2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

Grade 7 Standards

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibility.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
    - Learn to think critically, logically, ethically, and analytically
    - Learn to express ideas orally and in writing using correct mathematical terminology
    - Learn to apply the techniques of mathematics to real world situations
    - Understand that mathematics is important to function in today's world
    - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

GRADE SEVEN

***As life-long learners**, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.*

The Diocese of Wilmington has established the following mathematics Standards to clarify for teachers, students, and parents the knowledge, understanding, and skills students should attain in GRADE SEVEN:

Standard 1 — Number Sense

Developing number sense is the foundation of mathematics. Students extend this understanding to include irrational numbers, such as π and the square root of 2. They compare and order rational and irrational numbers and convert terminating decimals into fractions. They also use exponents to write whole numbers in scientific notation and to write the prime factorizations of numbers.

Standard 2 — Computation

Mastering computational skills is vital. Students add, subtract, multiply, and divide integers, fractions, and decimals. They solve problems using percentages, including calculating discounts, markups, and commissions. They use mental arithmetic to compute with simple fractions, decimals, and powers.

Standard 3 — Algebra and Functions

Understanding patterns, rules, and symbols is the foundation of Algebra. Students at this level use variables and other symbols to translate verbal descriptions into equations and formulas. They write and solve linear equations and inequalities, and write and use formulas to solve problems. They also use properties of the rational numbers to evaluate and simplify algebraic expressions, and they further extend their understanding of graphs by investigating rates of change for linear and nonlinear functions and by developing and using the concept of the slope of a straight line

Standard 4 — Geometry

Exploring shapes and developing spatial sense is the basis of Geometry. Students link geometry to coordinate graphs, using them to plot shapes, calculate lengths and areas, and find images under transformations. They understand the Pythagorean Theorem and use it to find lengths in right triangles. They also construct nets (two-dimensional patterns) for three-dimensional objects, such as prisms, pyramids, cylinders, and cones.

Standard 5 — Measurement

Using measurement is essential to everyday life. Students measure in order to compare lengths, areas, volumes, weights, times, temperatures, etc. They develop the concept of similarity and use it to make scale drawings and scale models and to solve problems relating to these drawings and models. They find areas and perimeters of two-dimensional shapes and volumes and surface areas of three-dimensional shapes, including irregular shapes made up of more basic shapes.

Standard 6 — Data Analysis and Probability

Analyzing data is a fundamental life skill. Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they learn how to display data in bar, line, and circle graphs and in stem-and-leaf plots. They analyze data displays to find whether they are misleading and analyze the wording of survey questions to tell whether these could influence the results. They find the probability of disjoint events. They also find the number of arrangements of objects using a tree diagram.

Standard 7 — Problem Solving

Solving problems is the practical application of mathematics. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with irrational numbers, analyzing graphs, or finding surface areas, for example, students move from simple ideas to more complex ones by taking logical steps that build a better understanding of mathematics.

Students should also develop the following learning skills by Grade 12 that are integrated throughout the National Council of Teachers of Mathematics (NCTM) Standards:

Communication

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and

produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Representation

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Standard 1
Number Sense

Students understand and use scientific notation and square roots. They convert between fractions and decimals.

7.1.1 Read, write, compare, and solve problems using whole numbers in scientific notation.

Example: Write 300,000 in scientific notation.

7.1.2 Compare and order rational and common irrational numbers and place them on a number line.

Example: Place in order: -2, $\frac{5}{8}$, -2.45, 0.9, π, $-1\frac{3}{4}$.

7.1.3 Identify rational and common irrational numbers from a list.

Example: Name all the irrational numbers in the list: -2, $\frac{5}{8}$, -2.45, 0.9, π, $-1\frac{3}{4}$.

7.1.4 Understand and compute whole number powers of whole numbers.

Example: $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = ?$

7.1.5 Find the prime factorization of whole numbers and write the results using exponents.

Example: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

7.1.6 Understand and apply the concept of square root.

Example: Explain how you can find the length of the hypotenuse of a right triangle with legs that measure 5 cm and 12 cm.

7.1.7 Convert terminating decimals into reduced fractions.

Example: Write 0.95 as a fraction.

Standard 2
Computation

Students solve problems involving integers, fractions, decimals, ratios, and percentages.

7.2.1 Solve addition, subtraction, multiplication, and division problems that use integers, fractions, decimals, and combinations of the four operations.

Example: The temperature one day is 5º. It then falls by 3º each day for 4 days and, after that, rises by 2º each day for 3 days. What is the temperature on the last day? Explain your method.

7.2.2 Calculate the percentage increase and decrease of a quantity.

Example: The population of a country was 36 million in 1990 and it rose to 41.4 million during the 1990s. What was the percentage increase in the population?

