DEPARTMENT OF EDUCATION

2022 Minnesota K–12 Academic Standards in Mathematics

Commissioner Approved Draft

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Introduction

The Minnesota K–12 Academic Standards in Mathematics represent the work of the Mathematics Standards Review Committee. This committee included K–12 teachers, administrators, college faculty and representatives of educational and community organizations. The committee membership, timeline and assumptions that guided their work are found on the Minnesota Department of Education (MDE) Mathematics webpage, (https://education.mn.gov/MDE/dse/stds/Math/) or scan the QR code to the right.

During the 2021–22 school year, the Mathematics Standards Review Committee reviewed the 2007 Minnesota K–12 Academic Standards in Mathematics, other states' recently revised standards, current academic research, K–12 instructional best practices and public feedback. This review process followed the guidelines in Minnesota Statutes 2023, section 120B.021, subdivision 4e. The standards and anchor standards provide a summary description of student learning. The kindergarten through grade 12 benchmarks identify a "specific knowledge or skill that a student must master to complete part of an academic standard by the end of the grade level or grade band" (Minnesota Statutes 2023, section 120.B.018).

Minnesota Statutes 2022, section 120B.021, requires that there be statements of standards and benchmarks. Minnesota Statutes, 120B.021, subdivision 4a states that "the commissioner must include the contributions of Minnesota American Indian tribes and communities as related to the academic standards during the review and revision of the required academic standards." The four Dakota and seven Anishinaabe/Ojibwe Tribal Nations, as well as Minnesota's significant Urban Indigenous communities, have been relatively invisible in Minnesota's academic standards, and Minnesota Statutes 120B.021 counteracts that invisibility and lack of representation in Minnesota's education system. This statute requirement, added by the legislature in 2007, demonstrates the commitment of the State of Minnesota to ensure that the contributions of the Tribal Nations in Minnesota are integrated into student academic learning and instruction. Minnesota's K–12 Academic Standards in Mathematics include the historical and current contributions of the eleven sovereign, federally recognized Tribal Nations in Minnesota and Minnesota's Urban Indigenous communities.

Career, College and Community Readiness Vision Statement

"Let's put our minds together and see what kind of future we can build for our children" (Tatanka Iyotake [Chief Sitting Bull], 1868).

Mathematics belongs to all of us. Mathematics is part of our everyday lives and is rooted in all cultures. Equitable access to mathematics for each and every student is accomplished by recognizing, respecting and attending to the diversity of our students with the aim of ensuring that all students are able to learn and thrive.

The goal of mathematics education is to prepare each and every student for effective participation in society, including in their career(s), post-secondary education and daily decision-making about everything from finances, personal health, civic discourse and policy-making to their ability to comprehend and analyze data. All students should learn mathematics "in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder and beauty of mathematics" (National Council of Teachers of Mathematics [NCTM], 2018).

Mathematics education needs to evolve with the constant cultural and technological changes in our society. The importance of math education, related to career, college and community readiness, is supported by the World Economic Forum's *The Future of Jobs Report* (World Economic Forum, 2022). The report lists complex problem-solving, critical thinking, reasoning, analytical thinking and active learning in its list of Top Ten Workplace Skills. These are all skills that are developed in the study of mathematics throughout a student's K–12 experience.

Mathematically literate students have the experiences, mindset, knowledge and skills to be career and college ready and engage as productive members in their community. They are empowered to use mathematics as a resource to successfully navigate pathways towards achieving their aspirations (MDE Career and College Readiness [CCR] Resource Guide).

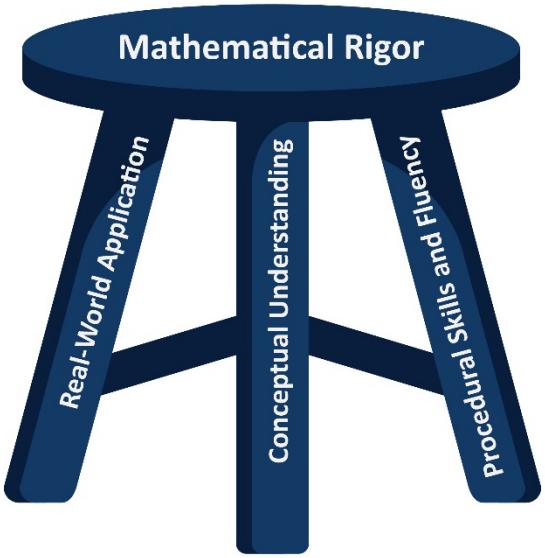
Minnesota career, college and community ready students will:

- Be curious, pose questions and seek patterns in order to make sense of their world.
- Communicate their mathematical thinking and contribute to high-level math discussions.
- Be persistent, flexible, collaborative and creative problem solvers.
- Make connections between mathematics concepts and other disciplines, experiences outside the classroom, interests and career aspirations, as well as the connections amongst mathematical ideas.

- Build conceptual understanding, thinking and reasoning in order to develop procedural fluency and flexible problem-solving strategies.
- Collaborate with cultural perspectives and traditions like and unlike one's own, allowing students to make sense of mathematical concepts and value various mathematical identities connected to lived experiences.
- Solve problems connected to place, story, cultural practices, language and perspectives relevant to historical and contemporary Dakota and Anishinaabe/Ojibwe communities (Minnesota Statutes 2021, section 120B.021).

In order to support students to become career, college and community ready, these standards and benchmarks:

- Pursue mathematical rigor with an equal intensity of conceptual understanding, application and procedural skill and fluency (Funderburk et al., 2016).
- Incorporate the eight Standards for Mathematical Practice (SMPs) to promote experiences that empower students to be "confident in themselves as doers, knowers, and sense makers of mathematics" (NCTM, 2020).
- Incorporate five contexts in Dimension 2 to use as a lens for the teaching and learning of the standards and benchmarks.
- Equip students to analyze information in a data-rich environment by including data science and computer science, including computational thinking, throughout the K–12 mathematics experience.
- Empower students to make informed financial decisions now and into the future. Students will learn mathematics which, when combined with the Minnesota K–12 Social Studies Standards, addresses the five key components of financial literacy: earn, spend, save and invest, borrow and protect.



Organization of the Standards

The organization and structure of the 2022 Minnesota K–12 Academic Standards in Mathematics communicate how mathematics education is conceptualized for Minnesota students. The standards are ordered by grade, beginning in kindergarten (represented by zero) and ending in high school (represented by 9). In each grade the standards are subdivided into strand, anchor standard and benchmark. Additional dimensions within benchmarks are mathematical practices and benchmark contexts, which serve to encourage integration of concepts through process and context.

The three strands—Data and Probability, Spatial Reasoning and Patterns and Relationships—organize the standards. Most grades have seven anchor standards that are consistent across kindergarten through grade 11. Each standard will have one or more benchmarks that are grade-level specific that define a specific knowledge or skill a student must master. The benchmarks are placed at the end of the grade level where mastery is expected, with the recognition that a progression of learning experiences in earlier grades builds the foundation for mastery later on.

Strands

The standards are organized into three strands common across all grade levels:

1. **Data and Probability** – "*Data are not merely numbers but numbers in context*" (Cobb and Moore, 1997). Students will develop questions about situations impacting their everyday lives, make a plan to produce data and provide answers by organizing, describing and summarizing the data. In addition, students will develop and evaluate inferences and make predictions. They will find patterns and order from random phenomena to analyze and understand important relationships and use the concept of variability to make sense of the world.
2. **Spatial Reasoning** – Students will actively make sense of relationships between and within geometric figures, generalize statements about the figures and develop arguments about these relationships.
3. **Patterns and Relationships** – Mathematics is the science of pattern and order. Students will organize and describe the world using representations of numbers and operations. They will learn actions that transform numbers as well as ways of thinking that bring them back to where they started. Students will analyze and describe relationships among variables by focusing on how variables change and stay the same.

Anchor Standards

Minnesota Statutes 2021, section 120B.021, requires that there be statements of standards and benchmarks. Anchor standards are a summary description of student learning that reflects a learning progression, spanning from kindergarten to graduation. There are seven anchor strands organized into three strands.

Anchor Standards by Strand

Strand 1: Data and Probability

1. Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.

2. Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.

Strand 2: Spatial Reasoning

3. Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.
4. Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.

