

Sequence of Grade 5 Modules Aligned with the Standards

Module 1: Place Value and Decimal Fractions

Module 2: Multi-Digit Whole Number and Decimal Fraction Operations

Module 3: Addition and Subtraction of Fractions

Module 4: Multiplication and Division of Fractions and Decimal Fractions

Module 5: Addition and Multiplication with Volume and Area

Module 6: Problem Solving with the Coordinate Plane

Summary of Year

Grade 5 mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Key Areas of Focus for 3–5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving

Required Fluency: 5.NBT.5 Multi-digit multiplication.

Major Emphasis Clusters

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Rationale for Module Sequence in Grade 5

Students' experiences with the algorithms as ways to manipulate place value units in Grades 2–4 really begin to pay dividends in Grade 5. In Module 1, whole number patterns with number disks on the place value chart are easily generalized to decimal numbers. As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate the value and the meaning of decimals. Students apply their work with place value to adding, subtracting, multiplying, and dividing decimal numbers with tenths and hundredths.

Module 2 begins by using place value patterns and the distributive and associative properties to multiply multi-digit numbers by multiples of 10 and leads to fluency with multi-digit whole number multiplication.⁷⁵ For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for teaching the multi-digit algorithm). While the multi-digit multiplication algorithm is a straightforward generalization of the one-digit multiplication algorithm, the division algorithm with two-digit divisors requires far more care to teach because students have to also learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

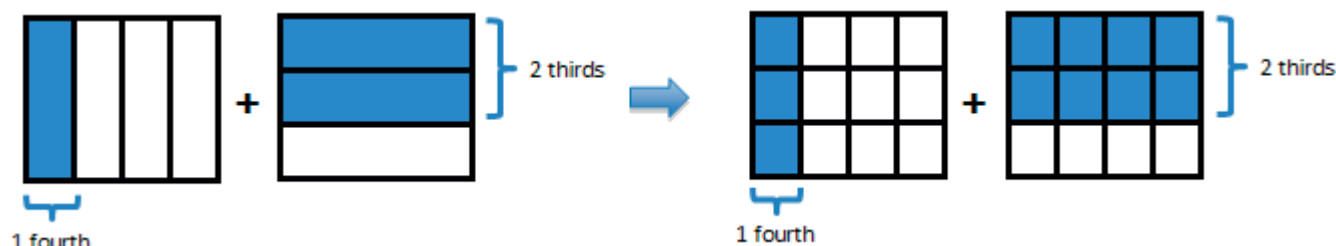
Work with place value units paves the path toward fraction arithmetic in Module 3 as elementary math's place value emphasis shifts to the larger set of fractional units for algebra. Like units are added to and subtracted from like units:

$$1.5 + 0.8 = 1\frac{5}{10} + \frac{8}{10} = 15 \text{ tenths} + 8 \text{ tenths} = 23 \text{ tenths} = 2 \text{ and } 3 \text{ tenths} = 2\frac{3}{10} = 2.3$$

$$1\frac{5}{9} + \frac{8}{9} = 14 \text{ ninths} + 8 \text{ ninths} = 22 \text{ ninths} = 2 \text{ and } 4 \text{ ninths} = 2\frac{4}{9}$$

The new complexity is that when units are not equivalent, they must be changed for smaller equal units so that they can be added or subtracted. Probably the best model for showing this is the rectangular fraction model pictured below. The equivalence is then represented symbolically as students engage in active meaning-making rather than obeying the perhaps mysterious command to “multiply the top and bottom by the same number.”

1 boy + 2 girls = 1 child + 2 children = 3 children
 1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

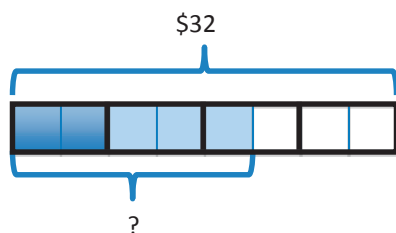


$$\frac{1}{4} + \frac{2}{3} = \left(\frac{1 \times 3}{4 \times 3}\right) + \left(\frac{2 \times 4}{3 \times 4}\right) = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

⁷⁵ Multi-digit decimal multiplication such as 4.1×3.4 and division such as $4.5 \div 1.5$ are studied in Module 4.

