2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

Grade 8/Pre-Algebra Standards

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibility.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
    - Learn to think critically, logically, ethically, and analytically
    - Learn to express ideas orally and in writing using correct mathematical terminology
    - Learn to apply the techniques of mathematics to real world situations
    - Understand that mathematics is important to function in today's world
    - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

GRADE EIGHT/PRE-ALGEBRA

***As life-long learners**, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.*

The Diocese of Wilmington has established the following mathematics Standards to clarify for teachers, students, and parents the knowledge, understanding, and skills students should attain in GRADE EIGHT/PRE-ALGEBRA:

Standard 1 — Number Sense
Developing number sense is the foundation of mathematics. Students extend their understanding of irrational numbers, such as π and the square root of 2, learning the relationship between the nature of the decimal of a number and whether it is rational or irrational. They use negative exponents to write decimals in scientific notation, and they use the inverse relationship between squaring and finding a square root to calculate approximate square roots.

Standard 2 — Computation
Mastering computational skills is vital. Students add, subtract, multiply, and divide rational numbers. They use percentages to calculate simple and compound interest. They also use mental arithmetic to compute with fractions, decimals, powers, and percentages.

Standard 3 — Algebra and Functions
Understanding patterns, rules, and symbols is the foundation of Algebra. Students at this level write and solve linear equations and inequalities, including solving pairs of linear equations by the substitution method. They use properties of the rational numbers to evaluate and simplify algebraic expressions. They further extend their understanding of the relationship between equations and graphs by connecting slopes to rates of change and by drawing graphs of quadratic functions and simple cubic functions.

Standard 4 — Geometry
Exploring shapes and developing spatial sense is the basis of Geometry. Students learn new concepts relating to shapes, such as altitudes, bisectors, and chords and perform constructions connected with them. They further develop their sense of three-dimensional space by investigating how objects intersect in space. They draw a wide range of transformations of shapes, and they apply the Pythagorean Theorem and its converse to problems in two- and three-dimensions.

Standard 5 — Measurement
Using measurement is essential to everyday life. Students convert common measurements for lengths, areas, volumes, weights, capacities, and times. They develop and use the concept of rate and derived measures — e.g., velocity and density. They apply the concepts of similarity, ratio, and proportion to problems involving scale factors, areas, and volumes. They find areas, perimeters, volumes, and surface areas, including those of irregular shapes made up of more basic shapes.

Standard 6 — Data Analysis and Probability
Analyzing data is a fundamental life skill. Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they evaluate whether claims based on data are reasonable and employ various sampling methods, analyzing their strengths and weaknesses. They understand the concepts of the median and quartiles and use these measures to draw and analyze box-and-whisker plots. They represent and analyze two-variable data using scatter plots. They understand the concept of equally likely events and use it to find probabilities. They also find the number of arrangements of objects using the Basic Counting Principle.

Standard 7 — Problem Solving
Solving problems is the practical application of mathematics. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with irrational numbers, analyzing graphs, or finding surface areas, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

Students should also develop the following learning skills by Grade 12 that are integrated throughout the National Council of Teachers of Mathematics (NCTM) Standards:

Communication
As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Reasoning and Proof
Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Connections
Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Representation
Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Standard 1
Number Sense

Students know the properties of rational and irrational numbers expressed in a variety of forms. They understand and use exponents, powers, and roots.

8PA.1.1 Read, write, compare, and solve problems using decimals in scientific notation.

Example: Write 0.00357 in scientific notation.

8PA.1.2 Know that every rational number is either a terminating or repeating decimal and that every irrational number is a non-repeating decimal.

Example: Recognize that 2.375 is a terminating decimal, 5.121212… is a repeating decimal, and that π = 3.14159265… is a non-repeating decimal. Name a rational number. Explain your reasoning.

8PA.1.3 Understand that computations with an irrational number and a rational number (other than zero) produce an irrational number.

Example: Tell whether the product of 7 and π is rational or irrational. Explain how you know that your answer is correct.

8PA.1.4 Understand and evaluate negative integer exponents.

Example: Write 2^{-3} as a fraction.

8PA.1.5 Use the laws of exponents for integer exponents.

Example: Write $2^2 \times 2^3$ as $2 \times 2 \times 2 \times 2 \times 2$ and then as a single power of 2. Explain what you are doing.

8PA.1.6 Use the inverse relationship between squaring and finding the square root of a perfect square integer.

Example: Find the value of $(\sqrt{144})^2$.

8PA.1.7 Calculate and find approximations of square roots.

Example: For an integer that is not a perfect square, find the two integers (one larger, one smaller) that are closest to its square root and explain your reasoning.

Standard 2
Computation

Students compute with rational numbers expressed in a variety of forms. They solve problems involving ratios, proportions, and percentages.

8PA.2.1 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.

Example: $-3.4 + 2.8 \times 5.75 = ?$, $1\frac{4}{5} + -\frac{3}{8} \times 2\frac{2}{9} = ?$, $81.04 \div 17.4 - 2.79 = ?$

8PA.2.2 Solve problems by computing simple and compound interest.

