Chapter 8

Mathematics Knowledge

In This Chapter

- Getting more terminology under your belt
- Revisiting high school: Algebra and geometry review
- Performing calculations without the calculator
- Perfecting your way to a higher score

Albert Einstein once said, "Do not worry about your problems with mathematics. I assure you mine are far greater." The good professor obviously never faced an upcoming ASVAB exam! Okay, just kidding. You don't have to be a mathematical theoretician to score well on the Mathematics Knowledge subtest. This subtest asks questions about basic high school mathematics. No college or graduate degrees needed.

The Mathematics Knowledge subtest consists of 25 questions, and you have 24 minutes to complete the subtest. You don't necessarily have to rush through each calculation, but the pace you need to set (a little less than a minute per question) doesn't exactly give you time to daydream. You have to focus and concentrate to solve each problem quickly and accurately. And no calculators allowed!

REMEMBER

The vast majority of questions on this subtest are expressed in mathematical terms, but you may see some word problems as well. Generally, such word problems are more direct than the problems you see on the Arithmetic Reasoning subtest (see Chapter 7). But most of the time, the Mathematics Knowledge subtest only contains one or two questions testing each specific mathematical concept. For example, one question may ask you to multiply fractions, the next may ask you to solve a mathematical inequality, and the question after that may ask you to find the value of an exponent. (If you're freaked out by the last sentence, calm down. These concepts are covered in this chapter.)

All this variety forces you to constantly shift your mental gears to quickly deal with different concepts. You can look at this situation from two perspectives. These mental gymnastics can be difficult and frustrating, especially if you know everything about solving for x but nothing about deriving a square root. But variety can also be the spice of life. If you don't know how to solve a specific type of problem, this oversight may only cause you to get one or two questions wrong.

REMEMBER

To qualify for certain jobs in the military, you have to score well on the Mathematics Knowledge subtest. You also have to do well on this subtest (which is part of the AFQT discussed in Chapter 1) in order to enlist. Turn to the Appendix to find out more about the subtest scores needed for specific military jobs.

Just When You Thought You Were Done with Vocab: Math Terminology

Like any science, math has its own vocabulary. In order to understand what each problem on the Mathematical Knowledge subtest asks you to do, you must understand certain mathematical terms:

- **Base:** A number that's used as a factor at least two times. For instance, the term 4^3 (which can be written $4 \times 4 \times 4$, and in which 4 is a factor three times) has a base of 4.
- **Factorial:** A factorial is represented by an exclamation point (!), and is figured by finding the product of a whole number and all the whole numbers less than it. So 6 factorial (6!) is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

  A factorial helps you determine *permutations* — all the different possible ways an event might turn out. For example, if you want to know how many different ways six runners could finish a race (permutation), you would solve for 6!
- **Reciprocal:** The number by which another number can be multiplied to produce 1. For example, the reciprocal of 3 is $\frac{1}{3}$. If you multiply 3 times $\frac{1}{3}$ you get 1. The reciprocal of $\frac{1}{6}$ is $\frac{6}{1}$ (which is the same thing as 6). $\frac{1}{6} \times 6 = 1$. Get the idea?
- **Root:** The square root of a number is the number, which, when multiplied by itself (*squared*), equals the original number. For example, the square root of 36 is 6. If you square 6, or multiply it by itself, you produce 36. (Check out "More about roots: Math roots, not the movie," later in this chapter.)
- **Rounding:** Limiting a number to a few (or no) decimal places. You perform rounding operations all the time — often without even thinking about it. If you have a $1.97 in change in your pocket, you may say, "I have about two dollars." The rounding process simplifies mathematical operations.

  Often, numbers are rounded to the nearest tenth. The ASVAB may ask you to do this. For any number 5 and over, round up; for any number under 5, round down. For example, 1.55 can be rounded up to 1.6, and 1.34 can be rounded down to 1.3.

  Many math problems require rounding. (Especially when you're doing all this without a calculator.) For example, pi (π) represents a number approximate to 3.1415926535897932384626433 83 (and on and on and on). However, in mathematical operations, it's common to round π to 3.14.

What Part of X Don't You Understand? Algebra Review

Some people may freak out just hearing the word *algebra.* But in actuality, algebra is just a way to put problems into mathematical language using the simplest mathematical terms possible. In fact, it's almost impossible to solve most word problems without some use of algebra.

In algebra, you often hear about "solving for *x*" or "solving for the unknown," but what's the unknown? The *unknown* is the answer you want find. Check out this example:

Rod's mom has worked up a powerful thirst solving a ton of math problems and asked Rod to run to the corner store and get her one of those super-duper gigantic nuclear soft drinks. If a regular-sized soft drink costs $0.50 and the super-duper gigantic nuclear size costs three times the cost of the regular size, how much will Rod have to spend?

