PARMA CITY SCHOOL DISTRICT
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MATHEMATICS K-5

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INTRODUCTION/BACKGROUND/PURPOSE/RATIONALE

In 2010, the State Board of Education adopted Ohio's Learning Standards in Mathematics as a guide to teaching and learning in the classroom. The kindergarten-grade 12 standards have been fully in use in Ohio classrooms since the start of the 2014-2015 school year. In early 2016, educators statewide began assisting the Ohio Department of Education in updating Ohio's Learning Standards in Mathematics. To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education. The board adopted the proposed revisions for Ohio's Learning Standards for English Language Arts in early winter 2017.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

KEY DESIGN CONSIDERATIONS: TWO SETS OF STANDARDS FOR MATHEMATICS

Ohio's Learning Standards for Mathematics are comprised of two sets of complementary standards: Standards for Mathematical Practice and Standards for Mathematical Content. BOTH sets of standards should be taught to and assessed for mastery in K-12 classrooms through a variety of evidence-based methods.

STANDARDS FOR MATHEMATICAL PRACTICE K-12

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$.

Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x2 + x + 1)$, and $(x - 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT K-12

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

FLUENCY IN MATHEMATICS

Fluency in mathematics as it pertains to the academic content standards is more than memorizing math facts or procedures and is more than understanding and being able to apply more than one procedure during problem solving. The National Council for Teachers of Mathematics (NCTM) holds the following position on fluency in mathematics

NCTM Position

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through distributed practice.

INTENDED LEARNING OUTCOMES & LEARNING PROGRESSIONS

The concept of learning progressions was critical in the development, review, and revision of Ohio's Learning Standards (OLS). Ohio's learning progressions were developed during Ohio's international benchmarking project and provided guidance to the writing and revision committees of Ohio's Learning Standards. Ohio believes that the concept of learning progressions is important for the understanding and coherence of mathematical topics within and across the grade levels. The Ohio Department of Education has reformatted Ohio's Learning Standards by domains to show the progression of concepts and skills across the grade levels. The document entitled *Ohio's K-8 Learning Progressions* February 2017 provides staff with clear indications of the progression of conceptual standards categories as well as the specific standards for each grade level that fall under each category. Staff should use this document when writing courses of study, curriculum maps and unit plans.

Ohio's Learning Standards for Mathematics include ***critical areas for instruction*** in the introduction to each grade, kindergarten through grade 8. The critical areas are designed to bring focus to the standards at each grade by describing the *big ideas that educators can use to build their curriculum and to guide instruction*. It is advised that K-5 staff teaching mathematics familiarize themselves with the scope and sequence of these critical areas of focus through the Ohio Department of Education Document entitled *Mathematics- K-8 Critical Areas of Focus.*

The grade-level introductions include at least two and no more than five critical areas for each grade. These critical areas identify the intended overall learning outcomes for grades K-8 and are cited within each grade level section standard detail use parenthetical notation (e.g. CA1, CA2, etc.).

EXPECTATIONS FOR MATHEMATICS INSTRUCTION

The *Teaching Math to Young Children* practice guide was developed by a panel of nationally recognized experts in early childhood education. The *Teaching Math to Young Children* practice guide presents five recommendations designed to help early education teachers capitalize on children's natural interest in math. The first two recommendations identify early math content areas that should be included in the preschool, prekindergarten, and kindergarten curricula. The last three recommendations focus on strategies and teaching techniques that incorporate math content into the classroom.

Recommendation 1: Teach number and operations using a developmental progression.
● Provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them. ● Promote accurate one-to-one counting as a means of identifying the total number of items in a collection. ● Provide opportunities for children to use number words and counting to compare quantities once children can recognize or count collections. ● Encourage children to label collections with number words and numerals. ● Encourage children to solve basic problems once children develop these fundamental number skills.
Recommendation 2: Teach geometry, patterns, measurement, and data analysis using a developmental progression.
● Help children to recognize, name, and compare shapes, and then teach them to combine and separate shapes. ● Encourage children to look for and identify patterns, then teach them to extend, correct, and create patterns. ● Promote children's understanding of measurement by teaching them to make direct comparisons and to use both informal and nonstandard (e.g., the child's hand or foot) and formal or standard (e.g., a ruler) units and tools. ● Help children collect and organize information, then teach them to represent that information graphically.
Recommendation 3: Use progress monitoring to ensure that math instruction builds on what each child knows.
● Use introductory activities, observations, and assessments to determine each child's existing math knowledge, or the level of understanding or skill he or she has reached on a developmental progression. ● Tailor instruction to each child's needs, and relate new ideas to his or her existing knowledge. ● Assess, record, and monitor each child's progress so that instructional goals and methods can be adjusted as needed.
Recommendation 4: Teach children to view and describe their world mathematically.
● Encourage children to use informal methods to represent math concepts, processes, and solutions. ● Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences. ● Use open-ended questions to prompt children to apply their math knowledge. ● Encourage children to recognize and talk about math in everyday situations.
Recommendation 5: Dedicate time each day to teaching math, and integrate math instruction throughout the school day.
● Plan daily instruction targeting specific math concepts and skills. ● Embed math in classroom routines and activities. ● Highlight math within topics of study across the curriculum. ● Create a math-rich environment where children can recognize and meaningfully apply math. ● Use games to teach math concepts and skills and to give children practice in applying them.

According to the IES practice guide entitled *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, the understanding of fractions is essential for understanding algebraic concepts and other higher levels of math (p. 6). Furthermore, knowledge of fraction concepts presents a larger gap for students in the United States than whole number concepts when compared to other countries such as East Asia (p. 6). The panel of this practice guide suggest these deficiencies are the result of the following conceptual misconceptions:

- Not viewing fractions as numbers at all, but rather as meaningless symbols that need to be manipulated in arbitrary ways to produce answers that satisfy a teacher.
- Focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number. Errors such as believing that 3/8 > 3/5 arise from comparing the two denominators and ignoring the essential relation between each fraction's numerator and its denominator.
- Confusing properties of fractions with those of whole numbers. This is evident in many high school students' claim that just as there is no whole number between 5 and 6, there is no number of any type between 5/7 and 6/7.

The panel recommends that teachers employ the following strategies and supports in student fractions concept development:

Recommendation 1: Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.
• Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole subjects. • Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions. • Build on students' informal understanding to develop more advanced understanding of proportional-reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.
Recommendation 2: Help student recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
• Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share. • Provide opportunities for students to locate and compare fractions on number lines. • Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are infinite number of fractions between any two fractions), and negative fractions. • Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.
Recommendation 3: Help students understand why procedures for computations with fractions make sense.
• Use area models, number lines, and other visual presentations to improve students' understanding of formal computational procedures. • Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions. • Address common misconceptions regarding computational procedures with fractions. • Present real-world contexts with plausible numbers for problems that involve computing with fractions.

Recommendation 4: Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.
● Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Building on students' developing strategies for solving ratio, rate, and proportion problems. ● Encourage students to use visual representations to solve ratio, rate, and proportion problems. ● Provide opportunities for students to use and discuss alternative strategies for solving rate, ratio, and proportion problems.
Recommendation 5: Professional development programs should place a high priority on improving teachers' understanding of fractions and how to teach them.
● Build teachers' depth of understanding of fractions and computational procedures involving fractions. ● Prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations. ● Develop teachers' ability to assess students' understandings and misunderstandings of fractions.

