2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

Grade 6 Standards

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibility.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
    - Learn to think critically, logically, ethically, and analytically
    - Learn to express ideas orally and in writing using correct mathematical terminology
    - Learn to apply the techniques of mathematics to real world situations
    - Understand that mathematics is important to function in today's world
    - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

GRADE SIX

***As life-long learners**, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.*

The Diocese of Wilmington has established the following mathematics Standards to clarify for teachers, students, and parents the knowledge, understanding, and skills students should attain in GRADE SIX:

Standard 1 — Number Sense
Developing number sense is the foundation of mathematics. Students continue to develop their understanding of the relationship between fractions and decimals. They extend the number system to include negative numbers. They also relate percentages to fractions and decimals and begin learning how to use ratios. They find multiples and factors of whole numbers, using the multiples and factors to solve problems involving fractions.

Standard 2 — Computation
Mastering computational skills is vital. Students add, subtract, multiply, and divide fractions, decimals, and both positive and negative integers. They solve problems using ratios, proportions, and percentages, including calculating discount and interest. They use mental arithmetic to add or subtract simple fractions and decimals.

Standard 3 — Algebra and Functions
Understanding patterns, rules, and symbols is the foundation of Algebra. Students at this level write and solve simple equations and inequalities, and write and use formulas to solve problems. They use parentheses in more complex expressions to show the order of operations. They also extend graphs of straight lines to include negative values.

Standard 4 — Geometry
Exploring shapes and developing spatial sense is the basis of Geometry. Students draw special types of angles and use them to solve problems. They find and use the sum of the angles of a triangle and of a quadrilateral. They identify shapes that are similar (the same shape but not necessarily the same size). They draw reflections and translations of shapes, and they also draw two-dimensional views of three-dimensional shapes.

Standard 5 — Measurement
Using measurement is essential to everyday life. Students measure in order to compare lengths, areas, volumes, weights, times, temperatures, etc. They learn about the number π and use it to calculate the circumference and area of circles. They construct models, find the volume and surface area of prisms and cylinders, and they convert temperatures between Celsius and Fahrenheit.

Standard 6 — Data Analysis and Probability
Analyzing data is a fundamental life skill. Data are all around us — in newspapers and magazines, in television news and commercials, in quality control for manufacturing — and students need to learn how to understand data. At this level, they learn how to display data in frequency tables and in stem-and-leaf plots. They compare the mean, median, and mode. They find probabilities for compound events and write them as fractions, decimals, and percentages. They also estimate the probabilities of future events.

Standard 7 — Problem Solving
Solving problems is the practical application of mathematics. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. As they develop their skills with negative numbers, calculating angles, or finding areas, for example, students move from simple to more complex ideas by taking logical steps that build a better understanding of mathematics.

Students should also develop the following learning skills by Grade 12 that are integrated throughout the National Council of Teachers of Mathematics (NCTM) Standards:

Communication
As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Reasoning and Proof
Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Representation

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Standard 1
Number Sense

Students compare and order positive and negative integers, decimals, fractions, and mixed numbers. They find multiples and factors.

6.1.1 Understand and apply the basic concept of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").

Example: The temperature this morning was -6º and now it is 3º. How much has the temperature risen? Explain your answer.

6.1.2 Interpret the absolute value of a number as the distance from zero on a number line and find the absolute value of real numbers.

Example: Use a number line to explain the absolute values of -3 and of 7.

6.1.3 Compare and represent on a number line: positive and negative integers, fractions, decimals (to hundredths), and mixed numbers.

Example: Find the positions on a number line of 3.56, -2.5, $1\frac{5}{6}$, and -4.

6.1.4 Convert between any two representations of numbers (fractions, decimals, and percents) without the use of a calculator.

Example: Write $\frac{5}{8}$ as a decimal and as a percent.