Strand 3: Patterns and Relationships

5. Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.
6. Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.
7. Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.

Benchmarks

A benchmark is a "specific knowledge or skill that a student must master to complete part of an academic standard by the end of the grade level or grade band" (Minnesota Statutes 2013, section 120B.018, subdivision 3). The benchmarks are placed at the grade level where mastery is expected, and with recognition that the progression of learning experiences in earlier grades builds the foundation for mastery later on. The benchmarks include notations of two cross-cutting dimensions described below. The benchmarks inform the graduation requirements for students, which read: *"three credits of mathematics, including an algebra II credit or its equivalent, sufficient to satisfy all of the academic standards in mathematics;" and "an algebra I credit by the end of 8th grade sufficient to satisfy all of the 8th grade*

standards in mathematics" (Minnesota Statutes 2019, section120B.024, subdivision 1).[1] The standards and benchmarks for grades 8 and 9–11 define the completion and mastery of Algebra 1 in grade 8 and Algebra 2 in grades 9–11.

Mathematical Dimensions Description and Notation

Practices used by mathematicians and authentic contexts for mathematical thinking serve as dimensions that interact with the grade level benchmarks. Standards of Mathematical Process and benchmark contexts are corresponding cross-cutting dimensions that are referenced in each benchmark. The dimensions encourage integration of benchmarks with process (dimension 1) and contributions and context (dimension 2). Mathematical connections to a coherent/comprehensive view of the world provide students access to Minnesota's Career, College and Community Readiness goal.

Dimension 1: Standards of Mathematical Practice (MP)

This dimension focuses on the Standards for Mathematical Practice.[2] These practices describe the processes, behaviors and habits of mind that are exhibited by students who are mathematically proficient. Mathematical understanding is the intersection of these practices and mathematics content. It is critical that the Standards for Mathematical Practice are embedded in daily mathematics instruction. The mathematical practices are:

MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

[1] This 2019 citation represents the statute and its wording at the time of the committee work. The current statute, updated in 2023, shifts this specificity to 120B.021, subdivision 1.
[2] Appendix 1

Dimension 2: Benchmark Contexts

This dimension focuses on using context to do and learn mathematics. All students should make sense of and persevere in mathematical problem-solving experiences using local contexts of the learning community, within the context of Minnesota and in the contexts of our nation and the global community. The benchmarks noted with the symbols below represent opportunities to utilize the named contexts to teach the mathematics within the benchmark and standard.

✜ **Minnesota Tribal Nations Contexts:** Connect mathematical problem-solving experiences and contributions to place, story, cultural practices, language and perspectives relevant to historical and contemporary Dakota and Anishinaabe/Ojibwe communities. The four directions symbol (✜) indicates benchmarks' instructional topics that could include contributions of Dakota and Anishinaabe/Ojibwe in current-day Minnesota.

$ **Financial Literacy Contexts:** Allow students to make sense of mathematics through the context of money to manage one's financial resources effectively for a lifetime of financial security. The dollar sign ($) represents financial literacy contexts.

\# **Computer Science Contexts:** Computer science is an independent field of study that focuses on learning how to think computationally and adapting/creating new technology. There are many connections with mathematics, and integrating the subjects can deepen student understanding in mathematics and provide more students with access to computer science. The number sign (#) represents computer science contexts.

μ **Modeling Contexts:** Students apply mathematical problem-solving experiences to connect with the Standards of Mathematical Practices meriting greater time, resources, innovative energies and focus to deeply explore the content of the benchmark. The mu symbol (μ) represents modeling contexts.

☼ **Real-world Contexts:** Apply mathematical problem-solving experiences to real-world situations in contexts connected to students' lived experiences, cultural perspectives and traditions. The sun symbol (☼) represents real-world contexts.

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How to Read the Standards, Benchmarks and Dimensions

Each benchmark has a 4-digit code, which includes the grade, strand, standard and benchmark numbers. See the examples below.