Relating different fractional units to one another requires extensive work with area and number line diagrams whereas tape diagrams are used often in word problems. Tape diagrams, which students began using in the early grades and which become increasingly useful as students applied them to a greater variety of word problems, hit their full strength as a model when applied to fraction word problems. At the heart of a tape diagram is the now-familiar idea of forming units. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic, as in the following example:

Jill had \$32. She gave $\frac{1}{4}$ of her money to charity and $\frac{3}{8}$ of her money to her brother. How much did she give altogether?



Solution with units:

$$\begin{aligned} 8 \text{ units} &= \$32 \\ 1 \text{ unit} &= \$4 \\ 5 \text{ units} &= \$20 \end{aligned}$$

Solution with arithmetic:

$$\begin{aligned} \frac{1}{4} + \frac{3}{8} &= \frac{2}{8} + \frac{3}{8} = \frac{5}{8} \\ \frac{5}{8} \times 32 &= 20 \end{aligned}$$

Jill gave \$20 altogether.

Near the end of Module 4, students know enough about fractions and whole number operations to begin to explore multi-digit decimal multiplication and division. In multiplying 2.1×3.8 , for example, students now have multiple skills and strategies that they can use to locate the decimal point in the final answer, including:

- Unit awareness: $2.1 \times 3.8 = 21 \text{ tenths} \times 38 \text{ tenths} = 798 \text{ hundredths}$
- Estimation (through rounding): $2.1 \times 3.8 \approx 2 \times 4 = 8$, so $2.1 \times 3.8 = 7.98$
- Fraction multiplication: $\frac{21}{10} \times \frac{38}{10} = 21 \times \frac{1}{10} \times 38 \times \frac{1}{10} = 21 \times 38 \times \frac{1}{100} = \frac{798}{100}$

Similar strategies enrich students' understanding of division and help them to see multi-digit decimal division as whole number division in a different unit. For example, we divide to find, "How many groups of 3 apples are there in 45 apples?" and write $45 \text{ apples} \div 3 \text{ apples} = 15$. Similarly, $4.5 \div 0.3$ can be written as $45 \text{ tenths} \div 3 \text{ tenths}$ with the same answer: *There are 15 groups of 0.3 in 4.5*. This idea was used to introduce fraction division earlier in the module, thus gluing division to whole numbers, fractions, and decimals together through an understanding of units.

Frequent use of the area model in Modules 3 and 4 prepares students for an in-depth discussion of area and volume in Module 5. But the module on area and volume also reinforces work done in the fraction module. Now, questions about how the area changes when a rectangle is scaled by a whole or fractional scale factor may be asked, and missing fractional sides may be found. Measuring volume once again highlights the unit theme, as a unit cube is chosen to represent a volume unit and used to measure the volume of simple shapes composed of rectangular prisms.

In this final module of *A Story of Units*, students connect plane geometry with numerical work to investigate relationships. They construct the coordinate plane, plot points and draw lines. For points on a given line, students discover a common relationship between the x and y coordinates, foreshadowing the proportional reasoning of Grade 6, and later, the slope of a line.

Alignment Chart⁷⁶

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
<p>Module 1: Place Value and Decimal Fractions (20 days)</p>	<p>Understand the place value system.</p> <p>5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.</p> <p>5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p>5.NBT.3 Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.</p> <p>b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>5.NBT.4 Use place value understanding to round decimals to any place.</p> <p>Perform operations with multi-digit whole numbers and with decimals to hundredths.⁷⁷</p> <p>5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>

⁷⁶ When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

⁷⁷ This standard is addressed again in Modules 2 and 4; the balance of this cluster is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>Convert like measurement units within a given measurement system.⁷⁸</p> <p>5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>
<p>Module 2: Multi-Digit Whole Number and Decimal Fraction Operations (35 days)</p>	<p>Write and interpret numerical expressions.⁷⁹</p> <p>5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</p> <p>5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i></p> <p>Understand the place value system.⁸⁰</p> <p>5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p> <p>5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.⁸¹</p> <p>5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using</p>

⁷⁸ The focus of this module is on the metric system to reinforce place value and writing measurements using mixed units.

