Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

3rd Grade – CCSS for Mathematics

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CCSS Where to Focus Grade 3 Mathematics

Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 3

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: Major Clusters   Supporting Clusters   Additional Clusters

3.OA.A	Represent and solve problems involving multiplication and division.
3.OA.B	Understand properties of multiplication and the relationship between multiplication and division.
3.OA.C	Multiply and divide within 100.
3.OA.D	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.NBT.A	Use place value understanding and properties of operations to perform multi-digit arithmetic.
3.NF.A	Develop understanding of fractions as numbers.
3.MD.A	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.B	Represent and interpret data.
3.MD.C	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.D	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.G.A	Reason with shapes and their attributes.

REQUIRED FLUENCIES FOR GRADE 3

3.OA.C.7	Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2	Add/subtract within 1000

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

3rd Grade – CCSS for Mathematics

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3.OA.A Represent and solve problems involving multiplication and division

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Essential Skills and Concepts:
- ☐ Multiplication
- ☐ Grouping
- ☐ Interpreting products
- ☐ Skip counting
- ☐ Rows and columns
- ☐ Arrays

Question Stems and Prompts:
- ✓ If you have 3 rows and there is 6 in each row, how many do you have?
- ✓ How many do you have when you have __ rows and ___ in each row?
- ✓ A group of ____students collected a total of ___pages of a notebook for recycling. If they each collected the same amount, how many pages did each student collect?

Vocabulary	**Spanish Cognates**
Tier 2	
• rows	
• columns	columnas
• product	producto

Standards Connections
3.OA.1 → 3.OA.2, 3.OA.3, 3.OA.5
3.OA.1 – 3.OA.6

3rd Grade – CCSS for Mathematics

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3.OA.A Represent and solve problems involving multiplication and division.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Essential Skills and Concepts:

- ☐ Partition equally in to shares
- ☐ Division
- ☐ Quotients
- ☐ Decomposing a number

Question Stems and Prompts:

- ✓ If you have ___ objects and ____ baskets. How many would each basket receive if the objects were share equally?
- ✓ Which multiplication fact can help you with this division problem?

Vocabulary	**Spanish Cognates**
Tier 2	
• partition	
• shares	
Tier 3	
• quotients	cociente
• division	división

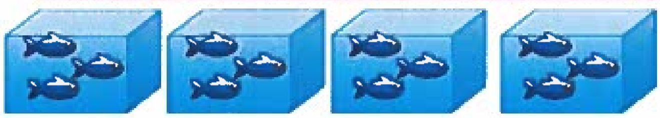
Standards Connections

3.OA.2 → 3.OA.3, 3.OA.5

Illustrative Tasks:

- **Fish Tanks,** **https://www.illustrativemathematics.org/illustrations/1531**

Markers in Boxes, **https://www.illustrativemathematics.org/illustrations/1540**



Suppose there are 4 tanks and 3 fish in each tank. The total number of fish in this situation can be expressed as $4 \times 3 = 12$.

a. Describe what is meant in this situation by $12 \div 3 = 4$

b. Describe what is meant in this situation by $12 \div 4 = 3$

- Presley has 18 markers. Her teacher gives her three boxes and asks her to put an equal number of markers in each box.
- Anthony has 18 markers. His teacher wants him to put 3 markers in each box until he is out of markers.

a. Before you figure out what the students should do, answer these questions:

What is happening in these two situations? How are they similar? How are they different?

b. Figure out how many markers Presley should put in each box. Show your work. Then figure out how many boxes Anthony should fill with markers. Show your work.

3^{rd} Grade – *CCSS for Mathematics*

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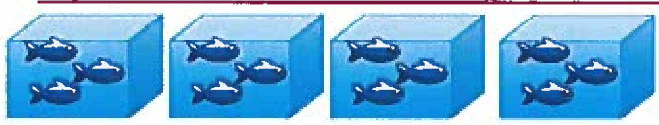
Markers in Boxes, **https://www.illustrativemathematics.org/illustrations/154**



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Standard Explanation
Students recognize multiplication as finding the total number of objects in a certain number of equal-sized groups (3.OA.1▲). Also, students recognize division in two different situations—partitive (or fair-share) division, which requires equal sharing (e.g., how many are in each group?), and quotitive (or measurement division), which requires determining how many groups (e.g., how many groups can you make?) (3.OA.2▲).

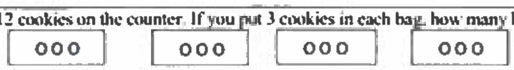
These two interpretations of division have important uses later when studying division of fractions, and both should be explored as representations of division. In grade three teachers should use the terms "number of shares" or "number of groups" with students rather than "partitive" or "quotitive" (CA *Mathematics Framework*, adopted Nov. 6, 2013).

3.OA.2 Examples:

Partitive Division (also known as Fair-Share or Group Size Unknown Division)	3.OA.2▲
The number of groups or shares to be made is known, but the number of objects in (or size of) each group or share is unknown. *Example:* There are 12 apples on the counter. If you are sharing the apples equally among 3 bags, how many apples will go in each bag?	
Quotitive Division (also known as Measurement or Number of Groups Unknown Division)	**3.OA.2▲**
The number of objects in (or size of) each group or share is known, but the number of groups or shares is unknown. *Example:* There are 12 apples on the counter. If you put 3 apples in each bag, how many bags will you fill?	

Partition model example
There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?

Measurement model example
There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?



3rd Grade – CCSS for Mathematics

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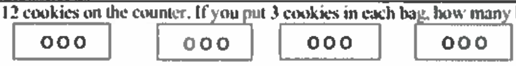

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3.OA.A Represent and solve problems involving multiplication and division

3. OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Essential Skills and Concepts:
- ☐ Multiplication
- ☐ Division
- ☐ Equal groups
- ☐ Arrays
- ☐ Symbol representation

Question Stems and Prompts:
- ✓ You have ____ objects. You put ____objects into each row. How many rows will you make?
- ✓ Aggie plays ____songs in his car. Each song takes ____ minutes to play. How long did it take Aggie to listen to all the songs?
- ✓ Separate the____ objects into _____even groups

Vocabulary

Vocabulary	Spanish Cognates
Tier 2	
• symbol	símbolo
• unknown	
Tier 3	
• array	

Standards Connections

3.OA.3 → 3.OA.8
3.OA.3 – 3.OA.4

Illustrative Tasks:
- Two Interpretations of Division, https://www.illustrativemathematics.org/illustrations/344

a. Maria cuts 12 feet of ribbon into 3 equal pieces so she can share it with her two sisters. How long is each piece?

b. Maria has 12 feet of ribbon and wants to wrap some gifts. Each gift needs 3 feet of ribbon. How many gifts can she wrap using the ribbon?

- Gifts from Grandma, Variation, https://www.illustrativemathematics.org/illustrations/262

a. Juanita spent $9 on each of her 6 grandchildren at the fair. How much money did she spend?

b. Nita bought some games for her grandchildren for $8 each. If she spent a total of $48, how many games did Nita buy?

c. Helen spent an equal amount of money on each of her 7 grandchildren at the fair. If she spent a total of $42, how much did each grandchild get?

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Vocabulary / Spanish Cognates
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Standard Explanation
Students are exposed to related terminology for multiplication (factor and product) and division (quotient, dividend, divisor, and factor). They use multiplication and division within 100 to solve word problems (3.OA.3▲) in situations involving equal groups, arrays and measurement quantities. Note that while "repeated addition" can be used as a strategy for finding whole number products in some cases, repeated addition should not be misconstrued as the meaning of multiplication. The intention of the standards in grade three is to move students beyond additive thinking to multiplicative thinking.

The three major common types of multiplication and division word problems are summarized in the following table (CA *Mathematics Framework*, adopted Nov. 6, 2013).

	Unknown Product	Group Size Unknown (Partitive or Fair Share Division)	Number of Groups Unknown (Quotitive or Measurement Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement Example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 plums to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area Example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement Example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement Example.* A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement Example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

3.OA.3 Examples:

Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24-4= 20	20-4= 16	16-4= 12	12-4= 8	8-4= 4	4-4= 0

3.OA.A Represent and solve problems involving multiplication and division

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3.OA.A Represent and solve problems involving multiplication and division.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$.

Essential Skills and Concepts:

- ☐ Multiplication
- ☐ Division
- ☐ Finding unknown numbers

Question Stems and Prompts:

- ✓ 7 times what makes 35?
- ✓ ___ divided by 5 equals 5?
- ✓ 44 divided by what number has a product of 11

Vocabulary	**Spanish Cognates**
Tier 2	
• Unknown	
Tier 3	
• Division	división

Standards Connections

3.OA.1 → 3.OA.2, 3.OA.3, 3.OA.5
3.OA.1 – 3.OA.6

3.OA.4 Example:

Example: (Number of Groups Unknown): Molly the zookeeper has 24 bananas to feed the monkeys. Each monkey needs to eat 4 bananas. How many monkeys can Molly feed?
Solution: ($? \times 4 = 24$) The student might simply draw on the remembered product $6 \times 4 = 24$ to say that the related quotient is 6. Alternatively, the student might draw on other known products—for example, if $5 \times 4 = 20$ is known, then since $20 + 4 = 24$, the student can reason that one more group of 4 will give the desired factor ($5 + 1 = 6$). Or, knowing that $3 \times 4 = 12$ and $12 + 12 = 24$, the student might reason that the desired factor is $3 + 3 = 6$. Any of these methods (or others) might be supported by an abstract drawing that shows the equal groups in the situation.

Illustrative Task:

- Finding the Unknown in a Division Equation, https://www.illustrativemathematics.org/illustrations/1814

Tehya and Kenneth are trying to figure out which number could be placed in the box to make this equation true.

Tehya insists that 12 is the only number that will make this equation true.

Kenneth insists that 3 is the only number that will make this equation true.

$$2 = \square \div 6$$

Who is right? Why? Draw a picture to support your idea.

3rd Grade – CCSS for Mathematics

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Molly the zookeeper has 24 bananas to feed the monkeys. Each monkey needs to eat 4 bananas. How many monkeys can Molly feed?
Solution: ($? \times 4 = 24$)
The student might simply draw on the remembered product $6 \times 4 = 24$ to say that the related quotient is 6. Alternatively, the student might draw on other known products—for example, if $5 \times 4 = 20$ is known, then since $20 + 4 = 24$, the student can reason that one more group of 4 will give the desired factor ($5 + 1 = 6$). Or, knowing that $3 \times 4 = 12$ and $12 + 12 = 24$, the student might reason that the desired factor is $3 + 3 = 6$. Any of these methods (or others) might be supported by an abstract drawing that shows the equal groups in the situation.

Illustrative Task:

- Finding the Unknown in a Division Equation, https://www.illustrativemathematics.org/illustrations/18

Tehya and Kenneth are trying to figure out which number could be placed in the box to make this equation true.

Tehya insists that 12 is the only number that will make this equation true.

Kenneth insists that 3 is the only number that will make this equation true.

$$2 = \square \div 6$$

Who is right? Why? Draw a picture to support your idea.

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3.OA.A Represent and solve problems involving multiplication and division.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$.*

Standard Explanation

In grade three, students focus on equal groups and array problems. Compare problems will be introduced in grade four. The more difficult problem structures include “Group Size Unknown” ($3 \times ? = 18$ or $18 \div 3 = 6$) or “Number of Groups Unknown” ($? \times 6 = 18$, $18 \div 6 = 3$). To solve problems, students determine the unknown whole number in a multiplication or division equation relating three whole numbers (3.OA.4▲). Students use numbers, words, pictures, physical objects, or equations to represent problems, explain their thinking, and show their work. (MP.1, MP.2, MP.4, MP.5) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

	Unknown Product	Group Size Unknown (Partitive or Fair Share Division)	Number of Groups Unknown (Quotitive or Measurement Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement Example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 plums to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area Example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement Example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement Example.* A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement Example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

3.OA.4 Examples:

When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Example: Solve the equations below:

24 = ? x 6 72 ÷ = 9 Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = m

3rd Grade – CCSS for Mathematics

3.OA.A Represent and solve problems involving multiplication and division.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$.