As student progress through the grade level standards in mathematics and apply their knowledge to different problem solving tasks, it is important for educators to work with student to improve problem solving skills. In the IES practice guide entitled *Improving Mathematical Problem Solving in Grade 4 through 8*, the panel recommends that teachers employ the following strategies and supports in student problem solving skill development:

Recommendation 1: Prepare problems and use them in whole-class instruction.
● Include both routine and non-routine problems in problem-solving activities (will vary based upon each student's level of experience with problem-solving). ○ *Routine:* can be solved using methods familiar to students by replicating previously learned methods in step-by-step fashion. ○ *Non-routine:* problems for which there are no predictable or well-rehearsed approaches nor an explicit suggestion of the pathway in the task, task instructions or a worked-out example. ● Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language. ● Consider students' knowledge of mathematical content when planning lessons.
Recommendation 2: Assist students with in monitoring and reflecting on the problem-solving process.
● Provide students with a list of prompts to help them monitor and reflect during the problem-solving process. ● Model how to monitor and reflect on the problem-solving process. ● Use student thinking about a problem to develop students' ability to monitor and reflect.
Recommendation 3: Teach students how to use visual representations.

● Select visual representations that are appropriate for students and the problems they are solving. ● Use think-alouds and discussions to teach students how to represent problems visually. ● Show students how to convert the visually represented information into mathematical notation.
Recommendation 4: Expose students to multiple problem-solving strategies.
● Provide instruction in multiple strategies. ● Provide opportunities for students to compare multiple strategies in worked examples. ● Ask students to generate and share multiple strategies for solving a problem.
Recommendation 5: Help students recognize and articulate mathematical concepts and notation.
● Describe mathematical concepts and notation, and related them to the problem-solving activity. ● Ask students to explain each step used to solve a problem in a worked example. ● Help students make sense of algebraic notation.

The Ohio Content Standards for Mathematics emphasize the importance of incorporating the recommendations in the Guide for Assessment and Instruction in Statistics Education (GAISE) report when teaching PK-12 mathematical concepts related to statistics. For grades K-5, the GAISE report recommends developmental levels of the learning process as applied to statistics education as follows:

Table 1: The Framework

Process Component	Level A	Level B	Level C
I. Formulate Question	**Beginning awareness of the *statistics question distinction*** Teachers pose questions of interest Questions restricted to the classroom	**Increased awareness of the *statistics question distinction*** Students begin to pose their own questions of interest Questions not restricted to the classroom	**Students can make the *statistics question distinction*** Students pose their own questions of interest Questions seek generalization
II. Collect Data	Do not yet *design for differences* Census of classroom Simple experiment	Beginning awareness of *design for differences* Sample surveys; begin to use random selection Comparative experiment; begin to use random allocation	Students make *design for differences* Sampling designs with random selection Experimental designs with randomization
III. Analyze Data	*Use* particular properties of *distributions* in the context of a specific example Display variability within a group Compare individual to individual Compare individual to group Beginning awareness of group to group Observe association between two variables	Learn to *use* particular properties of *distributions* as tools of analysis Quantify variability within a group Compare group to group in displays Acknowledge sampling error Some quantification of association; simple models for association	Understand and *use distributions* in analysis as a global concept Measure variability within a group; measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association; fitting of models for association

Process Component	Level A	Level B	Level C
IV. Interpret Results	Students do not look *beyond the data* No generalization beyond the classroom Note difference between two individuals with different conditions Observe association in displays	Students acknowledge that *looking beyond the data* is feasible Acknowledge that a sample may or may not be representative of the larger population Note the difference between two groups with different conditions Aware of distinction between observational study and experiment Note differences in strength of association Basic interepretation of models for association Aware of the distinction between association and cause and effect	Students are able to *look beyond the data* in some contexts Generalize from sample to population Aware of the effect of randomization on the results of experiments Understand the difference between observational studies and experiments Interpret measures of strength of association Interpret models of association Distinguish between conclusions from association studies and experiments
Nature of Variability	Measurement variability Natural variability Induced variability	Sampling variability	Chance variability
Focus on Variability	Variability within a group	Variability within a group and variability between groups Covariability	Variability in model fitting

It is expected that teachers of mathematics apply the recommendations contained in the GAISE report when planning and delivering lessons as well as assessing student mastery of statistics education concepts.

LEARNING TARGETS & ACADEMIC VOCABULARY

The following section outlines the specific state and local content standards each teacher should teach and assess for mastery as outlined by the required sequencing depicted in curriculum maps provided each year by the Department of Curriculum and Instruction. In addition, each teacher should teach and assess for mastery the target academic vocabulary words contained within the standards for each grade level/course as directed and provided by the Department of Curriculum and Instruction. The Department of Curriculum & Instruction will request feedback periodically from staff in regard to any suggested revisions to sequencing and/or local standards language or target academic vocabulary.

The standards offer a focus for instruction each year and help ensure that student gain adequate experience with a range of mathematical concepts and practice skills through a variety of appropriately aligned math tasks. *Students advancing through the grades should meet each year's specific standards and retain or further develop skills and understandings mastered in preceding grades.*

These standards encourage fostering students' understanding and working knowledge of mathematical concepts and practices. Educators should differentiate instruction; **the point is to teach students what they need to learn and not what they already know--- to discern when particular children or activities warrant more or less attention.**

It is expected that staff keep abreast of current evidence-based practices that have a strong and/or moderate evidence to support effectiveness of the strategy and/or resource. The teaching and reinforcement of academic vocabulary is paramount to student academic success and a promising practice for closing learning gaps with at risk subgroups of learners (Marzano, 2001; Marzano, 2005; Marzano, 2010; Marzano & Simms, 2013; Institute of Education Sciences, 2016).

KINDERGARTEN

In Kindergarten, instructional time should focus on two critical areas:
Critical Area 1 (CA1): Representing, relating, and operating on whole numbers, initially with sets of objects Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 − 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
Critical Area 2(CA2): Describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics Students describe their physical world using geometric ideas, e.g., shape, orientation, spatial relations, and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways, e.g., with different sizes and orientations, as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complicated shapes. They identify the measurable attributes of shapes.

COUNTING AND CARDINALITY ● ***Know number names and the count sequence.***
Standard
K.CC.1 Count to 100 by ones and by tens. **(CA1)**
K.CC.2 Count forward within 100 beginning from any given number other than 1. **(CA1)**
K.CC.3 Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **(CA1)**
● ***Count to tell the number of objects.***
K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies. **(CA1)** a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and that the number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger.
K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. **(CA1)**
● ***Compare numbers.***
K.CC.6 Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group. **(CA1)**
K.CC.7 Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals. **(CA1)**

OPERATIONS AND ALGEBRAIC THINKING ● ***Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.***
Standard
K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **(CA1)**
K.OA.2 Solve addition and subtraction problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem. **(CA1)**
K.OA.3 Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations. **(CA1)**
K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation. **(CA1)**
K.OA.5 Fluently add and subtract within 5. **(CA1)**
NUMBER AND OPERATIONS IN BASE TEN ● ***Work with numbers 11-19 to gain foundations for place value.***
K.NBT.1 Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **(CA1)**
MEASUREMENT AND DATA ● ***Identify, describe, and compare measurable attributes.***
K.MD.1 Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short. **(CA2)**
K.MD.2 Directly compare two objects with a measurable attribute in common to see which object has “more of” or “less of” the attribute, and describe the difference. *For example, directly compare the heights of two children, and describe one child as taller/shorter.* **(CA1)**
● ***Classify objects and count the number of objects in each category.***
K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies. **(CA1)**