Example 1: Benchmark 0.2.4.3

- The 0 represents the kindergarten grade level.
- The 2 represents the second strand: Spatial Reasoning.
- The 4 represents the fourth anchor standard: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.
- The 3 represents the third benchmark: Compose, decompose and name simple shapes. Recognize shapes regardless of their overall size and orientation. (MP1, MP2) µ

Grade	Strand	Anchor Standard	Code	Benchmark
K	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	0.2.4.3	Compose, decompose and name simple shapes. Recognize shapes regardless of their overall size and orientation. (MP1, MP2) µ

Example 2: Benchmark 9.3.7.10

- The 9 represents the high school, 9–11 grade band.
- The 3 represents the third strand: Patterns and Relationships
- The 7 represent**s** the seventh Anchor Standard: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.
- The 10 represents the 10th benchmark: Use the concept of a function as a connection between inputs and outputs to find function values and use function notation.

Grade	Strand	Anchor Standard	Code	Benchmark
9	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.10	Use the concept of a function as a connection between inputs and outputs to find function values and use function notation. (MP2)

Mathematics Standards Grade-Level Tables

Kindergarten

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
K	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	0.1.1.1	Notice and describe patterns in data-rich situations. (MP1, MP7) ✣ # ☼
K	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	0.1.1.2	Organize objects, draw pictures or use tally marks to represent data and communicate observations. (MP3, MP6) # µ

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
K	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	0.2.3.1	Compare objects with a measurable attribute in common, to see which object has “more of,” “less of” or the “same as” the attribute and explain the reasoning. (MP3, MP5) ✣ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
K	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	0.2.3.2	Describe several measurable attributes of objects such as length and weight. (MP4, MP6) ✣ ☼
K	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	0.2.4.1	Sort objects using characteristics such as shape, size, color and thickness. (MP1, MP3) ✣ ☼
K	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	0.2.4.2	Identify and compare two- and three-dimensional shapes such as squares, circles, triangles, rectangles, trapezoids, hexagons, cubes, cones, cylinders and spheres using informal language to describe their similarities, differences, parts and other attributes. (MP2) ✣ ☼
K	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	0.2.4.3	Compose, decompose and name simple shapes. Recognize shapes regardless of their overall size and orientation. (MP1, MP2) µ

Grade	Strand	Anchor Standard	Code	Benchmark
K	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	0.2.4.4	Describe objects in the environment using names of shapes. Describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to. (MP1, MP6) ✣ ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.1	Recognize that a number can be used to represent how many objects are in a set or to represent the position of an object in a sequence. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number with one and only one object. Understand that the last number said tells the number of objects counted. Understand that each successive number refers to a quantity that is one more. Name the position of an object in a sequence (ordinal count). (MP1, MP6) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.2	Count collections of objects up to 31 by grouping in 10s using ten-frames, cups or other tools. (MP6, MP7) ✣ $ ☼
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.3	Read, write, compare, order and represent whole numbers from 0 to at least 31 (with 0 representing the count of no objects) to answer the question “how many?” Representations may include numerals, pictures, real objects, picture graphs, spoken words and manipulatives, such as connecting cubes. The numbers from 11 to 19 are composed of a 10 and one, two, three, four, five, six, seven, eight or nine ones. (MP4, MP8) ✣
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.4	Count forward, with and without objects, to at least 31. Count backward from 20. (MP6) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.5	Find a number that is 1 more or 1 less than a given number. (MP7, MP8)
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.6	Solve and represent a variety of addition and subtraction contextual situation types using objects, drawings, mental images or equations within 10. (MP4, MP5) $ µ
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.7	Compose and decompose numbers less than or equal to 10 into pairs in more than one way with objects and pictures. Record each decomposition with a drawing or equation. (MP7)
K	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	0.3.5.8	Fluently add and subtract within 5. (MP2)

Grade	Strand	Anchor Standard	Code	Benchmark
K	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	0.3.6.1	Identify whether the number of objects in one group is greater than, less than or equal to the number of objects in another group (by using matching, counting strategies and a number line). (MP2, MP5) ☼
K	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	0.3.6.2	Recognize that the equal sign (=) is a comparison symbol of two math expressions of equal value number. (MP6)
K	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	0.3.7.1	Recognize, create, complete and extend simple patterns using shape, color, size, number, sounds and movements. Patterns may be repeating, growing or shrinking. (MP1, MP7) ✣ # ☼
K	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	0.3.7.2	Recognize patterns in counting. Skip count by 10s starting at zero up to 100. (MP7) ✣ $ ☼

