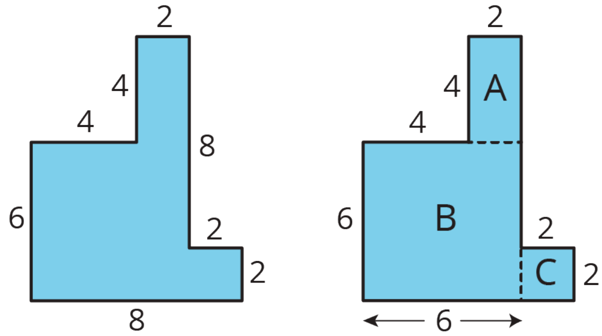
Before grade 6, your student learned to measure the **area** of a shape by finding the number of unit squares that cover the shape without gaps or overlaps. For example, the orange and blue shapes each have an area of 8 square units.

In grade 6, students learn to find the areas of more complicated shapes using two ideas:

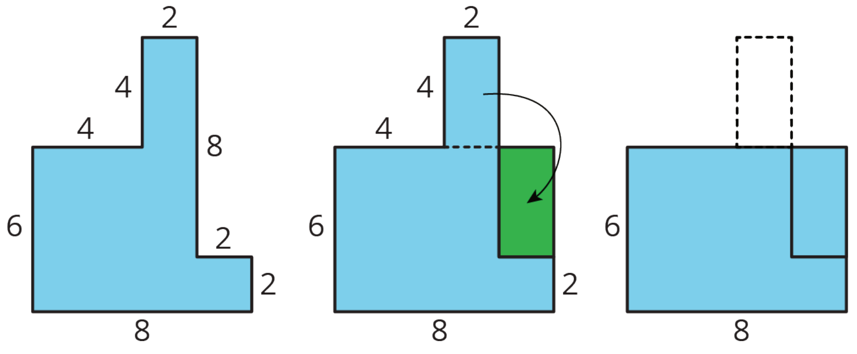
- Two shapes that "match up exactly" have the same area. For example, triangles A and B have the same area because Triangle A can be placed on Triangle B so they match up exactly.



- We can **decompose** (break) a shape into smaller pieces and find its area by adding the areas of the pieces. For example, the area of the shape on the left is equal to the area of Rectangle A, plus the area of Rectangle B, plus the area of Rectangle C.

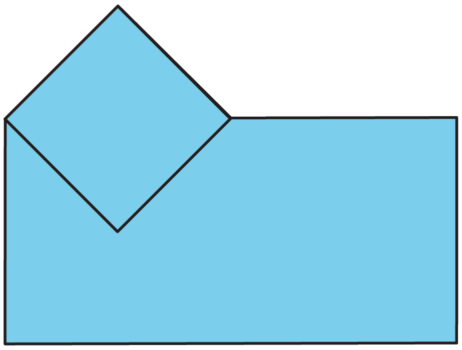


It is sometimes helpful to **rearrange** the pieces of a shape in order to find its area. For example, the rectangular piece that is 2 units by 4 units at the top of the shape can be broken and rearranged to make a simple rectangle that is 8 units and 6 units. We can easily find the area of this rectangle (48 square units, because $8 \times 6 = 48$).



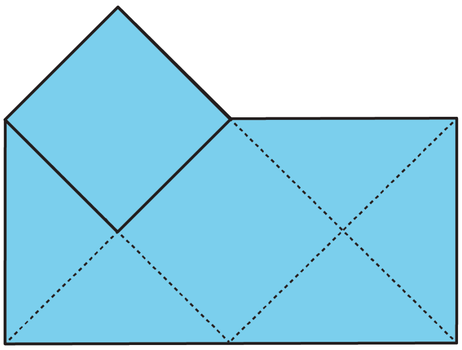
Here is a task to try with your student:

The area of the square is 1 square unit. Find the area of the entire shaded region. Show your reasoning.



Solution:

$4\frac{1}{2}$ square units. Sample reasoning: The rest of the region can be decomposed into a square and several triangles. Two triangles can be arranged to match up perfectly with a square, so each triangle has half the area of the square ($\frac{1}{2}$ square units). In the entire shape, there is a total of 2 squares (2 square units) and 5 triangles ($5 \times \frac{1}{2}$ or $2\frac{1}{2}$ square units). $2 + 2\frac{1}{2} = 4\frac{1}{2}$.

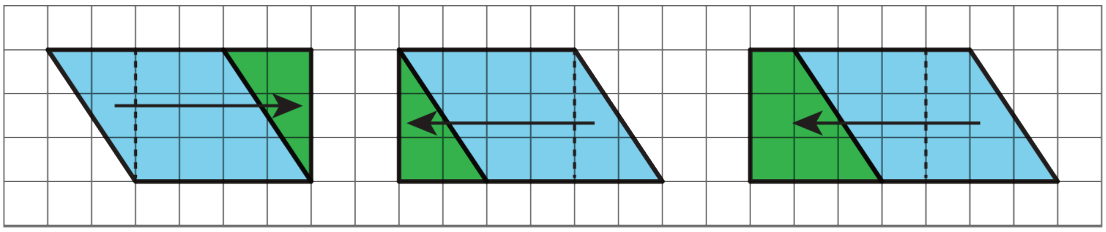


Parallelograms

Family Support Materials 2

This week, your student will investigate **parallelograms**, which are four-sided figures whose opposite sides are parallel.

We can find the **area of a parallelogram** by breaking it apart and rearranging the pieces to form a rectangle. The diagram shows a few ways of rearranging pieces of a parallelogram. In each one, the result is a rectangle that is 4 units by 3 units, so its area is 12 square units. The area of the original parallelogram is also 12 square units.

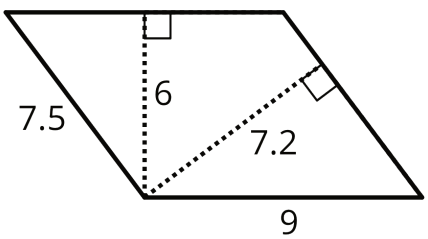


Using these strategies allows students to notice pairs of measurements that are helpful for finding the area of any parallelogram: a **base** and a corresponding **height**. The length of any side of a parallelogram can be used as a base. The height is the distance from the base to the opposite side, measured at a right angle. In the parallelogram shown here, we can say that the horizontal side that is 4 units long is the base and the vertical segment that is 3 units is the height that corresponds to that base.

The area of any parallelogram is $base \times height$.

Here is a task to try with your student:

Elena and Noah are investigating this parallelogram.



Elena says, “If the side that is 9 units is the base, the height is 7.2 units. If the side that is 7.5 units is the base, the corresponding height is 6 units.”

Noah says, “I think if the base is 9 units, the corresponding height is 6 units. If the base is 7.5 units, the corresponding height is 7.2 units.”

Do you agree with either one of them? Explain your reasoning.

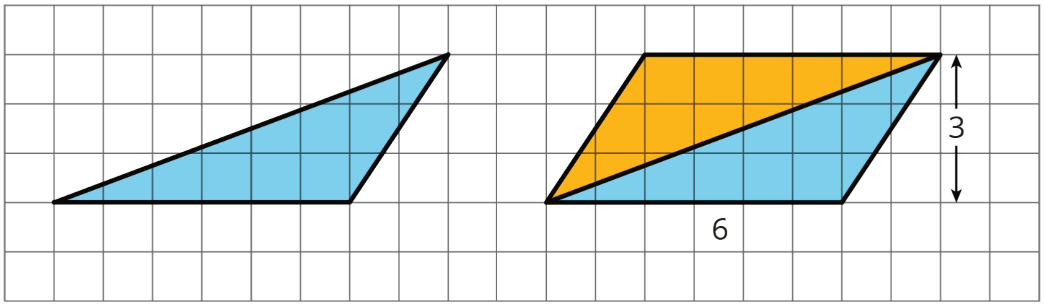
Solution:

Agree with Noah. Explanations vary. Sample explanation: A corresponding height must be perpendicular (drawn at a right angle) to the side chosen as the base. The dashed segment that is 6 units is perpendicular to the two parallel sides that are 9 units long. The dashed segment that is 7.2 units long is perpendicular to the two sides that are 7.5 units.

Triangles

Family Support Materials 3

Your student will now use their knowledge of the area of parallelograms to find the area of triangles. For example, to find the area of the blue triangle on the left, we can make a copy of it, rotate the copy, and use the two triangles to make a parallelogram.



This parallelogram has a base of 6 units, a height of 3 units, and an area of 18 square units. So the area of each triangle is half of 18 square units, which is 9 square units.

A triangle also has **bases** and corresponding **heights**. Any side of a triangle can be a base. Its corresponding height is the distance from the side chosen as the base to the opposite corner, measured at a right angle. In this example, the side that is 6 units long is the base and the height is 3 units.

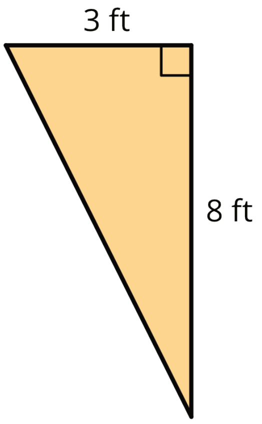
Because two copies of a triangle can always be arranged to make a parallelogram, the area of a triangle is always half of the area of a parallelogram with the same pair of base and height. We can use this formula to find the area of any triangle:

$$\frac{1}{2} \times base \times height$$

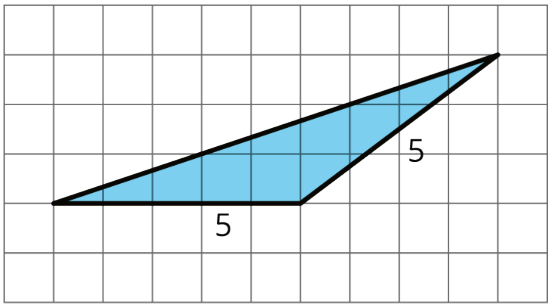
Here is a task to try with your student:

Find the area of each triangle. Show your reasoning.

1.



1.



Solution:

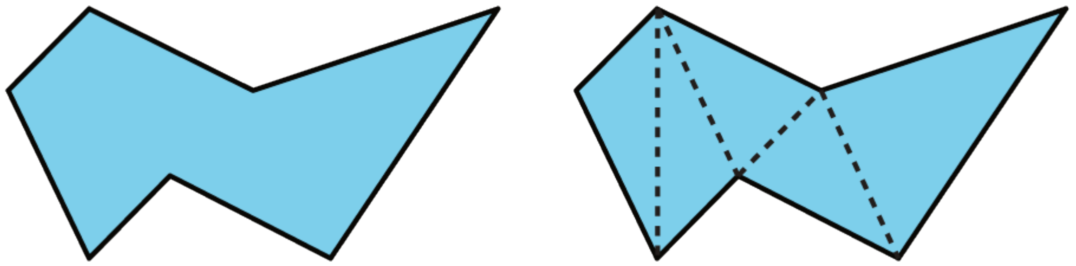
1. 12 square feet. Sample reasoning: The triangle is half of a rectangle that is 3 feet by 8 feet, which has an area of 24 square feet.
2. $\frac{15}{2}$ square units. Sample reasoning: The triangle is half of a parallelogram with a base of 5 units and a height of 3 units. $\frac{1}{2} \cdot 5 \cdot 3 = \frac{15}{2}$.

Polygons

Family Support Materials 4

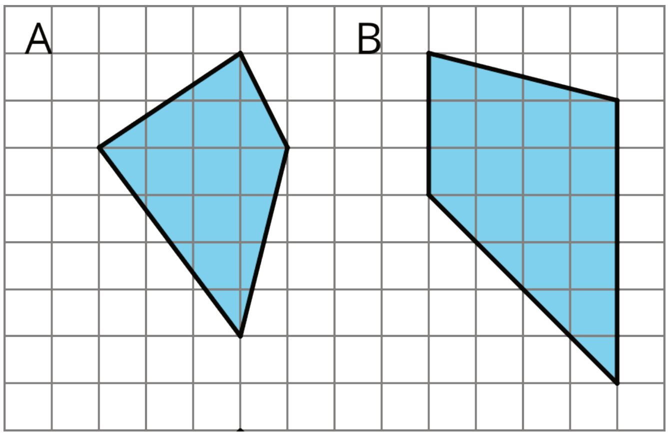
Knowing how to find the area of triangles allows your student to find the area of **polygons**, which are two-dimensional shapes made up of line segments. The line segments meet one another only at their end points. Triangles, quadrilaterals, pentagons, and hexagons are all polygons.