GEOMETRY ● ***Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones cylinders, and spheres).***
Standard
K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.* **(CA2)**
K.G.2 Correctly name shapes regardless of their orientations or overall size. **(CA2)**
K.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). **(CA2)**
● ***Describe, compare, create, and compose shapes.***
K.G.4 Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes. **(CA2)**
K.G.5 Model shapes in the world by building shapes from components, e.g., sticks and clay balls, and drawing shapes. **(CA2)**
K.G.6 Combine simple shapes to form larger shapes. **(CA2)**

GRADE 1

In Grade 1, instructional time should focus on four critical areas:

Critical Area 1(CA1): Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20 Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models, e.g., cubes connected to form lengths, to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction, e.g., adding two is the same as counting on two. They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties, e.g., "making tens", to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

Critical Area 2(CA2): Developing understanding of whole number relationships and place value, including grouping in tens and ones Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes. Students use money as a tool to reinforce concepts of place value using pennies (ones) and dimes (tens).

Critical Area 3(CA3): Developing understanding of linear measurement and measuring lengths as iterating length units Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

Critical Area 4(CA4): Reasoning about attributes of, and composing and decomposing geometric shapes Students compose and decompose plane or solid figures, e.g., put two triangles together to make a quadrilateral, and build understanding of part whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

OPERATIONS AND ALGEBRAIC THINKING ● ***Represent and solve problems involving addition and subtraction.***
Standard
1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **See Table 1, page 36**. **(CA1)**
1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **(CA1)**

● ***Understand and apply properties of operations and the relationship between addition and subtraction.***
1.OA.3 Apply properties of operations as strategies to add and subtract*. For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative Property of Addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative Property of Addition).* Students need not use formal terms for these properties. **(CA1)**
1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 − 8 by finding the number that makes 10 when added to 8. **(CA1)**
● ***Add and subtract within 20.***
Standard
1.OA.5 Relate counting to addition and subtraction, e.g., by counting on G 2 to add 2. **(CA1)**
1.OA.6 Add and subtract within 20, demonstrating fluency G with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., 8 + 6 = 8 + 2 + 4 =10 + 4 = 14; decomposing a number leading to a ten, e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13. **(CA1)**
● ***Work with addition and subtraction equations.***
1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6; 7 = 8 – 1; 5 + 2 = 2 + 5; 4 + 1 = 5 + 2.* **(CA1)**
1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations: 8 + = ☐ 11; 5 = ☐ -3; 6 + 6 ☐ =.*
NUMBER AND OPERATIONS IN BASE TEN
● ***Extend the counting sequence.***
1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **(CA2)**
● ***Understand place value.***
1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones — called a “ten;” the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **(CA2)**
1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
● ***Use place value understanding and properties of operations to add and subtract.***
1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten. **(CA1)**
1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **(CA1)**
1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(CA1)**

MEASUREMENT AND DATA ● ***Measure lengths indirectly and by iterating length units.***
Standard
1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. **(CA3)**
1.MD.2 Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **(CA3)**
● ***Work with time and money.***
1.MD.3 Work with time and money. **(CA2)** a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify pennies and dimes by name and value.
● ***Represent and interpret data.***
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **(CA3)**
GEOMETRY ● ***Reason with shapes and their attributes.***
1.G.1 Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus non-defining attributes, e.g., color, orientation, overall size; build and draw shapes that possess defining attributes. **(CA4)**
1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three- dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism." **(CA4)**
1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares. **(CA4)**

GRADE 2

In Grade 2, instructional time should focus on four critical areas:

Critical Area 1(CA1): Extending understanding of base-ten notation. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones, e.g., 853 is 8 hundreds + 5 tens + 3 ones.

Critical Area 2(CA2): Building fluency with addition and subtraction Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds. They apply their understanding of addition and subtraction to data represented in the picture and bar graphs.

Critical Area 3(CA3): Using standard units of measure. Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length. They also apply number concepts solving real-world problems.

Critical Area 4(CA4): Describing and analyzing shapes Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades. They apply number concepts in real-world problems.

OPERATIONS AND ALGEBRAIC THINKING ● ***Represent and solve problems involving addition and subtraction.***
Standard
2.OA.1 Use addition and subtraction within 100 to solve one- and two- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **See Table 1, page 36**. **(CA2)**
● ***Add and subtract within 20.***
2.OA.2 Fluently G add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies. **(CA2)**
● ***Work with equal groups of objects to gain foundations for multiplication.***
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. **(CA2)**
2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **(CA2)**

NUMBER AND OPERATIONS IN BASE TEN ● ***Understand place value.***
Standard
2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens - called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **(CA1)**
2.NBT.2 Count forward and backward within 1,000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5. **(CA1)**
2.NBT.3 Read and write numbers to 1,000 using base-ten numerals, number names, expanded form G, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc. **(CA1)**
2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **(CA1)**
● ***Use place value understanding and properties of operations to add and subtract.***
2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **(CA2)**
2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.**(CA2)**
2.NBT.7 Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds.**(CA2)**
2.NBT.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. **(CA2)**
2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects. **(CA2)**
MEASUREMENT AND DATA ● ***Measure and estimate lengths in standard units.***
2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **(CA3)**
2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **(CA3)**
2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters. **(CA3)**
2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **(CA3)**

● ***Relate addition and subtraction to length.***
Standard
2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **(CA2)**
2.MD.6 Represent whole numbers as lengths from 0 on a **number line diagram**[a] with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram. **(CA2)**
● ***Work with time and money.***
2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **(CA3)**
2.MD.8 Solve problems with money. **(CA2)** **a.** Identify nickels and quarters by name and value. **b.** Find the value of a collection of quarters, dimes, nickels, and pennies. **c.** Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ₵ symbols appropriately (not including decimal notation).
● ***Represent and interpret data.***
2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot G, where the horizontal scale is marked off in whole-number units. **(CA3)**
2.MD.10 Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. **See Table 1, page 36. (CA2)**
GEOMETRY ● ***Reason with shapes and their attributes.***
2.G.1 Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders. **(CA4)**
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **(CA4)**
2.G.3 Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape. **(CA4)**

[a] **Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

GRADE 3

In Grade 3, instructional time should focus on five critical areas:
Critical Area 1: Developing understanding of multiplication and division and strategies for multiplication and division within 100 Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
Critical Area 2: Developing understanding of fractions, especially unit fractions (fractions with numerator 1) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
Critical Area 3: Developing understanding of the structure of rectangular arrays and of area Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
Critical Area 4: Describing and analyzing two-dimensional shapes Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.
Critical Area 5: Solving multi-step problems Students apply previous understanding of addition and subtraction strategies and algorithms to solve multi-step problems. They reason abstractly and quantitatively by modeling problem situations with equations or graphs, assessing their processes and results, and justifying their answers through mental computation and estimation strategies. Students incorporate multiplication and division within 100 to solve multi-step problems with the four operations.