You can express this problem in terms of x, with x being the cost of the super-duper sized drink: x equals 3 (the price difference) × 50 cents. Written a bit more formally, the equation looks like this: $x = 3 \times .50$ *or* $3 \times .50 = x$.

What if you don't know how much the regular sized soft drink costs? You can express this missing piece of information in an equation as well: x (how much it will cost to buy a super-duper size) equals 3 (the cost increase) times p (the price of one regular sized drink). Once again, written a bit more formally, the equation looks like this: $x = 3 \times p$.

You can remove the multiplication symbol in algebraic expressions when using a combination of letters and numbers. Therefore, the equation $x = 3 \times p$ can also be written $x = 3p$. The multiplication symbol is implied.

The letters in an algebra problem are commonly called *variables,* meaning that the number they stand for *varies* or changes.

What? More vocabulary? Algebra-related terms

Special algebra terms are used to describe how numbers function and how they relate to each other. Knowing what these terms mean is important to your ASVAB success:

- **Composite number:** A whole number that can be divided evenly by itself and by 1, as well as by one or more other whole numbers, which means that it has more than two factors. Examples of composite numbers are 6, 8, and 9.
- **Exponents:** You can think of exponents as a shorthand method of indicating multiplication. For example, 15×15 can also be expressed as 15^2, which is also known as "15 squared" or "15 to the second power." The small number (2) written slightly above and to the right of a number is called the exponent. An exponent indicates the number of times you multiply the number it accompanies by itself — 15^2 (15×15) isn't the same as 15×2.

  To express $15 \times 15 \times 15$ using this shorthand method, simply write it as 15^3, which is also called "15 cubed" or "15 to the third power." Again, 15^3 isn't the same as 15×3.
- **Factors:** Numbers that can divide into a composite number. To *factor* a composite number, you simply determine the numbers that you can divide into it. For example, 8 can be divided by the numbers 2 and 4 (in addition to 1 and 8), so 2 and 4 are factors of 8.
- **Prime number:** A whole number that can be divided evenly by itself and by 1 but not by any other number, which means that it has exactly two *factors.* (Check out the definition of *factor* a bit earlier in this list.) Examples of prime numbers are 2, 5, and 11.

When all things are equal: The algebra equation

Algebra problems are equations, which means that the quantities on both sides of the equal sign are equal — they're the same. $2 = 2$. $1 + 1 = 2$. And $3 - 1 = 2$. In all these cases, the quantities are the same on both sides of the equal sign. So, if $x = 2$, then x *is* 2 because the equal sign says so.

Solving one-step equations involving addition and subtraction

If $x + 1 = 2$, then x must be 1, because only 1 added to 1 is 2. So far, so simple, so good. But what if the equation is a little more complicated:

$$x + 47{,}432 = 50{,}000$$

To find out what x equals, which solves the problem, you need to isolate x on one side of the equal sign. To get that job done, you have to move any other numbers on the x side of the equal sign to the other side of the equal sign.

By looking at the x side of the equation, you can see that it's an addition problem. To move the number on the x side to the opposite side, you have to perform the inverse operation. The inverse operation of addition is subtraction. (For a full rundown on inverse operations, check out Chapter 7.) So, to move 47,432 from the x side to the non-x side of the equation, simply subtract it from both sides:

$$x + 47{,}432 - 47{,}432 = 50{,}000 - 47{,}432$$

Performing these operations removes the 47,432 from the x side of the equation ($47{,}432 - 47{,}432 = 0$, so that side of the equation is $x + 0$ or simply x) and gives you 2,568 on the non-x side of the equation ($50{,}000 - 47{,}432 = 2{,}568$). You're left with the final answer:

$$x = 2{,}568$$

To double-check that this answer is correct, plug your answer into the original problem:

$$x + 47{,}432 = 50{,}000$$
$$2{,}568 + 47{,}432 = 50{,}000$$

If you plug the answer in and it doesn't work, you've made an error in your calculations. Start again; remember that you're trying to isolate x on one side of the equation.

REMEMBER

You can perform any calculation on either side of an equation as long as you do it to both sides of the equation. That keeps the equation *equal.*

Multiplying and dividing using integers

An *integer* is any positive or negative whole number or zero. The ASVAB often requires you to work with integers such as $-6x = 36$. (Don't forget, $6x$ is the same thing as $6 \times x$.)

TIP

In multiplication and division, if the two terms being operated on (on either side of the equal sign) are both positive numbers or both negative numbers, the answer is a positive number. If one number is negative and the other is positive, the answer is negative.

So to solve this problem, $-6x = 36$, you need to isolate x, so perform an inverse operation (remember, the inverse operation of multiplication is division):

$$-6x \div -6 = 36 \div -6$$
$$x = -6$$

The answer is a negative number because the two terms, 36 and –6, have different signs.

REMEMBER

In an algebra equation, if the same letter is used more than once, it stands for the same number. $3x + 2x = 10$, the first x will *never* be a different number from the second x. In this case, $x = 2$ (both times).