To find the area of *any* polygon, we can break it apart into rectangles and triangles. Here is a polygon with 7 sides and one way to break it apart into triangles. Finding the areas of all triangles and adding them gives the area of the original polygon.



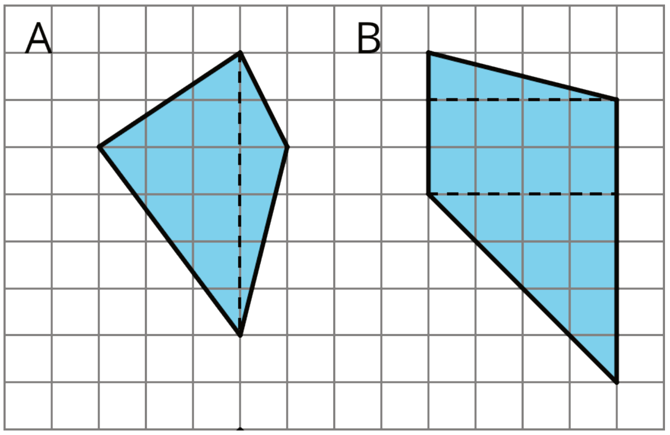
Here is a task to try with your student:

Find the area of polygons A and B. Explain or show your reasoning.



Solution:

A: 12 square units, B: 18 square units. Sample diagram and explanations:



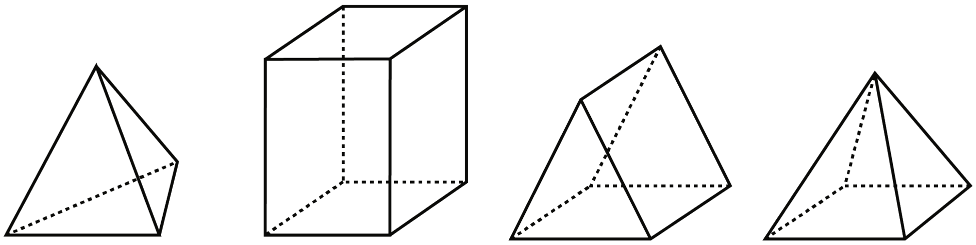
Polygon A can be broken into two triangles. The one on the left has base 6 units and height 3 units, so its area is 9 square units ($\frac{1}{2} \cdot 6 \cdot 3 = 12$). The one on the right has base 6 units and height 1 unit, so its area is 3 square units ($\frac{1}{2} \cdot 6 \cdot 1 = 3$). The total area is $9 + 3$ or 12 square units.

Polygon B can be broken into a rectangle and two triangles. The area of the top triangle is $\frac{1}{2} \cdot 4 \cdot 1$ or 2 square units. The rectangle is 8 square units. The area of the bottom triangle is $\frac{1}{2} \cdot 4 \cdot 4$ or 8 square units. $2 + 8 + 8 = 18$

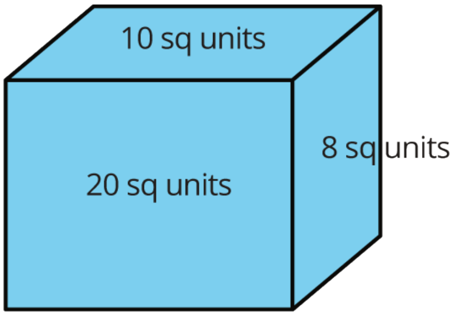
Surface Area

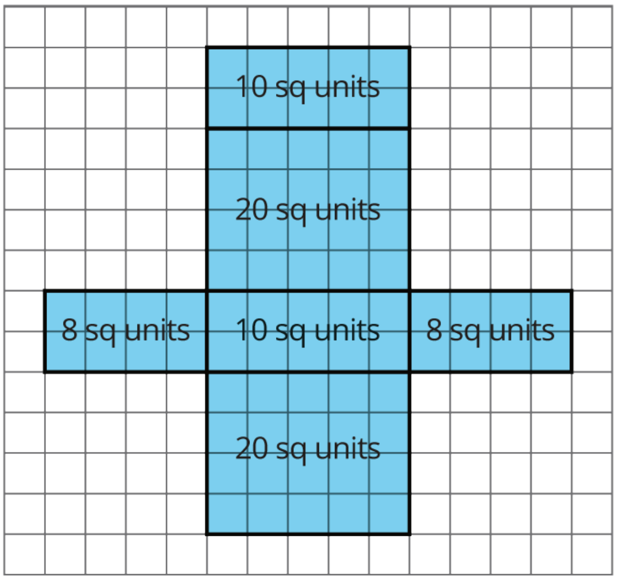
Family Support Materials 5

Imagine painting all of the sides of a box. The amount of surface to be covered with paint is the **surface area** of the box. Your student will focus on finding the surface areas of different three-dimensional objects such as the **prisms** and **pyramids** shown here.



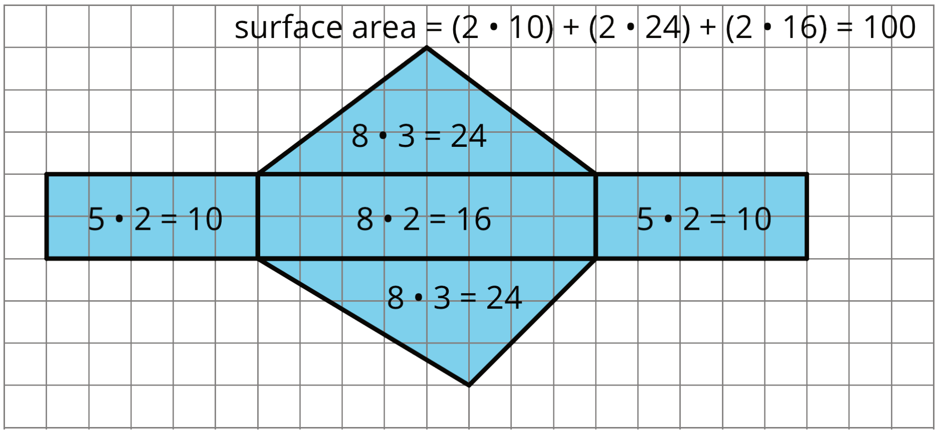
One way to find the surface area of a three-dimensional object is to draw its **net**, which shows all the **faces** of the object as a two-dimensional drawing. A net can be cut out and folded to make the object. To find the surface area of the object, we can find the area of each face (as shown on the net) and add them. The areas of the six rectangular faces shown add up to 76 square units because $10+20+10+20+8+8=76$, so the surface area of this box is 76 square units.





Here is a task to try with your student:

Andre drew a net of a triangular prism and calculated its surface area. He made an error in both the net drawing and in the calculation.



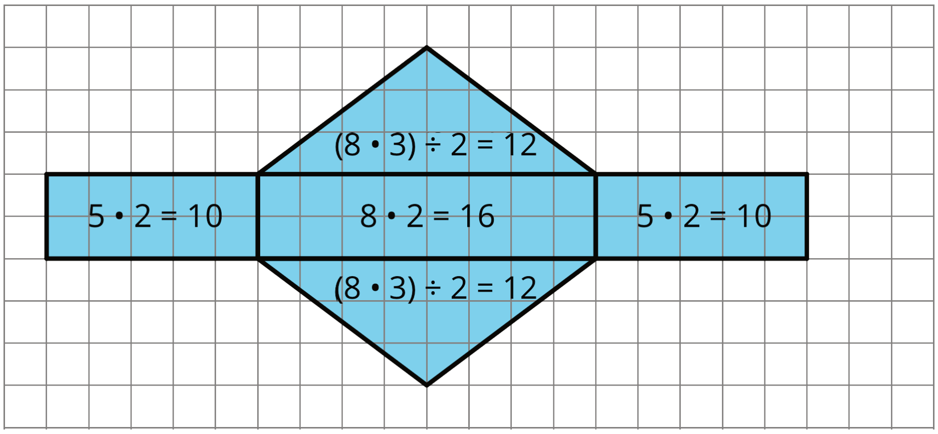
1. Identify Andre's errors.

2. Find the correct surface area for the prism. Show your reasoning.

Solution:

1. Net: The triangles in a triangular prism should be identical, but the net shows two different triangles. Calculation: There are a few errors. The area of each triangle should be $\frac{1}{2} \cdot 8 \cdot 3$ or 12 square units. Andre did not multiply the base and height by half. The wrong calculation is repeated for both triangles. In the calculation for the surface area, Andre doubled the area of the largest rectangle (which is 16 square units) while there is only one rectangle with that area.

2. The surface area should be 60 square units. The combined area of the two triangles should be $2(\frac{1}{2} \cdot 8 \cdot 3)$ or 24 square units. $10 + 10 + 16 + 24 = 60$. Sample corrected net:



Family Support Materials

Introducing Ratios

Here are the video lesson summaries for Grade 6, Unit 2 Introducing Ratios. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

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Grade 6, Unit 2: Introducing Ratios	Vimeo	YouTube
Video 1: What are Equivalent Ratios (Lessons 1–5)	Link	Link
Video 2: Double Number Line Diagrams (Lessons 6–8)	Link	Link
Video 3: Comparing Situations by Examining Ratios (Lessons 9–10)	Link	Link
Video 4: Tables of Equivalent Ratios (Lessons 11–14)	Link	Link
Video 5: Using Diagrams to Solve Ratio Problems (Lessons 15–16)	Link	Link

Video 1

Video 'VLS G6U2V1 What are Equivalent Ratios (Lessons 1–5)' available here: https://player.vimeo.com/video/455248778.

Video 2

Video 'VLS G6U2V2 Double Number Line Diagrams (Lessons 6–8)' available here: https://player.vimeo.com/video/457996610.

Video 3

Video 'VLS G6U2V3 Comparing Situations by Examining Ratios (Lessons 9–10)' available here: https://player.vimeo.com/video/457998155.

Video 4

Video 'VLS G6U2V4 Tables of Equivalent Ratios (Lessons 11–14)' available here: https://player.vimeo.com/video/458003339.

Video 5

Video 'VLS G6U2V5 Using Diagrams to Solve Ratio Problems (Lessons 15–16)' available here: https://player.vimeo.com/video/458004640.

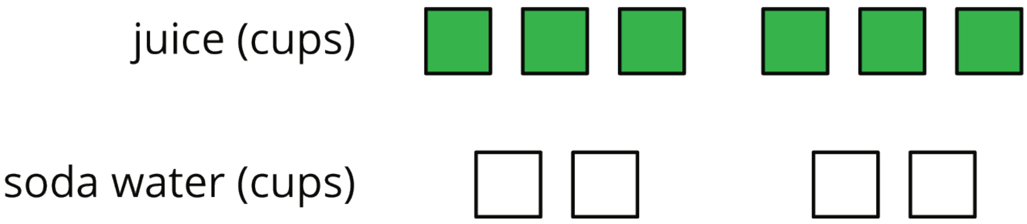
Connecting to Other Units

- *Coming soon*

What are Ratios?

Family Support Materials 1

A **ratio** is an association between two or more quantities. For example, say we have a drink recipe made with cups of juice and cups of soda water. Ratios can be represented with diagrams like those below.



Here are some correct ways to describe this diagram:

- The ratio of cups of juice to cups of soda water is $6:4$.
- The ratio of cups of soda water to cups of juice is 4 to 6.
- There are 3 cups of juice for every 2 cups of soda water.

The ratios $6:4$, $3:2$, and $12:8$ are **equivalent** because each ratio of juice to soda water would make a drink that tastes the same.

