Operations of Algebraic Thinking

1. Interpret and write equations for multiplicative comparison.
2. Solve word problems involving multiplication comparison using drawings and write equations to represent the problem, using a symbol for the unknown number.
3. Determine and justify solutions for multi-step word problems, including problems where remainders must be interpreted.
a. Write equations to show solutions for multi-step word problems with a letter standing for the unknown quantity.
b. Determine the reasonableness of answers for multi-step word problems, using mental computation and estimation strategies including rounding.
4. For whole numbers in the range 1 to 100, find all factor pairs, identifying a number as a multiple of each of its factors.
a. Determine whether a whole number in the range 1 to 100 is a multiple of a given one-digit number.
b. Determine whether a whole number in the range 1 to 100 is prime or composite.
5. Generate and analyze a number or shape pattern that follows a given rule.

Operations with Numbers: Base Ten

6. Using models and quantitative reasoning, explain that in a multi-digit whole number, a digit in any place represents ten times what it represents in the place to its right.
7. Read and write multi-digit whole numbers using standard form, word form, and expanded form.
8. Use place value understanding to compare two multi-digit numbers using >, =, and < symbols.
9. Round multi-digit whole numbers to any place using place value understanding.
10. Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.
11. Find the product of two factors (up to four digits by a one-digit number and two two-digit numbers), using strategies based on place value and the properties of operations.
a. Illustrate and explain the product of two factors using equations, rectangular arrays, and area models.
12. Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends.
a Illustrate and/or explain quotients using equations, rectangular arrays, and/or area models.

Operations with Numbers: Fractions

13 Using area and length fraction models, explain why one fraction is equivalent to another, taking into account that the number and size of the parts differ even though the two fractions themselves are the same size.
a. Apply principles of fraction equivalence to recognize and generate equivalent fractions.
Example: $\frac{a}{b}$ is equivalent to $\frac{n \times a}{n \times b}$.
14 Compare two fractions with different numerators and different denominators using concrete models, benchmarks (0, $\frac{1}{2}$, 1), common denominators, and/or common numerators, recording the comparisons with symbols >, =, or <, and justifying the conclusions.
a. Explain that comparison of two fractions is valid only when the two fractions refer to the same whole.
15 Model and justify decompositions of fractions and explain addition and subtraction of fractions as joining or separating parts referring to the same whole.
a Decompose a fraction as a sum of unit fractions and as a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
b. Add and subtract fractions and mixed numbers with like denominators using fraction equivalence, properties of operations, and the relationship between addition and subtraction.
c. Solve word problems involving addition and subtraction of fractions and mixed numbers having like denominators, using drawings, visual fraction models, and equations to represent the problem.
16. Apply and extend previous understandings of multiplication to multiply a whole number times a fraction.
a. Model and explain how a non-unit fraction can be represented by a whole number times the unit fraction.
Example: $\frac{9}{8} = 9 \times \frac{1}{8}$
b. Extend previous understanding of multiplication to multiply a whole number times any fraction less than one.
Example: $4 \times \frac{2}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$
c. Solve word problems involving multiplying a whole number times a fraction using visual fraction models and equations to represent the problem.
Examples: $3 \times \frac{1}{2}$; $3 \times \frac{1}{8}$
17. Express, model, and explain the equivalence between fractions with denominators of 10 and 100.
a. Use fraction equivalency to add two fractions with denominators of 10 and 100.
18. Use models and decimal notation to represent fractions with denominators of 10 and 100.
19. Use visual models and reasoning to compare two decimals to hundredths (referring to the same whole), recording comparisons using symbols >, =, or <, and justifying the conclusions.

Data Analysis

20. Interpret data in graphs (picture, bar, and line plots) to solve problems using numbers and operations.
a. Create a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, & $\frac{1}{8}$).
b. Solve problems involving addition and subtraction of fractions using information presented in line plots.