OPERATIONS AND ALGEBRAIC THINKING ● ***Represent and solve problems involving multiplication and division.***
Standard
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that an x b means a groups of b objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each). **(CA1)**

OPERATIONS AND ALGEBRAIC THINKING ● ***Represent and solve problems involving multiplication and division, continued.***
Standard
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. **(CA1)**
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **See Table 2, page 37.** Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **(CA1)**
3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × □□= 48; 5 = □□÷ 3; 6 × 6 = □□.* **(CA1)**
● ***Understand properties of multiplication and the relationship between multiplication and division.***
3.OA.5 Apply properties of operations as strategies to multiply and divide. *For example, if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative Property of Multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (Associative Property of Multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) =(8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive Property).* Students need not use formal terms for these properties. **(CA1)**
3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **(CA1)**
● ***Multiple and divide within 100.***
3.OA.7 Fluently G multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8, or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers. **(CA1)**
● ***Solve problems involving the four operations, and identify and explain patterns in arithmetic.***
3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected. **(CA1)(CA5)**
3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. **(CA1)**
NUMBER AND OPERATIONS IN BASE TEN ● ***Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.***
Standard
3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100. **(CA5)**
3.NBT.2 Fluently add and subtract within 1,000 using strategies and **algorithms**[b] based on place value, properties of operations, and/or the relationship between addition and subtraction. **(CA5)**
3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations. **(CA1)**

NUMBER AND OPERATIONS IN BASE - FRACTIONS

- ***Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.***

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. **(CA2)(CA4)**

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a **number line diagram**[a] **(CA2)**

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
b. Represent a fraction *a*/*b* (which may be greater than 1) on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **(CA2)(CA4)**

a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model G.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

MEASUREMENT AND DATA

- ***Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.***

Standard

3.MD.1 Work with time and money. **(CA5)**

a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.
b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ₵ symbol appropriately (not including decimal notation).

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; **See Table 2, page 37. (CA5)**

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[a]**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

[b]**Algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

● ***Represent and interpret data.***
Standard
3.MD.3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.* **(CA5)**
3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot G, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **(CA2)**
● ***Geometric measurement: understand concepts of area and relate to multiplication and to addition.***
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. **(CA3)** a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square Ft., and improvised units). **(CA3)**
3.MD.7 Relate area to the operations of multiplication and addition. **(CA1)(CA3)** a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model). d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.
● ***Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.***
3.MD.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **(CA3)**
GEOMETRY ● ***Reason with shapes and their attributes.***
3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles). **(CA4)**
3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape. **(CA3)**

GRADE 4

In Grade 4, instructional time should focus on three critical areas:

Critical Area 1(CA1): Developing an understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends as part of effectively and efficiently performing multi-digit arithmetic Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Students efficiently and effectively add and subtract multi-digit whole numbers.

Critical Area 2(CA2): Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal, e.g., 15/9 = 5/3, and they develop methods such as using models for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Students solve measurement problems involving conversion of measurements and fractions.

Critical Area 3(CA3): Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, and particular angle measures Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems.

OPERATIONS AND ALGEBRAIC THINKING ● ***Use the four operations with whole numbers to solve problems.***
Standard
4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **(CA1)**
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. **See Table 2, page 37.** Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **(CA1)**
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **(CA1)**

● ***Gain familiarity with factors and multiples.***	
	Standard
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **(CA1)**	
● ***Generate and analyze patterns.***	
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. **(CA2)**	
NUMBER AND OPERATIONS IN BASE TEN ● ***Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.***	
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. **(CA1)**	
4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form G. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. **(CA1)**	
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000. **(CA1)**	
● ***Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.***	
4.NBT.4 Fluently[c] add and subtract multi-digit whole numbers using a standard **algorithm**[b] . **(CA1)**	
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(CA1)**	
4.NBT.6 Find whole-number quotients and remainders with up to four- digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(CA1)**	
NUMBER AND OPERATIONS - FRACTIONS ● ***Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.***	
4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **(CA2)**	
4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **(CA2)**	

[b]**Algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

[c]**Fluently/Fluency.** The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

● ***Build fractions from unit fractions by applying an extending previous understanding of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).***
Standard
4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. **(CA2)** a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model G. *Examples: 3/8 = 1/8 + 1/8 + 1/8*; *3/8 = 1/8 + 2/8*; 2 *1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**(CA2)** a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4) or 5/4 = (1/4) + (1/4) + (1/4) + (1/4) + (1/4).* b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
● ***Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.***
4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. **(CA2)**
4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. **(CA2)**
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. **(CA2)**

MEASUREMENT AND DATA ● ***Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.***
Standard
4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300, **(CA2)**
4.MD.2 Solve real-world problems involving money, time, and metric measurement. **(CA1)(CA2)** a. Using models, add and subtract money and express the answer in decimal notation. b. Using number line diagrams G, clocks, or other models, add and subtract intervals of time in hours and minutes. c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.
4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter. **(CA1)**
● ***Represent and interpret data.***
4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots G) to solve problems using numbers and operations for this grade. **(CA2)**
● ***Geometric measurement: understand concepts of angle and measure angles.***
4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **(CA3)** a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. Understand an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **(CA3)**
4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. **(CA3)**
GEOMETRY ● ***Draw and identify lines and angles, and classify shapes by properties of their lines and angles.***
4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two- dimensional figures. **(CA3)**
4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. **(CA3)**

GRADE 5

In Grade 5, instructional time should focus on five critical areas:

***Critical Area 1(CA1)*:** Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. They apply their understanding of fractions to solve real-world problems. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Critical Area 2(CA2): Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Critical Area 3(CA3): Developing understanding of volume Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Critical Area 4(CA4): Modeling numerical relationships with the coordinate plane Based on previous work with measurement and number lines, students develop understanding of the coordinate plane as a tool to model numerical relationships. These initial understandings provide the foundation for work with negative numbers, and ratios and proportional relationships in Grade Six and functional relationships in further grades.

Critical Area 5(CA5): Classifying two-dimensional figures by properties Students build on their understanding of angle measures and parallel and perpendicular lines to explore the properties of triangles and quadrilaterals. They develop a foundation for classifying triangles or quadrilaterals by comparing the commonalities and differences of triangles or between types of quadrilaterals.

GRADE 5

OPERATIONS AND ALGEBRAIC THINKING ● ***Write and interpret numerical expressions.***
Standard
5.OA.1 Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. **(CA2)**
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product. **(CA2)**
● ***Analyze patterns and relationships.***
5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **(CA4)**
NUMBER AND OPERATIONS IN BASE TEN ● ***Understand the place value system.***
5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **(CA2)**
5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **(CA2)**
5.NBT.3 Read, write, and compare decimals to thousandths. **(CA2)** a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form G, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.4 Use place value understanding to round decimals to any place, millions through hundredths. **(CA2)**
● ***Perform operations with multi-digit whole numbers and with decimals to hundredths.***
5.NBT.5 Fluently[c] multiply multi-digit whole numbers using a standard **algorithm**[b] **(CA2)**
5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(CA2)**

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[b]**Algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

[c]**Fluently/Fluency.** The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