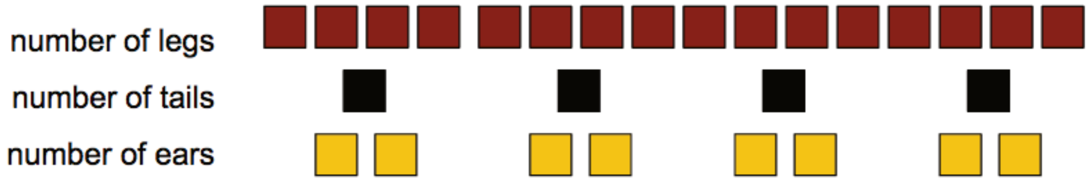
Here is a task to try with your student:

There are 4 horses in a stall. Each horse has 4 legs, 1 tail, and 2 ears.

1. Draw a diagram that shows the ratio of legs, tails, and ears in the stall.
2. Complete each statement.
    - The ratio of _______ to _______ to _______ is _______ : _______ : _______.
    - There are _______ ears for every tail. There are _______ legs for every ear.

Solution:

1. Answers vary. Sample response:



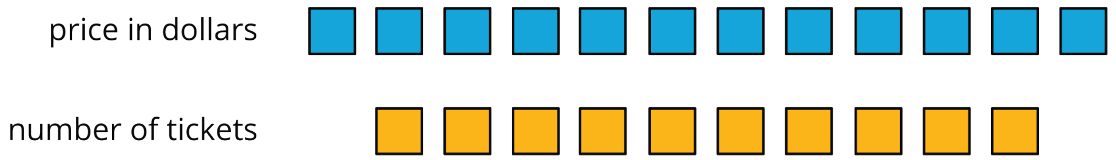
2. Answers vary. Sample response: The ratio of legs to tails to ears is $16 : 4 : 8$. There are 2 ears for every tail. There are 2 legs for every ear.

Representing Equivalent Ratios

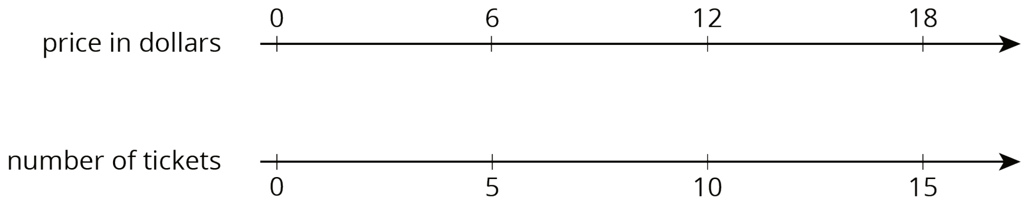
Family Support Materials 2

There are different ways to represent ratios.

Let’s say the 6th grade class is selling raffle tickets at a price of \$6 for 5 tickets. Some students may use diagrams with shapes to represent the situation. For example, here is a diagram representing 10 tickets for \$12.



Drawing so many shapes becomes impractical. Double number line diagrams are easier to work with. The one below represents the price in dollars for different numbers of raffle tickets all sold *at the same rate* of \$12 for 10 tickets.



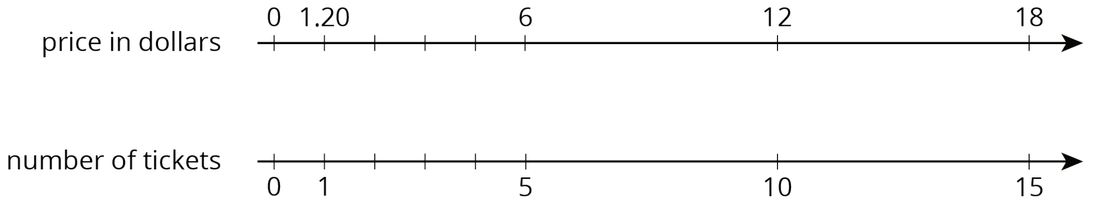
Here is a task to try with your student:

Raffle tickets cost \$6 for 5 tickets.

1. How many tickets can you get for \$90?
2. What is the price of 1 ticket?

Solution:

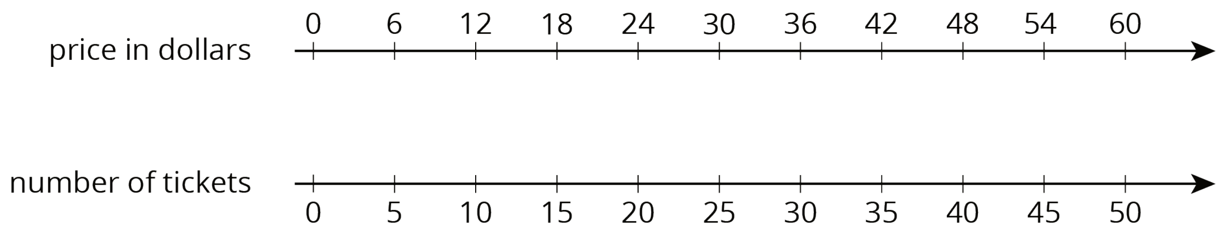
1. 75 tickets. Possible strategies: Extend the double number line shown and observe that \$90 is lined up with 75 tickets. Or, since 90 is 6 times 15, compute 5 times 15.
2. \$1.20. Possible strategies: Divide the number line into 5 equal intervals, as shown. Reason that the price in dollars of 1 ticket must be $6 \div 5$.


price in dollars
0
1.20
6
12
18
number of tickets
0
1
5
10
15

Solving Ratio and Rate Problems

Family Support Materials 3

Over the course of this unit, your student has learned to use the language of ratios and to work with ratios using representations like diagrams and double number lines. In the final sections of the unit, they use **tables** to organize equivalent ratios. Double number lines are hard to use in problems with large amounts. Let’s think about an example we saw before: the 6th grade class is selling raffle tickets at a price of \$6 for 5 tickets. If we tried to extend the double number line below to represent the price of 300 raffle tickets, it would take 5 times more paper!



A table is a better choice to represent this situation. Tables of equivalent ratios are useful because you can arrange the rows in any order. For example, a student may find the price for 300 raffle tickets by making the table shown.

price in dollars	number of tickets
6	5
1.20	1
360	300

÷5 (from row 6/5 to row 1.20/1, on both sides); •300 (from row 1.20/1 to row 360/300, on both sides)

Although students can choose any representation that helps them solve a problem, it is important that they get comfortable with tables because they are used for a variety of purposes throughout high school and college mathematics courses.

Here is a task to try with your student:

At a constant speed, a train travels 45 miles in 60 minutes. At this rate, how far does the train travel in 12 minutes? If you get stuck, consider creating a table.

Solution:

9 miles. Possible strategy:

time in minutes	distance in miles
60	45
1	0.75
12	9

Family Support Materials

Unit Rates and Percentages

Here are the video lesson summaries for Grade 6, Unit 3 Unit Rates and Percentages. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

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Grade 6, Unit 3: Unit Rates and Percentages	Vimeo	YouTube
Video 1: Converting Measurements (Lessons 2–4)	Link	Link
Video 2: Unit Rates (Lessons 5–8)	Link	Link
Video 3: Understanding Percentage (Lessons 10–13)	Link	Link
Video 4: Solving Percentage Problems (Lessons 14–16)	Link	Link

Video 1

Video 'VLS G6U3V1 Converting Measurements (Lessons 2–4)' available here: https://player.vimeo.com/video/469298365.

Video 2

Video 'VLS G6U3V2 Unit Rates (Lessons 5–8)' available here: https://player.vimeo.com/video/470623725.

Video 3

Video 'VLS G6U3V3 Understanding Percentage (Lessons 10–13)' available here: https://player.vimeo.com/video/469393213.

Video 4

Video 'VLS G6U3V4 Solving Percentage Problems (Lessons 14–16)' available here: https://player.vimeo.com/video/471578428.

Connecting to Other Units

- *Coming soon*

Units of Measurement

Family Support Materials 1

If you weighed four objects in pounds, then weighed the same four objects in kilograms, you might come up with this table.

weight (pounds)	weight (kilograms)
22	10
88	40
33	15
40.7	18.5

Students are using what they know about ratios and rates to reason about measurements in different *units of measurement* such as pounds and kilograms. In earlier grades, students converted yards to feet using the fact that 1 yard is 3 feet, and kilometers to meters using the fact that 1 kilometer is 1,000 meters. Now in grade 6, students convert units that do not always use whole numbers.

Here is a task to try with your student:

Explain your strategy for each question.

1. Which is heavier, 1 pound or 1 kilogram?
2. A canoe weighs 99 pounds. How many kilograms does it weigh?
3. A watermelon weighs 12 kilograms. How many pounds does it weigh?

Solution:

Any correct strategy that your student understands and can explain is acceptable. Sample strategies:

1. 1 kilogram is heavier than 1 pound. When we weigh the same object in pounds and kilograms, the number of pounds is more than the number of kilograms. It takes fewer kilograms to express the weight of the same object, so each kilogram must be heavier than each pound. Another example of this idea: if we measure the length of a table in both meters and inches, the number of inches is more than the number of meters. Therefore, 1 inch must be shorter than 1 meter.

2. 45. Using the table, we can reason that 11 pounds is 5 kilograms. Multiplying each of these by 9 shows that 99 pounds is 45 kilograms.

3. 26.4. Using the table, we can find that each kilogram is equal to about 2.2 pounds. This means if we know an object's weight in kilograms, we can multiply by 2.2 to find its weight in pounds. $12 \cdot (2.2) = 26.4$

Rates

Family Support Materials 2

Who biked faster: Andre, who biked 25 miles in 2 hours, or Lin, who biked 30 miles in 3 hours? One strategy would be to calculate a **unit rate** for each person. A unit rate is an equivalent ratio expressed as something "per 1." For example, Andre's rate could be written as "$12\frac{1}{2}$ miles in 1 hour" or "$12\frac{1}{2}$ miles *per 1 hour*." Lin's rate could be written "10 miles per 1 hour." By finding the unit rates, we can compare the distance each person went in 1 hour to see that Andre biked faster.

Every ratio has *two* unit rates. In this example, we could also compute *hours per mile*: how many hours it took each person to cover 1 mile. Although not every rate has a special name, rates in "miles per hour" are commonly called **speed** and rates in "hours per mile" are commonly called **pace**.

Andre:

distance (miles)	time (hours)
25	2
1	0.08
12.5	1

Lin:

distance (miles)	time (hours)
30	3
10	1
1	0.1

Here is a task to try with your student:

Dry dog food is sold in bulk: 4 pounds for $16.00.

1. At this rate, what is the cost *per pound* of dog food?
2. At this rate, what is the amount of dog food you can buy *per dollar*?

Solution:

1. \$4.00 per pound because $16 \div 4 = 4$.

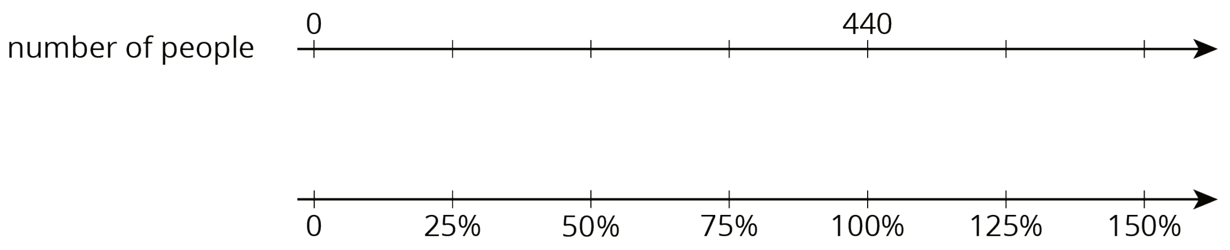
2. You get $\frac{1}{4}$ or 0.25 of a pound per dollar because $4 \div 16 = 0.25$.

dog food (pounds)	cost (dollars)
4	16
1	4
0.25	1

Percentages

Family Support Materials 3

Let’s say 440 people attended a school fundraiser last year. If 330 people were adults, what percentage of people were adults? If it’s expected that the attendance this year will be 125% of last year, how many attendees are expected this year? A double number line can be used to reason about these questions.