7.2.3 Solve problems that involve discounts, markups, sales tax, and commissions.

Example: A merchant buys CDs for $11 wholesale and marks up the price by 35%. What is the retail price?

7.2.4 Use estimation to decide whether answers are reasonable in problems involving fractions and decimals.

Example: Your friend says that $3\frac{3}{8} \times 2\frac{2}{9} = 10$. Without solving, explain why you think the answer is wrong.

7,2.5 Use mental arithmetic to compute with simple fractions, decimals, and powers.

Example: Find 3^4 without using pencil and paper.

Standard 3
Algebra and Functions

Students express quantitative relationships using algebraic terminology, expressions, equations, inequalities, and graphs.

7.3.1 Use variables and appropriate operations to write an expression, a formula, an equation, or an inequality that represents a verbal description.

Example: Use symbols to write the inequality: 5 less than twice the number is greater than 42.

7.3.2 Write and solve two-step linear equations and inequalities in one variable and check the answers.

Example: Solve the equation $4x - 7 = 12$ and check your answer in the original equation.

7.3.3 Use correct algebraic terminology, such as variable, equation, term, coefficient, inequality, expression, and constant.

Example: Name the variable, terms, and coefficient in this equation: $7x + 4 = 67$.

7.3.4 Evaluate numerical expressions and simplify algebraic expressions by applying the correct order of operations and the properties of rational numbers (e.g., identity, inverse, commutative, associative, distributive properties). Justify each step in the process.

Example: Simplify $3(4x + 5x - 1) + 2(x + 3)$ by removing the parentheses and rearranging. Explain each step you take.

7.3.5 Solve an equation or formula with two variables for a particular variable.

Example: Solve the formula $C = 2\pi r$ for r.

7.3.6 Define slope as vertical change per unit of horizontal change and recognize that a straight line has constant slope or rate of change.

Example: Examine a table of values and make a conjecture about whether the table represents a linear function.

7.3.7 Find the slope of a line from its graph.

Example: Draw the graph of $y = 2x - 1$. Choose two points on the graph and divide the change in y-value by the change in x-value. Repeat this for other pairs of points on the graph. What do you notice?

7.3.8 Draw the graph of a line given the slope and one point on the line, or two points on the line.

Example: Draw the graph of the equation with slope of 3 and passing through the point with coordinates (0, -2).

7.3.9 Identify functions as linear or nonlinear and examine their characteristics in tables, graphs, and equations.

Example: A plant is growing taller according to the formula $H = 2d + 3$, where H is the height after d days. Draw the graph of this function and explain what the point where it meets the vertical axis represents. Is this graph linear or nonlinear?

7.3.10 Identify and describe situations with constant or varying rates of change and know that a constant rate of change describes a linear function.

Example: In the last example, how will the graph be different if the plant's speed of growth changes?

Standard 4
Geometry

Students deepen their understanding of plane and solid geometric shapes by constructing shapes that meet given conditions and by identifying attributes of shapes.

7.4.1 Understand coordinate graphs and use them to plot simple shapes, find lengths and areas related to the shapes, and find images under translations (slides), rotations (turns), and reflections (flips).

Example: Draw the triangle with vertices (0, 0), (3, 0), and (0, 4). Find the lengths of the sides and the area of the triangle. Translate (slide) the triangle 2 units to the right. What are the coordinates of the new triangle?

7.4.2 Understand that transformations such as translations, reflections, and rotations preserve the length of segments, and that figures resulting from these translations are congruent to the original figures.

Example: In the last example, find the lengths of the sides and the area of the new triangle. Discuss your results.

7.4.3 Know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and the lengths of other line segments. Use direct measurement to test conjectures about triangles.

Example: Use the length and width of your classroom to calculate the distance across the room diagonally. Check by measuring.

7.4.4 Construct nets for three-dimensional objects, such as right prisms, pyramids, cylinders, and cones.

Example: Draw a rectangle and two circles that will fit together to make a cylinder.

Standard 5
Measurement

Students compare units of measure and use similarity to solve problems. They compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less regular objects.

7.5.1 Compare lengths, areas, volumes, weights, capacities, times, and temperatures within measurement systems.

Example: The area of a carpet is 200 sq. ft. How many square yards of carpet must be purchased to cover the floor? Explain your method.

7.5.2 Use experimentation and modeling to visualize similarity problems. Solve problems using similarity.

Example: At a certain time, the shadow of your school building is 36 feet long. At the same time, the shadow of a yardstick held vertically is 4 feet long. How high is the school building?

7.5.3 Read and create drawings made to scale, construct scale models, and solve problems related to scale.

Example: On a plan of your school, your classroom is 5 cm long and 3 cm wide. The actual classroom is 10 m long. How wide is it? Explain your answer.