Standard Explanation

In grade three, students focus on equal groups and array problems. Compare problems will be introduced in grade four. The more difficult problem structures include “Group Size Unknown” ($3 \times ? = 18$ or $18 \div 3 = 6$) or “Number of Groups Unknown” ($? \times 6 = 18$, $18 \div 6 = 3$). To solve problems, students determine the unknown whole number in a multiplication or division equation relating three whole numbers (3.OA.4▲). Students use numbers, words, pictures, physical objects, or equations to represent problems, explain their thinking, and show their work. (MP.1, MP.2, MP.4, MP.5) (CA Mathematics Framework, adopted Nov. 6, 2013).

3.OA.4 Examples:

When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
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- I know that 4 groups of 10 is 40 so the unknown number is 10
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Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Example: Solve the equations below:

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3.OA.B Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Essential Skills and Concepts:
- ☐ Multiplication
- ☐ Division
- ☐ Commutative property
- ☐ Associative property
- ☐ Distributive property

Question Stems and Prompts:
- ✓ Which property is being shown here?
- ✓ Which property must you follow to answer this question?
- ✓ This problem is a form of what property?
- ✓ What property is this an example of?
- ✓ 6x7=7x?
- ✓ 5x4 is the same as?

Vocabulary	**Spanish Cognates**
Tier 3	
• commutative property	propiedad conmutativa
• associative property	propiedad asociativa
• distributive property	propiedad distributiva

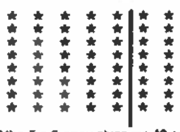
Standards Connections

3.OA.5 → 3.NBT.3, 3.OA.7, 3.OA.9

3.OA.5 – 3.MD.7c

3.OA.5 Example:

(Adapted from Arizona 2010)

Example: Students can use the distributive property to discover new products of whole numbers (such as 7 × 8) based on products they can find more easily.	
Strategy 1: By creating an array, I want to find how many total stars there are in 7 columns of 8 stars.	**Strategy 2:** By creating an array, I want to find how many total stars there are in 7 rows of 8 stars.
	
I see that I can arrange the 7 columns into a group of 5 rows and a group of 2 columns.	I see that I can arrange the 8 up-down rows of stars into two groups of 4 rows.
	
I know that the 5 × 8 array gives me 40 and the 2 × 8 array gives me 16. So altogether I have 5 × 8 + 2 × 8 = 40 + 16 = 56 stars.	I know that each new 4 × 7 array gives me 28 stars, and so altogether I have 4 × 7 + 4 × 7 = 28 + 28 = 56 stars.

[2] Students need not use formal terms for these properties.

3rd Grade – CCSS for Mathematics

3.OA.B Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

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- ☐ Multiplication
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- ☐ Associative property
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Vocabulary	**Spanish Cognates**
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• commutative property	propiedad conmutativa
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I see that I can arrange the 7 columns into a group of 5 rows and a group of 2 columns.	I see that I can arrange the 8 up-down rows of stars into two groups of 4 rows.
	
I know that the 5 × 8 array gives me 40 and the 2 × 8 array gives me 16. So altogether I have 5 × 8 + 2 × 8 = 40 + 16 = 56 stars.	I know that each new 4 × 7 array gives me 28 stars, and so altogether I have 4 × 7 + 4 × 7 = 28 + 28 = 56 stars.

[2] Students need not use formal terms for these properties.

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3.OA.B.5

Standard Explanation
In grade three, students apply properties of operations as strategies to multiply and divide (3.OA.5▲). At third grade students do not need to use the formal terms for these properties. Students use increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn about the relationship between multiplication and division.

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers, which appears in grade five. Third grade students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they do not know. (MP.2, MP.7) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Focus, Coherence, and Rigor

Arrays can be seen as equal-sized groups where objects are arranged by rows and columns, and they form a major transition to understanding multiplication as finding area (connection to 3.MD.7▲). For example, students can analyze the structure of multiplication and division (MP.7) through their work with arrays (MP.2) and work toward precisely expressing their understanding of the connections between area and multiplication (MP.6).

Illustrative Task:

- Valid Equalities (Part 2), https://www.illustrativemathematics.org/illustrations/1821

Decide if the equations are true or false. Explain your answer.

a. 4 x 5 = 20
b. 34 = 7 x 5
c. 3 x 6 = 9 x 2
d. 5 x 8 = 10 x 4
e. 6 x 9 = 5 x 10
f. 2 x (3 x 4) = 8 x 3
g. 8 x 6 = 7 x 6 + 6
h. 4 x (10 + 2) = 40 + 2

3rd Grade – CCSS for Mathematics

3.OA.B.5

Standard Explanation
In grade three, students apply properties of operations as strategies to multiply and divide (3.OA.5▲). At third grade students do not need to use the formal terms for these properties. Students use increasingly sophisticated strategie based on these properties to solve multiplication and divisi problems involving single-digit factors. By comparing a variety of solution strategies, students learn about the relationship between multiplication and division.

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h. 4 x (10 + 2) = 40 + 2

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3.OA.B Understand properties of multiplication and the relationship between multiplication and division.

3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Essential Skills and Concepts:
- ☐ Division
- ☐ Multiplication
- ☐ Inverse operations

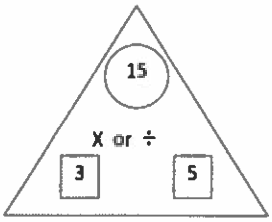
Question Stems and Prompts:
- ✓ What is the unknown-factor in the question 45 divided by 9?
- ✓ What number do you multiply by 9 to get 45?
- ✓ If 7x8 is 56, what is 56 divided by 8 ?
- ✓ Find the unknown-factor.

Vocabulary	**Spanish Cognates**
Tier 3	
• unknown factor	
• inverse operations	operaciones inversas

Standards Connections
3.OA.6 → 3.OA.7
3.OA.6 – 3.OA.1, 2, 3

3.OA.6 Examples:
A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning. Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:
- 3 x 5 = 15 5 x 3 = 15
- 15 ÷ 3 = 5 15 ÷ 5 = 3



3rd Grade – CCSS for Mathematics

3.OA.B Understand properties of multiplication and t relationship between multiplication and division.

3.OA.6 Understand division as an unknown-factor proble For example, find 32 ÷ 8 by finding the number that makes when multiplied by 8.

Essential Skills and Concepts:
☐ Division
☐ Multiplication
☐ Inverse operations

Question Stems and Prompts:
✓ What is the unknown-factor in the question 45 divided 9?
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✓ If 7x8 is 56, what is 56 divided by 8 ?
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Vocabulary Spanish Cognates
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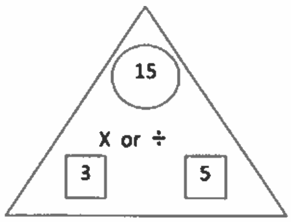

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3.OA.B.6

Standard Explanation

The connection between multiplication and division should be introduced early in the year. Students understand division as an unknown-factor problem (3.OA.6▲). For example, find 15 ÷ 3 by finding the number that makes 15 when multiplied by 3. Multiplication and division are inverse operations and students use this inverse relationship to compute and check results. Below are some general strategies that can be used to develop multiplication and division facts in grade three (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Strategies for learning multiplication facts include:

Patterns

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s); Doubling three times (8s)
- Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
- Five facts (knowing the five facts are half of the tens facts)

General Strategies

- "Count bys" (counting groups of __ and knowing how many groups have been counted). For example, students count by twos keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and can start within it.
- Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
- The principle of "Turn-around facts" (based on the Commutative Property – knowing 2 x 7 is the same as 7 x 2 reduces the total number of facts to memorize)

Other Strategies

- Square numbers (e.g., 6 x 6)
- Nines (e.g., understanding this is 10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3, or knowing 9 times a number results in a tens place that is one below the number and that the two digits in the tens and ones place will add to 9 -- 9 x 6 is 5 in the tens place and 4 in the ones place, which equal a sum of 9)

Strategies for learning division facts include:

- Unknown factors. Students can state a division problem as a unknown factor problem (e.g., 24÷ 4 = ? becomes 4 × ? = 24). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplications and divisions involving a particular number can be helpful.
- Related facts (e.g., 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24). Students know 6 × 4 = 24 and 4 × 6 = 24; and 4 × ? = 24 and 6 × ? = 24 are related facts).

(Adapted from Arizona 2010)

3.OA.6 Examples:

Example:

Sarah did not know the answer to 63 divided by 7. Are each of the following was an appropriate way for Sarah to think about the problem? Explain why or why not with a picture or words for each one.

- "I know that 7 x 9 = 63, so 63 divided by 7 must be 9."
- "I know that 7x10 = 70. If I take away a group of 7, that means that I have 7x9 = 63. So 63 divided by 7 is 9."
- "I know that 7x5 is 35. 63 minus 35 is 28. I know that 7x4 = 28. So if I add 7x5 and 7x4 I get 63. That means that 7x9 is 63, or 63 divided by 7 is 9."

3[rd] *Grade – CCSS for Mathematics*

3.OA.B.6

Standard Explanation

The connection between multiplication and division should be introduced early in the year. Students understand divisio as an unknown-factor problem (3.OA.6▲). For example, find 15 ÷ 3 by finding the number that makes 15 when multiplied by 3. Multiplication and division are inverse operations and students use this inverse relationship to compute and check results. Below are some general strategies that can be used to develop multiplication and division facts in grade three (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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General Strategies

- "Count bys" (counting groups of __ and knowing how many groups have been counted). For example, students count by twos keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and can start within it.
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Strategies for learning division facts include:

- Unknown factors. Students can state a division problem as a unknown factor problem (e.g., 24÷ 4 = ? becomes 4 × ? = 24). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplications and divisions involving a particular number can be helpful.
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3.OA.C Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Essential Skills and Concepts:
- ☐ Inverse operations
- ☐ Multiply fluently
- ☐ Divide fluently

Question Stems and Prompts:
- ✓ What is 7 x 8? 8 x 7?
- ✓ What is 56 divided by 8? 56 divided by 7?

Vocabulary	**Spanish Cognates**
Tier 2	
• product	producto
• fluently	
Tier 3	
• Inverse operations	operacion inversa

Standards Connections
3.OA.7 – 3.OA.4, 3.OA.8

3rd Grade – CCSS for Mathematics

3.OA.C Multiply and divide within 100.

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Essential Skills and Concepts:
- ☐ Inverse operations
- ☐ Multiply fluently
- ☐ Divide fluently

Question Stems and Prompts:
- ✓ What is 7 x 8? 8 x 7?
- ✓ What is 56 divided by 8? 56 divided by 7?

Vocabulary / **Spanish Cognates**
Tier 2
• product — producto
• fluently
Tier 3
• Inverse operations — operacion inversa

Standards Connections
3.OA.7 – 3.OA.4, 3.OA.8

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3.OA.C.7

Standard Explanation

Students in grade three use various strategies to fluently multiply and divide within 100 (3.OA.7 ▲). The following are some general strategies that can be used to help students know from memory all products of two one-digit numbers.

Multiplication and division are new concepts in grade three, and reaching fluency with these operations within 100 represents a major portion of students' work. By the end of grade three, students also know all products of two one-digit numbers from memory (3.OA.7 ▲). Organizing practice to focus most heavily on products and unknown factors that are understood but not yet fluent in students can speed learning and support fluency with multiplication and division facts. Practice and extra support should continue all year for those who need it to attain fluency (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Strategies for Learning Multiplication Facts 3.OA.7▲

Patterns

- Multiplication by zeros and ones
- Doubles (twos facts), doubling twice (fours), doubling three times (eights)
- Tens facts (relating to place value, 5×10 is 5 tens, or 50)
- Fives facts (knowing the fives facts are half of the tens facts)

General Strategies

- Use skip-counting (counting groups of specific numbers and knowing how many groups have been counted). For example, students count by twos, keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and are able to start within it.
- Decompose into known facts (e.g., 6×7 is 6×6 plus one more group of 6).
- Use "turn-around facts" (based on the commutative property—for example, knowing that 2×7 is the same as 7×2 reduces the total number of facts to memorize).