Students use their understanding of “rates per 1” to find **percentages**, which we can think of as “rates per 100.” Double number lines and tables continue to support their thinking. The example about attendees of a fundraiser could also be organized in a table:

number of people	percentage
440	100%
110	25%
330	75%
550	125%

Toward the end of the unit, students develop more sophisticated strategies for finding percentages. For example, you can find 125% of 440 attendees by computing $\frac{125}{100} \cdot 440$. With practice, students will use these more efficient strategies and understand why they work.

Here is a task to try with your student:

For each question, explain your reasoning. If you get stuck, try creating a table or double number line for the situation.

1. A bottle of juice contains 16 ounces, and you drink 25% of the bottle. How many ounces did you drink?
2. You get 9 questions right in a trivia game, which is 75% of the questions. How many questions are in the game?
3. You planned to walk 8 miles, but you ended up walking 12 miles. What percentage of your planned distance did you walk?

Solution:

Any correct reasoning that a student understands and can explain is acceptable. Sample reasoning:

1. 4. 25% of the bottle is $\frac{1}{4}$ of the bottle, and $\frac{1}{4}$ of 16 is 4.
2. 12. If 9 questions is 75%, we can divide each by 3 to know that 3 questions is 25%. Multiplying each by 4 shows that 12 questions is 100%.
3. 150%. If 8 miles is 100%, then 4 miles is 50%, and 12 miles is 150%.

Family Support Materials

Dividing Fractions

Here are the video lesson summaries for Grade 6, Unit 4: Dividing Fractions. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 6, Unit 4: Dividing Fractions	Vimeo	YouTube
Video 1: Meanings of Division (Lessons 1–3)	Link	Link
Video 2: Using Diagrams to Divide Fractions (Lessons 5–9)	Link	Link
Video 3: Using an Algorithm to Divide Fractions (Lessons 10–12)	Link	Link
Video 4: Area and Volume with Fractions (Lessons 13–15)	Link	Link

Video 1

Video 'VLS G6U4V1 Meanings of Division (Lessons 1–3)' available here: https://player.vimeo.com/video/481745482.

Video 2

Video 'VLS G6U4V2 Using Diagrams to Divide Fractions (Lessons 5–9)' available here: https://player.vimeo.com/video/481403959.

Video 3

Video 'VLS G6U4V3 Using an Algorithm to Divide Fractions (Lessons 10–12)' available here: https://player.vimeo.com/video/486045903.

Video 4

Video 'VLS G6U4V4 Area and Volume with Fractions (Lessons 13–15)' available here: https://player.vimeo.com/video/486048726.

Connecting to Other Units

- *Coming soon*

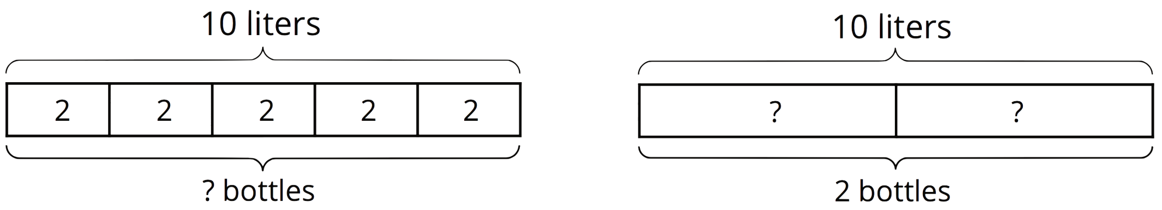
Making Sense of Division

Family Support Materials 1

This week, your student will be thinking about the meanings of division to prepare to learn about division of fraction. Suppose we have 10 liters of water to divide into equal-size groups. We can think of the division $10 \div 2$ in two ways, or as the answer to two questions:

- “How many bottles can we fill with 10 liters if each bottle has 2 liters?”
- “How many liters are in each bottle if we divide 10 liters into 2 bottles?”

Here are two diagrams to show the two interpretations of $10 \div 2$:



In both cases, the answer to the question is 5, but it could either mean “there are 5 bottles with 2 liters in each” or “there are 5 liters in each of the 2 bottles.”

Here is a task to try with your student:

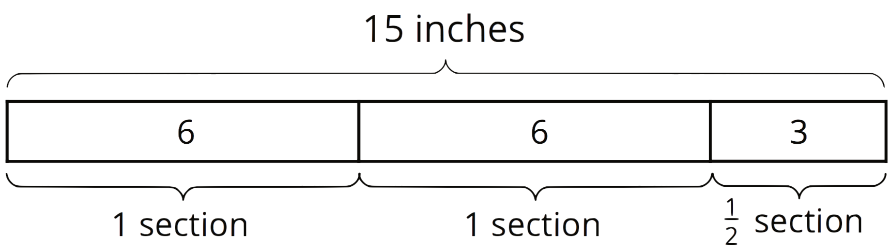
1. Write two different questions we can ask about $15 \div 6$.
2. Estimate the answer: Is it less than 1, equal to 1, or greater than 1? Explain your estimate.
3. Find the answer to one of the questions you wrote. It might help to draw a picture.

Solution:

1. Questions vary. Sample questions:
    - A ribbon that is 15 inches long is divided into 6 equal sections. How long (in inches) is each section?
    - A ribbon that is 15 inches is divided into 6-inch sections. How many sections are there?
2. Greater than 1. Sample explanations:
    - $12 \div 6$ is 2, so $15 \div 6$ must be greater than 2.

- If we divide 15 into 15 groups ($15 \div 15$), we get 1. So if we divide 15 into 6, which is a smaller number of groups, the amount in each group must be greater than 1.

3. $2\frac{1}{2}$. Sample diagram:



Meanings of Fraction Division

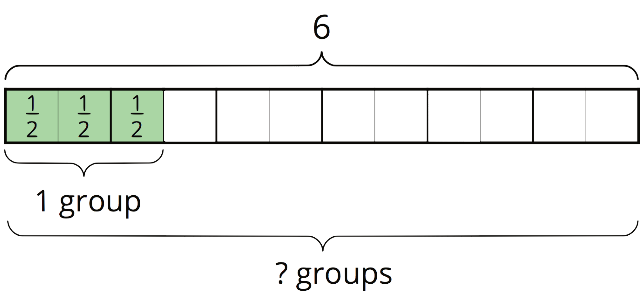
Family Support Materials 2

Earlier, students learned that a division such as $10 \div 2 = ?$ can be interpreted as "*how many groups* of 2 are in 10?" or "*how much is in each group* if there are 10 in 2 groups?" They also saw that the relationship between 10, 2 and the unknown number ("?") can also be expressed with multiplication:

$$2 \cdot ? = 10$$
$$? \cdot 2 = 10$$

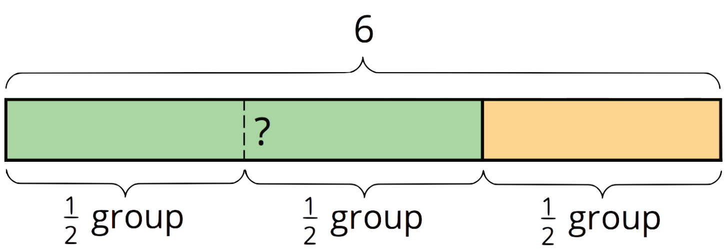
This week, they use these ideas to divide fractions. For example, $6 \div 1\frac{1}{2} = ?$ can be thought of as "how many groups of $1\frac{1}{2}$ are in 6?" Expressing the question as a multiplication and drawing a diagram can help us find the answer.

$$? \cdot 1\frac{1}{2} = 6$$



From the diagram we can count that there are 4 groups of $1\frac{1}{2}$ in 6.

We can also think of $6 \div 1\frac{1}{2} = ?$ as "how much is in each group if there are $1\frac{1}{2}$ equal groups in 6?" A diagram can also be useful here.



From the diagram we can see that if there are three $\frac{1}{2}$-groups in 6. This means there is 2 in each $\frac{1}{2}$ group, or 4 in 1 group.

In both cases $6 \div 1\frac{1}{2} = 4$, but the 4 can mean different things depending on how the division is interpreted.

Here is a task to try with your student:

1. How many groups of $\frac{2}{3}$ are in 5?
   a. Write a division equation to represent the question. Use a "?" to represent the unknown amount.

   b. Find the answer. Explain or show your reasoning.

2. A sack of flour weighs 4 pounds. A grocer is distributing the flour into equal-size bags.
   a. Write a question that $4 \div \frac{2}{5} = ?$ could represent in this situation.

   b. Find the answer. Explain or show your reasoning.

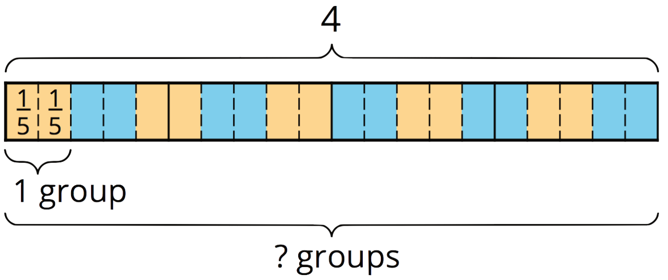
Solution:

1. a. $5 \div \frac{2}{3} = ?$

   b. $7\frac{1}{2}$. Sample reasoning: There are 3 thirds in 1, so there are 15 thirds in 5. That means there are half as many two-thirds, or $\frac{15}{2}$ two-thirds, in 5.

2. a. 4 pounds of flour are divided equally into bags of $\frac{2}{5}$-pound each. How many bags will there be?

   b. 10 bags. Sample reasoning: Break every 1 pound into fifths and then count how many groups of $\frac{2}{5}$ there are.



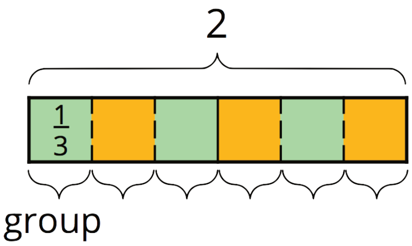
Algorithm for Fraction Division

Family Support Materials 3

Many people have learned that to divide a fraction, we "invert and multiply." This week, your student will learn why this works by studying a series of division statements and diagrams such as these:

- $2 \div \frac{1}{3} = ?$ can be viewed as "how many $\frac{1}{3}$s are in 2?"

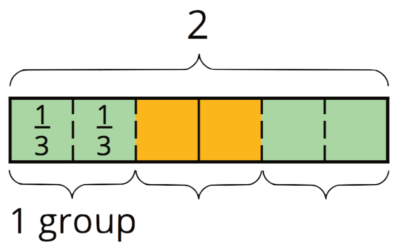
$$2 \div \frac{1}{3} = ?$$



Because there are 3 thirds in 1, there are $(2 \cdot 3)$ or 6 thirds in 2. So dividing 2 by $\frac{1}{3}$ has the same outcome as multiplying 2 by 3.

- $2 \div \frac{2}{3} = ?$ can be viewed as "how many $\frac{2}{3}$s are in 2?"

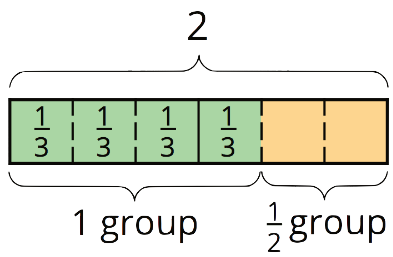
$$2 \div \frac{2}{3} = ?$$



We already know that there are $(2 \cdot 3)$ or 6 thirds in 2. To find how many $\frac{2}{3}$s are in 2, we need to combine every 2 of the thirds into a group. Doing this results in half as many groups. So $2 \div \frac{2}{3} = (2 \cdot 3) \div 2$, which equals 3.

- $2 \div \frac{4}{3} = ?$ can be viewed as "how many $\frac{4}{3}$s are in 2?"

$$2 \div \frac{4}{3} = ?$$



Again, we know that there are $(2 \cdot 3)$ thirds in 2. To find how many $\frac{4}{3}$s are in 2, we need to combine every 4 of the thirds into a group. Doing this results in one fourth as many groups. So $2 \div \frac{4}{3} = (2 \cdot 3) \div 4$, which equals $1\frac{1}{2}$.