Example: You leave $100 in each of three bank accounts paying 5% interest per year. One account pays simple interest, one pays interest compounded annually, and the third pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after one year, two years, three years, ten years, and twenty years. Compare the results in the three accounts and explain how compounding affects the balance in each account.

8PA.2.3 Use estimation techniques to decide whether answers to computations on a calculator are reasonable.

Example: Your friend uses his calculator to find 15% of $25 and gets $375. Without solving, explain why you think the answer is wrong.

8PA.2.4 Use mental arithmetic to compute with common fractions, decimals, powers, and percents.

Example: Find 20% of $50 without using pencil and paper.

Standard 3
Algebra and Functions

Students solve simple linear equations and inequalities. They interpret and evaluate expressions involving integer powers. They graph and interpret functions. They understand the concepts of slope and rate.

8PA.3.1 Write and solve linear equations and inequalities in one variable, interpret the solution or solutions in their context, and verify the reasonableness of the results.

Example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be least $100. Write an inequality for the number of sales you need to make, solve it, and check that your answer is reasonable.

8PA.3.2 Interpret positive integer powers as repeated multiplication and negative integer powers as repeated division or multiplication by the multiplicative inverse.

Example: Use a spreadsheet to explore the relationship between positive and negative integer powers by making a table of values of powers of 3, from 3^{-5} to 3^{5}.

8PA.3.3 Use the correct order of operations to find the values of algebraic expressions involving powers.

Example: Use a scientific calculator to find the value of $3(2x + 5)^2$ when $x = -35$.

8PA.3.4 Identify and graph linear functions and identify lines with positive and negative slope.

Example: Draw the graphs of $y = 2x - 1$, $y = 3x - 1$, $y = -2x - 1$, and $y = -3x - 1$. Find the slope of each graph. What do you notice?

8PA.3.5 Find the slope of a linear function given the equation and write the equation of a line given the slope and any point on the line.

Example: Write an equation of the line with slope 2 and *y*-intercept -4.

8PA.3.6 Demonstrate an understanding of rate as a measure of one quantity with respect to another quantity.

Example: A car moving at a constant speed travels 90 km in 2 hours, 135 km in 3 hours, 180 km in 4 hours, etc. Draw a graph of distance as a function of time and find the slope of the graph. Explain what the slope tells you about the movement of the car.

8PA.3.7 Demonstrate an understanding of the relationships among tables, equations, verbal expressions, and graphs of linear functions.

Example: Write an equation that represents the verbal description: "the perimeter of a square is four times the side length." Construct a table of values for this relationship and draw its graph.

Standard 4
Geometry

Students deepen their understanding of plane and solid geometric shapes and properties by constructing shapes that meet given conditions, by identifying attributes of shapes, and by applying geometric concepts to solve problems.

8PA.4.1 Identify and describe basic properties of geometric shapes: altitudes, diagonals, angle and perpendicular bisectors, central angles, radii, diameters, and chords.

Example: Describe a central angle of a circle in words and draw a diagram.

8PA.4.2 Perform simple constructions, such as bisectors of segments and angles, copies of segments and angles, and perpendicular segments. Describe and justify the constructions.

Example: Explain the procedures used to construct the three angle bisectors of a triangle.

8PA.4.3 Identify properties of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more figures intersect in a plane or in space.

Example: Find two lines in your classroom that are not parallel, yet do not meet.

8PA.4.4 Draw the translation, rotation, reflection, and dilation (stretches and shrinks) of shapes.

Example: Draw a rectangle and slide it 3 inches horizontally across your page. Then rotate it clockwise through 90º about the bottom left vertex. Draw the new rectangle in a different color.

8PA.4.5 Use the Pythagorean Theorem and its converse to solve problems in two and three dimensions.

Example: Measure the dimensions of a shoe box and calculate the length of a diagonal from the top right to the bottom left of the box. Measure with a string to evaluate your solution.

Standard 5
Measurement

Students convert between units of measure and use rates and scale factors to solve problems. They compute the perimeter, area, and volume of geometric objects. They investigate how perimeter, area, and volume are affected by changes of scale.

8PA.5.1 Convert common measurements for length, area, volume, weight, capacity, and time to equivalent measurements within the same system.

Example: The area of a hall is 40 square yards. What is the area in square feet?

8PA.5.2 Solve simple problems involving rates and derived measurements for attributes such as velocity and density.

Example: A car travels at 60 mph for 20 minutes. How far does it travel? What units are appropriate for distance? Explain your answer.

8PA.5.3 Solve problems involving scale factors, area, and volume using ratio and proportion.

Example: Calculate the volume and surface area of cubes with side 1 cm, 2 cm, 3 cm, etc. Make a table of your results and describe any patterns in the table.

8PA.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, prisms, cylinders, spheres, cones, and pyramids.

Example: Find the total surface area of a right triangular prism 14 feet high and with a base that measures 8 feet by 6 feet.

8PA.5.5 Estimate and compute the area of irregular two-dimensional shapes and the volume of irregular three-dimensional objects by breaking them down into more basic geometric objects.

Example: Find the volume of a dog house that has a rectangular space that is 3 ft by 2 ft by 5 ft and has a triangular roof that is 1.5 ft higher than the walls of the house.