Other Strategies

- Know square numbers (e.g., 6×6).
- Use arithmetic patterns to multiply. Nines facts include several patterns. For example, using the fact that $9 = 10 - 1$, students can use the tens multiplication facts to help solve a nines multiplication problem.
  $9 \times 4 = 9$ fours $= 10$ fours $- 1$ four $= 40 - 4 = 36$
  Students may also see this as:
  $4 \times 9 = 4$ nines $= 4$ tens $- 4$ ones $= 40 - 4 = 36$

Strategies for Learning Division Facts

- Turn the division problem into an unknown-factor problem. Students can state a division problem as an unknown-factor problem (e.g., $24 \div 4 = ?$ becomes $4 \times ? = 24$). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplication and division involving a particular number can be helpful.
- Use related facts (e.g., $6 \times 4 = 24$; $24 \div 6 = 4$; $24 \div 4 = 6$; $4 \times 6 = 24$).

Adapted from ADE 2010.

FLUENCY

California's Common Core State Standards for Mathematics (K–6) set expectations for fluency in computation (e.g., "*Fluently* multiply and divide within 100 . . ." [3.OA.7▲]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word *fluent* is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies.

Adapted from UA Progressions Documents 2011a.

3rd Grade – CCSS for Mathematics

3.OA.C.7

Standard Explanation

Students in grade three use various strategies to fluently multiply and divide within 100 (3.OA.7 ▲). The following are some general strategies that can be used to help students know from memory all products of two one-digit numbers.

Multiplication and division are new concepts in grade three, and reaching fluency with these operations within 100 represents a major portion of students' work. By the end of grade three, students also know all products of two one-digit numbers from memory (3.OA.7 ▲). Organizing practice to focus most heavily on products and unknown factors that are understood but not yet fluent in students can speed learning and support fluency with multiplication and division facts. Practice and extra support should continue all year for those who need it to attain fluency (CA Mathematics Framework, adopted Nov. 6, 2013).

Strategies for Learning Multiplication Facts 3.OA.7▲

Patterns

- Multiplication by zeros and ones
- Doubles (twos facts), doubling twice (fours), doubling three times (eights)
- Tens facts (relating to place value, 5 × 10 is 5 tens, or 50)
- Fives facts (knowing the fives facts are half of the tens facts)

General Strategies

- Use skip-counting (counting groups of specific numbers and knowing how many groups have been counted). For example, students count by twos, keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and are able to start within it.
- Decompose into known facts (e.g., 6 × 7 is 6 × 6 plus one more group of 6).
- Use "turn-around facts" (based on the commutative property—for example, knowing that 2 × 7 is the same as 7 × 2 reduces the total number of facts to memorize).

Other Strategies

- Know square numbers (e.g., 6 × 6).
- Use arithmetic patterns to multiply. Nines facts include several patterns. For example, using the fact that 9 = 10 − 1, students can use the tens multiplication facts to help solve a nines multiplication problem.
  9 × 4 = 9 fours = 10 fours − 1 four = 40 − 4 = 36
  Students may also see this as:
  4 × 9 = 4 nines = 4 tens − 4 ones = 40 − 4 = 36

Strategies for Learning Division Facts

- Turn the division problem into an unknown-factor problem. Students can state a division problem as an unknown-factor problem (e.g., 24 ÷ 4 = ? becomes 4 × ? = 24). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplication and division involving a particular number can be helpful.
- Use related facts (e.g., 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24).

Adapted from ADE 2010.

FLUENCY

California's Common Core State Standards for Mathematics (K–6) set expectations for fluency in computation (e.g., "Fluently multiply and divide within 100 . . ." [3.OA.7▲]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word fluent is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies.

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3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Essential Skills and Concepts:

- ☐ Two-step problem word problems
- ☐ Knowing the four operations
- ☐ Letter representation for unknown
- ☐ Mental math
- ☐ Estimation skills
- ☐ Rounding

Question Stems and Prompts:

✓ ______

Vocabulary	**Spanish Cognates**
Tier 2	
• operation	operación
• product	producto
• reasonableness	
• property	propiedad
Tier 3	
• multiply	multiplicar
• divide	dividir
• mental computation	

Standards Connections
3.OA.8 – 3.OA.7

Illustrative Tasks:

- The Stamp Collection, https://www.illustrativemathematics.org/illustrations/13

Masha had 120 stamps. First, she gave her sister half of the stamps and then she used three to mail letters. How many stamps does Masha have left?

- The Class Trip, https://www.illustrativemathematics.org/illustrations/1301

Mrs. Moore's third grade class wants to go on a field trip to the science museum.

- The cost of the trip is $245.
- The class can earn money by running the school store for 6 weeks.
- The students can earn $15 each week if they run the store.

a. How much more money does the third grade class still need to earn to pay for their trip?

b. Write an equation to represent this situation.

3rd Grade – CCSS for Mathematics

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✓ ______

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3.OA.D.8

Standard Explanation

Students in third grade begin the step towards formal algebraic language by using a letter for the unknown quantity in expressions or equations when solving one and two-step word problems (3.OA.8▲). Students are not formally solving algebraic equations at this grade level. Students know to perform operations in the conventional order when there are not parentheses to specify a particular order (order of operations). Students use estimation during problem solving and then revisit their estimates to check for reasonableness (CA *Mathematics Framework*, adopted Nov. 6, 2013).

3.OA.8 Examples:

Example 1: Chicken Coop. There are five nests in the chicken coop with 2 eggs in each nest. If the farmer wants 25 eggs, how many more eggs does she need?

Solution: Students might create a picture representation of this situation using a tape-like diagram:

2	2	2	2	2	*m*
25					

Students might solve this by seeing that when the 5 nests with 2 eggs are added up, they have 10 eggs. To make 25 eggs the farmer would need 25 – 10 = 15 more eggs. A simple equation that represents this situation could be $5 \times 2 + m = 25$, where *m* is how many more eggs the farmer needs.

Example 2: Soccer Club. The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Both wristbands are the same price. Find the price of one of the wristbands. Write an equation that represents this situation.

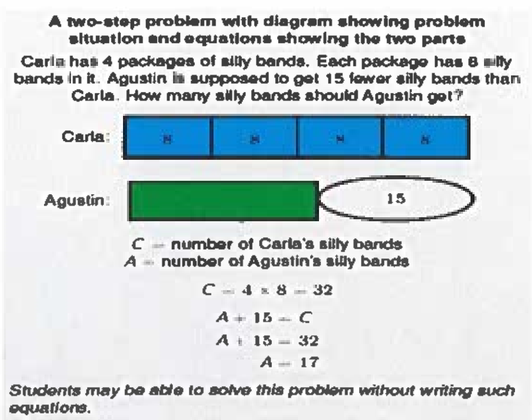
Solution: Students might solve the problem by seeing that the cost of the two tickets must be $63 – $13 = $50.

w	*w*	$13
$63		

Therefore the cost of one of the wristbands must be $50 ÷ 2 = $25. Equations that represents this situation is $w + w + 13 = 63$ or $63 = w + w + 13$.

Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, 3rd Grade Flipbook, and NCDPI 2013b.

Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution (2 x 5 + m = 25).



3^{rd} *Grade – CCSS for Mathematics*

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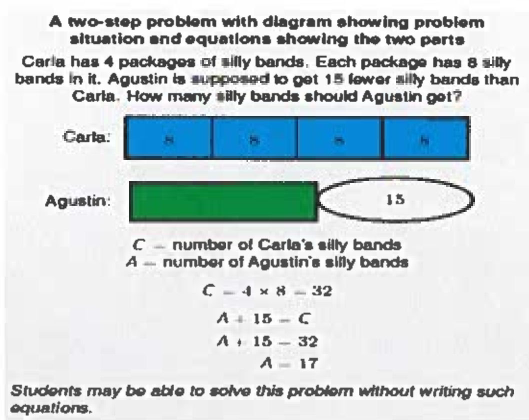

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3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Essential Skills and Concepts:

- ☐ Arithmetic patterns
- ☐ Understand a multiplication/addition table
- ☐ Multiples

Question Stems and Prompts:

- ✓ What do you notice about the numbers highlighted in the multiplication table?
- ✓ What patterns do you notice in this addition table?
- ✓ What patterns do you notice in this multiplication table? Explain why the pattern works this way?

Vocabulary	**Spanish Cognates**
Tier 2	
• decompose	descomponer
• multiples	múltiplos
Tier 3	
• arithmetic patterns	
• properties of operations	propiedades de operaciones

Standards Connections

3.OA.9 → 4.OA.5

Illustrative Tasks:

- Addition Patterns, https://www.illustrativemathematics.org/illustrations/13

Below is a table showing addition of numbers from 1 through 5.

+	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

- Symmetry of the Addition Table, https://www.illustrativemathematics.org/illustrations/954

Below is a table showing how to add numbers from 1 to 3:

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

Cut out the table and fold it over the dotted line. Notice that the blue squares match up and so do the orange squares. Notice that the squares that match up have the same numbers in them. We say that the squares that match up when you fold along the line are "mirror images" of each other.

3rd Grade – CCSS for Mathematics

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Essential Skills and Concepts:
☐ Arithmetic patterns
☐ Understand a multiplication/addition table
☐ Multiples

Question Stems and Prompts:
✓ What do you notice about the numbers highlighted in th multiplication table?
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Vocabulary **Spanish Cognates**
Tier 2
• decompose descomponer
• multiples múltiplos
Tier 3
• arithmetic patterns
• properties of operations propiedades de operacione

Standards Connections
3.OA.9 → 4.OA.5

Illustrative Tasks:
• Addition Patterns, https://www.illustrativemathematics.org/illustrations/1
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• Symmetry of the Addition Table, https://www.illustrativemathematics.org/illustrations/9
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3.OA.D.9

Standard Explanation
In grade three, students identify arithmetic patterns and explain them using properties of operations (3.OA.9▲). Students can investigate addition and multiplication tables in search of patterns (MP.7) and explain or discuss why these patterns make sense mathematically and how they are related to properties of operations (e.g., why is the multiplication table symmetric about its diagonal from the upper left to the lower right?) [MP.3] (CA *Mathematics Framework*, adopted Nov. 6, 2013).

3.OA.9 Examples:
Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.
For Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
20	0	20

3rd Grade – CCSS for Mathematics

3.OA.D.9

Standard Explanation
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3	17	20
4	16	20
20	0	20

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3rd Grade – CCSS for Mathematics

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3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

Essential Skills and Concepts:
- ☐ Rounding
- ☐ Deep understanding of place value

Question Stems and Prompts:
- ✓ Round 567 to the nearest 10? Nearest 100?
- ✓ Which was more appropriate to the nearest ten or to the nearest hundred?

Vocabulary	**Spanish Cognates**
Tier 3	
• place value	
• round	redondo

Standards Connections
3.NBT.1 → 4.NBT.3

Illustrative Tasks:
- Rounding to 50 or 500, https://www.illustrativemathematics.org/illustrations/745

When rounding to the nearest ten:

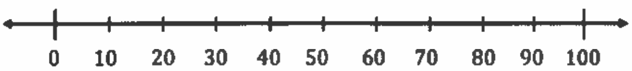
a. What is the smallest whole number that will round to 50?
b. What is the largest whole number that will round to 50?
c. How many different whole numbers will round to 50?

When rounding to the nearest hundred:

d. What is the smallest whole number that will round to 500?
e. What is the largest whole number that will round to 500?
f. How many different whole numbers will round to 500?