Notice that each division problem above can be answered by multiplying 2 by the denominator of the divisor and then dividing it by the numerator. So $2 \div \frac{a}{b}$ can be solved with $2 \cdot b \div a$, which can also be written as $2 \cdot \frac{b}{a}$. In other words, dividing 2 by $\frac{a}{b}$ has the same outcome as multiplying 2 by $\frac{b}{a}$. The fraction in the divisor is "inverted" and then multiplied.

Here is a task to try with your student:

1. Find each quotient. Show your reasoning.
    a. $3 \div \frac{1}{7}$

    b. $3 \div \frac{3}{7}$

    c. $3 \div \frac{6}{7}$

    d. $\frac{3}{7} \div \frac{6}{7}$

2. Which has a greater value: $\frac{9}{10} \div \frac{9}{100}$ or $\frac{12}{5} \div \frac{6}{25}$? Explain or show your reasoning.

Solution:

1. a. 21. Sample reasoning: $3 \div \frac{1}{7} = 3 \cdot \frac{7}{1} = 21$

    b. 7. Sample reasoning: $3 \div \frac{3}{7} = 3 \cdot \frac{7}{3} = 7$

    c. $3\frac{1}{2}$. Sample reasoning: $3 \div \frac{1}{7} = 3 \cdot \frac{7}{6} = \frac{7}{2}$. The fraction $\frac{6}{7}$ is two times $\frac{3}{7}$, so there are half as many $\frac{6}{7}$s in 3 as there are $\frac{3}{7}$s.

    d. $\frac{1}{2}$. Sample reasoning: $\frac{3}{7} \div \frac{6}{7} = \frac{3}{7} \cdot \frac{7}{6} = \frac{3}{6}$

2. They have the same value. Both equal 10. $\frac{9}{10} \div \frac{9}{100} = \frac{9}{10} \cdot \frac{100}{9} = 10$ and $\frac{12}{5} \div \frac{6}{25} = \frac{12}{5} \cdot \frac{25}{6} = 10$.

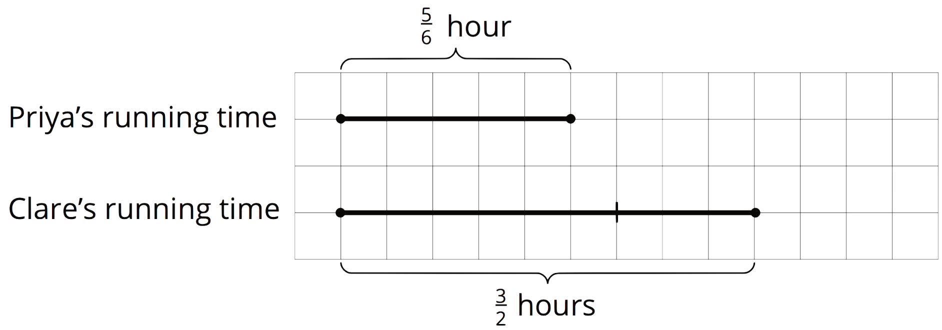
Fractions in Lengths, Areas, and Volumes

Family Support Materials 4

Over the next few days, your student will be solving problems that require multiplying and dividing fractions. Some of these problems will be about comparison. For example:

- If Priya ran for $\frac{5}{6}$ hour and Clare ran for $\frac{3}{2}$ hours, what fraction of Clare's running time was Priya's running time?

  We can draw a diagram and write a multiplication equation to make sense of the situation.

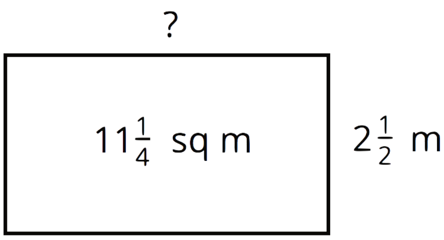


$$(\text{fraction}) \cdot (\text{Clare's time}) = (\text{Priya's time})$$
$$? \cdot \frac{3}{2} = \frac{5}{6}$$

We can find the unknown by dividing. $\frac{5}{6} \div \frac{3}{2} = \frac{5}{6} \cdot \frac{2}{3}$, which equals $\frac{10}{18}$. So Priya's running time was $\frac{10}{18}$ or $\frac{5}{9}$ of Clare's.

Other problems your students will solve are related to geometry—lengths, areas, and volumes. For examples:

- What is the length of a rectangular room if its width is $2\frac{1}{2}$ meters and its area is $11\frac{1}{4}$ square meters?



We know that the area of a rectangle can be found by multiplying its length and width ($? \cdot 2\frac{1}{2} = 11\frac{1}{4}$), so dividing $11\frac{1}{4} \div 2\frac{1}{2}$ (or $\frac{45}{4} \div \frac{5}{2}$) will give us the length of the room. $\frac{45}{4} \div \frac{5}{2} = \frac{45}{4} \cdot \frac{2}{5} = \frac{9}{2}$. The room is $4\frac{1}{2}$ meters long.

- What is the volume of a box (a rectangular prism) that is $3\frac{1}{2}$ feet by 10 feet by $\frac{1}{4}$ foot?

  We can find the volume by multiplying the edge lengths. $3\frac{1}{2} \cdot 10 \cdot \frac{1}{4} = \frac{7}{2} \cdot 10 \cdot \frac{1}{4}$, which equals $\frac{70}{8}$. So the volume is $\frac{70}{8}$ or $8\frac{6}{8}$ cubic feet.

Here is a task to try with your student:

1. In the first example about Priya and Clare's running times, how many times as long as Priya's running time was Clare's running time? Show your reasoning.
2. The area of a rectangle is $\frac{20}{3}$ square feet. What is its width if its length is $\frac{4}{3}$ feet? Show your reasoning.

Solution:

1. $\frac{9}{5}$. Sample reasoning: We can write $? \cdot \frac{5}{6} = \frac{3}{2}$ to represent the question "how many times of Priya's running time was Clare's running time?" and then solve by dividing. $\frac{3}{2} \div \frac{5}{6} = \frac{3}{2} \cdot \frac{6}{5} = \frac{18}{10}$. Clare's running time was $\frac{18}{10}$ or $\frac{9}{5}$ as long as Priya's.
2. 5 feet. Sample reasoning: $\frac{20}{3} \div \frac{4}{3} = \frac{20}{3} \cdot \frac{3}{4} = \frac{20}{4} = 5$

Family Support Materials

Arithmetic in Base Ten

Warming Up to Decimals

Family Support Materials 1

This week, your student will add and subtract numbers using what they know about the meaning of the digits. In earlier grades, your student learned that the 2 in 207.5 represents 2 *hundreds*, the 7 represents 7 *ones*, and the 5 represents 5 *tenths*. We add and subtract the digits that correspond to the same units like *hundreds* or *tenths*. For example, to find $10.5 + 84.3$, we add the tens, the ones, and the tenths separately, so $10.5 + 84.3 = 90 + 4 + 0.8 = 94.8$.

Any time we add digits and the sum is greater than 10, we can "bundle" 10 of them into the next higher unit. For example, $0.9 + 0.3 = 1.2$.

To add whole numbers and decimal numbers, we can arrange $0.921 + 4.37$ vertically, aligning the decimal points, and find the sum. This is a convenient way to be sure we are adding digits that correspond to the same units. This also makes it easy to keep track when we bundle 10 units into the next higher unit (some people call this "carrying").

$$\begin{array}{r} {\scriptstyle 1} \\ 0.921 \\ +\ 4.37 \\ \hline 5.291 \end{array}$$

Here is a task to try with your student:

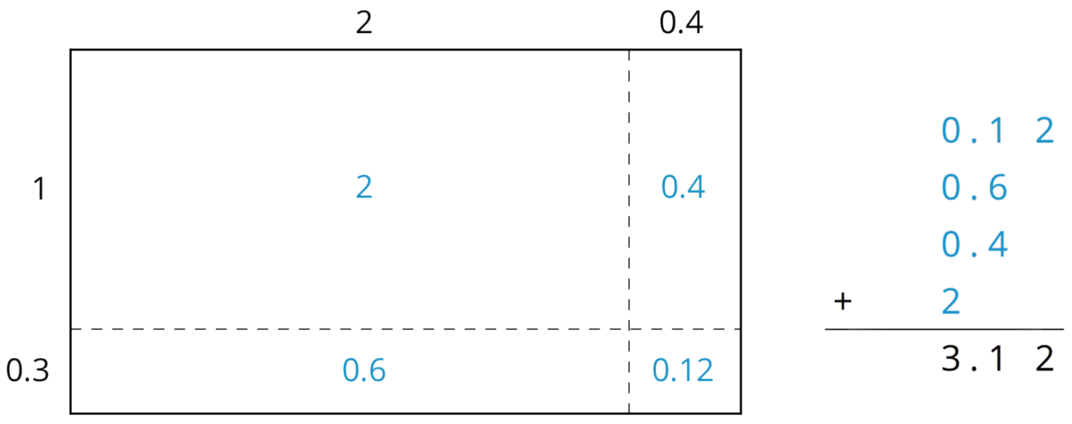
Find the value of $6.54 + 0.768$.

Solution: 7.308. Sample explanation: there are 8 thousandths from 0.768. Next, the 4 hundredths from 6.54 and 6 hundredths from 0.768 combined make 1 tenth. Together with the 5 tenths from 6.54 and the 7 tenths from 0.768 this is 13 tenths total or 1 and 3 tenths. In total, there are 7 ones, 3 tenths, no hundredths, and 8 thousandths.

Multiplying Decimals

Family Support Materials 2

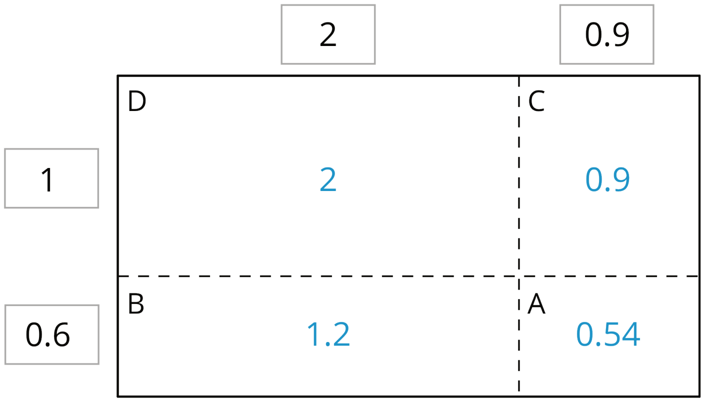
This week, your student will multiply decimals. There are a few ways we can multiply two decimals such as $(2.4) \cdot (1.3)$. We can represent the product as the area of a rectangle. If 2.4 and 1.3 are the side lengths of a rectangle, the product $(2.4) \cdot (1.3)$ is its area. To find the area, it helps to decompose the rectangle into smaller rectangles by breaking the side lengths apart by place value. The sum of the areas of all of the smaller rectangles, 3.12, is the total area.



Here is a task to try with your student:

Find $(2.9) \cdot (1.6)$ using an area model and partial products.

Solution: 4.64. The area of the rectangle (or the sum of the partial products) is:
$2 + 0.9 + 1.2 + 0.54 = 4.64$



Dividing Decimals

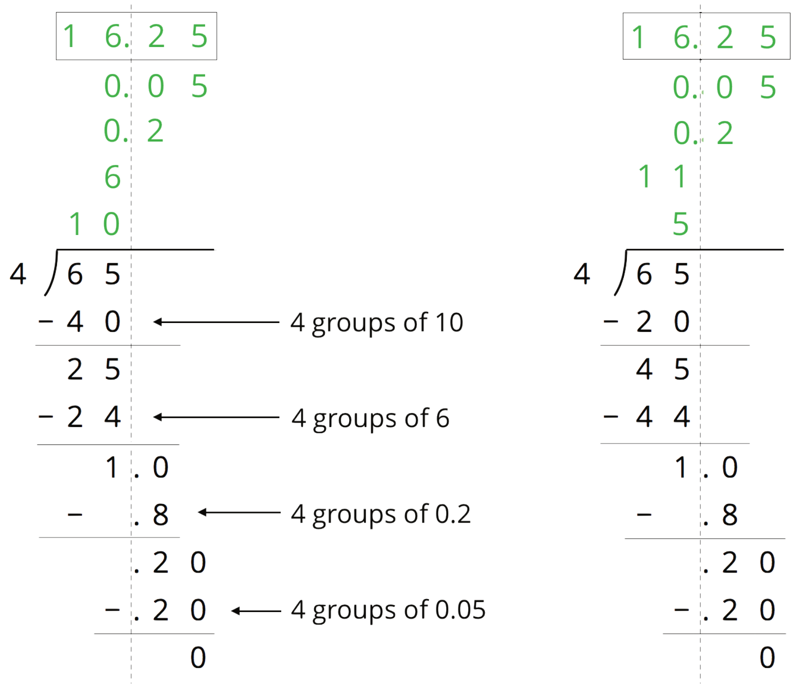
Family Support Materials 3

This week, your student will divide whole numbers and decimals. We can think about division as breaking apart a number into equal-size groups.