6.1.5 Recognize decimal equivalents for commonly used fractions without the use of a calculator.

Example: Know that $\frac{1}{3}$ = 0.333… , $\frac{1}{2}$ = 0.5, $\frac{2}{5}$ = 0.4, etc.

6.1.6 Use models to represent ratios.

Example: Divide 27 pencils to represent the ratio 4:5.

Standard 2
Computation

Students solve problems involving addition, subtraction, multiplication, and division of integers. They solve problems involving fractions, decimals, ratios, proportions, and percentages.

6.2.1 Add and subtract positive and negative integers.

Example: 17 + -4 = ?, -8 – 5 = ?.

6.2.2 Multiply and divide positive and negative integers.

Example: Continue the pattern: 3×2 = ?, 2×2 = ?, 1×2 = ?, 0×2 = ?, -1×2 = ?, -2×2 = ?, etc.

6.2.3 Find the least common multiple and the greatest common factor of whole numbers. Use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Example: Find the smallest number that both 12 and 18 divide into. How does this help you add the fractions $\frac{5}{12}$ and $\frac{7}{18}$?

6.2,4 Multiply and divide decimals.

Example: $3.265 \times 0.96 = ?$, $56.79 \div 2.4 = ?$.

6.2.5 Explain how to multiply and divide positive fractions and perform the calculations.

Example: Explain why $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \div \frac{16}{15} = \frac{2}{3}$.

6.2.6 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

Example: You want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door $27\frac{1}{2}$ inches wide. How far from each edge should you place the bar? Explain your method.

6.2.7 Interpret and use ratios to show the relative sizes of two quantities. Use the notations: *a/b*, *a* to *b*, *a:b*.

Example: A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time and use it to find how far the car will travel in 5 hours.

6.2.8 Understand proportions and use them to solve problems.

Example: Sam made 8 out of 24 free throws. Use a proportion to show how many free throws Sam would probably make out of 60 attempts.

6.2.9 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, sales tax, and tips.

Example: In a sale, everything is reduced by 20%. Find the sale price of a shirt whose pre-sale price was $30.

6.2.10 Use estimation to decide whether answers are reasonable in decimal problems.

Example: Your friend says that $56.79 \div 2.4 = 2.36625$. Without solving, explain why you think the answer is wrong.

6.2.11 Use mental arithmetic to add or subtract simple fractions and decimals.

Example: Subtract $\frac{1}{6}$ from $\frac{1}{2}$ without using pencil and paper.

Standard 3
Algebra and Functions

Students write verbal expressions and sentences as algebraic expressions and equations. They evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results. They investigate geometric relationships and describe them algebraically.

6.3.1 Write and solve one-step linear equations and inequalities in one variable and check the answers.

Example: The area of a rectangle is 143 cm^2 and the length is 11 cm. Write an equation to find the width of the rectangle and use it to solve the problem. Describe how you will check to be sure that your answer is correct.

6.3.2 Write and use formulas with up to three variables to solve problems.

Example: You have *P* dollars in a bank that gives *r*% simple interest per year. Write a formula for the amount of interest you will receive in one year. Use the formula to find the amount of interest on $80 at 6% per year for one year.

6.3.3 Interpret and evaluate expressions that use symbols of inclusion.

Example: Find the values of 10 – (7 – 3) and of 2(10 – 7)(3 + 1).

6.3.4 Write an expression using parentheses to indicate which operation to perform first.

Example: Write in symbols: add 19 and 34 and double the result.

6.3.5 Use variables in expressions describing geometric quantities.

Example: Let *l*, *w*, and *P* be the length, width, and perimeter of a rectangle. Write a formula for the perimeter in terms of the length and width.

6.3.6 Apply the correct order of operations and the properties of real numbers to evaluate numerical expressions. Justify each step in the process.

Example: Simplify 3(4 – 1) + 2. Explain your method.

6.3.7 Identify and graph ordered pairs in the four quadrants of the coordinate plane.

Example: Plot the points (3, -1), (-6, 2) and (9, -3). What do you notice?

