

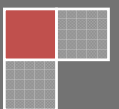
2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

Algebra I Standards

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
 - Learn to think critically, logically, ethically, and analytically
 - Learn to express ideas orally and in writing using correct mathematical terminology
 - Learn to apply the techniques of mathematics to real world situations
 - Understand that mathematics is important to function in today's world
 - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

ALGEBRA I

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

The Diocese of Wilmington has established the following mathematics Standards to clarify for teachers, students, and parents the knowledge, understanding, and skills students should attain in ALGEBRA I:

Standard 1 — Operations with Real Numbers

Students deepen their understanding of real numbers by comparing expressions involving square roots and exponents. They use the properties of real numbers to simplify algebraic formulas, and they convert between different measurement units using dimensional analysis.

Standard 2 — Linear Equations and Inequalities

Students solve linear equations to find the value of the variable and they rearrange formulas. They solve linear inequalities by using order properties of the real numbers, and they solve word problems involving linear equations, inequalities, and formulas.

Standard 3 — Relations and Functions

Students draw and interpret graphs of relations. They understand the concept of a function, find domains and ranges, and link equations to functions.

Standard 4 — Graphing Linear Equations and Inequalities

Students draw graphs of straight lines and relate their equations to their slopes and intercepts. They model situations with linear equations and use them to make predictions, and they graph linear inequalities in two variables.

Standard 5 — Pairs of Linear Equations and Inequalities

Students solve pairs of linear equations in two variables using both graphs and algebraic methods. They use pairs of linear equations to solve word problems, and they use graphs to solve pairs of linear inequalities in two variables.

Standard 6 — Polynomials

Students operate with polynomials, adding, subtracting, multiplying, dividing, and raising to powers. They find factors of polynomials, learning special techniques for factoring quadratics. They understand the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial.

Standard 7 — Algebraic Fractions

Students simplify algebraic fractions, using what they have learned about factoring polynomials. They solve algebraic proportions.

Standard 8 — Quadratic, Cubic, and Radical Equations

Students draw graphs of quadratic, cubic, and rational functions. They derive the formula for solving quadratic equations and solve these equations by using the formula, by factoring, and by completing the square. They also solve equations that contain radical expressions and use graphing calculators to find approximate solutions of equations.

Standard 9 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false

Students should also develop the following learning skills by Grade 12 that are integrated throughout the National Council of Teachers of Mathematics (NCTM) Standards:

Communication

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Representation

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Standard 1

Operations With Real Numbers

Students simplify and compare expressions. They use rational exponents and simplify square roots.

A.1.1 Compare real number expressions.

Example: Which is larger: 2^3 or $\sqrt{49}$?

A.1.2 Simplify square roots using factors.

Example: Explain why $\sqrt{48} = 4\sqrt{3}$.

A.1.3 Understand and use the distributive, associative, and commutative properties.

Example: Simplify $(6x^2 - 5x + 1) - 2(x^2 + 3x - 4)$ by removing the parentheses and rearranging. Explain why you can carry out each step.

A.1.4 Use the laws of exponents for rational exponents.

Example: Simplify $25^{\frac{3}{2}}$.

A.1.5 Use dimensional (unit) analysis to organize conversions and computations.

Example: Convert 5 miles per hour to feet per second: $\frac{5 \text{ mi}}{1 \text{ hr}} \leq \frac{1 \text{ hr}}{3600 \text{ sec}} \leq \frac{5280 \text{ ft}}{1 \text{ mi}} \approx 7.3$ ft/sec.

Standard 2

Linear Equations and Inequalities

Students solve linear equations and inequalities in one variable. They solve word problems that involve linear equations, inequalities, or formulas.

A.2.1 Solve linear equations.

Example: Solve the equation $7a + 2 = 5a - 3a + 8$.

A.2.2 Solve equations and formulas for a specified variable.

Example: Solve the equation $q = 4p - 11$ for p .

A.2.3 Find solution sets of linear inequalities when possible numbers are given for the variable.

Example: Solve the inequality $6x - 3 > 10$ for x in the set $\{0, 1, 2, 3, 4\}$.