For example, consider $65 \div 4$. We can image that we are sharing 65 grams of gold equally among 4 people. Here is one way to think about this:

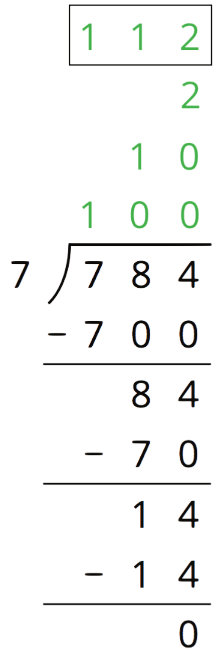
- First give everyone 10 grams. Then 40 grams have been shared out, and 25 grams are left over. We can see this in the first example.
- If we give everyone 6 more grams, then 24 grams have been shared out, and 1 gram is left.
- If we give everyone 0.2 more grams, then 0.8 grams are shared out and 0.2 grams are left.
- If everyone gets 0.05 more grams next, then all of the gold has been shared equally.

Everyone gets $10 + 6 + 0.2 + 0.05 = 16.25$ grams of gold.



The calculation on the right shows different intermediate steps, but the quotient is the same. This approach is called the **partial quotients** method for dividing.

Here is a task to try with your student:



Here is how Jada found $784 \div 7$ using the partial quotient method.

1. In the calculation, what does the subtraction of 700 represent?
2. Above the dividend 784, we see the numbers 100, 10, and 2. What do they represent?
3. How can we check if 112 if the correct quotient for $784 \div 7$?

Solution

1. Subtraction of 7 groups of 100 from 784.
2. 100, 10, and 2 are the amounts distributed into each group over 3 rounds of dividing.
3. We can multiply $7 \cdot 112$ and see if it produces 784.

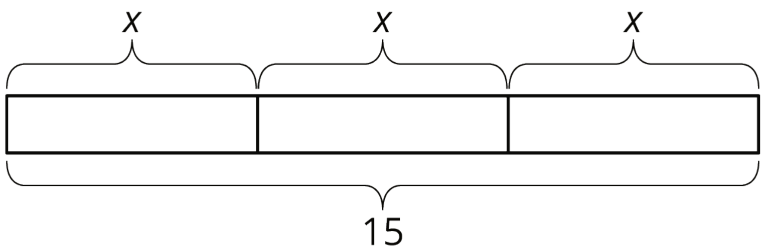
Family Support Materials

Expressions and Equations

Equations in One Variable

Family Support Materials 1

This week your student will be learning to visualize, write, and solve equations. They did this work in previous grades with numbers. In grade 6, we often use a letter called a **variable** to represent a number whose value is unknown. Diagrams can help us make sense of how quantities are related. Here is an example of such a diagram:



Since 3 pieces are labeled with the same variable x, we know that each of the three pieces represent the same number. Some equations that match this diagram are $x + x + x = 15$ and $15 = 3x$.

A **solution** to an equation is a number used in place of the variable that makes the equation true. In the previous example, the solution is 5. Think about substituting 5 for x in either equation: $5 + 5 + 5 = 15$ and $15 = 3 \cdot 5$ are both true. We can tell that, for example, 4 is *not* a solution, because $4 + 4 + 4$ does not equal 15.

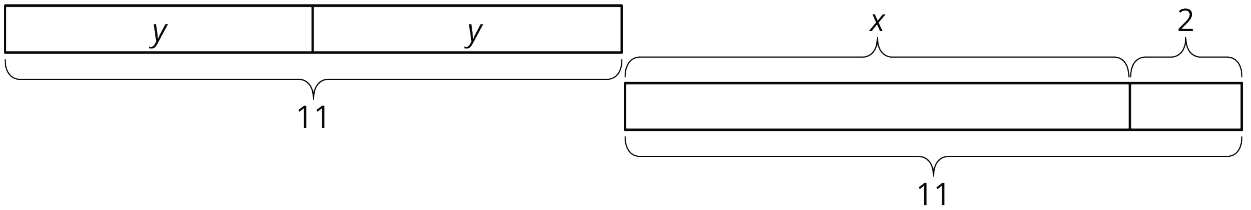
Solving an equation is a process for finding a solution. Your student will learn that an equation like $15 = 3x$ can be solved by dividing each side by 3. Notice that if you divide each side by 3, $15 \div 3 = 3x \div 3$, you are left with $5 = x$, the solution to the equation.

Here is a task to try with your student:

Draw a diagram to represent each equation. Then, solve each equation.

$2y = 11$ $\qquad$ $11 = x + 2$

Solution:



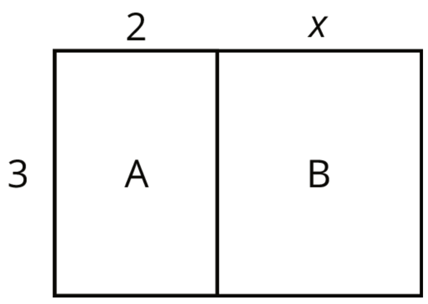
$y = 5.5$ or $y = \frac{11}{2}$

$x = 9$

Equal and Equivalent

Family Support Materials 2

This week your student is writing mathematical expressions, especially expressions using the distributive property.



In this diagram, we can say one side length of the large rectangle is 3 units and the other is $x + 2$ units. So, the area of the large rectangle is $3(x + 2)$. The large rectangle can be partitioned into two smaller rectangles, A and B, with no overlap. The area of A is 6 and the area of B is $3x$. So, the area of the large rectangle can also be written as $3x + 6$. In other words,

$$3(x + 2) = 3x + 3 \cdot 2$$

This is an example of the distributive property.

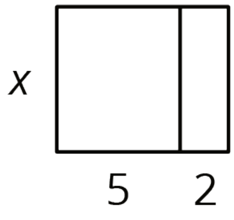
Here is a task to try with your student:

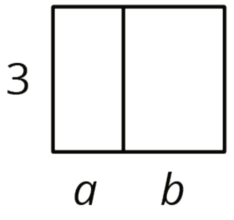
Draw and label a partitioned rectangle to show that each of these equations is always true, no matter the value of the letters.

- $5x + 2x = (5 + 2)x$
- $3(a + b) = 3a + 3b$

Solution:

Answers vary. Sample responses:





Expressions with Exponents

Family Support Materials 3

This week your student will be working with **exponents**. When we write an expression like 7^n, we call n the exponent. In this example, 7 is called the **base**. The exponent tells you how many factors of the base to multiply. For example, 7^4 is equal to $7 \cdot 7 \cdot 7 \cdot 7$. In grade 6, students write expressions with whole-number exponents and bases that are

- whole numbers like 7^4
- fractions like $\left(\frac{1}{7}\right)^4$
- decimals like 7.7^4
- variables like x^4

Here is a task to try with your student:

Remember that a solution to an equation is a number that makes the equation true. For example, a solution to $x^5 = 30 + x$ is 2, since $2^5 = 30 + 2$. On the other hand, 1 is not a solution, since 1^5 does not equal $30 + 1$. Find the solution to each equation from the list provided.

List: $0, 0.008, \frac{1}{2}, \frac{9}{16}, \frac{6}{8}, 0.8, 1, 2, 3, 4, 5, 6, 7$

1. $n^2 = 49$
2. $4^n = 64$
3. $4^n = 4$
4. $\left(\frac{3}{4}\right)^2 = n$
5. $0.2^3 = n$
6. $n^4 = \frac{1}{16}$
7. $1^n = 1$
8. $3^n \div 3^2 = 3^3$

Solution:

1. 7, because $7^2 = 49$. (Note that -7 is also a solution, but in grade 6 students aren't expected to know about multiplying negative numbers.)
2. 3, because $4^3 = 64$
3. 1, because $4^1 = 4$
4. $\frac{9}{16}$, because $\left(\frac{3}{4}\right)^2$ means $\left(\frac{3}{4}\right) \cdot \left(\frac{3}{4}\right)$
5. 0.008, because 0.2^3 means $(0.2) \cdot (0.2) \cdot (0.2)$
6. $\frac{1}{2}$, because $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$
7. Any number! $1^n = 1$ is true no matter what number you use in place of n.
8. 5, because this can be rewritten $3^n \div 9 = 27$. What would we have to divide by 9 to get 27? 243, because $27 \cdot 9 = 243$. $3^5 = 243$.

Relationships Between Quantities

Family Support Materials 4

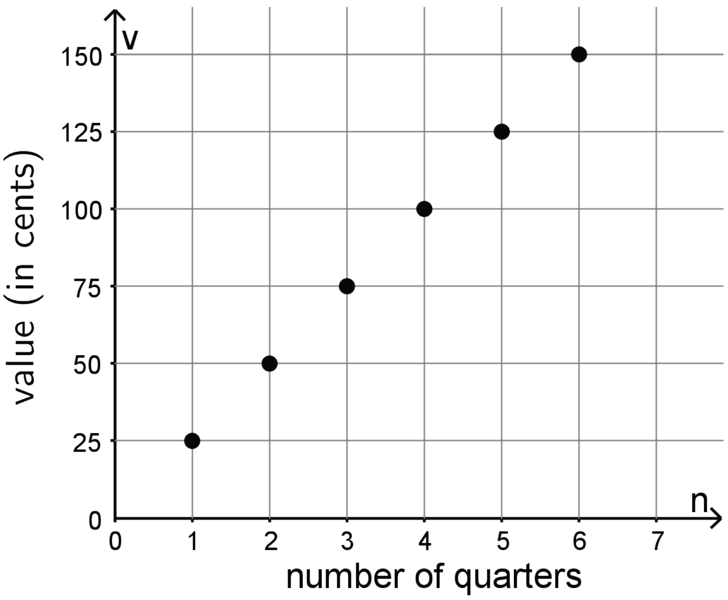
This week your student will study relationships between two quantities. For example, since a quarter is worth 25¢, we can represent the relationship between the number of quarters, n, and their value v in cents like this:

$$v = 25n$$

We can also use a table to represent the situation:

n	v
1	25
2	50
3	75

Or we can draw a graph to represent the relationship between the two quantities:



Here is a task to try with your student:

A shopper is buying granola bars. The cost of each granola bar is $0.75.

1. Write an equation that shows the cost of the granola bars, c, in terms of the number of bars purchased, n.
2. Create a graph representing associated values of c and n.
3. What are the coordinates of some points on your graph? What do they represent?

Solution:

1. $c = 0.75n$. Every granola bar costs $0.75 and the shopper is buying n of them, so the cost is $0.75n$.
2. Answers vary. One way to create a graph is to label the horizontal axis with "number of bars" with intervals, 0, 1, 2, 3, etc, and label the vertical axis with "total cost in dollars" with intervals 0, 0.25, 0.50, 0.75, etc.
3. If the graph is created as described in this solution, the first coordinate is the number of granola bars and the second is the cost in dollars for that number of granola bars. Some points on such a graph are $(2, 1.50)$ and $(10, 7.50)$

Family Support Materials

Rational Numbers

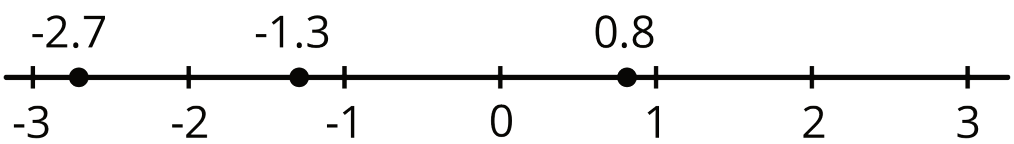
Negative Numbers and Absolute Value

Family Support Materials 1

This week, your student will work with signed numbers, or positive and negative numbers. We often compare signed numbers when talking about temperatures. For example, -30 degrees Fahrenheit is colder than -10 degrees Fahrenheit. We say "-30 is less than -10" and write: $-30 < -10$.