● ***Perform operations with multi-digit whole numbers and with decimals to hundredths, continued.***
Standard
5.NBT.7 Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **(CA2)** a. Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). c. Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*
NUMBER AND OPERATIONS - FRACTIONS ● ***Use equivalent fractions as a strategy to add and subtract fractions. (Fractions need not be simplified).***
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators*. For example, use visual models and properties of operations to show 2/3 + 5/4 = 8/12 + 15/12 = 23/12. In general, a/b + c/d = (a/b x d/d) + (c/d x b/b) = (ad + bc)/bd.* **(CA1)**
5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models G or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. **(CA1)**
● ***Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified).***
5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? **(CA1)**
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **(CA1)(CA3)** a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

● ***Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified), continued.***
Standard
5.NF.5 Interpret multiplication as scaling (resizing). **(CA1)(CA3)** a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **(CA1)**
● ***Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified), continued.***
5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. **(CA1)** a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = (1/12) because (1/12) × 4 = (1/3).* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) =4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?*
MEASUREMENT AND DATA ● ***Convert like measurement units within a given measurement system.***
5.MD.1 Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems. **(CA1)**
● ***Represent and interpret data.***
5.MD.2 Display and interpret data in graphs (picture graphs, bar graphs, and line plots G) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions 1/2, 1/4, 1/8, or decimals. **(CA1)**
● ***Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.***
5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **(CA3)** a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

● ***Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition, continued.***
Standard
5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic Ft., and improvised units. **(CA3)**
5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **(CA3)** a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication. b. Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.
GEOMETRY ● ***Graph points on the coordinate plane to solve real-world and mathematical problems.***
5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., x-axis and x-coordinate, y-axis and y-coordinate. **(CA4)**
5.G.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **(CA4)**
5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). **(CA5)**
5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids G, and rhombuses. **(CA5)**

TABLE 1. COMMON ADDITION ADDITION AND SUBTRACTION SITUATIONS.

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[1]**
PULL TOGETHER/ TAKE APART[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	DIFFERENCE UNKNOWN	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
COMPARE[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "makes" or "results in" but always does mean "is the same number as."

[2] Either addend can be unknown, so there are three variations of these problem situations. *Both Addends Unknown* is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the *Bigger Unknown* or *Smaller Unknown* situations, one version directs the correct operation (the version using more for the *Bigger Unknown* and using less for the *Smaller Unknown*). The other versions are more difficult.

TABLE 2. COMMON MULTIPLICATION AND DIVISION SITUATIONS[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN (HOW MANY IN EACH GROUP? DIVISION)	NUMBER OF GROUPS UNKNOWN (HOW MANY GROUPS? DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? **Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? **Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? **Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	▯▯ × ▯▯ = ?	▯▯ × ? = ▯▯, and ▯▯ ÷ ▯▯ = ?	? × ▯▯ = ▯▯, and ▯▯ ÷ ▯▯ = ?

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

PCSD
Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 3

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 3 Mathematics. A student at this level has a **consistent ability** to demonstrate strategies of multiplication and division, show understanding of fractions, solve problems involving measurement and data, and recognize and distinguish between the area and perimeter of a shape.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 3 Mathematics. A student at this level has a **superior ability** to demonstrate strategies of multiplication and division, show understanding of fractions, solve problems involving measurement and data, and recognize and distinguish between the area and perimeter of a shape.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 3 Mathematics. A student at this level has a **sophisticated ability** to demonstrate strategies of multiplication and division, show understanding of fractions, solve problems involving measurement and data, and recognize and distinguish between the area and perimeter of a shape.
Round whole numbers to the nearest 10 or 100; 3.NTB.1 (DOK 1)		
Demonstrate an understanding of multiplication and division of whole numbers including using equal-sized groups, arrays, and area models; Calculate the product of any two single-digit whole numbers; Multiply and divide within 100 to solve one-step problems; Use the relationship between multiplication and division to solve problems; Use equations, models, tables or graphs to solve routine problems; Use the properties of multiplication and division to solve problems; 3.OA.1,2,3,4,5,6,7 (DOK 1,2)	Use equal-sized groups, arrays, area models, the properties of multiplication and division, and place value to solve a variety of problems; Apply understanding of place value, properties, relationships among operations to solve multiplication and division problems; Use mental computation and estimation strategies to review thinking and results; 3.0A.8,9 (DOK 2,3)	Use equations with unknowns, models and graphs to solve problems; Apply efficient understanding of place value, properties, relationships among operations to solve multiplication and division problems; Use mental computation and estimation strategies to analyze thinking and results; 3.0A.8,9 (DOK 2,3)
Develop an understanding of fractions as numbers; Use fractions along with visual fraction models to represent parts of a whole; Understand that the size of a fractional part is relative to the size of the whole; Use fractions to represent numbers equal to, less than, and greater than one; Solve problems that involve comparing fractions by using visual fraction models; 3.NF.1,2 (DOK 1)	Apply an understanding of fractions to solve routine problems; Use the understanding that the size of a fractional part is relative to the size of the whole to solve problems; Compare fractions using visual fraction models and < > = . 3.NF.2 (DOK 2)	Apply an understanding of fractions to solve problems unfamiliar problems; Explain how the size of a fractional part is relative to the size of the whole; Compare fractions using <, > = . • Solve problems that involve comparing fractions using visual fraction models and strategies based on noticing equal numerators & denominators; 3.NF.2,3 (DOK 2,3)
Understand perimeter and area and their differences; Solve problems involving perimeter;	Solve complex problems involving perimeter and area; Compute the area of a shape composed of non-overlapping rectangles; 3.MD.7,8 (DOK 2)	Understand perimeter and area and explain their differences; Analyze rectangles with the same perimeter and different areas or rectangles with the same area and different perimeters;

Measure the area of a shape: by counting unit squares or by using the relationship of rectangular arrays and multiplication; 3.MD.5,6,7,8 (DOK 1,2)		Write equations to solve problems involving perimeter and/or area; 3.MD.8 (DOK 2)
Solve one-step problems involving measurement units of time, liquid volume, and mass; 3.MD.1,2 (DOK 2)	Solve one- and two-step problems involving measurement units of time, liquid volume, and mass; 3.MD.1,2 (DOK 2)	Solve multi-step problems involving measurement units of time, liquid volume and mass; 3.MD.1,2 (DOK 2)
Add and subtract using strategies and algorithms to solve multi-step problems; 3.NBT.2 (DOK 2)	Add and subtract using strategies and algorithms to solve multi-step problems; 3.NBT.2 (DOK 2)	Add and subtract using efficient strategies and/or algorithms to solve multi-step problems; 3.NBT.2 (DOK 2)
Represent and solve routine one- and two-step problems that may involve up to two different operations: +, -, x, ÷; 3.0A.8 (DOK 2)	Represent and solve routine one- and two-step problems that may involve up to two different operations: +, -, x, ÷; 3.0A.8 (DOK 2)	Represent and solve non-routine multi-step problems involving more than one of the four operations +, - x, ÷; 3.OA.8 (DOK 2)
Compare and classify shapes by their sides and angles; Relate fractions to geometry by naming the area of part of a shape as a fraction of the whole; 3.G.1,2 (DOK 2)	Describe, analyze, and compare properties of quadrilaterals; 3.G.1 (DOK 2)	Analyze the properties of two-dimensional shapes to explain classifications; 3.G.1 (DOK 2)
Create line plots for measurement data with scales of wholes, halves, and fourths of an inch; 3.MD.4 (DOK 2) Represent data on scaled picture graphs and scaled bar graphs; Solve problems using information presented in a scaled bar graph; 3.MD.3 (DOK 2)	Represent data on scaled picture graphs and scaled bar graphs to solve problems; Solve non-routine problems using information presented in a scaled bar graph; 3.MD.3 (DOK 2)	Create scaled picture graphs and scaled bar graphs to represent data and solve problems; 3.MD.3 (DOK 2)

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 3 Mathematics. A student at this level has a **general ability** to demonstrate strategies of multiplication and division, show understanding of fractions, solve problems involving measurement and data, and recognize and distinguish between the area and perimeter of a shape.