- Rounding to the Nearest Ten and Hundred, https://www.illustrativemathematics.org/illustrations/1805

Plot 8, 32, and 79 on the number line.

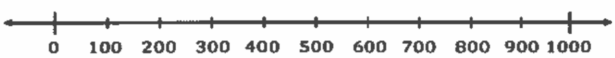


a. Round each number to the nearest 10. How can you see this on the number line?
b. Round each number to the nearest 100. How can you see this on the number line?

- Rounding to the Nearest 100 and 1000 https://www.illustrativemathematics.org/illustrations/1806

Plot the following numbers on the number line:

80
328
791



a. Round each number to the nearest 100. How can you see this on the number line?
b. Round each number to the nearest 1000. How can you see this on the number line?

[4] A range of algorithms may be used.

3rd Grade – CCSS for Mathematics

3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

Essential Skills and Concepts:
- ☐ Rounding
- ☐ Deep understanding of place value

Question Stems and Prompts:
- ✓ Round 567 to the nearest 10? Nearest 100?
- ✓ Which was more appropriate to the nearest ten or to the nearest hundred?

Vocabulary / Spanish Cognates
Tier 3
• place value
• round — redondo

Standards Connections
3.NBT.1 → 4.NBT.3

Illustrative Tasks:
- Rounding to 50 or 500, https://www.illustrativemathematics.org/illustrations/745

When rounding to the nearest ten:

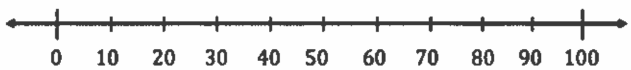
a. What is the smallest whole number that will round to 50?
b. What is the largest whole number that will round to 50?
c. How many different whole numbers will round to 50?

When rounding to the nearest hundred:

d. What is the smallest whole number that will round to 500?
e. What is the largest whole number that will round to 500?
f. How many different whole numbers will round to 500?

- Rounding to the Nearest Ten and Hundred, https://www.illustrativemathematics.org/illustrations/1805

Plot 8, 32, and 79 on the number line.



a. Round each number to the nearest 10. How can you see this on the number line?
b. Round each number to the nearest 100. How can you see this on the number line?

- Rounding to the Nearest 100 and 1000 https://www.illustrativemathematics.org/illustrations/1806

Plot the following numbers on the number line:

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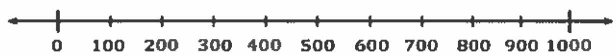

a. Round each number to the nearest 100. How can you see this on the number line?
b. Round each number to the nearest 1000. How can you see this on the number line?

[4] A range of algorithms may be used.

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3.NBT.A.1

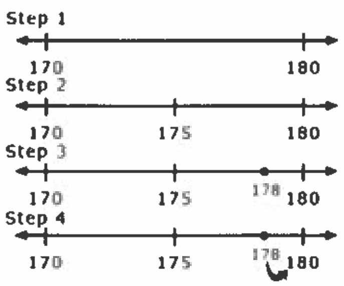
Standard Explanation
In grade three, students are introduced to the concept of rounding whole numbers to the nearest 10 or 100 (3.NBT.1), an important prerequisite for working with estimation problems. Students can use a number line or a hundreds chart as tools to support their work with rounding. They learn when and why to round numbers and extend their understanding of place value to include whole numbers with four digits (CA *Mathematics Framework*, adopted Nov. 6, 2013).

This standard refers to place value understanding, which extends beyond an algorithm or memorized procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Teaching Strategies:
- Using/drawing number lines
- Hundreds charts
- Place value charts

3.NBT.1 Examples:
Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. When she told her husband she estimated how many miles to the nearest 10 before adding the total. When she told her sister she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?

Example: Round 178 to the nearest 10.



Step 1: The answer is either 170 or 180.

Step 2: The halfway point is 175.

Step 3: 178 is between 175 and 180.

Step 4: Therefore, the rounded number is 180.

3rd Grade – CCSS for Mathematics

3.NBT.A.1

Standard Explanation
In grade three, students are introduced to the concept of rounding whole numbers to the nearest 10 or 100 (3.NBT.1), an important prerequisite for working with estimation problems. Students can use a number line or a hundreds chart as tools to support their work with rounding. They learn when and why to round numbers and extend their understanding of place value to include whole numbers with four digits (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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Teaching Strategies:
- Using/drawing number lines
- Hundreds charts
- Place value charts

3.NBT.1 Examples:
Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. When she told her husband she estimated how many miles to the nearest 10 before adding the total. When she told her sister she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?

Example: Round 178 to the nearest 10.

Step 1: The answer is either 170 or 180.

Step 2: The halfway point is 175.

Step 3: 178 is between 175 and 180.

Step 4: Therefore, the rounded number is 180.

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3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Essential Skills and Concepts:
- ☐ Place value
- ☐ Addition/subtraction
- ☐ Addition/subtraction properties

Question Stems and Prompts:
- ✓ How do properties work in subtraction problems?
- ✓ How does knowing associative, commutative, and identity property help us add/subtract numbers efficiently?

Vocabulary | **Spanish Cognates**

Tier 3
- addend

Standards Connections

3.NBT.2 → 4.NBT.4, 5, 6

Illustrative Task:
- Classroom Supplies https://www.illustrativemathematics.org/content-standards/3/NBT/A/2/tasks/1315

Your teacher was just awarded $1,000 to spend on materials for your classroom. She asked all 20 of her students in the class to help her decide how to spend the money. Think about which supplies will benefit the class the most.

Books and maps	
A set of 20 books about science	$250
A set of books about the 50 states	$400
A story book (there are 80 to choose from)	$3
A map: there is one of your city, one for every state, one of the country, and one of the world to choose from	$45

a. Write down the different items and how many of each you would choose. Find the total for each category.
- Supplies
- Books and maps
- Puzzles and games
- Special items

b. Create a bar graph to represent how you would spend the money. Scale the vertical axis by $100. Write all of the labels.

c. What was the total cost of all your choices? Did you have any money left over? If so, how much?

d. Compare your choices with a partner. How much more or less did you choose to spend on each category than your partner? How much more or less did you choose to spend in total than your partner?

[4] A range of algorithms may be used.

3rd Grade – CCSS for Mathematics

3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Essential Skills and Concepts:
☐ Place value
☐ Addition/subtraction
☐ Addition/subtraction properties

Question Stems and Prompts:
✓ How do properties work in subtraction problems?
✓ How does knowing associative, commutative, and identity property help us add/subtract numbers efficiently?

Vocabulary | Spanish Cognates
Tier 3
addend

Standards Connections
3.NBT.2 → 4.NBT.4, 5, 6

Illustrative Task:
Classroom Supplies https://www.illustrativemathematics.org/content-standards/3/NBT/A/2/tasks/1315

Your teacher was just awarded $1,000 to spend on materials for your classroom. She asked all 20 of her students in the class to help her decide how to spend the money. Think about which supplies will benefit the class the most.

Books and maps
A set of 20 books about science $250
A set of books about the 50 states $400
A story book (there are 80 to choose from) $3
A map: there is one of your city, one for every state, one of the country, and one of the world to choose from $45

a. Write down the different items and how many of each you would choose. Find the total for each category.
Supplies
Books and maps
Puzzles and games
Special items

b. Create a bar graph to represent how you would spend the money. Scale the vertical axis by $100. Write all of the labels.

c. What was the total cost of all your choices? Did you have any money left over? If so, how much?

d. Compare your choices with a partner. How much more or less did you choose to spend on each category than your partner? How much more or less did you choose to spend in total than your partner?

4 A range of algorithms may be used.

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3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standard Explanation

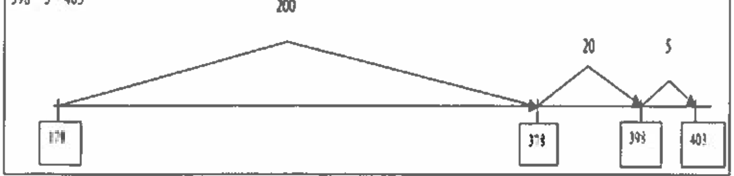
Third-grade students continue to add and subtract within 1000 and achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction (3.NBT.2). They use addition and subtraction methods developed in grade two, where they began to add and subtract within 1000 without the expectation of full fluency and used at least one method that generalizes readily to larger numbers—so this is a relatively small and incremental expectation for third-graders. Such methods continue to be the focus in grade three, and thus the extension at grade four to generalize these methods to larger numbers (up to 1,000,000) should also be relatively easy and rapid (CA *Mathematics Framework*,

3.NBT.2 Examples:

There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1	Student 2	Student 3
100 + 200 = 300 70 + 20 = 90 8 + 5 = 13 300 + 90 + 13 = 403 students	I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.	I know the 75 plus 25 equals 100. I then added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

Student 4
178 + 225 = ?
178 + 200 = 378
378 + 20 = 398
398 + 5 = 403



(North Carolina Department of Public Instruction, 2014)

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3.NBT.A Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Essential Skills and Concepts:
- ☐ Patterns are evident when multiplying a number by ten or a multiple of ten.
- ☐ The distributive property of multiplication allows us to find partial products and then find their sum.

Question Stems and Prompts:
- ✓ How can I model multiplication by ten?
- ✓ How can multiplication be represented?

Vocabulary Tier 3	**Spanish Cognates**
• factor	factor
• associative property of multiplication	propiedad asociativa de multiplicación
• commutative property of multiplication	propiedad conmutativa de multiplicación

Standards Connections
3.NBT.3 → 4.NBT.5

Illustrative Task:
- How Many Colored Pencils?, https://www.illustrativemathematics.org/illustrations/1445

There are 6 tables in Mrs. Potter's art classroom. There are 4 students sitting at each table. Each student has a box of 10 colored pencils.

(A) How many colored pencils are at each table?

(B) How many colored pencils do Mrs. Potter's students have in total?

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3rd Grade – CCSS for Mathematics

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Standard Explanation

Third grade students also multiply one-digit whole numbers by multiples of 10 (3.NBT.3) in the range 10–90, using strategies based on place value and properties of operations (e.g., "I know 5 × 90 = 450 because 5 × 9 = 45 and so 5 × 90 should be ten times as much."). Students also interpret 2 × 40 as 2 groups of 4 tens or 8 groups of ten. They understand 5 × 60 is 5 groups of 6 tens or 30 tens, and they know 30 tens is 300. After developing this understanding students begin to recognize the patterns in multiplying by multiples of 10 (ADE 2010). The ability to multiply one-digit numbers by multiples of 10 can support later student learning of standard algorithms for multiplication of multi-digit numbers (CA *Mathematics Framework*, adopted Nov. 6, 2013).

This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens, and that twenty tens equals 200.

3.NBT.3 Examples:

Grade 3 explanations for "15 tens is 150"

- Skip-counting by 50. 5 tens is 50, 100, 150.
- Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150.
- Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150.
- Decomposing 15.

$$\begin{aligned} 15 \times 10 &= (10+5) \times 10 \\ &= (10 \times 10) + (5 \times 10) \\ &= 100 + 50 \\ &= 150 \end{aligned}$$

All of these explanations are correct. However, skip counting and counting on become more difficult to use accurately as numbers become larger, e.g., in computing 5 × 90 or explaining why 15 tens is 150, and needs modification for products such as 4 × 90. The first does not indicate any place value understanding.

[4] A range of algorithms may be used.

3rd Grade – CCSS for Mathematics

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3.NBT.3 Examples:

Grade 3 explanations for "15 tens is 150"

- Skip-counting by 50. 5 tens is 50, 100, 150.
- Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150.
- Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150.
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3.NF.A Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Essential Skills and Concepts:
- ☐ Fractional parts are equal shares of a whole
- ☐ When the numerator and denominator are the same, the fraction equals one whole

Question Stems and Prompts:
- ✓ How can I use fractions to name parts of a whole?
- ✓ What is a fraction?