You can only combine like terms when operating on algebraic expressions: $3x + 3x = 6x$, but $3x + 3y$ doesn't equal $6xy$, nor does $x^2 + x^3 = x^5$ (see the section "Explaining exponents," later in this chapter to find out more about algebra involving exponents).

Solving multistep equations

Not all algebra problems have one-step solutions. (That would be too easy, and you wouldn't sweat nearly as much.) Solving algebra problems on the ASVAB often requires you to perform several steps.

An example of a multistep equation is when x shows up on both sides of the equal sign. Then you have to get rid of x from one side of the equation by moving an x from one side to the other. You do this by performing the inverse operation.

Suppose you want to solve this equation:

$$3x + 3 = 9 + x$$

Follow these steps:

1. **To remove the x from one side of the equation, perform the inverse operation:**

   $3x + 3 - x = 9 + x - x$. This equation can also be stated as $3x + 3 - 1x = 9 + 0$.

2. **Perform the subtraction operation.**

   $2x + 3 = 9$

3. **To finish solving the problem, subtract 3 from each side of the equation.**

   $2x + 3 - 3 = 9 - 3$

   $2x = 6$

4. **Divide both sides of the equation by 2.**

   $2x \div 2 = 6 \div 2$

   $x = 3$

REMEMBER: When you have a variable by itself, such as x, it's always equal to $1 \times$ that variable (or one of that variable), like $1x$, even if the 1 isn't written out. In fact, any number is equal to 1 times itself, so you could also say $2 = 2 \times 1$. Sometimes this comes in handy when you're solving those algebra problems.

Explaining exponents

Exponents are an easy way to show that a number is to be multiplied by itself a certain number of times. For example, 5^2 is the same as 5×5. y^3 is the same as $y \times y \times y$. The number or variable that's multiplied by itself is called the *base*, and the number or variable showing how many times it is to be multiplied by itself is called the *exponent* or *power*.

Here are important rules when working with exponents:

- Any base raised to the power of one equals itself. Example: $x^1 = x$.
- Any base raised to the zero power (except 0) equals 1. Example: $x^0 = 1$.
- To multiply terms with the same base, you add the exponents. Example: $x^2 \times x^3 = x^5$.
- To divide terms with the same base, you subtract the exponents. Example x^5 (divided by) $x^2 = x^3$.

- If a base has a negative exponent, it's equal to its reciprocal with a positive exponent. Example: $x^{-3} = \frac{1}{x^3}$.
- When a product has an exponent, each factor is raised to that power. Example: $(xy)^3 = x^3 \times y^3$.

A note about scientific notation

Scientific notation is a compact format for writing very large or very small numbers. While its most often used in scientific fields, you may find a question or two on the Mathematics Knowledge subtest of the ASVAB, asking you to covert a number to scientific notation or vice-versa.

Scientific notation separates a number into two parts: a decimal fraction, usually between 1 and 10, and a power of ten. Therefore 1.25×10^4 means 1.25×10 to the fourth power or 12,500; 5.79×10^{-8} means $5.79 \div$ by 10 to the eighth power or 0.0000000579.

More about roots: Math roots, not the movie

A *square root* is the factor (see the "What? More vocabulary? Algebra-related terms" section earlier in this chapter) of a number that, when multiplied by itself, produces the number. Take the number 36, for example. One of the factors of 36 is 6. If you multiply 6 by itself (6×6), you come up with 36, so 6 is the square root of 36. The number 36 has other factors such as 18. But, if you multiply 18 by itself (18×18), you get 324, not 36. So 18 isn't the square root of 36.

REMEMBER

One number can only have one square root.

All numbers are grouped into one of two camps when it comes to roots:

- **Perfect squares:** Only a few numbers, called *perfect squares*, have exact square roots.
- **Irrational numbers:** All the rest have square roots that include decimals that go on forever and have no pattern that repeats (nonrepeating, nonterminating decimals), so they're called *irrational numbers.*

REMEMBER

The sign for a square root is called the radical sign. It looks like this: $\sqrt{}$. Here's how you use it: $\sqrt{36}$ means "the square root of 36" — in other words, 6.

Perfect squares

Square roots can be difficult to find at times without a calculator, but because you can't use a calculator during the test, you're going to have to use your mind and some guessing methods. To find the square root of a number without a calculator, make an educated guess and then verify your results.