A student whose performance falls within the **Basic Level** typically can:

- Carry out routine procedures;
- Solve simple problems using visual representations;
- Compute accurately some grade level numbers and operations;
- Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:

- Demonstrate an understanding of multiplication of whole numbers using equal-sized groups and arrays;
- Use multiplication fact family understanding to solve simple problems;
- Solve routine one-step problems involving any of the four operations +, -, x, ÷;
- Demonstrate a general understanding of fractions (denominators of 2, 3, 4, 6, 8) using visual fraction models;
- Place unit fractions (denominators of 2, 3, 4, 6, 8) on a number line;
- Use fractions to represent numbers less than one and equal to one;

• Compare the visual fraction models of two common fractions (with denominator pairs of 2 and 4, 2 and 8, 3 and 6, 4 and 8);
• Understand how to find perimeter;
• Solve simple problems involving perimeter;
• Measure the area of a shape by counting unit squares;
• Describe and compare sides and angles of common quadrilaterals;
• Write the fraction name for a shaded part of a rectangular whole;
• Add and subtract using strategies to solve routine one-step problems;
• Given models, solve simple problems involving liquid volume or mass;
• Round simple whole numbers to the nearest 10 or 100;
• Complete a scaled picture graph or scaled bar graph;
• Answer simple questions about information presented in a scaled bar graph.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 3 Mathematics. A student at this level has an **emerging ability** to demonstrate strategies of multiplication and division, show understanding of fractions, solve problems involving measurement and data, and recognize and distinguish between the area and perimeter of a shape.

A student whose performance lies within the **Limited Level** typically can:
• Carry out some routine procedures to solve straight forward one-step problems;
• Recognize solutions to some simple computation, straight forward problems;
• Compute a few grade level numbers and operations accurately;
• Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:
• Demonstrate some understanding of multiplication using equal-sized groups and/or arrays;
• Use multiplication fact family understanding or models to solve simple multiplication and division problems;
• Solve familiar one-step problems involving one of the four operations +, -, x, ÷ and basic fact calculation;
• Demonstrate minimal understanding of fractions by naming common unit fractions (denominators of 2, 3, 4, 6, 8) from visual models;
• Complete visual fraction models to represent parts of a whole (denominators of 2, 3, 4, 6, 8);
• Compare the visual fraction models of two common fractions (denominator with pairs of 2 and 4, 3 and 6, 4 and 8);
• Find the perimeter of a simple shape given all the measurements in simple whole numbers;
• Count the unit squares to find the area of a gridded shape;
• Recognize common quadrilaterals (square, rectangle, rhombus, and trapezoid) by the sides or the angles;
• Use strategies to solve straight forward one-step problems involving simple addition and subtraction within 1000;
• Solve simple linear measurement problems;
• Given models, solve straight forward problems involving liquid volume or mass;
• Complete a picture graph or bar graph that uses a unit scale;
• Answer straightforward questions about information presented in a scaled picture graph or scaled bar graph.

PCSD
Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 4

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 4 Mathematics. A student at this level has a **consistent ability** to demonstrate understanding of multi-digit multiplication, demonstrate understanding of addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers, and classify and draw geometric figures and shapes based on their characteristics.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 4 Mathematics. A student at this level has a **superior ability** to demonstrate understanding of multi-digit multiplication, demonstrate understanding of addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers, and classify and draw geometric figures and shapes based on their characteristics.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 4 Mathematics. A student at this level has a **sophisticated ability** to demonstrate understanding of multi-digit multiplication, demonstrate understanding of addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers, and classify and draw geometric figures and shapes based on their characteristics.
Solve routine multi-step word problems posed with whole numbers and whole-number answers using the four operations, including problems in which remainders must be interpreted; 4.OA.3 (DOK 2) Accurately add and subtract multi-digit numbers; 4.NBT.4 (DOK 1) Recognize that in a multi-digit whole number a digit in one place represents ten times what it represents in the place to its right; 4.NBT.1 (DOK 1)	Solve routine multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent the problems using equations with a letter standing for the unknown quantity; 4.OA.3 (DOK2)	Use equations to solve non-routine multi-step word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted; 4.OA.3 (DOK 3)
Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form; 4.NBT.2 (DOK 1) Compare two multi-digit whole numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons; 4.NBT.2 (DOK 1)	Apply place value understanding to read, write, and compare multi-digit whole numbers less than or equal to 1,000,000; 4.NBT.2 (DOK 1)	Compare multi-digit whole numbers using <, >, and =; then justify answers using place value understanding; 4.NBT.2 (DOK 1)
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations; 4.NBT.5 (DOK 1) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication & division;	Use efficient procedures for multiplying whole numbers to solve routine problems; use understanding of place value and properties of operations to explain why the procedures work; 4.NBT.5 (DOK 2) Use efficient procedures by applying understanding of models for division, place value, properties of operations, and the relationship of division to multiplication to find quotients involving multi-digit dividends; and use them to solve routine problems;	Use efficient procedures to accurately multiply multi-digit whole numbers and to find quotients involving multi-digit dividends in solving non-routine problems; 4.NBT.5, 4.NBT.6 (DOK 3)

4.NBT.6 (DOK 1)	4.NBT.6 (DOK 2)	
	Select and accurately apply mental computation and estimation strategies to solve problems and/or assess reasonableness of answers; 4.OA.3 (DOK 3)	Select and accurately apply mental computation and estimation strategies to solve problems, assess reasonableness of answers or to interpret remainders; 4.OA.3, 4.NBT.6 (DOK 3) Use number patterning to solve problems and generalize rules; 4.OA.5 (DOK 2)
Use the four operations to solve routine word problems (metric and US standard units where applicable) involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit; 4.MD.1 (DOK 1) Make conversions within a given measurement system by expressing measurements in a larger unit in terms of a smaller unit; 4.MD.1 (DOK 1)	Know and be able to use conversions in multiple measurement units to solve real world problems; 4.MD.2 (DOK 2)	Use the four operations to solve non-routine word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit; 4.MD.2 (DOK 3)
Apply the formulas for area and perimeter to rectangles with whole number sides in real-world problems; 4.MD.3 (DOK 2)	Apply perimeter and area formulas to solve routine real-world problems. 4.MD.3 (DOK 2)	Apply perimeter and area formulas to solve routine real-world problems. 4.MD.3 (DOK 2)
Identify and generate equivalent forms of a fraction including mixed numbers with like denominators using visual models; 4.NF.1 (DOK 2) Compare two fractions with different numerators and different denominators using visual models or by creating common denominators or numerators; record using the symbols <, >, and =; 4.NF.2 (DOK 3)	Compare two fractions with different numerators and different denominators using the symbols <, >, And =; 4.NF.2 (DOK 3)	Solve problems by comparing two fractions with different numerators and different denominators using the symbols <, >, and =, and justifying the conclusion using a visual fraction model; 4.NF.2 (DOK 3)
Compare two decimals to the hundredths place by using place value understanding, models, or number lines; record using <, >, and =. 4.NF.7 (DOK 1)	Compare two decimals to the hundredths place using the symbols <, >, and =; 4.NF.7 (DOK 1)	
Add and subtract to solve routine word problems involving like denominators in reference to the same whole using visual models and/or equations; 4.NF.3D (DOK 2) Create a line plot to represent a data set using the fractions 12, 14, and 18; then solve problems involving addition and subtraction; 4.MD.4 (DOK 2) • Solve routine word problems involving multiplication of a fraction by a whole number;		Interpret line plot data including fractions to solve non-routine real-world problems; 4.MD.4 (DOK 3)