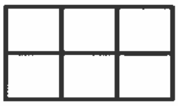
Vocabulary Tier 3	**Spanish Cognates**
• unit fraction	fracción unitaria
• numerator	numerador
• denominator	denominador

Standards Connections
3.NF.1 → 3.G.2, 3.NF.3
3.NF.1 – 3.NF.2

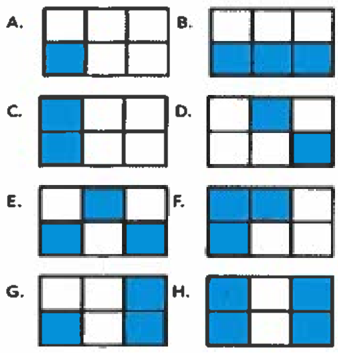
Illustrative Tasks:
- Halves, Thirds, and Sixths

https://www.illustrativemathematics.org/illustrations/1502

a. A small square is a square unit. What is the area of this rectangle? Explain.



b. What fraction of the area of each rectangle is shaded blue? Name the fraction in as many ways as you can. Explain your answers.



c. Shade $\frac{1}{2}$ of the area of rectangle in a way that is different from the rectangles above.

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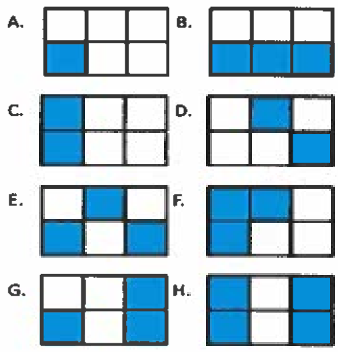

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3.NF.A.1

Standard Explanation

In grade three students develop an understanding of fractions as numbers, beginning with unit fractions by building on the idea of partitioning a whole into equal parts. Student proficiency with fractions is essential for success in more advanced mathematics such as percentages, ratios and proportions, and in algebra at later grades.

In grades one and two, students partitioned circles and rectangles into two, three, and four equal shares and used fraction language (e.g., halves, thirds, half of, a third of). In grade three, students begin to enlarge their concept of number by developing an understanding of fractions as numbers (Adapted from PARCC 2012).

Grade three students understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts and the fraction a/b as the quantity formed by a parts of size 1/b. (3.NF.1▲).

To understand fractions, students build on the idea of partitioning (dividing) a whole into equal parts.
Students begin their study of fractions with unit fractions (fractions with the numerator 1), which are formed by partitioning a whole into equal parts (the number of equal parts becomes the denominator). One of those parts is a unit fraction. An important goal is for students to see unit fractions as the basic building blocks of all fractions, in the same sense that the number 1 is the basic building block of whole numbers. Students make the connection that, just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions (adapted from UA Progressions Documents 2013a). They explore fractions first, using concrete models such as fraction bars and geometric shapes, and this culminates in understanding fractions on the number line (CA *Mathematics Framework*, adopted Nov. 6, 2013).

3.NF.1 Examples:

Examples 3.NF.1▲

Teacher: Show fourths by folding the piece of paper into equal parts.

Student: I know that when the number on the bottom is 4, I need to make four equal parts. By folding the paper in half once and then again, I get four parts, and each part is equal. Each part is worth $\frac{1}{4}$.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Teacher: Shade $\frac{3}{4}$ using the fraction bar you created.

Student: My fraction bar shows fourths. The 3 tells me I need three of them, so I'll shade them. I could have shaded any three of them and I would still have $\frac{3}{4}$.

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3.NF.A Develop understanding of fractions as numbers.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Essential Skills and Concepts:

☐ Fractions are numbers on a number line

Question Stems and Prompts:

✓ What fractions are on the number line between 0 and 1?

Vocabulary	**Spanish Cognates**
Tier 3	
• number line	línea numérica

Standards Connections

3.NF.2 → 3.NF.3

3.NF.2 – 3.NF.1, 3.MD.4

Illustrative Tasks:

- Locating Fractions Less than One on the Number Line, https://www.illustrativemathematics.org/illustrations/168

a. Mark and label the points $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ on the number line. Be as exact as possible.

b. Mark and label the point $\frac{2}{3}$ on the number line. Be as exact as possible.

c. Mark and label the points $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ on the number line. Be as exact as possible.

- Which is Closer to 1?,
- https://www.illustrativemathematics.org/illustrations/172

Which is closer to 1 on the number line, $\frac{4}{5}$ or $\frac{5}{4}$? Explain.

- Find 2/3, https://www.illustrativemathematics.org/illustrations/170

Label the point where $\frac{2}{3}$ belongs on the number line. Be as exact as possible.

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3.NF.A.2

Standard Explanation
Students build on the idea of partitioning or dividing a whole into equal parts to understand fractions. Students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts (the number of equal parts becomes the denominator) and taking one of those parts. An important goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers. Students make the connection that just as every whole number is obtained by combining a sufficient number of 1s; every fraction is obtained by combining a sufficient number of unit fractions (Adapted from Progressions 3-5 NF 2012). They explore fractions first using concrete models such as fraction bars and geometric shapes, which will culminate in understanding fractions on the number line.

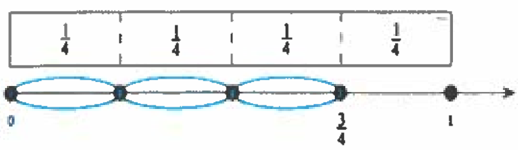
Eventually, students represent fractions by dividing a number line from 0 to 1 into equal parts and recognize that each segmented part represents the same length (MP.2, MP.4, MP.7). Stacking fraction bars and number lines can help students see how the unit length has been divided into equal parts. Important is that students "mark off" lengths of 1/b when locating fractions on the number line. Notice the difference between how the fraction bar and number line are labeled in the example shown below (3.NF.2a-b).

Third grade students need opportunities to place fractions on a number line and understand fractions as a related component of the ever-expanding number system.
The number line reinforces the analogy between fractions and whole numbers. Just as 5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so is the point obtained by marking off 5 times the length of a different interval as the basic unit of length, namely the interval from 0 to 1/3.

3.NF.2 Examples:

Teacher: Explain how you know your mark is in the right place. 3.NF.2b▲

Student (Solution): When I use my fraction strip as a measuring tool, it shows me how to divide the unit interval into four equal parts (since the denominator is *4*). Then I start from the mark that has *0* and measure off three pieces of $\frac{1}{4}$ each. I circled the pieces to show that I marked three of them. This is how I know I have marked $\frac{3}{4}$.



3rd Grade – CCSS for Mathematics

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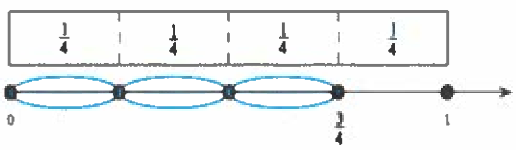
Teacher: Explain how you know your mark is in the right place. 3.NF.2b▲

Student (Solution): When I use my fraction strip as a measuring tool, it shows me how to divide the unit interval into four equal parts (since the denominator is 4). Then I start from the mark that has 0 and measure off three pieces of $\frac{1}{4}$ each. I circled the pieces to show that I marked three of them. This is how I know I have marked $\frac{3}{4}$.



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3.NF.A Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Essential Skills and Concepts:

- ☐ Fractional parts must be the same size
- ☐ The number of equal parts tells how many make a whole
- ☐ As the number of equal pieces in the whole increases, the size of the fractional pieces decreases
- ☐ Common benchmark numbers such as 0, 1/2, ¾ and 1 can be used to determine if an unknown fraction is greater or smaller than a benchmark fraction.

Question Stems and Prompts:

✓ What does the 3 and the 4 represent in the fraction ¾?

Vocabulary	Spanish Cognates
Tier 3	
• equivalent fractions	fracción equivalente

Standards Connections

3.NF.3 → 4.NF.1

Illustrative Task:

- **Jon and Charlie's Run**

https://www.illustrativemathematics.org/content-standards/3/NF/A/3/tasks/871

Jon and Charlie plan to run together. They are arguing about how far to run. Charlie says, "I run 3/6 of a mile each day."
Jon says, "I can only run ½ of a mile."

If Charlie runs 3/6 of a mile and Jon runs ½ of a mile, explain why it is silly for them to argue. Draw a picture or a number line to support your reasoning.

3rd Grade – CCSS for Mathematics

3.NF.A Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
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✓ What does the 3 and the 4 represent in the fraction ¾?

Vocabulary — Spanish Cognates
Tier 3
• equivalent fractions — fracción equivalente

Standards Connections
3.NF.3 → 4.NF.1

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3rd Grade – CCSS for Mathematics

3.NF.A.3

Standard Explanation

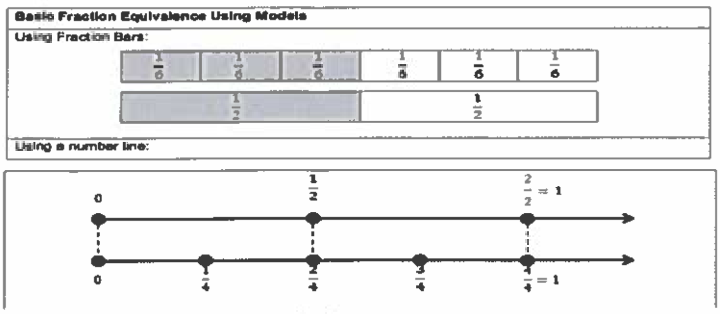
Students develop an understanding of fractions as they use visual models and a number line to represent, explain, and compare unit fractions, equivalent fractions (e.g., 1/2 = 2/4), whole numbers as fractions (e.g., 3 = 3/1), and fractions with the same numerator (e.g., 4/3 and 4/6) or the same denominator (e.g., 4/8 and 5/8). (NF.2-3▲).

Students develop an understanding of order in terms of position on a number line. Given two fractions—thus two points on the number line—students understand that the one to the left is said to be smaller, and the one to the right is said to be larger (Adapted from Progressions 3-5 NF 2012).

Students learn that when comparing fractions they need to look at the size of the parts and the number of the parts. For example, is smaller than because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole of the same size is cut into 2 pieces.

To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts is different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces (Adapted from Arizona 2012 and KATM FlipBook 2012).

Students develop an understanding of equivalent fractions as they compare fractions using a variety of visual fraction models and justify their conclusions (MP.3). Through opportunities to compare fraction models with the same whole divided into different numbers of pieces, students identify fractions that show the same amount or name the same number, and learn that they are equal (or equivalent) (CA *Mathematics Framework*, adopted Nov. 6, 2013).



(Adapted from Progressions 3-5 NF 2012)

Illustrative Task:

- Ordering Fractions,

  https://www.illustrativemathematics.org/illustrations/460

Arrange the fractions in order from least to greatest. Explain your answer with a picture.

a. $\frac{1}{5}, \frac{1}{7}, \frac{1}{3}$

3.NF.A.3

Standard Explanation

Students develop an understanding of fractions as they use visual models and a number line to represent, explain, and compare unit fractions, equivalent fractions (e.g., 1/2 = 2/4), whole numbers as fractions (e.g., 3 = 3/1), and fractions with the same numerator (e.g., 4/3 and 4/6) or the same denominator (e.g., 4/8 and 5/8). (NF.2-3▲).