We also use signed numbers when referring to elevation, or height relative to the sea level. An elevation of 2 feet (which means 2 feet above sea level) is higher than an elevation of -4 feet (which means 4 feet below sea level). We say "2 is greater than -4" and write $2 > -4$.

We can plot positive and negative numbers on the number line. Numbers to the left are always less than numbers to the right.



We can see that -1.3 is less than 0.8 because -1.3 is to the left of 0.8, but -1.3 is greater than -2.7 because it is to the right of -2.7.

We can also talk about a number in terms of its **absolute value**, or its distance from zero on the number line. For example, 0.8 is 0.8 units away from zero, which we can write as $|0.8| = 0.8$, and -2.7 is 2.7 units away from zero, which we can write as $|-2.7| = 2.7$. The numbers -3 and 3 are both 3 units from 0, which we can write as $|3| = 3$ and $|-3| = 3$.

Here is a task to try with your student:

1. A diver is at the surface of the ocean, getting ready to make a dive. What is the diver's elevation in relation to sea level?

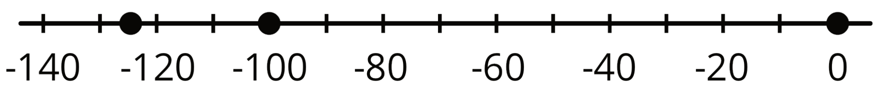
2. The diver descends 100 feet to the top of a wrecked ship. What is the diver's elevation now?

3. The diver descends 25 feet more toward the ocean floor. What is the absolute value of the diver's elevation now?

4. Plot each of the three elevations as a point on a number line. Label each point with its numeric value.

Solution:

1. 0, because sea level is 0 feet above or below sea level
2. -100, because the diver is 100 feet *below* sea level
3. The new elevation is -125 feet or 125 feet *below* sea level, so its absolute value is 125 feet.
4. A number line with 0, -100, and -125 marked, as shown:

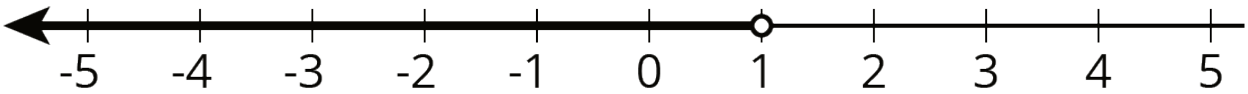


Inequalities

Family Support Materials 2

This week, your student will compare positive and negative numbers with inequalities symbols (< and >). They will also graph inequalities in one variable, such as $x < 1$ or $1 > x$, on the number line.

For example, to represent the statement “the temperature in Celsius (x) is less than 1 degree,” we can write the inequality $x < 1$ and draw a number line like this:



The diagram shows all numbers to the left of 1 (or less than 1) being possible values of x.

We call any value of x that makes an inequality true a **solution to the inequality**.

This means x values that are greater than -8 are solutions to the inequality $x > \text{-}8$. Likewise, x values that are less than 15 could be a solution to the inequality $x < 15$. Depending on the context, however, the solutions may include only positive whole numbers (for example, if x represents the number of students in a class), or any positive and negative numbers, not limited to whole numbers (for example, if x represents temperatures).

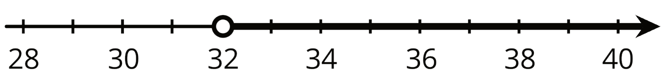
Here is a task to try with your student:

A sign at a fair says, “You must be taller than 32 inches to ride the Ferris wheel.” Write and graph an inequality that shows the heights of people who are tall enough to ride the Ferris wheel.

Solution:

If x represents the height of a person in inches, then the inequality $x > 32$ represents the heights of people who can ride the ferris wheel. We can also write the inequality $32 < x$.

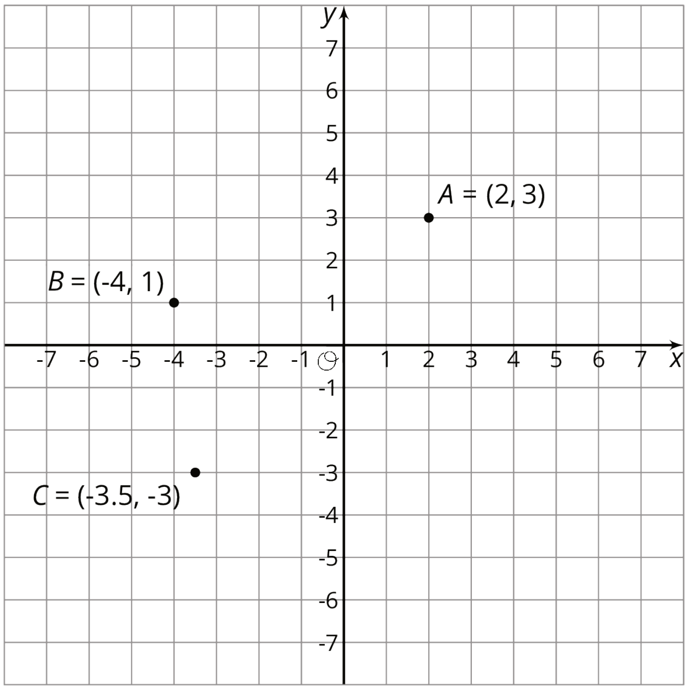
The graph of the inequality is:



The Coordinate Plane

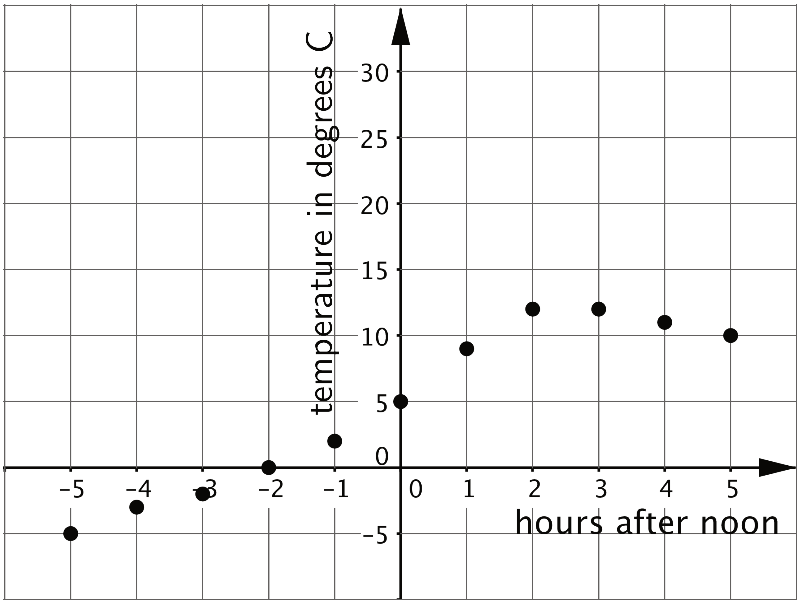
Family Support Materials 3

This week, your student will plot and interpret points on the coordinate plane. In earlier grades, they plotted points where both coordinates are positive, such as point A in the figure. They will now plot points that have positive and negative coordinates, such as points B and C.



To find the distance between two points that share the same horizontal line or the same vertical lines, we can simply count the grid units between them. For example, if we plot the point $(2, \text{-}4)$ on the grid above (try it!), we can tell that the point will be 7 units away from point $A = (2, 3)$.

Points on a coordinate plane can also represent situations that involve positive and negative numbers. For instance, the points on this coordinate plane shows the temperature in degrees Celsius every hour before and after noon on a winter day. Times before noon are negative and times after noon are positive.



For example, the point $(5, 10)$ tells us that 5 hours after noon, or 5:00 p.m, the temperature was 10 degrees Celsius.

Here is a task to try with your student:

In the graph of temperatures above:

1. What was the temperature at 7 a.m.?

2. For which recorded times was it colder than 5 degrees Celsius?

Solution:

1. It was -5 degrees Celsius at 7:00 a.m. You can see this at the point $(-5, -5)$.

2. It was 5 degrees Celsius right at noon, and for the times recorded before that, it was colder.

Common Factors and Common Multiples

Family Support Materials 4

This week, your student will solve problems that involve **factors** and **multiples**. Because $2 \cdot 6 = 12$, we say that 2 and 6 are factors of 12, and that 12 is a multiple of both 2 and 6. The number 12 has other factors: 1, 3, 4, and 12 itself.

Factors and multiples were studied in earlier grades. The focus here is on **common factors** and **common multiples** of two whole numbers. For example, 4 is a factor of 8 and a factor of 20, so 4 is a common factor of 8 and 20. 80 is a multiple of 8 and a multiple of 20, so 80 is a common multiple of those two numbers.

One way to find the common factors of two numbers is to list all of the factors for each number and see which factors they have in common. Sometimes we want to find the *greatest* common factor. To find the greatest common factor of 18 and 24, we first list all the factors of each number and look for the greatest one they have in common.

- Factors of 18: **1**, **2**, **3**, **6**, 9,18
- Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

To find the common multiples of two numbers, we can do the same. Sometimes we want to find the *least* common multiple. Let's find the least common multiple of 18 and 24.

- Multiples of 18: 18, 36, 54, **72**, 90, 108, 126, **144**, . . .
- Multiples of 24: 24, 48, **72**, 96, 120, **144**, 168, 192, . . .

The first two common multiples are 72 and 144. We can see that 72 is the least common multiple.

Here is a task to try with your student:

A cook is making cheese sandwiches to sell. A loaf of bread can make 10 sandwiches. A package of cheese can make 15 sandwiches. How many loaves of bread and how many packages of cheese should the cook buy so that he can make cheese sandwiches without having any bread or any cheese left over?

Solution:

If he is using up the entire loaf of bread, then the number of sandwiches he can make will be a multiple of 10: 10, 20, **30**, 40, 50, **60**, 70, 80, **90**, 100, . . .

If he is using up all of the cheese in each package, then the number of sandwiches he can make will be a multiple of 15: 15, **30**, 45, **60**, 75, **90**, 105, . . .

30, 60, and 90 are some of the common multiples.

- To make 30 sandwiches, he will need 3 loaves of bread ($3 \cdot 10 = 30$) and 2 packages of cheese ($2 \cdot 15 = 30$).
- To make 60 sandwiches, he will need 6 loaves of bread and 4 packages of cheese.
- To make 90 sandwiches, he will need 9 loaves of bread and 6 packages of cheese.

There are other solutions as well! If he wants to buy the fewest number of loaves and cheese packages, then the first solution is the least.