4.NF.4C (DOK 2)		
Determine the measure of an angle by using the sum of two angle parts; 4.MD.7 (DOK 1)	Solve addition and subtraction problems to find unknown angles in a diagram with a symbol for the unknown angle measure; 4.MD.7 (DOK 2) Measure and/or draw angles in whole number degrees using a protractor to solve problems; 4.MD.6 (DOK 1)	Demonstrate an understanding of the concepts of angles by determining the measure of complex angles using appropriate strategies (protractors, equations); 4.MD.6 4.MD.7 (DOK 2) Write and solve equations to find the measure of angles including those with multiple parts or missing parts from real world problems; 4.MD.7 (DOK 2)
Draw and identify in two dimensional figures; lines, line segments, rays, perpendicular and parallel lines and angles; 4.G.1 (DOK 1) Classify two-dimensional figures (e.g. squares, rectangles and right triangles) based on the properties of parallel or perpendicular lines and/or angle sizes; 4.G.2 (DOK 2)	Classify two-dimensional shapes by the properties of their lines and angles; 4.G.2 (DOK 2)	

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 4 Mathematics. A student at this level has a **general ability** to demonstrate understanding of multi-digit multiplication, demonstrate understanding of addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers, and classify and draw geometric figures and shapes based on their characteristics.
A student whose performance falls within the **Basic Level** typically can:
• Carry out routine procedures;
• Solve simple problems using visual representations;
• Compute accurately some grade level numbers and operations;
• Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:
• Solve one-step problems involving addition, subtraction, multiplication or division and an unknown number;
• Solve two-step word problems using visual representations;
• Using a place value chart, determine the relationship of a digit to the digit to its right;
• Multiply four digit by one digit numbers using manipulatives, place value strategies, or visual representations;
• Given a place value chart, compare two multi-digit whole numbers based on the meanings of the digits in each place, using <, >, = symbols to record the results of the comparisons;
• Divide 4 digit dividends by 1 digit divisors with remainders using manipulatives, strategies, or visual representations;
• Compare two fractions with different numerators and different denominators using the symbols <, >, and = with the assistance of visual models (denominators limited to 2, 4, 8, 10, and 100);
• Using visual models, add and subtract fractions with like denominators in reference to the same whole;
• Solve mathematical problems involving multiplication of a fraction by a whole number, with the assistance of a visual model;
• Write fractions with denominators of 10 and 100 as decimals;
• Solve simple addition and subtraction problems involving fractions from data in a line plot;
• Measure and/or draw angles in whole number degrees using a protractor;
• Identify points, perpendicular and parallel lines, right, acute and obtuse angles in two-dimensional figures;

• Sort two-dimensional figures by their perpendicular and parallel sides and angle sizes.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 4 Mathematics. A student at this level has an **emerging ability** to demonstrate understanding of multi-digit multiplication, demonstrate understanding of addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers, and classify and draw geometric figures and shapes based on their characteristics.

A student whose performance lies within the **Limited Level** typically can:

• Carry out some routine procedures to solve straightforward one-step problems;
• Recognize solutions to some simple computation, straightforward problems;
• Compute accurately a few grade level numbers and operations;
• Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

• Use the four operations to solve one-step problems;
• Solve straightforward one-step word problems using basic multiplication and division facts;
• Divide up to four digit dividends and one digit divisors with no remainders using manipulatives, strategies, or visual representations;
• Solve straightforward real-world area and perimeter problems of rectangles involving basic computations;
• Use visual models to determine whether two common fractions are equivalent;
• Add and subtract fractions with like denominators using visual models;
• Compare two decimals using visual models;
• Identify points, perpendicular and parallel lines, right, acute and obtuse angles;
• Measure angles in whole number degrees using a protractor;
• Sort two-dimensional figures by perpendicular or parallel sides and presence or absence of right angles.

PCSD
Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 5

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 5 Mathematics. A student at this level has a consistent ability to perform all four arithmetic operations with fractions, use understanding of place values to perform all four arithmetic operations with whole numbers and decimal numbers, and solve problems involving volumes of rectangular prisms with whole-number edge lengths.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 5 Mathematics. A student at this level has a superior ability to perform all four arithmetic operations with fractions, use understanding of place values to perform all four arithmetic operations with whole numbers and decimal numbers, and solve problems involving volumes of rectangular prisms with whole-number edge lengths.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 5 Mathematics. A student at this level has a sophisticated ability to perform all four arithmetic operations with fractions, use understanding of place values to perform all four arithmetic operations with whole numbers and decimal numbers, and solve problems involving volumes of rectangular prisms with whole-number edge lengths.
Solve most routine and straightforward problems accurately;	Accurately solve routine and straightforward problems;	Accurately solve routine and straightforward problems;
	Accurately solve routine and straightforward problems;	Accurately solve routine and straightforward problems;
Compute accurately with most grade level numbers and operations;	Compute accurately and efficiently with familiar numbers;	Compute accurately and efficiently with familiar numbers;
Apply most grade level mathematical concepts, terms and properties, and use informal (visual representation and language) and some formal reasoning;	Recognize connections between mathematical concepts, terms and properties, and use informal and some formal reasoning with symbolic representation;	Recognize connections between mathematical concepts, terms and properties, and use informal and some formal reasoning with symbolic representation;
Solve routine word problems by finding the sum or difference of fractions and mixed numbers with unlike denominators;		Represent the sum or difference of fractions and mixed numbers in a form that is appropriate for the real-world situation;
Multiply a proper fraction by a mixed number to solve routine real-world problems; NF.4 & 6 (DOK 1 & 2)	Multiply a mixed number by a mixed number and use to solve real-world problems; NF.4 & 6 (DOK 1 & 2) Create contexts for multiplying two fractions, solve with visual models; NF.4 & 6 (DOK 3)	Create contexts for multiplying two fractions, solve without visual models; NF.4 & 6 (DOK 3)
Solve a routine real-world problem involving multiplying two fractions by using visual models; NF.6 (DOK 1 & 2)	Use visual models when multiplying two factions that are larger than 1: NF.6 (DOK 1 & 2)	Create visual models when multiplying two fractions that are larger than 1; NF.6 (DOK 3)
Know the effect that a fraction has on another fraction when multiplied (both fractions less than 1 and both fractions greater than 1); NF.5 (DOK 2 & 3)	Understand and use the fact that a fraction multiplied by 1 in the form of a/a is equivalent to the original fraction; NF.5 (DOD 2 & 3) Interpret and perform division of a unit fraction by a whole number in solving real-world problems; NF.7 (DOK 1 & 2)	