To use the educated-guess method, you have to know the square roots of a few perfect squares. One good way to do this is to study the squares of the square roots 1 through 12:

- 1 is the square root of 1 ($1 \times 1 = 1$)
- 2 is the square root of 4 ($2 \times 2 = 4$)
- 3 is the square root of 9 ($3 \times 3 = 9$)

- 4 is the square root of 16 ($4 \times 4 = 16$)
- 5 is the square root of 25 ($5 \times 5 = 25$)
- 6 is the square root of 36 ($6 \times 6 = 36$)
- 7 is the square root of 49 ($7 \times 7 = 49$)
- 8 is the square root of 64 ($8 \times 8 = 64$)
- 9 is the square root of 81 ($9 \times 9 = 81$)
- 10 is the square root of 100 ($10 \times 10 = 100$)
- 11 is the square root of 121 ($11 \times 11 = 121$)
- 12 is the square root of 144 ($12 \times 12 = 144$)

Irrational numbers

When the ASVAB asks you to figure square roots of numbers that don't have perfect squares, the task gets a bit more difficult. If you have to find the square root of a number that isn't a perfect square, the ASVAB usually asks you to find the square root to the nearest tenth.

Suppose you run across this problem:

$$\sqrt{54}$$

Think about what you know:

- You know from the preceding section that the square root of 49 is 7, and 54 is slightly greater than 49.
- You also know that the square root of 64 is 8, and 54 is slightly less than 64.
- So, if the number 54 is somewhere between 49 and 64, the square root of 54 is somewhere between 7 and 8.
- Because 54 is closer to 49 than to 64, the square root will be closer to 7 than to 8, so you can try 7.3 as the square root of 54:
  1. **Multiply 7.3 by itself.**

     $7.3 \times 7.3 = 53.29$, which is very close to 54.
  2. **Try multiplying 7.4 by itself to see if it's any closer to 54.**

     $7.4 \times 7.4 = 54.76$, which isn't as close to 54 as 53.29.
  3. **So 7.3 is the square root of 54 to the nearest tenth without going over.**

Exponential roots

The wonderful world of math is also home to concepts like *cube roots, fourth roots, fifth roots,* and so on. These roots are a factor of a number, which, when cubed (multiplied by itself three times), taken to the fourth power (multiplied by itself four times), and so on, produce the original number. A couple of examples seem to be in order:

- The cube root of 27 is 3. If you cube 3 (also known as raising it to the third power or multiplying $3 \times 3 \times 3$), the product is 27.
- The fourth root of 16 is that number which, when multiplied by itself four times, equals 16. Any guesses? Drumroll, please: 2 is the fourth root of 16 because $2 \times 2 \times 2 \times 2 = 16$.

Looking at Math from a Different Angle: Geometry Review

Geometry is the branch of mathematics that makes grown adults cry — end of discussion. What? You want a more specific explanation of geometry than that? Okay, geometry is the branch of mathematics concerned with measuring things and defining the properties of and relationships between and among shapes, lines, points, angles, and other such objects. Hey, don't blame us; you asked for it.

Before you read any further, you should note a few things to remember:

- Arcs, circles, triangles, and angles are measured in degrees and (not very often) in minutes (which are smaller than degrees).
- A circle has 360 degrees (360°).
- A *quadrilateral* (shapes with four sides like a square or rectangle) has 360°.
- Any arc or angle that isn't a complete circle or quadrilateral measures less than 360°.

Outlining angles

Angles are formed when two lines intersect at a point. Angles are measured in degrees. The greater the number of degrees, the wider the angle is:

- A *straight line* is 180°.
- A *right angle* is exactly 90°.
- An *acute angle* is more than 0° and less than 90°.
- An *obtuse angle* is more than 90° but less than 180°.
- *Complementary angles* are two angles that equal 90° when added together.
- *Supplementary angles* are two angles that equal 180° when added together.

Take a look at the different types of angles in Figure 8-1.

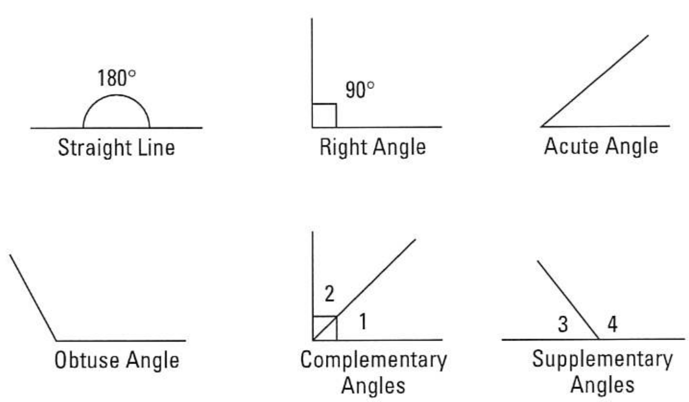

Figure 8-1: A diagram of the different types of angles.

Pointing out triangle types

A *triangle* consists of three straight lines whose three angles always add up to 180°. The sides of a triangle are called *legs.* Triangles can be classified according to the relationship between their angles or the relationship between their sides or some combination of these relationships:

- **Isosceles triangle:** Has two equal sides, and the angles opposite the equal sides are also equal
- **Equilateral triangle:** Has three equal sides, and all the angles measure 60°
- **Right triangle:** Has one right angle (90°); therefore, the remaining two angles are complementary (add up to 90°)

  The side opposite the right angle is called the *hypotenuse,* which is the longest side of a right triangle.