Grade 1 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
1	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	1.1.1.1	Notice and describe patterns in data-rich situations and create statistical investigative questions with teacher guidance. (MP6, MP7) ☼
1	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	1.1.1.2	Collect and use data to consider and decide what data will answer a question. Represent the data as a drawing, tally marks and frequency bar graph and digitally communicate observations. (MP1, MP3) # µ
1	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	1.1.2.1	Describe outcomes of events as impossible, possible or certain. (MP1, MP6) ☼

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
1	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	1.2.3.1	Order three objects by length. Compare the lengths of two objects indirectly by using a third object. (MP2, MP5) ✣ ☼
1	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	1.2.3.2	Measure the length of an object in terms of non-standard units. (MP5) ✣ ☼
1	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	1.2.3.3	Identify pennies, nickels and dimes. Find the value of a group of these coins, up to one dollar. (MP1, MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
1	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	1.2.4.1	Describe attributes of two- and three-dimensional objects, such as triangles, squares, rectangles, circles, rectangular prisms, cylinders, cones and spheres. (MP7, MP8) ✣ ☼
1	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	1.2.4.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles and quarter-circles) to create a composite shape. Decompose composite shapes into triangles, rectangles, squares and sectors. (MP3, MP7) µ
1	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	1.2.4.3	Describe objects in the environment using names of shapes and describe the relative positions of these objects using left and right. (MP6) ☼
1	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	1.2.4.4	Identify shapes regardless of their orientations. (MP1, MP6) ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.1	Count collections of objects up to 120 using groups of 5s or 10s. (MP1, MP5) ✣ $
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.2	Read, write, compare, order and represent whole numbers from 0 to 120. Representations may include numerals, expanded notation, addition and subtraction, pictures, tally marks, number lines and manipulatives such as bundles of sticks, ten frames and base 10 blocks. The numbers 10, 20, 30, 40, 50, 60, 70, 80 and 90 refer to one, two, three, four, five, six, seven, eight or nine groups of 10s. (MP7, MP8) ✣ $
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.3	Count, with or without objects, forward and backward from any given number up to 120. (MP7) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.4	Use models, pictures or numbers to recognize and describe the place value of numbers between 10 and 120 as a relationship of n groups of 10 plus an amount represented by a single digit (n x 10 + a). (MP4, MP7) $
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.5	Estimate amounts up to 120 using benchmarks of 5s and 10s. (MP1, MP2) $ ☼
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.6	Solve contextual situations, up to and including 20, using addition and subtraction strategies of adding to, taking from, part-part-whole, difference between and comparing. Solve for unknowns in contextual situations using objects, drawings and equations with unknowns represented by a symbol in all positions (result, change, start). (MP2, MP4) $ µ

Grade	Strand	Anchor Standard	Code	Benchmark
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.7	Add within 100, including adding a two-digit number with a one-digit number and adding a two-digit number with a multiple of 10 using concrete models, place value language and properties of operations. Understand that in adding two-digit numbers, sometimes it is necessary to compose a new ten. (MP2, MP4) $
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.8	Decompose numbers less than or equal to 10 into pairs, in more than one way, using objects or drawings. Record each decomposition with a drawing or equation. (MP7)
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.9	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten, decomposing a number leading to a ten using the relationship between addition and subtraction and creating equivalent but easier or known sums. (MP2, MP4) $ # µ
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.10	Use combinations of 10 to add to the next decade through 100. (MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.11	Determine the double of any single-digit number. (MP8)
1	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	1.3.5.12	Represent and solve contextual equal sharing situations where a whole number of items is shared equally among 2 groups. Name the fractional amount using the word “half.” (MP3, MP2) $
1	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	1.3.6.1	Compare two two-digit numbers based on the meaning of the tens and ones digits. (MP3, MP4) $
1	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	1.3.6.2	Determine if equations involving addition and subtraction are true or false, including those with operations on both sides. (MP2) $
1	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	1.3.6.3	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. (MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
1	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	1.3.7.1	Create simple patterns using objects, pictures, numbers and rules. Identify possible rules to complete or extend patterns. Patterns may be repeating, growing or shrinking. Calculators can be used to create and explore patterns. (MP5, MP7) ✣ # ☼
1	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	1.3.7.2	Recognize patterns in counting. Skip count by 2s and 5s starting at zero up to 120. Skip count by 10s up to 120 starting at a non-zero number. (MP7) ✣ ☼
1	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	1.3.7.3	Describe what is changing and what is staying the same in a visual growing pattern. (MP1, MP8) ✣ μ ☼