Measurement

21. Select and use an appropriate unit of measurement for a given attribute (length, mass, liquid volume, time) within one system of units: metric - km, m, cm; kg, g, l, ml; customary - lb, oz; time - hr, min, sec.
a. Within one system of units, express measurements of a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.
22. Use the four operations to solve measurement word problems with distance, intervals of time, liquid volume, mass of objects, and money.
a. Solve measurement problems involving simple fractions or decimals.
b. Solve measurement problems that require expressing measurements given in a larger unit in terms of a smaller unit.
c. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
23. Apply area and perimeter formulas for rectangles in real-world and mathematical situations.
24. Identify an angle as a geometric shape formed wherever two rays share a common endpoint.
25. Use a protractor to measure angles in whole-number degrees and sketch angles of specified measure.
26. Decompose an angle into non-overlapping parts to demonstrate that the angle measure of the whole is the sum of the angle measures of the parts.
a. Solve addition and subtraction problems on a diagram to find unknown angles in real-world or mathematical problems.

Geometry

27. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines, and identify these in two-dimensional figures.
28. Identify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
a. Describe right triangles as a category, and identify right triangles.
29. Define a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts.
a. Identify line-symmetric figures and draw lines of symmetry.

Quarter 1 August -October

Standards: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, & 23

Academic Vocabulary: whole number, operations, unknown quantity, equations, reasonableness, estimation, multi-digit whole number, digit place value, digit, base-ten numerals, number names, expanded form, compare symbols, round, fluently, properties of operations, & standard algorithm

Resources used: Eureka, AMSTI supplemental materials, Illustrative Mathematics, Howard County Resources, Math by the Book, Math in Practice

Standards: 6, 7, 8, & 9
Big Idea: Place Value & Rounding

Topics Covered:
-Place Value of Multi-Digit Whole Numbers
-Comparing Multi-Digit Whole Numbers
-Rounding Multi-Digit Whole Numbers

Standards: 10
Big Idea: Addition & Subtraction

Topics Covered:
-Multi-Digit Whole Number Addition
-Multi-Digit Whole Numbers Subtraction
-Addition & Subtraction Word Problems

Standards: 1, 2, 3, 4, 11, & 23
Big Idea: Multiplication Comparison/Multiplication

Topics Covered:
-Multiplicative Comparison Word Problems
-Multiplication by 10, 100, & 1,000
-Multiplication of up to 4 digits by single digit numbers
Note the US Algorithm for multiplication is not used. Use other strategies including proportional area models and partial products.
-Multiplication Word Problems
-Multiplication of 2-digit by 2 digit numbers

Manipulatives & Visual Models

Addition: Partial Sums

Many times it is easier to break apart addends. Often it makes sense to break them apart by their place value. Consider 248 + 345

248 = 200 + 40 + 8
345 = 300 + 40 + 5
500 + 80 + 13 = 593

Sometimes we might use partial sums in different ways to make an easier problem. Consider 484 + 276

484 = 400 + 84
276 = 260 + 16
660 + 100 = 760

Addition: Adjusting

We can adjust addends to make them easier to work with. We can adjust by giving a value from one addend to another.

Consider 326 + 274. We can take 1 from 326 and give it to 274.

More Friendly Problem → 326 + 274 / -1 +1 / 325 + 275 = 600

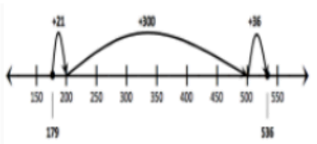
Consider 173 + 389. We can take 27 from 389 and give it to 173 to make 200.

More Friendly Problem → 173 + 389 / +27 -27 / 200 + 362 = 562

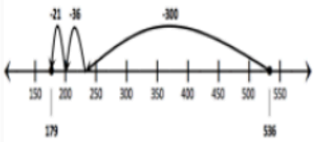
Subtraction: Count Up or Count Back

When subtracting, we can count back to find the difference of two numbers. In many situations, it is easier to count up.

Consider 536 – 179



We can count up from one number to the other. The difference is 300 + 21 + 36 or 357. (above)



We can count back from one number to the other. The difference is -300 (land at 236), -36 (land at 200), – 21 (end at 179).

Subtraction: Adjusting

We can use "friendlier numbers" to solve problems. 4,000 – 563 can be challenging to regroup. But the difference between these numbers is the same as the difference between 3,999 – 562. Now, we don't need to regroup.