Check out Figure 8-2 to see what these triangles look like.

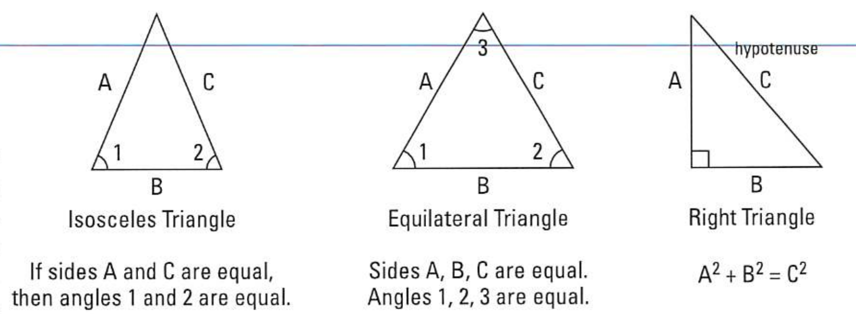

Figure 8-2: The different triangles of geometry.

When working with triangles, there are a few other terms and concepts you need to know:

- You can find the *perimeter* — the distance around a shape — of a triangle by adding together the length of the three sides.
- The *area* — the space within a shape — of a triangle is one-half the product of the base (the bottom or the length) and the height (the tallest point of the triangle) or $1/2bh$.
- The *Pythagorean theorem* states that if you know the length of two sides of a right triangle, the length of the third side can be determined, using the formula $a^2 + b^2 = c^2$, where *a* and *b* represent the length of the two known sides.

Back to square one: Quadrilaterals

Quadrilaterals — shapes with four sides — all contain angles totaling 360°. Many different types of quadrilaterals exist:

- **Parallelograms** have opposite sides that are parallel, and their opposite sides and angles are equal.
- **Rectangles** have all right angles.
- **Rhombuses** have four sides of equal length, but the angles don't have to be right angles.

- **Squares** have four sides of equal length, and all the angles are right angles.
- **Trapezoids** have at least two sides that are parallel.

See Figure 8-3 for the illustration of these quadrilaterals.

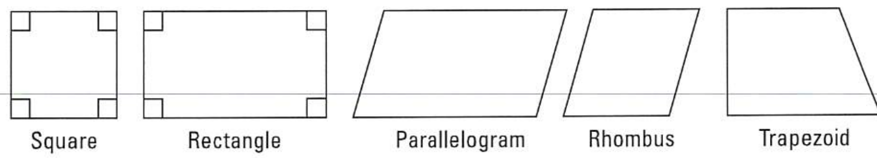

Figure 8-3: An illustration of quadrilaterals.

To determine the *perimeter* of a quadrilateral, simply add the length of all the sides. And to figure the *area* of a rectangle (including squares) multiply length × width.

Going around in circles

A *circle* is formed when the points of a closed line are all located equal distances from its center. A circle always has 360°. The closed line of a circle is called its perimeter or *circumference.* The *radius* of a circle is the measurement from the center of the circle to any point on the circumference of the circle. The *diameter* of the circle is measured as a line passing through the center of the circle, from a point on one side of the circle all the way to a point on the other side of the circle. The diameter of a circle is always twice as long as the radius of a circle, or $d = 2r$. (See Figure 8-4, which shows you the parts of a circle.)

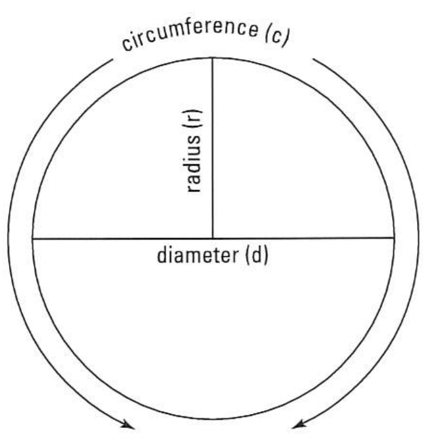

Figure 8-4: Checking out the parts of circles.

Navigating the circumference

To measure the *circumference* of a circle, use the number pi (π). Although π is a lengthy number, when used in geometry, it's generally rounded to 3.14 or $^{22}/_{7}$. Because π is rounded to 3.14 or $^{22}/_{7}$, when you solve a problem using π, the equal sign isn't used because the answer isn't exactly equal to the equation (due to the rounding). A symbol called the *approximation symbol* ($\approx$) is used.