A.2.4 Solve linear inequalities using properties of order.

Example: Solve the inequality $8x - 7 \leq 2x + 5$, explaining each step in your solution.

A.2.5 Solve combined linear inequalities.

Example: Solve the inequalities $-7 < 3x + 5 < 11$.

A.2.6 Solve word problems that involve linear equations, formulas, and inequalities.

Example: You are selling tickets for a play that cost \$3 each. You want to sell at least \$50 worth. Write and solve an inequality for the number of tickets you must sell.

Standard 3 Relations and Functions

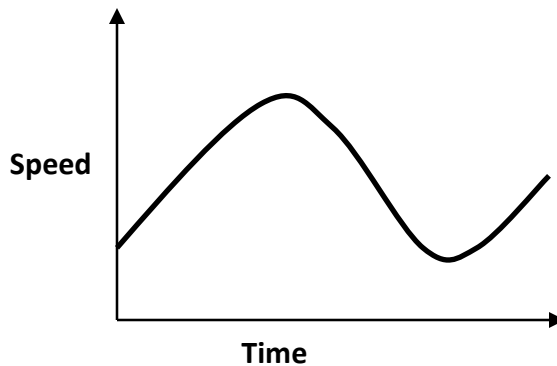
Students sketch and interpret graphs representing given situations. They understand the concept of a function and analyze the graphs of functions.

A.3.1 Sketch a reasonable graph for a given relationship.

Example: Sketch a reasonable graph for a person's height from age 0 to 25.

A.3.2 Interpret a graph representing a given situation.

Example: Jessica is riding a bicycle. The graph below shows her speed as it relates to the time she has spent riding. Describe what might have happened to account for such a graph.



A.3.3 Understand the concept of a function, decide if a given relation is a function, and link equations to functions.

Example: Use either paper or a spreadsheet to generate a list of values for x and y in $y = x^2$. Based on your data, make a conjecture about whether or not this relation is a function. Explain your reasoning.

A.3.4 Find the domain and range of a relation.

Example: Based on the list of values from the last example, what are the domain and range of $y = x^2$?

Standard 4

Graphing Linear Equations and Inequalities

Students graph linear equations and inequalities in two variables. They write equations of lines and find and use the slope and y-intercept of lines. They use linear equations to model real data.

A.4.1 Graph a linear equation.

Example: Graph the equation $3x - y = 2$.

A.4.2 Find the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

A.4.3 Write the equation of a line in slope-intercept form. Understand how the slope and y-intercept of the graph are related to the equation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

A.4.4 Write the equation of a line given appropriate information.

Example: Find an equation of the line through the points (1, 4) and (3, 10), then find an equation of the line through the point (1, 4) perpendicular to the first line.

A.4.5 Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. A graph of time and distance shows that the relation is approximately linear. Write the equation of the line that fits your data. Predict the time for a journey of 50 miles. What does the slope represent?

A.4.6 Graph a linear inequality in two variables.

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

Standard 5

Pairs of Linear Equations and Inequalities

Students solve pairs of linear equations using graphs and using algebra. They solve pairs of linear inequalities using graphs. They solve word problems involving pairs of linear equations.

A.5.1 Use a graph to estimate the solution of a pair of linear equations in two variables.

Example: Graph the equations $3y - x = 0$ and $2x + 4y = 15$ to find where the lines intersect.

A.5.2 Use a graph to find the solution set of a pair of linear inequalities in two variables.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

A.5.3 Understand and use the substitution method to solve a pair of linear equations in two variables.

Example: Solve the equations $y = 2x$ and $2x + 3y = 12$ by substitution.

A.5.4 Understand and use the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use subtraction to solve the equations: $3x + 4y = 11$ and $3x + 2y = 7$.

A.5.5 Understand and use multiplication with the addition or subtraction method to solve a pair of linear equations in two variables.

Example: Use multiplication with the subtraction method to solve the equations: $x + 4y = 16$ and $3x + 2y = -3$.