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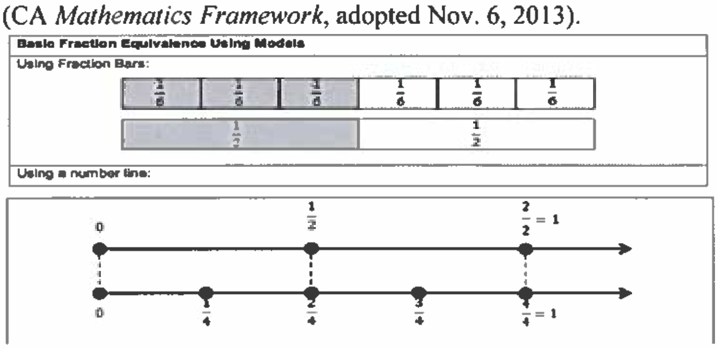

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3.MD.A Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Essential Skills and Concepts:
- ☐ Tell and write time to the nearest minute
- ☐ Measure time intervals in minutes
- ☐ Duration of an event is called elapsed time and it can be measured

Question Stems and Prompts:
- ✓ How can I use what I know about number lines to help me figure how much time has passed between two events?
- ✓ How can you prove to your parents you do not spend too much time watching television?

Vocabulary Tier 3	**Spanish Cognates**
• time intervals	intervalo de tiempo
• elapsed time	
• minute	minuto
• hour	hora

3rd Grade – CCSS for Mathematics

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Vocabulary Tier 3 / **Spanish Cognates**
- time intervals — intervalo de tiempo
- elapsed time
- minute — minuto
- hour — hora

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3.MD.A.2

Standard Explanation
Students begin to understand the concept of continuous measurement quantities and they add, subtract, multiply or divide to solve one-step word problems involving such quantities. Multiple opportunities to weigh classroom objects and fill containers will help students develop a basic understanding of the size and weight of a liter, a gram, and a kilogram (3.MD.2▲) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

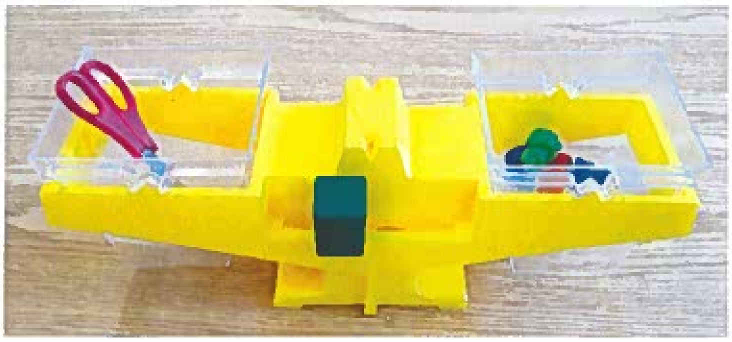
Focus, Coherence, and Rigor

Students' understanding and work with measuring and estimating continuous measurement quantities, such as liquid volume and mass (3.MD.2▲), are an important context for the fraction arithmetic they will experience in later grade levels.

This standard asks for students to reason about the units of mass and volume using the units g, kg, and L. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter emphasizing the relationship between smaller units to larger units in the same system. Word problems should only be one-step and include the same units. Students are not expected to do conversions between units, but reason as they estimate, using benchmarks to measure weight and capacity.

Illustrative Task:

- How Heavy?, https://www.illustrativemathematics.org/illustrations/1929



3rd Grade – CCSS for Mathematics

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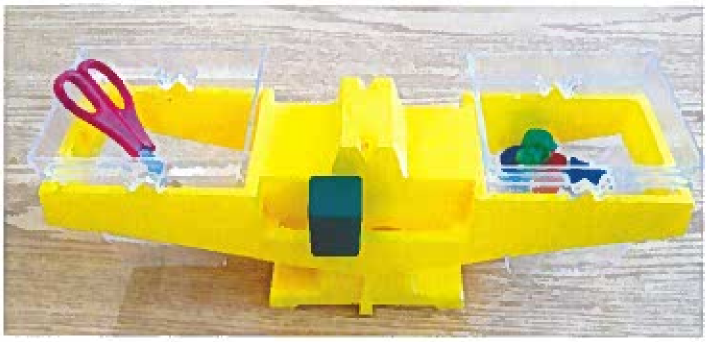

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3rd Grade – CCSS for Mathematics

3.MD.B Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Essential Skills and Concepts:
- ☐ One way to compare data is through the use of graphs
- ☐ Picture graphs and bar graphs may be used to display data

Question Stems and Prompts:
- ✓ How can graphs be used to display data?
- ✓ How do I decide what increments to use for my scale?

Vocabulary Tier 3	**Spanish Cognates**
• scaled picture graph	pictográfica escalada
• scaled bar graph	gráfica de barros escalado

Standards Connections
3.MD.3 – 3.OA.8

Illustrative Task:
- **Classroom Supplies**

https://www.illustrativemathematics.org/content-standards/3/MD/B/3/tasks/1315

a. Write down the different items and how many of each you would choose. Find the total for each category.
- Supplies
- Books and maps
- Puzzles and games
- Special items

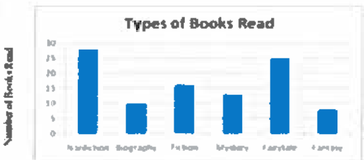
b. Create a bar graph to represent how you would spend the money. Scale the vertical axis by $100. Write all of the labels.

c. What was the total cost of all your choices? Did you have any money left over? If so, how much?

d. Compare your choices with a partner. How much more or less did you choose to spend on each category than your partner? How much more or less did you choose to spend in total than your partner?

3.MD.3 Examples:

Single Bar Graphs. Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data
- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?

(Adapted from N. Carolina 2012)

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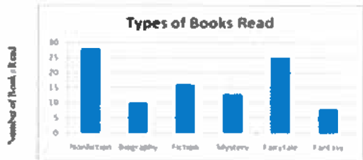

Analyze and Interpret data
• How many more nonfiction books where read than fantasy books?
• Did more people read biography and mystery books or fiction and fantasy books?
• About how many books in all genres were read?

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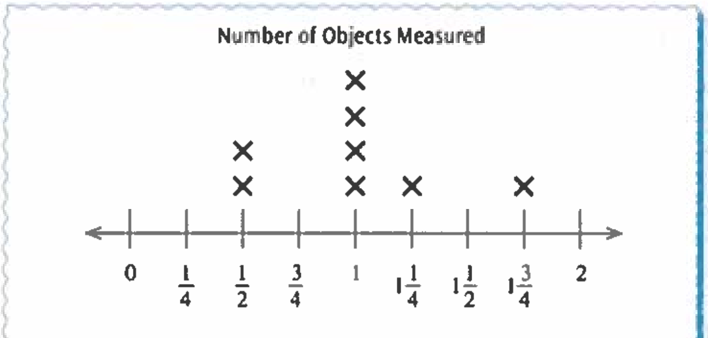
3.MD.B Represent and interpret data.

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Standard Explanation
Students use their knowledge of fractions and number lines to work with measurement data involving fractional measurement values. They generate data by measuring lengths using rulers marked with halves and fourths of an inch and create a line plot to display their findings (3.MD.4) (adapted from UA Progressions Documents 2011b). For example, students might use a line plot to display data. (CA *Mathematics Framework*, adopted Nov. 6, 2013).

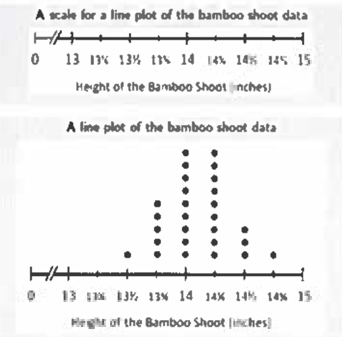
3.MD.4 Examples:



Adapted from NCDPI 2013b.

Students' measurements of a statue and of a bamboo shoot

Statue measurements: Student's initials	Student's measured value (inches)	Bamboo shoot measurements: Student's initials	Height value (inches)
W.B.	64	W.B.	13 3/4
D.W.	65	D.W.	14 1/2
H.D.	65	H.D.	14 1/4
G.W.	65	G.W.	14 3/4
V.Y.	67	V.Y.	14 1/4
T.T.	66	T.T.	14 1/2
D.F.	67	D.F.	14
B.H.	65	B.H.	13 1/2
H.H.	63	H.H.	14 1/4
V.H.	64	V.H.	14 1/4
I.O.	64	I.O.	14 1/4
W.N.	65	W.N.	14
B.P.	69	B.P.	14 1/2
V.A.	65	V.A.	13 3/4
H.L.	66	H.L.	14
O.M.	64	O.M.	13 3/4
L.E.	65	L.E.	14 1/4
M.J.	66	M.J.	13 3/4
T.D.	66	T.D.	14 1/4
K.P.	64	K.P.	14
H.N.	65	H.N.	14
W.M.	67	W.M.	14
C.Z.	64	C.Z.	13 3/4
J.I.	66	J.I.	14
M.S.	66	M.S.	14 1/4
T.C.	65	T.C.	14
G.V.	67	G.V.	14
O.F.	65	O.F.	14 1/4



(K – 3, Categorical Data; Grades 2 – 5, Measurement Data* June 20, 2011)

3rd Grade – CCSS for Mathematics

3.MD.B Represent and interpret data.

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Standard Explanation
Students use their knowledge of fractions and number lines to work with measurement data involving fractional measurement values. They generate data by measuring lengths using rulers marked with halves and fourths of an inch and create a line plot to display their findings (3.MD.4) (adapted from UA Progressions Documents 2011b). For example, students might use a line plot to display data. (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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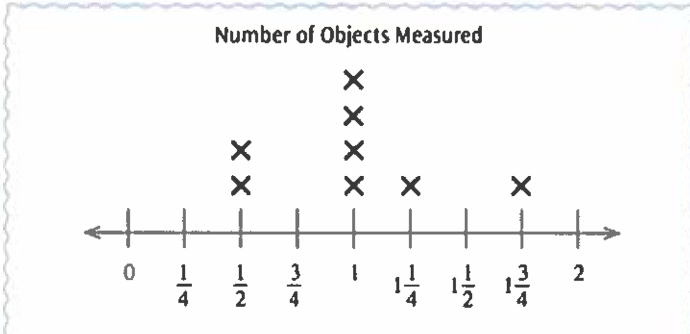
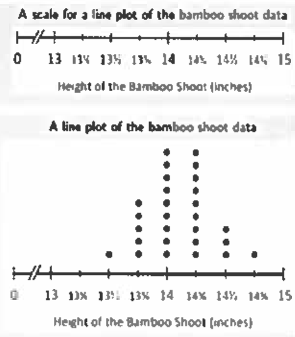

Adapted from NCDPI 2013b.



(K – 3, Categorical Data; Grades 2 – 5, Measurement Data* June 20, 2011)

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3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Essential Skills and Concepts:

- ☐ Area models are related to addition and multiplication.
- ☐ Area covers a certain amount of space using square units.

Question Stems and Prompts:

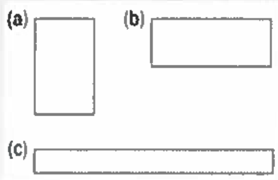
- ✓ How do rectangle dimensions impact the area of the rectangle?
- ✓ How does knowing the area of a square or rectangle relate to knowing multiplication facts?

Vocabulary	Spanish Cognates
Tier 3	
• area model	modelo de área
• square units	

Standards Connections
3.MD.5 → 3.MD.6, 3.MD.7d, 3.MD.8

3.MD.5 Example:

Which rectangle covers the most area?