Standard 6
Data Analysis and Probability

Students collect, organize, represent, and interpret relationships in data sets that have one or more variables. They determine probabilities and use them to make predictions about events.

8PA.6.1 Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim.

Example: A study shows that teenagers who use a certain brand of toothpaste have fewer cavities than those using other brands. Describe how you can test this claim in your school.

8PA.6.2 Identify different methods of selecting samples, analyzing the strengths and weaknesses of each method, and the possible bias in a sample or display.

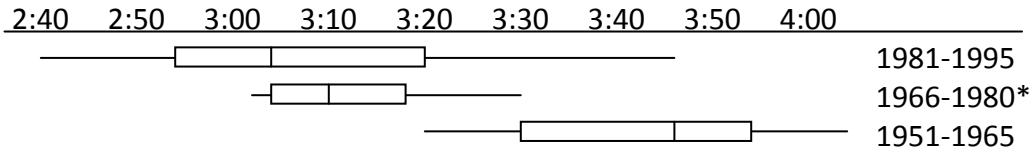
Example: Describe possible bias in the following survey: A local television station has a daily call-in poll. Viewers of the morning and noon newscasts are asked to call one telephone number to answer “yes” and a different telephone number to answer “no.” The results are reported on the six-o’clock newscast.

8PA.6.3 Understand the meaning of, and be able to identify or compute the minimum value, the lower quartile, the median, the upper quartile, the interquartile range, and the maximum value of a data set.

Example: Arrange a set of test scores in increasing order and find the lowest and highest scores, the median, and the upper and lower quartiles.

8PA.6.4 Analyze, interpret, and display single- and two-variable data in appropriate bar, line, and circle graphs; stem-and-leaf plots; and box-and-whisker plots and explain which types of display are appropriate for various data sets.

Example: The box-and-whisker plots below show winning times (hours:minutes) for the Indianapolis 500 race in selected years:



*Except 1967, 1973, 1975, and 1976.

In the years from 1951-1965, the slowest time was 3 h 57 min. Explain how the slowest time changed through the years 1951-1995. How did winning times change during that period? How did the median times change in the same period?

8PA.6.5 Represent two-variable data with a scatter plot on the coordinate plane and describe how the data points are distributed. If the pattern appears to be linear, draw a line that appears to best fit the data and write the equation of that line.

Example: Survey some of the students at each grade level in your school, asking them how much time they spend on homework. Plot the grade level and time of each student as a point (grade, time) on a scatter diagram. Describe and justify any relationship between grade and time spent on homework.

8PA.6.6 Understand and recognize equally likely events.

Example: When you roll a number cube, what is the probability that the number on the top face will be a 6? Explain your answer.

8PA.6.7 Find the number of possible arrangements of several objects by using the Basic Counting Principle.

Example: You are planning to place four pictures in a line on a shelf. Find the number of ways you can arrange the four pictures.

Standard 7
Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

8PA.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "For computers, binary numbers are great because they are simple to work with and they use just two values of voltage, magnetism, or other signal. This makes hardware easier to design and more noise resistant. Binary numbers let you represent any amount you want using just two digits: 0 and 1. The number you get when you count ten objects is written 1010. In expanded notation, this is $1 \leq 2^3 + 0 \leq 2^2 + 1 \leq 2^1 + 0 \leq 2^0$. Write the number for thirteen in the binary (base 2) system." Decide to make an organized list.

8PA.7.2 Make and justify mathematical conjectures based on a general description of a mathematical question or problem.

Example: In the first example, if you have only two symbols, 0 and 1, then one object: 1, two objects: 10, three objects: 11, four objects: 100. Predict the symbol for five objects.

8PA.7.3 Decide when and how to divide a problem into simpler parts.

Example: In the first example, write expanded notation for the number five in base 2; begin with the fact that 5 = 4 + 1.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

8PA.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, write the first five numbers in base 2 notation and look for a pattern.

8PA.7.5 Make and test conjectures using inductive reasoning.

Example: In the first example, predict the base 2 notation for six objects, then use expanded notation to test your prediction.

8PA.7.6 Express solutions clearly and logically using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, explain how you will find the base two notation for thirteen objects.

8PA.7.7 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Measure the length and width of a basketball court. Use the Pythagorean Theorem to calculate the length of a diagonal. How accurately should you give your answer?

8PA.7.8 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Use a calculator to find the cube of 15. Check your answer by finding the cubes of 10 and 20.

8PA.7.9 Use graphing to estimate solutions and check the estimates with analytic approaches.

Example: Draw the straight line $x + y = 10$. Use this to estimate solutions of the inequality $x + y > 10$ by testing points on each side of the line.

8PA.7.10 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, list the first thirteen numbers in base 2 notation. Use patterns or expanded notation to confirm your list.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

8PA.7.11 Decide whether a solution is reasonable in the context of the original situation.

Example: In the basketball court example, does the accuracy of your answer depend on your initial measuring?

8PA.7.12 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: In the first example, use your list of base 2 numbers to add numbers in base 2. Explain exactly how your addition process works.