(Original problem) **4,000** - **563** =
(Compensation) - 1 - 1
3,999 - 562 = 3,437

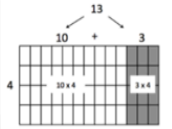
Expanded Notation

	Thousands	Hundreds	Tens	Ones
2,708 =	(2 x 1000) +	(7 x 100) +	(0 x 10) +	(8 x 1)
372 =		(3 x 100) +	(7 x 10) +	(2 x 1)

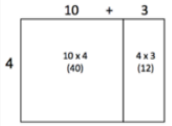
Multiplication: Area/Array

The area/array model for multiplication and the distributive property are used to solve multiplication problems

13 x 4 =
(10 x 4) + (3 x 4) =
40 + 12 =
52



This is the same model without grid lines. It is considered an "open model."



40 + 12 = 52

Another example, consider 4 x 327. We can break 327 into (300 + 20 + 7) then multiply.

4 x 300 = 1,200
4 x 20 = 80
+ 4 x 7 = 28
1,308

So, 4 x 327 = 1,308

	20	+	9
10	200 (20 x 10)		90 (9 x 10)
+			
4	80 (20 x 4)		36 (9 x 4)

200 + 90 + 80 + 36 = 406 So, 29 x 14 = 406

Quarter 2 October -December

Standards: 12, 21, 22, 24, 25, and 26

Academic Vocabulary: remainders, quotients, convert, kilometer, mass, milliliter, mixed units, algorithm, capacity, distance, equivalent, kilogram, length, liter, meter, centimeter, mixed units, weight, table, convert, foot gram, hour, inch, interval, length, liter, meter, minute, seconds, yard, quart, pound, pint, ounce, gallon, angle, acute angle, acute triangle, adjacent angle, arc, complementary angles, degree, & diagonal

Resources used: Eureka, AMSTI supplemental materials, Illustrative Mathematics, Howard County Resources, Math by the Book, Math in Practice

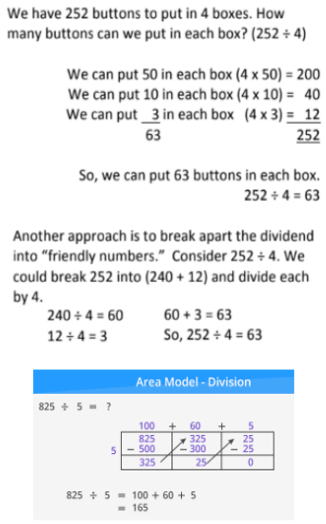
Standard: 12
Big Idea: Division

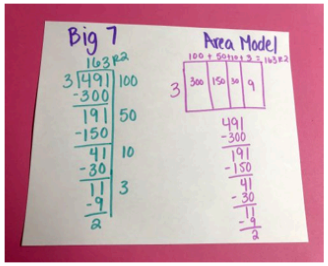
Topics Covered:
-Division of 10s & 1s with Successive Remainders
-Reasoning with Divisibility
-Division of 1,000s, 100s, 10s and 1s

Standards: 21, 22, 24, 25, 26, 27, 28, & 29
Big Idea: Measurement/Geometry

Topics Covered:
-Lines & Angles
-Angle Measurement
-Problem Solving with the Addition of Angle Measures
-Metric Unit Conversions
-Application of Metric Unit Conversions
-Measurement Conversion Tables
-Problem Solving with Measurement

Manipulatives & Visual Models

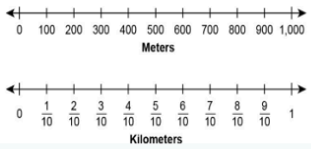


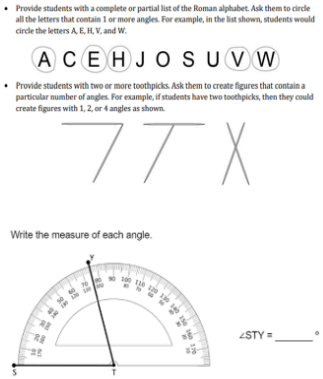


Create Tables to Show Conversions

Hours	Minutes	Seconds
1	60	3,600
2	120	7,200
3	180	10,800
4	240	14,400
5	300	18,000

Use a Double Number Line





Quarter 3 January-March

Standards: 5, 13, 14, 15, 16, 17, 19, & 20

Academic Vocabulary: number pattern, shape pattern, sequence pattern rule, compare, comparison, fractions, denominators, numerator, common denominator, mixed number, fraction greater than one

(denominators:2, 3, 4, 5, 6, 8, 12, 10, 100)

Resources used: Eureka, AMSTI supplemental materials, Illustrative Mathematics, Howard County Resources, Math by the Book, Math in Practice