Family Support Materials

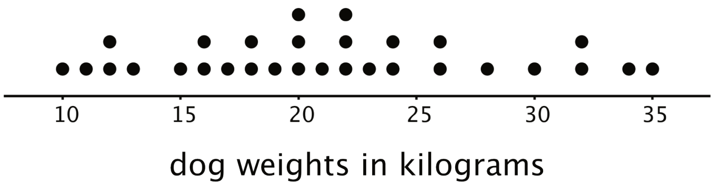
Data Sets and Distributions

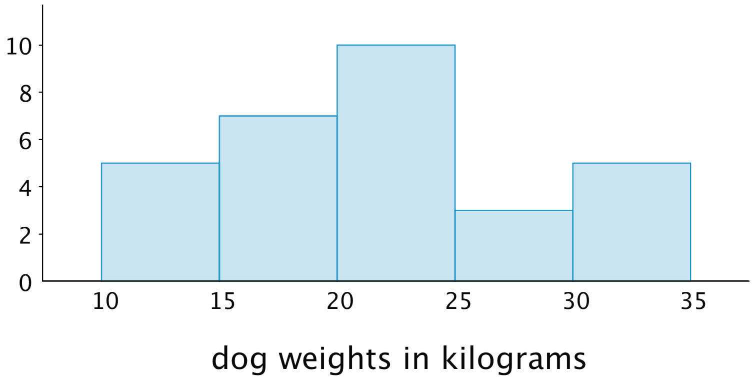
Data, Variability, and Statistical Questions

Family Support Materials 1

This week, your student will work with data and use data to answer **statistical questions**. Questions such as “Which band is the most popular among students in sixth grade?” or “What is the most common number of siblings among students in sixth grade?” are statistical questions. They can be answered using data, and the data are expected to vary (i.e. the students do not all have the same musical preference or the same number of siblings).

Students have used bar graphs and line plots, or **dot plots**, to display and interpret data. Now they learn to use **histograms** to make sense of numerical data. The following dot plot and histogram display the distribution of the weights of 30 dogs.



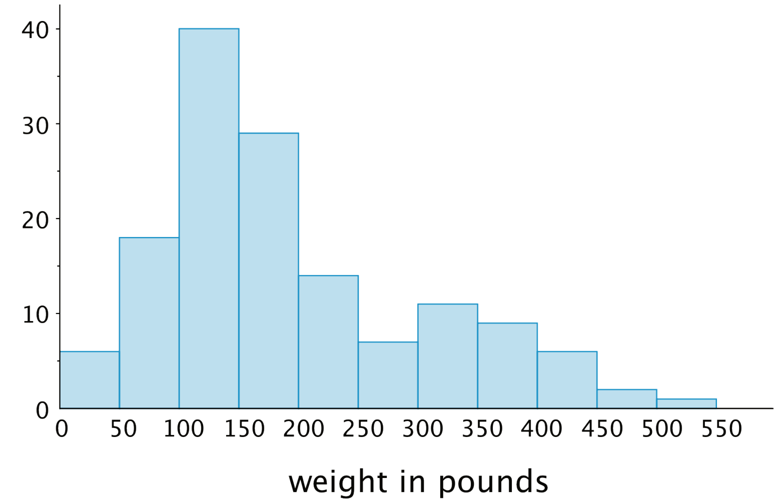


A dot plot shows individual data values as points. In a histogram, the data values are grouped. Each group is represented as a vertical bar. The height of the bar shows how many values are in that group. The tallest bar in this histogram shows that there are 10 dogs that weigh between 20 and 25 kilograms.

The shape of a histogram can tell us about how the data are distributed. For example, we can see that more than half of the dogs weigh less than 25 kilograms, and that a dog weighing between 25 and 30 kilograms is not typical.

Here is a task to try with your student:

This histogram shows the weights of 143 bears.



1. About how many bears weigh between 100 and 150 pounds?

2. About how many bears weigh less than 100 pounds?

3. Noah says that because almost all the bears weigh between 0 and 500 pounds, we can say that a weight of 250 pounds is typical for the bears in this group. Using the histogram, explain why this is incorrect.

Solution:

1. About 40 bears. This is the height of the tallest bar of the histogram.

2. About 24 bears. The two leftmost bars represent the bears that weigh less than 100 pounds. Add the heights of these two bars.

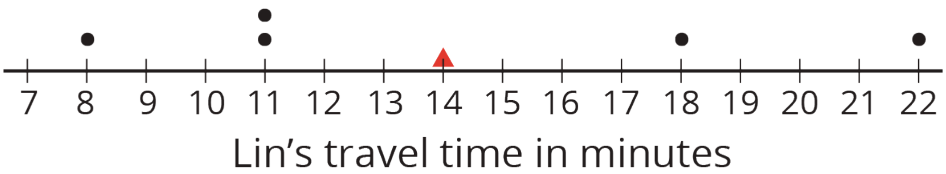
3. We can visually tell from the histogram that most bears weigh less than 250 pounds: the bars to the left of 250 are taller than those to the right. If we add the heights of bars, fewer than 40 bears weigh more than 250 pounds, while over 100 bears weigh less than 250 pounds, so it is not accurate to say that 250 pounds is a typical weight.

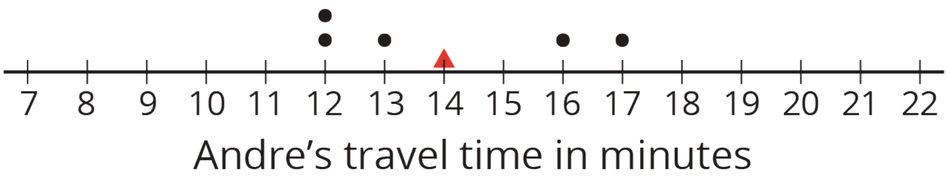
Measures of Center and Variability

Family Support Materials 2

This week, your student will learn to calculate and interpret the **mean**, or the average, of a data set. We can think of the mean of a data set as a fair share—what would happen if the numbers in the data set were distributed evenly. Suppose a runner ran 3, 4, 3, 1, and 5 miles over five days. If the total number of miles she ran, 16 miles, was distributed evenly across five days, the distance run per day, 3.2 miles, would be the mean. To calculate the mean, we can add the data values and then divide the sum by how many there are.

If we think of data points as weights along a number line, the mean can also be interpreted as the balance point of the data. The dots show the travel times, in minutes, of Lin and Andre. The triangles show each mean travel time. Notice that the data points are "balanced" on either side of each triangle.





Your student will also learn to find and interpret the **mean absolute deviation** or the **MAD** of data. The MAD tells you the distance, on average. of a data point from the mean. When the data points are close to the mean, the distances between them and the mean are small, so the average distance—the MAD—will also be small. When data points are more spread out, the MAD will be greater.

We use mean and MAD values to help us summarize data. The mean is a way to describe the center of a data set. The MAD is a way to describe how spread out the data set is.

Here is a task to try with your student:

1. Use the data on Lin's and Andre's dot plots to verify that the mean travel time for each student is 14 minutes.

2. Andre says that the mean for his data should be 13 minutes, because there are two numbers to the left of 13 and two to the right. Explain why 13 minutes cannot be the mean.

3. Which data set, Lin's or Andre's, has a higher MAD (mean absolute deviation)? Explain how you know.

Solution:

1. For Lin's data, the mean is $\frac{8+11+11+18+22}{5} = \frac{70}{5}$, which equals 14. For Andre's data, the mean is $\frac{12+12+13+16+17}{5} = \frac{70}{5}$, which also equals 14.

2. Explanations vary. Sample explanations:

    - The mean cannot be 13 minutes because it does not represent a fair share.

    - The mean cannot be 13 minutes because the data would be unbalanced. The two data values to the right of 13 (16 and 17) are much further away from the two that are to the left (12 and 12).

3. Lin's data has a higher MAD. Explanations vary. Sample explanations:

    - In Lin's data, the points are 6, 3, 3, 4, and 8 units away from the mean of 14. In Andre's data, the points are 2, 2, 1, 2, and 3 units away from the mean of 14. The average distance of Lin's data will be higher because those distances are greater.

    - The MAD of Lin's data is 4.8 minutes, and the MAD of Andre's data is 2 minutes.

    - Compared to Andre's data points, Lin's data points are farther away from the mean.

Median and IQR

Family Support Materials 3

This week, your student will learn to use the **median** and **interquartile range** or **IQR** to summarize the distribution of data.

The median is the middle value of a data set whose values are listed in order. To find the median, arrange the data in order from least to greatest, and look at the middle of the list.

Suppose nine students reported the following numbers of hours of sleep on a weeknight.

6	7	7	8	9	9	10	11	12

The middle number in 9, so the median number of hours of sleep is 9 hours. This means that half of the students slept for less than or equal to 9 hours, and the other half slept for greater than or equal to 9 hours.

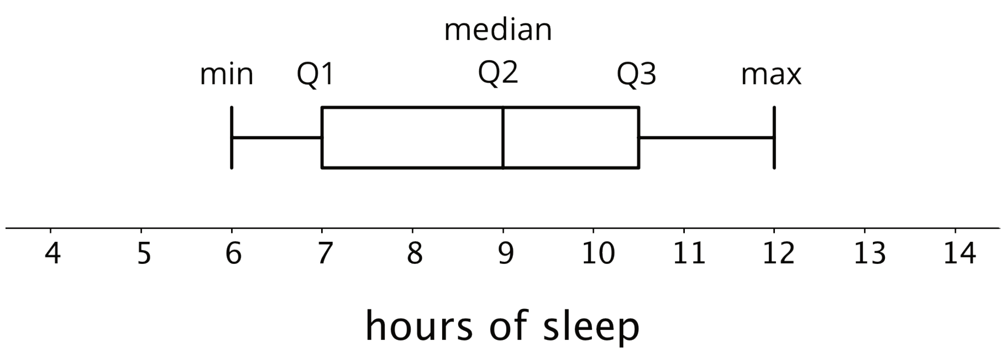
Suppose eight teachers reported these numbers of hours of sleep on a weeknight.

5	6	6	6	7	7	7	8

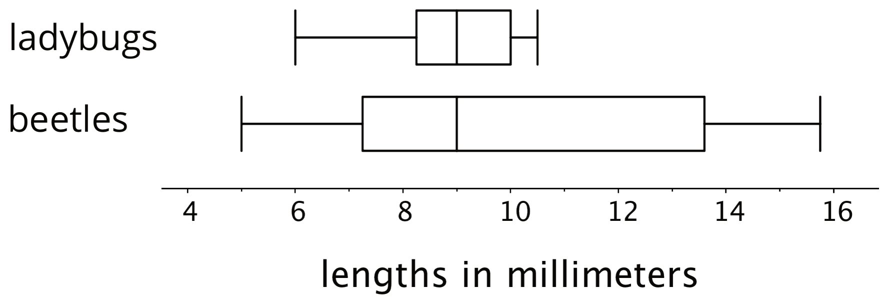
This data set has an even number of values, so there are two numbers in the middle—6 and 7. The median is the number exactly in between them: 6.5. In other words, if there are two numbers in the middle of a data set, the median is the average of those two numbers.

The median marks the 50th percentile of sorted data. It breaks a data set into two halves. Each half can be further broken down into two parts so that we can see the 25th and 75th percentiles. The 25th, 50th, and 75th percentiles are called the first, second, and third **quartiles** (or Q1, Q2, and Q3).

A **box plot** is a way to represent the three quartiles of a data set, along with its maximum and minimum. This box plot shows those five numbers for the data on the students' hours of sleep.



The distance between the first and third quartiles is the **interquartile range** or the **IQR** of data. It tells us about the middle half of the data and is represented by the "width" of the box of the box plot. We can use it to describe how alike or different the data values are. Box plots are especially useful for comparing the distributions of two or more data sets.



The box plots show that the smallest measured beetle is 5 millimeters long, and that half of the beetles are between approximately 7 and 14 millimeters long.

Here is a task to try with your student:

1. Look at the box plots for the ladybugs and beetles.
    a. Which group has a greater IQR: ladybugs or beetles? Explain how you know.

    b. Which group shows more variation in lengths: ladybugs or beetles? Explain how you know.

2. Here is data showing the number of points Jada scored in 10 basketball games.

    10 14 6 12 38 12 8 7 10 23

    What is her median score?

Solution:

1. a. Beetles have a greater IQR. For ladybugs, the IQR (the distance from the first quartile to the third quartile) is about 1.7 millimeters. For beetles, the IQR is about 6.3 millimeters.

   b. Beetles show more variation in lengths. Ladybugs are much more alike in their lengths. The IQR for ladybugs is a smaller number and the box in the plot is narrower, which mean that their lengths are fairly close to one another.

2. 11 points. First, sort the data: 6, 7, 8, 10, 10, 12, 12, 14, 23, 38. Then look at the middle of the list: the numbers 10 and 12 are the fifth and sixth numbers in the list. The median is the average of these numbers: $\frac{10+12}{2} = 11$.