

Beliefs about teaching and learning mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning and discourse.
All students need to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding and mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

Characteristics of Mathematical Tasks

Levels of Demands

<p><u>Lower-level demands</u> <u>(memorization):</u></p> <ul style="list-style-type: none">• Reproducing previously learned facts, rules, formulas, definitions or committing them to memory• Cannot be solved with a procedure• Have no connection to concepts or meaning that underlie the facts rules, formulas or definitions	<p><u>Lower-level demands</u> <u>(procedures without connections):</u></p> <ul style="list-style-type: none">• Are algorithmic• Require limited cognitive demand• Have no connection to the concepts or meaning that underlie the procedure• Focus on producing correct answers instead of understanding• Require no explanations
<p><u>Higher-level demands</u> <u>(procedures with connections):</u></p> <ul style="list-style-type: none">• Use procedure for deeper understanding of concepts• Broad procedures connected to ideas instead of narrow algorithms• Usually represented in different ways• Require some degree of cognitive effort; procedures may be used but not mindlessly	<p><u>Higher-level demands</u> <u>(doing mathematics):</u></p> <ul style="list-style-type: none">• Require complex non-algorithmic thinking• Require students to explore and understand the mathematics• Demand self-monitoring of one's cognitive process• Require considerable cognitive effort and may involve some level of anxiety b/c solution path isn't clear

Leinwand, S., D. Brahier, and D. Huinker, *Principles to Action*, Reston, VA: National Council of Teachers of Mathematics, 2014 (pg 18)