A.5.6 Use pairs of linear equations to solve word problems.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10,000$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

Standard 6 Polynomials

Students add, subtract, multiply, and divide polynomials. They also factor quadratics.

A.6.1 Add and subtract polynomials.

Example: Simplify $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$.

A.6.2 Multiply and divide monomials.

Example: Simplify $a^2b^5 \div ab^2$.

A.6.3 Find powers and roots of monomials (only when the answer has an integer exponent).

Example: Find the square root of a^2b^6 .

A.6.4 Multiply polynomials.

Example: Multiply $(n + 2)(4n - 5)$.

A.6.5 Divide polynomials by monomials.

Example: Divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

A.6.6 Find a common monomial factor in a polynomial.

Example: Factor $36xy^2 + 18xy^4 - 12x^2y^4$.

A.6.7 Factor the difference of two squares and other quadratics.

Example: Factor $4x^2 - 25$ and $2x^2 - 7x + 3$.

A.6.8 Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x -intercepts of a graph, and the factors of a polynomial expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x -intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

Standard 7

Algebraic Fractions

Students simplify algebraic ratios and solve algebraic proportions.

A.7.1 Simplify algebraic ratios.

Example: Simplify $\frac{x^2 - 16}{x^2 + 4x}$.

A.7.2 Solve algebraic proportions.

Example: Create a tutorial to be posted to the school's Web site to instruct beginning students in the steps involved in solving an algebraic proportion. Use $\frac{x+5}{4} = \frac{3x+5}{7}$ as an example.

Standard 8

Quadratic, Cubic, and Radical Equations

Students graph and solve quadratic and radical equations. They graph cubic equations.

A.8.1 Graph quadratic, cubic, and radical equations.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

A.8.2 Solve quadratic equations by factoring.

Example: Solve the equation $x^2 - 3x + 2 = 0$ by factoring.

A.8.3 Solve quadratic equations in which a perfect square equals a constant.

Example: Solve the equation $(x - 7)^2 = 64$.

A.8.4 Complete the square to solve quadratic equations.

Example: Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.

A.8.5 Derive the quadratic formula by completing the square.

Example: Prove that the equation $ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

A.8.6 Solve quadratic equations using the quadratic formula.

Example: Solve the equation $x^2 - 7x + 9 = 0$.

A.8.7 Use quadratic equations to solve word problems.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

A.8.8 Solve equations that contain radical expressions.

Example: Solve the equation $\sqrt{x+6} = x$.

A.8.9 Use graphing technology to find approximate solutions of quadratic and cubic equations.

Example: Use a graphing calculator to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth.

Standard 9

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems.

A.9.1 Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards.

Example: Fran has scored 16, 23, and 30 points in her last three games. How many points must she score in the next game so that her four-game average does not fall below 20 points?

A.9.2 Decide whether a solution is reasonable in the context of the original situation.

Example: John says the answer to the problem in the first example is 10 points. Is his answer reasonable? Why or why not?

Students develop and evaluate mathematical arguments and proofs.

A.9.3 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Given an argument (such as $3x + 7 > 5x + 1$, and therefore $-2x > -6$, and therefore $x > 3$), provide a visual presentation of a step-by-step check, highlighting any errors in the argument.

A.9.4 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: Try “solving” the equations $x + 3y = 5$ and $5x + 15y = 25$ simultaneously. Explain what went wrong.

A.9.5 Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, or inequalities).

Example: Is the statement $x^2 - 5x + 2 = x^2 + 5x + 2$ true for all x , for some x , or for no x ? Explain your answer.

A.9.6 Distinguish between inductive and deductive reasoning, identifying and providing examples of each.

Example: What type of reasoning are you using when you look for a pattern?

A.9.7 Identify the hypothesis and conclusion in a logical deduction.

Example: What is the hypothesis and conclusion in this argument: If there is a number x such that $2x + 1 = 7$, then $x = 3$?

A.9.8 Use counterexamples to show that statements are false, recognizing that a single counter example is sufficient to prove a general statement false.

Example: Use the demonstration-graphing calculator on an overhead projector to produce an example showing that this statement is false: all quadratic equations have two different solutions.