⁷⁹ These skills are also applied to fractions in this module.

⁸⁰ The balance of this cluster is addressed in Module 1.

⁸¹ From this point forward, fluency practice is part of students’ on-going experience.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>equations, rectangular arrays, and/or area models.</p> <p>5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.⁸²</p> <p>Convert like measurement units within a given measurement system.</p> <p>5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>
<p>Module 3: Addition and Subtraction of Fractions (22 days)</p>	<p>Use equivalent fractions as a strategy to add and subtract fractions.⁸³</p> <p>5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. <i>For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)</i></p> <p>5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. <i>For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.</i></p>
<p>Module 4: Multiplication and Division of Fractions and Decimal Fractions (38 days)</p>	<p>Write and interpret numerical expressions.</p> <p>5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</p>

⁸² Focus on decimal multiplication of a single-digit, whole number factor times a multi-digit number with up to 2 decimal places (e.g., 3×64.98). Restrict decimal division to a single-digit whole number divisor with a multi-digit dividend with up to 2 decimal places (e.g., $64.98 \div 3$). The balance of the standard is addressed in Module 4.

⁸³ Examples in this module also include tenths and hundredths in fraction and decimal form.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i></p> <p>Perform operations with multi-digit whole numbers and with decimals to hundredths.⁸⁴</p> <p>5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p> <p>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.⁸⁵</p> <p>5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i></p> <p>5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. <i>For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)</i></p>

⁸⁴ The balance of this cluster is addressed in Module 2. Teach problems such as 2.7×2.1 and $4.5 \div 1.5$. See the Progression Document “K–5, Number and Operations in Base Ten” pp. 17–18 (http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf).

⁸⁵ 5.NF.4b is addressed in Module 5. Include problems involving decimal fractions throughout the cluster.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>5.NF.5 Interpret multiplication as scaling (resizing), by:</p> <ol style="list-style-type: none"> Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. <p>5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p> <p>5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)</p> <ol style="list-style-type: none"> Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. <i>For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.</i> Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.</i> Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?</i>

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>Convert like measurement units within a given measurement system.⁸⁶</p> <p>5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p> <p>Represent and interpret data.</p> <p>5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p>
<p>Module 5: Addition and Multiplication with Volume and Area (25 days)</p>	<p>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.⁸⁷</p> <p>5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p> <p>5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p>

⁸⁶ The focus of 5.MD.1 in this module is on the customary system of units as a means of introducing fractions (e.g., 1 inch is $\frac{1}{12}$ foot, 1 foot is $\frac{1}{3}$ yard).

⁸⁷ The balance of this cluster is addressed in Module 4. In this module, 5.NF.4b is applied to multiplying to find volume and area. 5.NF.4b includes decimal fraction side lengths of sides of a rectangle (in both fraction and decimal form).

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</p> <p>5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</p> <ol style="list-style-type: none"> A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. <p>5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</p> <p>5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</p> <ol style="list-style-type: none"> Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. <p>Classify two-dimensional figures into categories based on their properties.</p> <p>5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i></p>

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	<p>5.G.4 Classify two-dimensional figures in a hierarchy based on properties.</p>
<p>Module 6: Problem Solving with the Coordinate Plane (40 days)</p>	<p>Write and interpret numerical expressions.</p> <p>5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.</i></p> <p>Analyze patterns and relationships.</p> <p>5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i></p> <p>Graph points on the coordinate plane to solve real-world and mathematical problems.</p> <p>5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p> <p>5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>