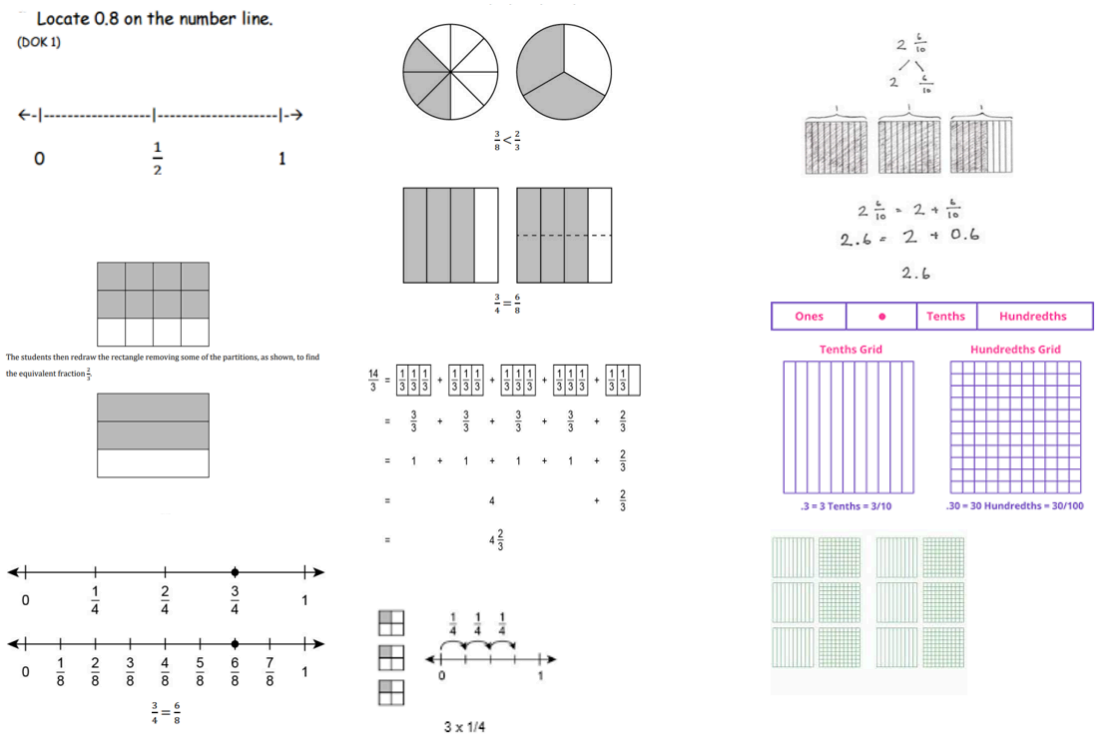
Standards: 5, 13, 14, 15, 16, & 20
Big Idea: Fractions

Topics Covered:
-Decomposition & Fraction Equivalence
-Fraction Equivalence Using Multiplication & Division
-Fraction Comparison
-Fractions Addition & Subtraction *note the standard for 4th grade is adding and subtracting like denominators.
-Extending Fractions Equivalence for Fractions Greater than 1
-Repeated Addition Fractions as Multiplication

Standards: 17 & 19
Big Idea: Decimals

Topics Covered:
-Exploration of Tenths
-Tenths & Hundredths
-Decimal Comparison
-Addition with Tenths & Hundredths
-Money Amounts as Decimal Numbers

Manipulatives & Visual Models



Quarter 4 March-May

Standards:15, 27, 28, & 29

Academic Vocabulary: equilateral, triangle, figure, interior of an angle, intersecting lines, isosceles triangle, length of an arc, line, line of symmetry, line segment, obtuse angle, parallel, perpendicular, point, protractor, ray, right angle, right triangle, scalene triangle, vertex, and vertical angles

Resources used: Eureka, AMSTI supplemental materials, Illustrative Mathematics, Howard County Resources, Math by the Book, Math in Practice

Standards: 27, 28, & 29
Big Idea: Geometry

Topics Covered:
-Two-Dimensional Figures & Symmetry

Standards:15
BIG Idea: Mixed Numbers

Topics Covered:
-Addition & Subtraction of Fraction by Decomposition
-Investigation of Measurements Expressed as Mixed Numbers

Manipulatives & Visual Models