6.3.8 Solve problems involving linear functions with integer values, create a table and graph the resulting ordered pairs of integers on a coordinate plane.

Example: A plant is 3 cm high the first time you measure it (on Day 0). Each day after that the plant grows by 2 cm.

6.3.9 Investigate how a change in one variable relates to a change in a second variable.

Example: In the last example, what do you notice about the shape of the graph?

Standard 4
Geometry

Students identify, describe, and classify the properties of plane and solid geometric shapes and the relationships between them.

6.4.1 Identify and draw vertical, adjacent, complementary, and supplementary angles and describe these angle relationships.

Example: Draw two parallel lines with another line across them. Identify all pairs of supplementary angles.

6.4.2 Use the properties of complementary, supplementary, and vertical angles to solve problems involving an unknown angle. Justify solutions.

Example: Find the size of the supplement to an angle that measures 122º. Explain how you obtain your answer.

6.4.3 Draw quadrilaterals and triangles from given information about them.

Example: Draw a quadrilateral with equal sides but no right angles.

6.4.4 Understand that the sum of the interior angles of any triangle is 180º and that the sum of the interior angles of any quadrilateral is 360º. Use this information to solve problems.

Example: Find the size of the third angle of a triangle with angles of 73º and 49º.

6.4.5 Identify and draw two-dimensional shapes that are similar.

Example: Draw a rectangle similar to a given rectangle, but twice the size.

6.4.6 Draw combinations of translations (slides), reflections (flips), and rotations (turns) of shapes.

Example: Draw a block letter, then slide it 3 inches horizontally across your page and then rotate it 90° on one corner.

6.4.7 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

Example: Draw a picture of an arrangement of rectangular blocks from the top, front, and right-hand side.

Standard 5
Measurement

Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems. They calculate with temperature and money, and choose appropriate units of measure in other areas.

6.5.1 Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.

Example: A triangular sheet of metal is about 1 foot across. Describe the units and tools you would use to measure its weight, its angles, and the lengths of its sides.

6.5.2 Understand and use larger units for measuring length by comparing miles to yards and kilometers to meters.

Example: How many meters are in a kilometer?

6.5.3 Understand and use larger units for measuring area by converting square units.

Example: How many square meters are in a square kilometer?

6.5.4 Understand the concept of the constant π as the ratio of the circumference to the diameter of a circle. Develop and use the formulas for the circumference and area of a circle.

Example: Measure the diameter and circumference of several circular objects. (Use string to find the circumference.) With a calculator, divide each circumference by its diameter. What do you notice about the results?

6.5.5 Know common estimates of π (3.14, $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles. Compare with actual measurements.

Example: Find the area of a circle of radius 15 cm.

6.5.6 Understand the concept of significant figures and round answers to an appropriate number of significant figures.

Example: You measure the diameter of a circle as 2.47 m and use the approximation 3.14 for π to calculate the circumference. Is it reasonable to give 7.7558 m as your answer? Why or why not?

6.5.7 Construct a cube and rectangular box from nets and use these patterns to compute the surface area of these objects.

Example: Find the total surface area of a shoe box with length 30 cm, width 15 cm, and height 10 cm.

6.5.8 Use strategies to find the surface area and volume of right prisms and cylinders using appropriate units.

Example: Find the volume of a cylindrical can 15 cm high and with a diameter of 8 cm.

6.5.9 Use a formula to convert temperatures between Celsius and Fahrenheit.

Example: What is the Celsius equivalent of 100ºF? Explain your method.

Standard 6
Data Analysis and Probability

Students compute and analyze statistical measures for data sets. They determine theoretical and experimental probabilities and use them to make predictions about events.

6.6.1 Organize and display single-variable data in appropriate graphs and stem-and-leaf plots, and explain which types of graphs are appropriate for various data sets.