Use this formula:

Circumference = $\pi \times$ diameter

or

$C = \pi d$

Because the *radius* of a circle is half its diameter, you can also use the radius to determine the circumference of a circle. Here's the formula:

$C = 2\pi r$

Suppose that you know that the pie you just baked has a diameter of 9 inches. You can determine its circumference by using the circumference formula:

$C = \pi d$

$C \approx 3.14 \times 9$

$C \approx 28.26$ inches

Mapping out the area

Determining the area of a circle also requires the use of π.

Area = $\pi \times$ the square of the circle's radius

or

$A = \pi r2$

To determine the area of a 9-inch-diameter pie, multiply π by the square of 4.5. Why 4.5 and not 9? Remember, the radius is always half the diameter, and the diameter is 9 inches.

$A = \pi r^2$

$A \approx 3.14 \times 4.5^2$

$A \approx 3.14 \times 4.5 \times 4.5$

$A \approx 3.14 \times 20.25$

$A \approx 63.585$ inches

Filling 'er up: Calculating volume

Volume is the space a solid (three-dimensional) shape takes up. You can think of volume as how much a shape would hold if you poured water into it. Volume is measured in cubic units.

The formula for finding volume depends on the object:

- For rectangular objects, you multiply length $\times$ width (depth) $\times$ height. This is possible because the length, width, and height of a rectangle are consistent throughout the whole shape. The formula looks like this: $V = lwh$.

  For a box that measures 5-feet long, 6-feet deep, and 2-feet tall, you simply multiply $5 \times 6 \times 2$ to arrive at a volume of 60 cubic feet.

- For a cylinder that has two circles for its bases, the calculation is $V = (\pi r^2)h$ or, volume = pi × the radius squared × height.

  For a cylinder that has a radius of 2 inches and a height of 10 inches, here's the deal: Multiply the value of pi (3.14) times 4 (which is the radius squared) times 10, or $3.14 \times 4 \times 10 = 125.6$ cubic inches.

Calculating without a Calculator: All You Need to Know

Too bad those ASVAB honchos don't allow you to use a calculator on the test. That would make it a breeze. Remember, though, the Mathematics Knowledge subtest of the ASVAB is based on arithmetic you most likely studied in high school. In this section, you come up to speed on how to solve problems that the Mathematics Knowledge subtest commonly throws at its victims, um, test takers.

Factoring to find original numbers

Now and then, the ASVAB gives you a *product* (the answer to a multiplication problem), and you have to find the original numbers that were multiplied together to produce that product. This process is called *factoring.* You use factors when you combine like terms and add fractions.

Take, for example, this product:

$$4xy + 2x^2$$

To factor this product, follow these steps:

1. **Find the *highest common factor* — the highest number that evenly divides all the terms in the expression.**

   In this case, the highest number that divides into both terms is 2.

2. **Then figure out the common factors for the variables too.**

   In this case, the highest variable that divides into both xy and x^2 is x.

3. **Okay — take what you know to this point, and you can see that the highest common factor is $2x$.**

   So far, so good.

4. **Now divide $2x$ into both terms in the expression.**

   The resulting terms are 2y + x.

5. **Finally, multiply the entire expression by 2x to set the equation equal to its original value.**

   Doing so produces the factors of $2x(2y + x)$.

Time to try something a little more complicated: factoring a *trinomial* (a problem with three terms). Look at the below example:

$$x^2 - 12x + 20$$

To factor this product, follow these steps:

1. **Find the factors of the first term of the trinomial.**

   The factors of x^2 are x and x ($x \times x = x^2$). Put those factors (x and x) on the left side of two sets of parentheses:

   $(x\ )(x\ )$

2. **Determine whether the two expressions will be positive or negative.**

   You can see that the last term in the trinomial (+ 20) has a plus sign. That means the resulting factors must be either plus or minus, because two pluses result in a positive number and two minuses result in a positive number. Because the second term ($-12x$) is a negative number, both of the factors must be negative. (Because two negative numbers multiplied equals a positive number.)

   $(x - \ )(x - \ )$

3. **Find the number that'll be used as the second term in the resulting factors.**

4. **Plug the two numbers into the right side of the parentheses.**

   This part can be tricky. The factors of the third term, when added or subtracted together must equal the second term of the trinomial. The factors of 20 (the third term) which combines with 12 are 2 and 10 because $2 \times 10 = 20$ (the third term) and $2 + 10 = 12$ (the second term).

   $(x - 2)(x + 10)$

   The factors of $x^2 - 12x + 20$ are $x - 2$ and $x + 10$.

Making alphabet soup: The quadratic equation

Algebra questions often ask you to solve for x or solve for an unknown. These questions can be expressed, for example, as $x = 2 + 3$. You simply isolate the unknown on one side of the equation and solve the other side to learn what x equals. In this case, x equals 5. The topic of solving for unknowns is covered in more depth in the section, "What Part of *X* Don't You Understand? Algebra Review," earlier in this chapter.