Grade 2 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
2	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	2.1.1.1	Notice and describe patterns in data-rich situations and create statistical investigative questions. (MP6, MP7) ✣ ☼
2	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	2.1.1.2	Determine what counts as data to answer a statistical investigative question. Recognize that people collect data to answer questions and that data can vary. (MP1) ☼
2	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	2.1.1.3	Collect and use data to consider and decide what data will answer a question. Represent the data as drawings, picture graphs, dot plots (a.k.a. line graphs or line plots) and with technology. Communicate observations. (MP3, MP5) # µ
2	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	2.1.1.4	Make predictions using patterns from data visualizations. (MP7, MP8) # µ

Grade	Strand	Anchor Standard	Code	Benchmark
2	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	2.1.2.1	Describe the difference between possible and likely. (MP1, MP6)

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.1	Estimate lengths using units of inches, feet, centimeters and meters. (MP2, MP5) ✣ ☼
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.2	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit. Relate addition and subtraction to length. (MP5) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.3	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. (MP5, MP6) ✣ ☼
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.4	Represent whole numbers as lengths from 0 on a number line with equally spaced points corresponding to the numbers 0, 1, 2, … Represent whole-number sums and differences, within 100, on a number line. (MP4, MP5)
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.5	Partition a rectangle into rows and columns of same-size squares and count the total number. (MP8)
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.6	Use addition and subtraction, within 100, to solve contextual situations involving lengths that are given in the same units using drawings (such as rulers) and equations with a symbol for the unknown number to represent the situation. (MP1, MP4) $

Grade	Strand	Anchor Standard	Code	Benchmark
2	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	2.2.3.7	Identify pennies, nickels, dimes and quarters. Find the value of a group of coins and determine combinations of coins that equal a given amount, using $ and ¢ symbols appropriately. (MP7) $
2	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	2.2.4.1	Classify two- and three-dimensional figures according to the number and shape of faces and the number of sides, edges and vertices. (MP1, MP2) ✜ ☼
2	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	2.2.4.2	Create a representation for basic two-dimensional shapes such as squares, circles, triangles, rectangles, trapezoids and hexagons. (MP5, MP6) ☼
2	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	2.2.4.3	Describe the location of an object in relation to another object. (MP1, MP6) ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.1	Count collections of objects using groups of 10s and 100s to 1,000. Represent the counting strategy and the total using words, symbols and pictures. (MP1) ✣ $ ☼
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.2	Read, write, compare, order and represent whole numbers up to 1,000. Representations may include numerals, expanded notation, addition, subtraction, multiplication, words, pictures, tally marks, number lines and manipulatives such as bundles of sticks, ten frames and base 10 blocks. (MP4) ✣ $ ☼
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.3	Given a three-digit number, mentally find 10 more or 10 less and 100 more or 100 less than the number. Justify reasoning by referencing a model. (MP2, MP3) $

Grade	Strand	Anchor Standard	Code	Benchmark
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.4	Recognize and describe the place value of numbers between 10 and 1,000 as a relationship of groups of tens, hundreds and thousands plus an amount of a single digit. Know that 100 is 10 tens and a thousand is 10 hundreds or 100 tens. (MP4, MP7) $
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.5	Estimate sums and differences of two-digit numbers. (MP1) $
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.6	Use addition and subtraction, within 1,000, to solve contextual situations using concrete models or drawings based on place value, properties of operations and/or the relationship between addition and subtraction. Relate the strategy to a written method. (MP1, MP4) $