Example: This stem-and-leaf diagram shows a set of test scores for your class:

Stem	Leaf
6	2 3 7
7	1 5 5 6 8 9
8	0 1 1 2 3 3 5 7 8 8
9	1 2 2 3 3 4

Find your score of 85 in this diagram. Are you closer to the top or the bottom of the class on this test?

6.6.2 Make frequency tables for numerical data, grouping the data in different ways to investigate how different groupings describe the data. Understand and find relative and cumulative frequency for a data set. Use histograms of the data and of the relative frequency distribution, and a broken line graph for cumulative frequency, to interpret the data.

Example: A bag contains pens in three colors. Nine students each draw a pen from the bag without looking, then record the results in the frequency table shown. Complete the column showing relative frequency.

Color	Frequency	Relative Frequency
Red	2	$\frac{2}{9}$
Green	4	
Purple	3	

6.6.3 Compare the mean, median, and mode for a set of data and explain which measure is most appropriate in a given context.

Example: Twenty students were given a science test and the mean, median and mode were as follows:

mean = 8.5, median = 9, mode = 10.

What does the difference between the mean and the mode suggest about the twenty quiz scores?

6.6.4 Show the sample space (all possible outcomes) for compound events in an organized way and find the theoretical probability of each outcome.

Example: A box contains four cards with the numbers 1 through 4 written on them. Show a list of all the possible outcomes if you draw two cards from the box without looking. What is the theoretical probability that you will draw the numbers one and two? Explain your answer.

6.6.5 Use data to estimate the probability of future events.

Example: Teams A and B have played each other 3 times this season and Team A has won twice. When they play again, what is the probability of Team B winning? How accurate do you think this estimate is?

6.6.6 Understand and represent probabilities as all forms of ratios, measures of relative frequency, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable.

Example: What is the probability of rolling a prime number on a number cube? Represent your answer in all three ratio forms.

Standard 7
Problem Solving

Students make decisions about how to approach problems and communicate their ideas.

6.7.1 Analyze problems by identifying relationships, telling relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Example: Solve the problem: "Develop a method for finding all the prime numbers up to 100." Notice that any numbers that 4, 6, 8, … divide into also divide exactly by 2, and so you do not need to test 4, 6, 8, … .

6.7.2 Make and justify mathematical conjectures based on a general description of a mathematical question or problem.

Example: In the first example, decide that you need to test only the prime numbers as divisors, and explain it in the same way as for 4, 6, 8, … .

6.7.3 Decide when and how to break a problem into simpler parts.

Example: In the first example, decide to find first those numbers not divisible by 2.

Students use strategies, skills, and concepts in finding and communicating solutions to problems.

6.7.4 Apply strategies and results from simpler problems to solve more complex problems.

Example: In the first example, begin by finding all the prime numbers up to 10.

6.7.5 Express solutions clearly and logically by using the appropriate mathematical terms and notation. Support solutions with evidence in both verbal and symbolic work.

Example: In the first example, use a hundreds chart to cross off all multiples of 2 (except 2), then all multiples of 3 (except 3), then all multiples of 5 (except 5), etc. Explain why you are doing this.

6.7.6 Recognize the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Example: Calculate the perimeter of a rectangular field that needs to be fenced. How accurate should you be: to the nearest kilometer, meter, centimeter, or millimeter? Explain your answer.

6.7.7 Select and apply appropriate methods for estimating results of rational-number computations.

Example: Measure the length and height of the walls of a room to find the total area. Estimate an answer by imagining meter squares covering the walls.

6.7.8 Make precise calculations and check the validity of the results in the context of the problem.

Example: In the first example, check whether some of the numbers not crossed out are in fact primes.

Students determine when a solution is complete and reasonable and move beyond a particular problem by generalizing to other situations.

6.7.9 Decide whether a solution is reasonable in the context of the original situation.

Example: In the first example, decide whether your method was a good one — did it find all the prime numbers efficiently?

6.7.10 Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.

Example: Use a hundreds chart to find all the numbers that are multiples of both 2 and 3.