These rectangles are formed from unit squares (tiles students have used) although students are not informed of this or the rectangle's dimensions: (a) 4 by 3, (b) 2 by 6, and (c) 1 row of 12. Activity from Lehrer, et al., 1998, "Developing understanding of geometry and space in the primary grades," in R. Lehrer & D. Chazan (Eds.), *Designing Learning Environments for Developing Understanding of Geometry and Space*, Lawrence Erlbaum Associates.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 16)

3rd Grade – CCSS for Mathematics

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Vocabulary Spanish Cognates
Tier 3
• area model modelo de área
• square units

Standards Connections
3.MD.5 → 3.MD.6, 3.MD.7d, 3.MD.8

3.MD.5 Example:

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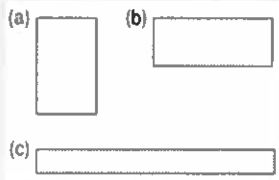

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Standard Explanation

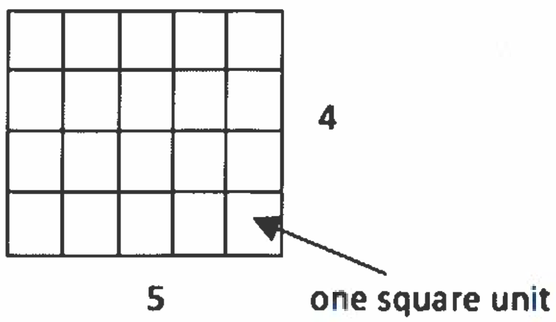
A critical area of instruction at grade three is for students to develop an understanding of the structure of rectangular arrays and of area measurement.

Students recognize area as an attribute of plane figures, and they develop an understanding of concepts of area measurement (3.MD.5▲). They discover a square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area.

Students measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units) (3.MD.6▲). Students develop an understanding of using square units to measure area by using different sized square units, filling in an area with the same sized square units, and then counting the number of square units (CA *Mathematics Framework*, adopted Nov. 6, 2013).

The standards call for students to explore the concept of covering a region with “unit squares,” which could include square tiles or shading on grid or graph paper. Based on students’ development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.

3.MD.5 Example:



3rd Grade – CCSS for Mathematics

3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Standard Explanation

A critical area of instruction at grade three is for students to develop an understanding of the structure of rectangular arrays and of area measurement.

Students recognize area as an attribute of plane figures, and they develop an understanding of concepts of area measurement (3.MD.5▲). They discover a square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area.

Students measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units) (3.MD.6▲). Students develop an understanding of using square units to measure area by using different sized square units, filling in an area with the same sized square units, and then counting the number of square units (CA *Mathematics Framework*, adopted Nov. 6, 2013).

The standards call for students to explore the concept of covering a region with “unit squares,” which could include square tiles or shading on grid or graph paper. Based on students’ development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.

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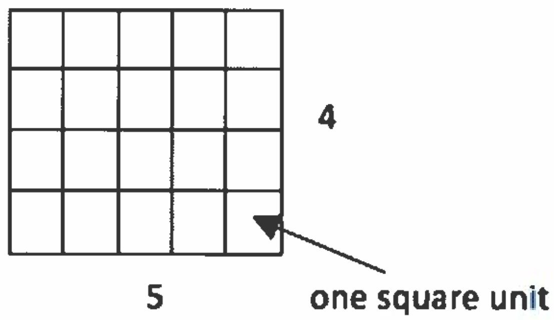

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3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).

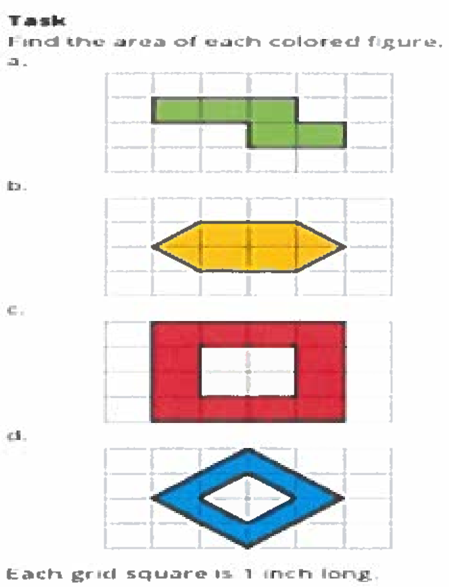
Essential Skills and Concepts:
- ☐ Understanding of arrays
- ☐ Rearranging an area such as 24 sq. units based on its dimensions or factors does NOT change the amount of area being covered.

Question Stems and Prompts:
- ✓ Can the same area measurement produce different size rectangles?

Vocabulary	**Spanish Cognates**
Tier 3	
• area model	modelo de área
• unit squares	
• dimensions	dimensiones

Standards Connections
3.MD.6 → 3.MD.7a

Illustrative Task:
- Finding the Area of Polygons, https://www.illustrativemathematics.org/illustrations/1515



3rd Grade – CCSS for Mathematics

3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

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- ☐ Understanding of arrays
- ☐ Rearranging an area such as 24 sq. units based on its dimensions or factors does NOT change the amount of area being covered.

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Vocabulary	**Spanish Cognates**
Tier 3	
• area model	modelo de área
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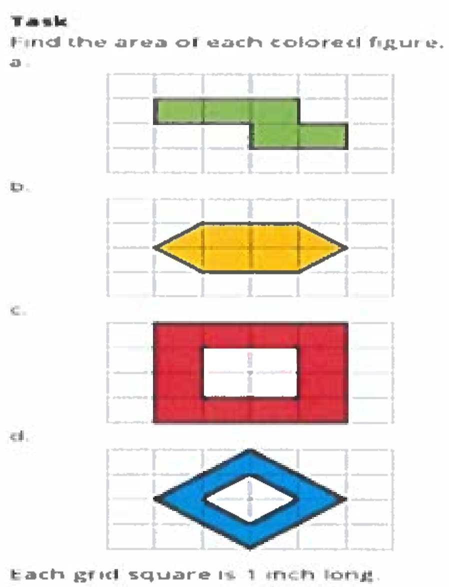

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3rd Grade – CCSS for Mathematics

3.MD.C.6

Standard Explanation
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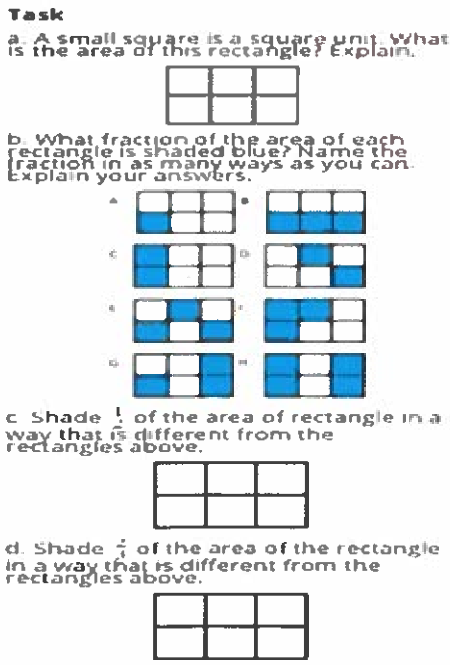
Students recognize area as an attribute of plane figures, and they develop an understanding of concepts of area measurement (3.MD.5▲). They discover a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area.

Students measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units) (3.MD.6▲). Students develop an understanding of using square units to measure area by using different sized square units, filling in an area with the same sized square units, and then counting the number of square units (CA *Mathematics Framework*, adopted Nov. 6, 2013).

The standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. Based on students' development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.

Illustrative Task:
- Halves, Thirds, and Sixths

https://www.illustrativemathematics.org/content-standards/3/MD/C/6/tasks/1502



3.MD.C.6

Standard Explanation
A critical area of instruction at grade three is for students to develop an understanding of the structure of rectangular arrays and of area measurement.

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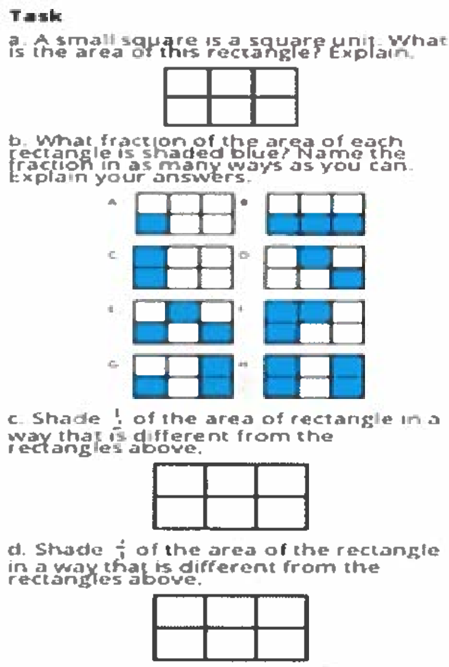

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3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Essential Skills and Concepts:

- ☐ Student use area model to represent the distributive property

Question Stems and Prompts:

- ✓ How does understanding the area model help us multiply large numbers?

Vocabulary	**Spanish Cognates**
Tier 3	
• commutative property of multiplication	propiedad conmutativa de multiplicación

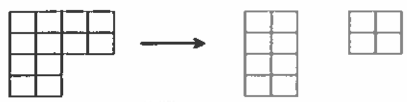
Standards Connections

3.MD.7a → 3.MD.7b, 3.MD.7c
3.MD.7b → 3.MD.7c, 4.MD.3
3.MD.7c → 3.MD.7d
3.MD.7c – 3.OA.5
3.MD.7d – 3.OA.8

3.MD.7 Example:

The standards mention rectilinear figures. A *rectilinear figure* is a polygon with only right angles. Such figures can be decomposed into rectangles to find their areas.

Example:



Area of Rectilinear Figure? 4 × 2 2 × 2

By breaking the figure into two pieces, it becomes easier to see that the area of the figure is 8 + 4 = 12 square units.

(Adapted from N Carolina)

3rd Grade – CCSS for Mathematics

3.MD.C Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

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a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
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c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
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Essential Skills and Concepts:
☐ Student use area model to represent the distributive property

Question Stems and Prompts:
✓ How does understanding the area model help us multiply large numbers?

Vocabulary / Spanish Cognates
Tier 3
• commutative property of multiplication — propiedad conmutativa de multiplicación

Standards Connections
3.MD.7a → 3.MD.7b, 3.MD.7c
3.MD.7b → 3.MD.7c, 4.MD.3
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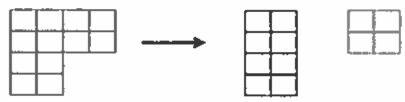

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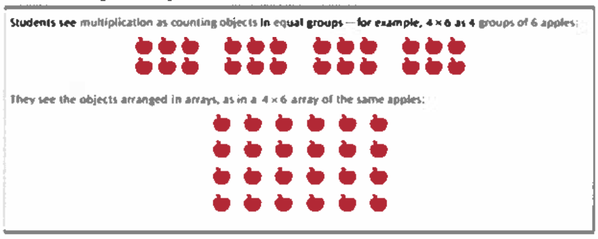
3.MD.C.7

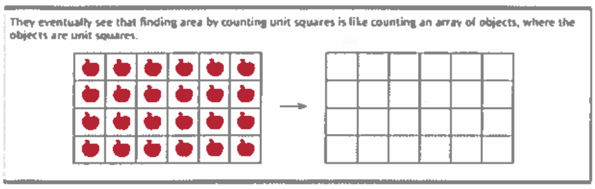
Standard Explanation

Students relate the concept of area to the operations of multiplication and addition and show that the area of a rectangle can be found by multiplying the side lengths (3.MD.7▲). Students make sense of these quantities as they learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are (adapted from UA Progressions Documents 2012a).

Students need opportunities to tile a rectangle with square units and then multiply the side lengths to show that they both give the area. For example, to find the area, a student could count the squares or multiply 4 × 3 = 12.

The transition from counting unit squares to multiplying side lengths to find area can be aided when students see the progression from multiplication as equal groups to multiplication as a total number of objects in an array, and then see the area of a rectangle as an array of unit squares. An example is presented below.





Students use area models to represent the distributive property in mathematical reasoning. For example, the area of a 6 × 7 figure can be determined by finding the area of a 6 × 5 and 6 × 2 and adding the two sums. Students recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping parts (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Focus, Coherence, and Rigor

The use of area models (3.MD.7▲) also supports multiplicative reasoning, a major focus in grade three in the domain "Operations and Algebraic Thinking" (3.OA.1–9▲). Students must begin work with multiplication and division at or near the start of the school year to allow time for understanding and to develop fluency with these skills. Because area models for products are an important part of this process (3.MD.7▲), work on concepts of area (3.MD.5–6▲) should begin at or near the start of the year as well (adapted from PARCC 2012).