So what's a quadratic equation? Sounds a little scary, huh? The Mathematical Knowledge subtest may ask you to solve one of these equations, but have no fear. You've come to the right place. This section can help.

REMEMBER

A *quadratic equation* is an equation that includes the square of an unknown. The exponent in these equations is never higher than 2 (because it would then no longer be the *square* of an unknown, but a cube or something else). Here are some examples of quadratic equations:

$$x^2 - 4x = -4$$

$$2x^2 = x + 6$$

$$x^2 = 36$$

REMEMBER

Simple quadratic equations (those that consist of just one squared term and a number) can be solved by using the *square root rule:*

If $x^2 = \sqrt{k}$, then $x = \pm k$, as long as k isn't a negative number.

Remember to include the ± sign, which indicates the answer is a positive or negative number. Take the following simple quadratic equation:

$$7y^2 = 28$$

1. **First get rid of the pesky 7 by dividing both sides by 7.**

   The result is $y^2 = 4$.

2. **Using the square root rule, you then take the square root of both sides of the equation.**

   $\sqrt{y^2} = y$ and $\sqrt{4} = 2$

   $y = \pm 2$

The above steps work with simple quadratic equations, but when you're solving a complex quadratic equation, you put all the terms on one side of the equal sign, making the equation equal zero. In other words, get the quadratic equation into this form: $ax^2 + bx + c = 0$, where *a, b,* and *c* are numbers and *x* is unknown. Take a look at the following equation:

$$x^2 - 2x = 15$$

You can convert this equation to quadratic form by subtracting 15 from both sides of the equation.

$$x^2 - 2x - 15 = 0$$

The most efficient way to solve most quadratic equations is by factoring the equation and then setting each separate factor equal to zero. See the section "Factoring to find original numbers" earlier in this chapter.

Look at this equation again:

$$x^2 - 2x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$x - 5 = 0$ and $x + 3 = 0$

$x = 5$ and $x = -3$

The solution for $x^2 - 2x - 15$ is 5, –3

All math isn't created equal: Solving inequalities

Some algebra problems state that two numbers aren't equal to each other (thus they're inequalities). In an inequality, the first number is either greater than (≥) or less than (≤) the second.

Just like with equations, the solution to an inequality is a value that makes the inequality true. For the most part, you solve inequalities the same as you would solve a normal equation. There are some facts of inequality life you need to keep in mind, however. Short and sweet, here they are:

- *Negative numbers* are less than zero and less than positive numbers.
- *Zero* is less than positive numbers but greater than negative numbers.
- *Positive numbers* are greater than negative numbers and greater than zero.

A regular algebraic equation includes the equal sign (=), because the very basis of the equation is that one side of the equation must equal the other. Quite the opposite is true with inequalities, and they have their own special symbols, used to express the differences:

- $\neq$ means *does not equal* in the way that 3 *does not equal* 4 or $3 \neq 4$.
- $>$ means *greater than* in the way that 4 *is greater than* 3 or $4 > 3$.
- $<$ means *less than* in the way the 3 *is less than* 4, or $3 < 4$.
- $\leq$ means *less than or equal to* in the way that x may be *less than or equal to* 4 or $x \leq 4$.
- $\geq$ means *greater than or equal to* in the way that x may be *greater than or equal to* 3 or $x \geq 3$.

To solve an inequality, you follow the same rules as you would for solving any other equation. For example, check out this inequality:

$$3 + x \geq 4$$

To solve it, simply isolate x by subtracting 3 from both sides of the equation:

$$3 + x - 3 \geq 4 - 3$$

or

$$x \geq 1$$

TIP: The only exception to this rule is when you multiply or divide both sides of the inequality by a negative number. In that case, the inequality sign is reversed. So, if you multiply both sides of the inequality $3 < 4$ by -4, your answer is $-12 > -16$.

Test-Taking Techniques for Your Mathematical Journey

As with most of the other subtests on the ASVAB, guessing on the Mathematical Knowledge subtest doesn't count against you. So scribble in an answer, any answer, on your answer sheet because, if you don't, your chances of getting that answer right are zero. But, if you take a shot at it, your chances increase to 25%, or 1 in 4. In the following sections, you find some tips that can help you improve those odds, even when you don't know how to solve the problem.

If you're not confident in your math skills you may wish to invest some extra study time. Check out *Algebra For Dummies* and *Algebra II For Dummies* by Mary Jane Sterling, *Geometry For Dummies* by Wendy Arnone, *Calculus For Dummies* by Mark Ryan, and *SAT II Math For Dummies* by Scott Hatch — all published by Wiley Publishing, Inc.

Knowing what the question is asking

This subtest presents most of the questions as straightforward math problems, not word problems, so knowing what the question is asking you to do is easier. However, reading each question carefully, paying particular attention to plus (+) and minus (–) signs (which can really change the answer to a question) is still important. Finally, make sure you do *all* the calculations needed to produce the correct answer. Check out this example:

EXAMPLE

Find the value of $\sqrt{(81^2)}$.