7.5.4 Use formulas for finding the perimeter and area of basic two-dimensional shapes and the surface area and volume of basic three-dimensional shapes, including rectangles, parallelograms, trapezoids, triangles, circles, right prisms, and cylinders.

Example: Find the surface area of a cylindrical can 15 cm high and with a diameter of 8 cm.

7.5.5 Estimate and compute the area of more complex or irregular two-dimensional shapes by dividing them into more basic shapes.

Example: A room to be carpeted is a rectangle 5 m $\times$ 4 m. A semicircular fireplace of diameter 1.5 m takes up some of the floor space. Find the area to be carpeted.

7.5.6 Use objects and geometry modeling tools to compute the surface area of the faces and the volume of a three-dimensional object built from rectangular solids.

Example: Build a model of an apartment building with blocks. Find its volume and total surface area.

Standard 6
Data Analysis and Probability

Students collect, organize, and represent data sets and identify relationships among variables within a data set. They determine probabilities and use them to make predictions about events.

7.6.1 Analyze, interpret, and display data in appropriately: graphs (bar, line, histogram, and circle) or plots (stem-and-leaf, box-and-whisker) and justify the choice of display.

Example: You survey the students in your school to find which of three designs for a magazine cover they prefer. To display the results, which would be more appropriate: a bar chart or a circle graph? Explain your answer.

7.6.2 Make predictions from statistical data.

Example: Record the temperature and weather conditions (sunny, cloudy, or rainy) at 1 p.m. each day for two weeks. In the third week, use your results to predict the temperature from the weather conditions.

7.6.3 Describe how additional data, particularly outliers, added to a data set may affect the mean, median, and mode.

Example: You measure the heights of the students in your grade on a day when the basketball team is playing an away game. Later you measure the players on the team and include them in your data. What kind of effect will including the team have on the mean, median, and mode? Explain your answer.

7.6.4 Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.

Example: On a bar graph of a company's sales, it appears that sales have more than doubled since last year. Then you notice that the vertical axis starts at $5 million and can see that sales have in fact increased from $5.5 million to $6.2 million.

7.6.5 Know that if P is the probability of an event occurring, then $1 - P$ is the probability of that event not occurring.

Example: The weather forecast says that the probability of rain today is 0.3. What is the probability that it won't rain?

7.6.6 Understand that the probability of either one or the other of two disjoint events occurring is the sum of the two individual probabilities.

Example: Find the probability of rolling 9 with two number cubes. Also find the probability of rolling 10. What is the probability of rolling 9 or 10?

7.6.7 Find the number of possible arrangements of several objects using a tree diagram.

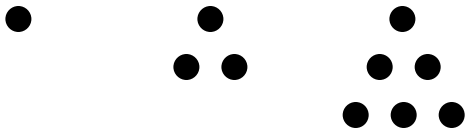
Example: A state's license plates contain 6 digits and one letter. How many different license plates can be made if the letter must always be in the third position and the first digit cannot be a zero?

Standard 7
Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

7.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "The first three triangular numbers are shown in the diagram below. Find an expression to calculate the *n*th triangular number."



1 3 6

Decide to look for patterns.

7.7.2 Make and justify mathematical conjectures based on a general description of a mathematical question or problem.

Example: In the first example, notice that three dots make an equilateral triangle for the number 3 and six dots make the next equilateral triangle.

7.7.2 Decide when and how to divide a problem into simpler parts.

Example: In the first example, decide to make a diagram for the fourth and fifth triangular numbers.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

7.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, list the differences between any two triangular numbers.

7.7.5 Make and test conjectures by using inductive reasoning.

Example: In the first example, predict the difference between the fifth and sixth numbers and use this to predict the sixth triangular number. Make a diagram to test your conjecture.

7.7.6 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.
Example: In the first example, use words, numbers, and tables to summarize your work with triangular numbers.

7.7.7 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Calculate the amount of aluminum needed to make a can with diameter 10 cm that is 15 cm high and 1 mm thick. Take π as 3.14 and give your answer to appropriate accuracy.

7.7.8 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Measure the dimensions of a swimming pool to find its volume. Estimate an answer by working with an average depth.

7.7.9 Use graphing to estimate solutions and check the estimates with analytic approaches.

Example: Find the intersecting point of the straight lines
$y = 2x + 3$ and $x + y = 10$. Confirm your answer by checking it in the equations.

7.7.10 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check that your later results fit with your earlier ones. If they do not, repeat the calculations to make sure.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

7.7.11 Decide whether a solution is reasonable in the context of the original situation.

Example: In the first example, calculate the 10th triangular number and draw the triangle of dots that goes with it.

7.7.12 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Use your method from the first example to investigate pentagonal numbers.