3rd Grade – CCSS for Mathematics

3.MD.C.7

Standard Explanation

Students relate the concept of area to the operations of multiplication and addition and show that the area of a rectangle can be found by multiplying the side lengths (3.MD.7▲). Students make sense of these quantities as they learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are (adapted from UA Progressions Documents 2012a).

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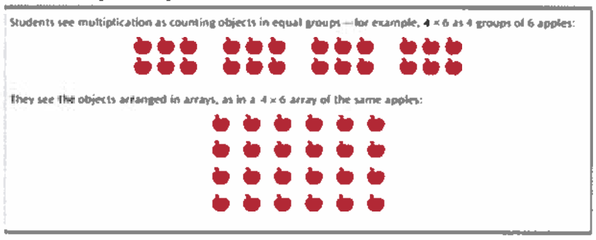
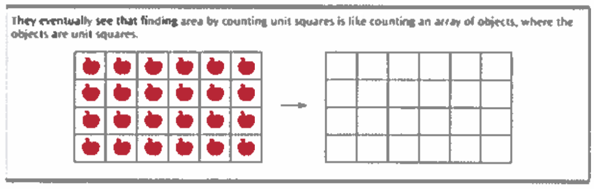

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3rd Grade – CCSS for Mathematics

3.MD.D Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Essential Skills and Concepts:

- ☐ The length around a polygon can be calculated by adding the lengths of its sides.

Question Stems and Prompts:

- ✓ How can I demonstrate my understanding of the measurement of area and perimeter?

Vocabulary	**Spanish Cognates**
Tier 3	
• area	área
• perimeter	perímetro

Standards Connections
3.MD.8 – 3.OA.8

3.MD.8 Example:
Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

3.MD.D Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

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Question Stems and Prompts:

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Vocabulary	**Spanish Cognates**
Tier 3	
• area	área
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Standards Connections
3.MD.8 – 3.OA.8

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Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find th rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12

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3.MD.D.8

Standard Explanation

In grade three, students solve real-world and mathematical problems involving perimeters of polygons (3.MD.8). Students can develop an understanding of the concept of perimeter as they walk around the perimeter of a room, use rubber bands to represent the perimeter of a plane figure with whole number side lengths on a geoboard, or trace around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. They explain their reasoning to others. Given a perimeter and a length or width, students use objects or pictures to find the unknown length or width.
They justify and communicate their solutions using words, diagrams, pictures, and numbers (adapted from ADE 2010) (CA Mathematics Framework, adopted Nov. 6, 2013).

Progression Information:

A perimeter is the boundary of a two-dimensional shape. For a polygon, the length of the perimeter is the sum of the lengths of the sides. Initially, it is useful to have sides marked with unit length marks, allowing students to count the unit lengths. Later, the lengths of the sides can be labeled with numerals. As with all length tasks, students need to count the length-units and not the end-points. Next, students learn to mark off unit lengths with a ruler and label the length of each side of the polygon. For rectangles, parallelograms, and regular polygons, students can discuss and justify faster ways to find the perimeter length than just adding all of the lengths. Rectangles and parallelograms have opposite sides of equal length, so students can double the lengths of adjacent sides and add those numbers or add lengths of two adjacent sides and double that number. A regular polygon has all sides of equal length, so its perimeter length is the product of one side length and the number of sides. Perimeter problems for rectangles and parallelograms often give only the lengths of two adjacent sides or only show numbers for these sides in a drawing of the shape. The common error is to add just those two numbers. Having students first label the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult “missing measurements” problems and other types of perimeter problems (Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 16).

3rd Grade – CCSS for Mathematics

3.MD.D.8

Standard Explanation

In grade three, students solve real-world and mathematical problems involving perimeters of polygons (3.MD.8). Students can develop an understanding of the concept of perimeter as they walk around the perimeter of a room, use rubber bands to represent the perimeter of a plane figure with whole number side lengths on a geoboard, or trace around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. They explain their reasoning to others. Given a perimeter and a length or width, students use objects or pictures to find the unknown length or width.
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3.G.A Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Essential Skills and Concepts:
- ☐ Sort geometric figures and identify squares, rectangles, and rhombuses as quadrilaterals.
- ☐ Geometric figures can be classified according to their properties.
- ☐ The broad category "Quadrilaterals" includes all types of parallelograms, trapezoids and other four-sided figures.
- ☐ How are the quadrilaterals alike/different?

Question Stems and Prompts:
- ✓ Do you think shapes could be grouped together in the same family or classification? Explain.
- ✓

Vocabulary	Spanish Cognates
Tier 3	
• quadrilateral	cuadrilátero
• rhombus	rombo

Standards Connections
3.G.1 → 4.G.1

3rd Grade – CCSS for Mathematics

3.G.A Reason with shapes and their attributes.

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Essential Skills and Concepts:
☐ Sort geometric figures and identify squares, rectangles, and rhombuses as quadrilaterals.
☐ Geometric figures can be classified according to their properties.
☐ The broad category "Quadrilaterals" includes all types of parallelograms, trapezoids and other four-sided figures.
☐ How are the quadrilaterals alike/different?

Question Stems and Prompts:
✓ Do you think shapes could be grouped together in the same family or classification? Explain.
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Vocabulary / Spanish Cognates
Tier 3
• quadrilateral cuadrilátero
• rhombus rombo

Standards Connections
3.G.1 → 4.G.1

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3.G.A Reason with shapes and their attributes.

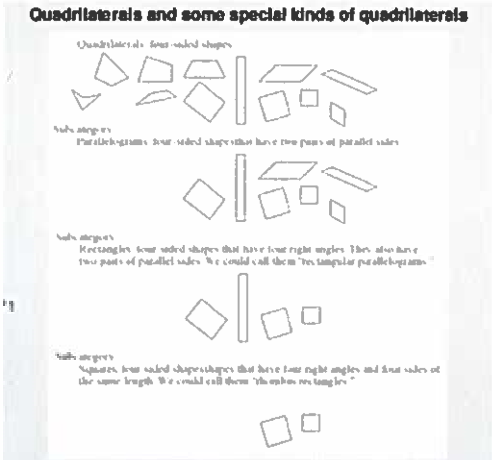
3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Standard Explanation

A critical area of instruction at grade three is for students to describe and analyze two-dimensional shapes. Students compare common geometric shapes (e.g., rectangles and quadrilaterals) based on common attributes, such as four sides (3.G.1). In earlier grades, students informally reasoned about particular shapes through sorting and classifying based on geometric attributes. Students also built and drew shapes given the number of faces, number of angles, and number of sides. In grade three students describe properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. For example, students could start by identifying shapes with right angles, explain and discuss why the remaining shapes do not fit this category, and determine common characteristics of the remaining shapes (CA Mathematics Framework, adopted Nov. 6, 2013).

Progression Information:

Students can form larger, categories, such as the class of all shapes with four sides, or quadrilaterals, and recognize that it includes other categories, such as squares, rectangles, rhombuses, parallelograms, and trapezoids. They also recognize that there are quadrilaterals that are not in any of those subcategories. (K – 6, Geometry, June 23, 2012)



The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).

3rd Grade – CCSS for Mathematics

3.G.A Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Standard Explanation

A critical area of instruction at grade three is for students to describe and analyze two-dimensional shapes. Students compare common geometric shapes (e.g., rectangles and quadrilaterals) based on common attributes, such as four sides (3.G.1). In earlier grades, students informally reasoned about particular shapes through sorting and classifying based on geometric attributes. Students also built and drew shapes given the number of faces, number of angles, and number of sides. In grade three students describe properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. For example, students could start by identifying shapes with right angles, explain and discuss why the remaining shapes do not fit this category, and determine common characteristics of the remaining shapes (CA Mathematics Framework, adopted Nov. 6, 2013).

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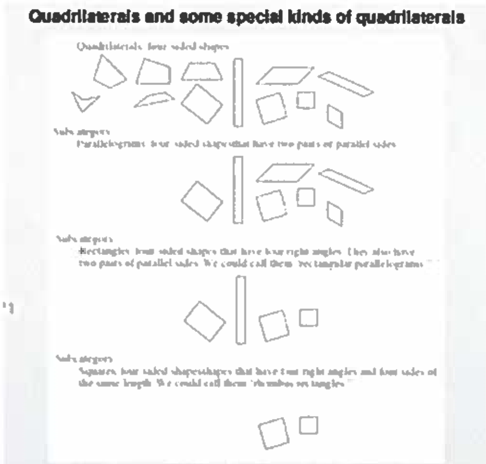

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3.G.A Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Essential Skills and Concepts:

- ☐ Shapes can be partitioned with equal areas in a variety of ways to show halves, thirds, fourths, sixths, and eighths.

Question Stems and Prompts:

- ✓ Can all shapes be split into halves, thirds, fourths, sixths and eighths? Prove it
- ✓ Describe what a fraction looks like in a shape?

Vocabulary	**Spanish Cognates**
Tier 2	
• Partition	
• Equal area	área igual

Standards Connections

3.G.2 → 3.G.1
3.G.2 → 3.MD.5
3.G.2 → 3.MD.6
3.G.2 → 3.MD.7

Illustrative Task:

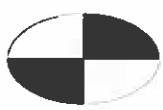
- Representing Half of a Circle
  https://www.illustrativemathematics.org/content-standards/3/G/A/2/tasks/1014

For each picture, decide whether one half of the circle is shaded or not. Explain how you know.

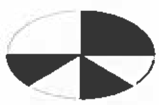
a.

b.

c.



d.



3rd Grade – CCSS for Mathematics

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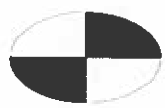
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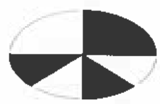

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Students relate their work with fractions to geometry as they partition shapes into parts with equal areas and represent each part as a unit fraction of the whole (3.G.2) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Focus, Coherence, and Rigor

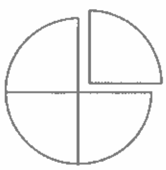
As students partition shapes into parts with equal areas (3.G.2), they also reinforce concepts of area measurement and fractions that are part of the major work at the grade in the clusters "Geometric measurement: understand concepts of area and relate area to multiplication and to addition" (3.MD.5–7▲) and "Develop understanding of fractions as numbers" (3.NF▲).

Adapted from PARCC 2012.

3.G.2 Example:

Example 3.G.2

The figure below was partitioned (divided) into four equal parts. Each part is $\frac{1}{4}$ of the total area of the figure.



Adapted from NCDPI 2013b.

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3rd Grade – CCSS for Mathematics

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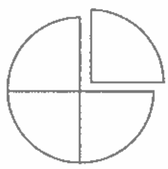

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Resources for the CCSS 3rd Grade Bookmarks

California *Mathematics Framework*, adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts)

- K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking (2011, May 29)
- K – 5, Number and Operations in Base Ten (2012, April 21)
- K – 3, Categorical Data; Grades 2 – 5, Measurement Data* (2011, June 20)
- K – 5, Geometric Measurement (2012, June 23)
- K – 6, Geometry (2012, June 23)
- Number and Operations – Fractions, 3 – 5 (2013, September 19)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

3rd Grade – CCSS for Mathematics

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Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

Howard County Public School System, Elementary Mathematics Office, Standards for Mathematical Practice for Parents, Draft 2011, https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP for Parents.docx/286906254/SFMP for Parents.docx

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Long Beach Unified School District, Math Cognates, retrieved on 7/14/14, http://www.lbschools.net/Main_Offices/Curriculum/Areas/Mathematics/XCD/ListOfMathCognates.pdf

A Graph of the Content Standards, Jason Zimba, June 7, 2012, http://tinyurl.com/ccssmgraph

3rd Grade – CCSS for Mathematics

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