Solve problems involving information presented in line plots; MD.2 (DOK 1 & 2)	Draw conclusions from line plots; MD.2 (DOK 3)	
Write and evaluate numerical expressions having any number of non-nested sets of parentheses; OA.1 & 2 (DOK 2)	Write and evaluate numerical expressions with non-nested parentheses, use to solve problems; OA.1 & 2 (DOK 2)	Write, evaluate, and interpret complex numerical expressions having any number of non-nested sets of parentheses, brackets, or braces, also use to solve problems; OA.1 & 2 (DOK 3)
Determine the value of a digit when multiplied or divided by 10 and powers of 10; (DOK 1 & 2)		
Read, write, and compare two decimals, to the thousandths, based on the meaning of the digits in each place, record using the symbols >, =, < and supported with place value charts and/or decimal models; NBT.1 & 2 (DOK 1)	Recognize the place value relationships between digits in multi-digit numbers and use them to read, write and compare decimals to the thousandths place and record with <, >, =; NBT.1 & 2 (DOK 2)	Explain the relationship of exponents to the number of zeroes when multiplying by powers of 10; NBT.1 & 2 (DOK 2 & 3)
Use strategies to multiply multi-digit numbers to solve mathematical and routine real-world problems; NBT.5 (DOK 1)	Use efficient strategies to multiply multi-digit numbers to solve mathematical and routine real-world problems; NBT.5 (DOK 2 & 3)	
Use a variety of strategies based on place value, properties of operations, and the relationship of multiplication to division to divide up to four-digit dividends by one- or two-digit divisors; NBT.6 (DOK 1)	Use a variety of strategies based on place value, properties of operations, and the relationship of multiplication to division to divide up to four-digit dividends by two-digit divisors; NBT.6 (DOK 1) Explain calculations of whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors; NBT.6 (DOK 3)	
Perform the four operations on decimals to the hundredths place and demonstrate a partial explanation of reasoning; NBT.7 (DOK 1 & 2)	Use efficient and accurate strategies to perform the four operations on decimals to the hundredths place and explain reasoning; NBT.7 (DOK 3)	
Convert and apply measurement units within a given measurement system to solve routine problems; MD.1 (DOK 1)	Use measurement conversions within a measurement system in solving real-world problems; MD.1 (DOK 2)	Use measurement conversions within a measurement system in solving multi-step, real-world problems; MD.1 (DOK 2)
Use the formulas $v = l \times w \times h$ and $v = b \times h$ to find the volume of right rectangular prisms; MD.3-5 (DOK 1)	Find the volume of figures composed of two or more non-overlapping right rectangular prisms in routine real-world problems; MD.3-5 (DOK 2)	Find the volume of figures composed of two or more non-overlapping right rectangular prisms in routine and non-routine real-world problems; MD.3-5 (DOK 3)
Compare and analyze two related numerical patterns within sequences of ordered pairs, and graph the ordered pairs on the coordinate plane; G.1 & 2 (DOK 2)	Solve routine real-world and mathematical problems by graphing points in the first quadrant of a coordinate grid; G.1 & 2 (DOK 2)	Solve non-routine real-world and mathematical problems by graphing points in the first quadrant of a coordinate grid; G.1 & 2 (DOK 3)
	Classify two-dimensional figures into multiple categories and subcategories by their attributes or properties; G.3 & 4 (DOK 2)	

Foundation PLDs not included in Proficient Level but required for success:

- Divide a whole number by 1/2 or 1/3 using visual models (from Limited)
- Divide a unit fraction by a whole number using visual models (from Basic)
- Represent multi-digit numbers in expanded form, including decimals to the thousandths place (from Basic)
- Round decimals to the hundredths place (from Basic)
- Classify two-dimensional figures into categories by their attributes or properties (from Basic; doesn't reappear in PLDs until the Accelerated Level)

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Grade 5 Mathematics. A student at this level has a general ability to perform all four arithmetic operations with fractions, use understanding of place values to perform all four arithmetic operations with whole numbers and decimal numbers, and solve problems involving volumes of rectangular prisms with whole-number edge lengths.
A student whose performance falls within the Basic Level typically can:
• Carry out routine procedures;
• Solve simple problems using visual representations;
• Compute accurately some grade level numbers and operations;
• Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:
• Find the sum or difference of common fractions with unlike denominators using visual models to solve straightforward problems;
• Multiply a whole number by a mixed number using visual models to solve straightforward problems;
• Divide a unit fraction by a whole number using visual models;
• Find a sum or difference involving information presented in a line plot;
• Evaluate numerical expressions having two non-nested sets of parentheses;
• Read and write decimals to the thousandths place using word names;
• Represent multi-digit numbers in expanded form, including decimals to the thousandths place;
• Round decimals to the hundredths place;
• Using strategies find the product of multi-digit factors;
• Use base-ten models to divide up to four digit dividends by one or two digit divisors;
• Use strategies and models to add, subtract and multiply decimals to the hundredths place;
• Convert different-sized measurement units within one measurement system;
• Understand the concept that the volume of a right rectangular prism packed with unit cubes is related to the edge lengths; determine the volume;
• Graph whole number coordinate pairs in the first quadrant of a coordinate grid to solve routine problems;
• Classify two-dimensional figures into categories by their attributes or properties.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 5 Mathematics. A student at this level has an emerging ability to perform all four arithmetic operations with fractions, use understanding of place values to perform all four arithmetic operations with whole numbers and decimal numbers, and solve problems involving volumes of rectangular prisms with whole-number edge lengths.
A student whose performance falls within the **Limited Level** typically can:
• Carry out some routine procedures to solve straightforward one-step problems;
• Recognize solutions to some simple computation, straightforward problems;
• Compute accurately a few grade level numbers and operations;
• Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

- Solve straightforward word problems involving addition and subtraction of fractions with common unlike denominators multiples, 2, 4; 3, 6, under 12;
- Multiply a whole number by a fraction to solve straightforward problems using visual models;
- Divide a whole number by 12 or 13 using visual models;
- Make a line plot and display data sets involving halves, quarters, and eighths;
- Evaluate numerical expressions with one set of parentheses;
- Given a place value chart, read decimals to the thousandths place using number names;
- Represent multi-digit whole numbers in expanded form;
- Round decimals to the whole number;
- Multiply one- and two-digit whole numbers and find whole-number quotients of whole numbers with up to three-digit dividends and one-digit divisors;
- Add, subtract, and multiply decimals to tenths using concrete models or drawings;
- Complete a chart converting different-sized, standard whole-number measurement units within one measurement system: inches to feet, cups to quarts, etc.;
- Use unit cubes to find the volume of right rectangular prisms with whole number edge lengths; Count the cubes to determine the volume;
- Graph whole-number coordinate pairs in the first quadrant of a coordinate plane.

INSTRUCTION, MATERIALS, AND RESOURCE REQUIREMENTS AND RECOMMENDATIONS

The content contained within this course guide outlines the district expectations for the sequencing of teaching and assessment activities throughout each school year for each grade/course. Specific material and/or resource recommendations will be communicated through the Dept. of Curriculum & Instruction at the beginning of each school year. Staff may also consult the Ohio Department of Education Model Curricula page by visiting the ODE web site and searching “Model Curriculum” for the subject specific documents.

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