Grade	Strand	Anchor Standard	Code	Benchmark
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.7	Use a range of strategies and algorithms based on knowledge of place value and equality to flexibly add and subtract two-digit numbers. Strategies may include decomposition, expanded notation and partial sums and differences. Use place value and properties of operations to explain why strategies work. (MP1, MP7) µ
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.8	Fluently add and subtract, within 20, using mental strategies that include incrementing, compensation or fact families. (MP7) $
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.9	Use landmarks of 10 to fluently add to 100 and subtract from 100. (MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
2	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	2.3.5.10	Represent and solve contextual equal sharing situations where a whole number of items is shared equally among 2 or 4 groups. Name the fractional amount using the words “halves” and “fourths.” Recognize that equal shares of identical wholes need not have the same shape. (MP3, MP7) $ µ ☼
2	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	2.3.6.1	Compare two- and three-digit numbers based on meanings of the hundreds, tens and ones digits. (MP2, MP7) ☼
2	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	2.3.6.2	Use number sentences involving addition, subtraction and unknowns to represent given situations. Use the relationship of addition and subtraction to find values for the unknowns that make the number sentences true. (MP4) $ ☼
2	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	2.3.6.3	Make conjectures and justifications involving subtraction and addition with true/false and open number equations. (MP3) $

Grade	Strand	Anchor Standard	Code	Benchmark
2	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	2.3.7.1	Identify, create and describe simple number patterns involving repeated addition or subtraction, skip counting and arrays of objects such as counters or tiles. Use patterns to solve situations in various contexts. (MP7, MP8) ✣ $ # ☼
2	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	2.3.7.2	Recognize patterns in counting. Skip count by 2s and 5s from any given number up to 120. (MP7) $
2	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	2.3.7.3	Use numeric expressions to describe a visual growing pattern. (MP8) ✣ µ ☼

Grade 3 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
3	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	3.1.1.1	Notice and describe patterns in data-rich situations or given data sets. Ask statistical questions that can be answered with data. (MP7) ✣ ☼
3	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	3.1.1.2	Describe how data can be collected, including from surveys, grouping of items and measurement, to answer a statistical investigative question. (MP7) ☼
3	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	3.1.1.3	Collect and organize data to answer a statistical question using various tools and addressing missing or incomplete data. Represent data in a variety of ways including technology. (MP1, MP5) # μ ☼
3	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	3.1.1.4	Make predictions and recognize that the amount and source of the data impacts the accuracy of predictions. (MP4, MP8)

Grade	Strand	Anchor Standard	Code	Benchmark
3	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	3.1.1.5	Critically analyze data visualizations, including frequency tables, bar graphs, picture graphs or number line plots having a variety of scales, to support a claim and solve situations. (MP1, MP3) $ #
3	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	3.1.2.1	Describe outcomes of events as impossible, certain, likely, unlikely and equally likely. (MP1, MP4)