(A) 9

(B) 18

(C) 81

(D) 6,561

If you're in a hurry, you may put 9 down as an answer because you remember that the square root of 81 is 9. Or, in a rush, you could multiply 9 (the square root of 81) by 2 instead of squaring it, as the exponent indicates you should. Or, you might just multiply 81 by 81 to get 6,561 without remembering that you also need to then find the square root, which gives you the correct answer: Choice (C). So make sure you perform all the operations needed (and that you perform the *correct* operations) to find the right answer.

Figuring out what you're solving for

Even though getting artistic with your answer sheet can be fun, the techniques in this section help you try to first improve your chances of guessing the right answer. Right out of the gate, read the question carefully. Some questions can seem out of your league at first glance, but if you look at them again, a light may go on in your brain. Suppose you get this question:

EXAMPLE

s number of students are in a classroom. $\frac{2}{5}$ of the students are enlisted personnel. $\frac{1}{2}$ of the enlisted personnel are privates. How many privates are in the audience?

(A) $2\frac{1}{2}s$

(B) $2s$

(C) $\frac{1}{5}s$

(D) $\frac{1}{10}s$

At first glance, you may think, "Oh, no! Solve for an unknown, *s*. I don't remember how to do that!" But, if you look at the question again, you may see that you're not solving for *s* at all. You're simply multiplying a fraction. So you take $\frac{2}{5}$ times $\frac{1}{2}$ and arrive at $\frac{2}{10}$, but you should reduce that fraction to get $\frac{1}{5}$. The correct answer is Choice (C). (See Chapter 6 for a refresher on multiplying fractions.)

Solving what you can and guessing the rest

Sometimes a problem requires multiple operations for you to arrive at the correct answer. If you don't know how to do all of the operations, don't give up. You can still narrow your guess down by doing what you can.

TIP

Because the Mathematical Knowledge subtest doesn't penalize you for guessing, mark the answer sheet even if you're clueless. You can even make a pretty design on your answer sheet and still have a one-in-four chance of getting each answer right.

Suppose this question confronts you:

What's the value of $(0.03)^3$?

(A) 0.0027

(B) 0.06

(C) 0.000027

(D) 0.0009

Say you don't remember how to multiply decimals. All isn't lost! If you remember how to use exponents, you'll remember that you have to multiply $0.03 \times 0.03 \times 0.03$. So, if you simplify the problem and just multiply $3 \times 3 \times 3$, without worrying about those pesky zeroes, your answer will have a 27 in it. With this pearl of wisdom in mind, you can see that Choice (B), which adds 0.03 to 0.03, is wrong. It also means that Choice (D), which multiplies 0.03 and 0.03, is wrong. Now you have two possible answers, and you've improved your chances of guessing the right one to 50 percent! By multiplying $3 \times 3 \times 3$ to get 27, don't forget to put the decimal points back in. You have six places to make up, so move the decimal from 27.0 six places to the left to get 0.000027. The correct answer is Choice (C).

Don't forget to use that scratch paper! Suppose you run across this question:

> A child is building a tower of blocks. Each block is a cube. Some blocks are white, and some blocks are red. Red blocks surround each white block. How many red blocks surround each white block?

This problem may be difficult to figure out until you sketch a six-sided block (a cube) on your scratch paper and realize that the block must be surrounded by six other blocks. Sometimes drawing that visual helps you solve the problem.

Using the process of elimination

Another method (besides guessing) you can use when you run into questions where you draw a total blank is to plug the possible answers into the equation and see which one works. Say the following problem is staring you right in the eyes:

Solve for x: $x - 5 = 32$

(A) $x = 5$

(B) $x = 32$

(C) $x = -32$

(D) $x = 37$

You're not sure what to do. If you're totally stumped and can't think of any possible way of approaching this problem, simply plugging in each of the four answers to see which one is correct is your best bet.

- **Answer A:** $5 - 5 = 32$, which you know is wrong
- **Answer B:** $32 - 5 = 32$, which is wrong
- **Answer C:** $-32 - 5 = 32$, which is wrong
- **Answer D:** $37 - 5 = 32$, which is correct

Don't forget that plugging in all the answers is time consuming, so save this procedure until you've answered all the problems you can answer. If you're taking the computer version, you can't skip a question, so remember to budget your time wisely. If you don't have much time, just make a guess and move on. You may be able to solve the next question easily.

Double-checking your work

Although you don't have a ton of time to complete the Mathematical Knowledge subtest, you do have about a minute per problem. Although a minute doesn't allow for a lot of head scratching, it's more time than you think. So double-check your answers before putting your pencil down (or before going on to the next problem on the computer).

You can go over your calculations again to make sure that you didn't make an error. You can also plug your answer into the original equation to make sure that it's the correct answer. Then move along, private!