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
3	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	3.2.3.1	Measure lengths to the nearest fourth when measuring with standard units. (MP2, MP5, MP6) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
3	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	3.2.3.2	Compare and contrast the relative sizes of measurement units within one system (such as inches, feet, and yards; centimeters and meters; grams and kilograms; ounces and pounds; cups and gallons; quarters and dollars). (MP5, MP6) $ ☼
3	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	3.2.3.3	Calculate the perimeter of a polygon with whole number side lengths. (MP5, MP6)
3	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	3.2.3.4	Use addition and subtraction with whole numbers, within 100, to calculate change up to one dollar in several different ways, using $ and ¢ symbols appropriately. (MP7, MP8) $
3	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	3.2.4.1	Create representations of regular and irregular polygons with a given number of sides, including triangles, quadrilaterals, pentagons, hexagons and octagons. (MP2, MP6) ✜ ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.1	Given a value, mentally find 100 more or 100 less, 1,000 more or 1,000 less and 10,000 more or 10,000 less than the number. Justify reasoning by referencing a model. (MP2, MP3)
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.2	Recognize and describe the place value of numbers between 10 and 100,000 as a relationship of groups of ten, hundreds and thousands plus an amount of a single digit. Know that 10,000 is 100 hundreds, 1,000 is 10 hundreds or 100 tens. (MP7)
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.3	Compare and order whole numbers up to 100,000 justifying with place value language, number lines, and other tools using >, = and < symbols to record the results of comparisons. (MP3, MP4) $ ☼
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.4	Estimate sums and differences up to 1,000 using strategies based on benchmarks and place value language. (MP2) ✣ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.5	Use a range of strategies and algorithms based on knowledge of place value and equality to flexibly add and subtract within 1,000. Strategies may include decomposition, expanded notation and partial sums and differences. Explain how the strategies work using place value and the properties of operations. (MP1, MP7) $ µ
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.6	Represent and solve contextual situations involving multiplication, measurement division and partitive division with single-digit factors using visual models. (MP1, MP4) $ µ ☼
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.7	Multiply and divide within 144, using strategies such as equal groups, repeated addition, the relationship between multiplication and division or properties of operations. Develop fluency with facts of 2s, 5s, 10s and square products. (MP7)
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.8	Multiply one-digit whole numbers by multiples of 10 and 100 using strategies such as decomposition of factors of ten, place value language, repeated addition and properties of operations. (MP7, MP8) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.9	Partition a whole into halves, thirds, fourths and eighths. Wholes can be circles, rectangles and the distance between 0 and 1 on a number line. (MP4, MP6) ☼
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.10	Use pictures and symbols to represent non-unit fractions up to 2 as sums of unit fractions using halves, fourths, thirds and eighths. (MP8)
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.11	Generate equivalent forms of one-half and 1 using fractions with denominators of 2, 4 and 8 and justify why these forms are equivalent using a visual model. (MP2, MP3)
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.12	Compare and order unit fractions using visual models and describe how the size of the fraction changes as the denominator changes. (MP1, MP7) µ

Grade	Strand	Anchor Standard	Code	Benchmark
3	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	3.3.5.13	Use addition and subtraction with estimated whole numbers to create short-term and long-term spending and saving goals based on planned and unplanned financial decisions. (MP2) $
3	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	3.3.6.1	Use relational thinking to find a missing value in an open number sentence with up to three-digit whole number addition and subtraction expressions. Determine if the equation is true or false. Justify your reasoning. (MP3) $ #
3	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	3.3.6.2	Make conjectures and justifications about multiplication and division involving 0 and 1 with true/false and open number equations. (MP3)
3	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	3.3.6.3	Make conjectures and justifications using the commutative and associative properties of addition and multiplication with true/false and open number equations. (MP1, MP3) µ

Grade	Strand	Anchor Standard	Code	Benchmark
3	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	3.3.7.1	Create, describe and apply single-operation input-output rules involving addition, subtraction and multiplication to solve situations in various contexts, including when x and y are 0. (MP7) ☼
3	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	3.3.7.2	Create the next two terms and the previous term in a visual pattern, growing or shrinking, and justify reasoning. (MP1, MP7) ✜ μ

Grade 4 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
4	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	4.1.1.1	Notice and describe patterns in data-rich situations or two given related data sets that are descriptive and comparative. Ask meaningful statistical questions that can be answered with data. (MP3, MP7) ✣
4	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	4.1.1.2	Collect and organize data to answer a statistical question, analyze variability and address missing, incomplete and bias in data. Represent data in a variety of ways, including technology. (MP1, MP5) #
4	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	4.1.1.3	Make predictions and recognize that how the data was collected impacts the reliability of predictions. (MP1, MP8)
4	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	4.1.1.4	Critically analyze data visualizations, including tables, double bar graphs, timelines, line plots or spreadsheets to support a claim and solve contextual situations. (MP3, MP7) $ # µ

Grade	Strand	Anchor Standard	Code	Benchmark
4	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	4.1.2.1	Classify probability events involving dice, coins, spinners with equal and unequal partitions and blocks in a bag as impossible, certain, likely, unlikely and equally likely. (MP7)
4	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	4.1.2.2	Use a number line to connect the values of 0 to impossible, 1/2 to equally likely, and 1 to certain. Approximate locations on the number line where likely and unlikely would occur based on the situation. (MP3, MP4)

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.1	Classify angles as acute, right and obtuse by estimation, comparison with a right angle and by measurement. (MP6) ✣

Grade	Strand	Anchor Standard	Code	Benchmark
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.2	Determine lengths to the nearest sixteenth of an inch when measuring with inches and to the nearest tenth of a centimeter when measuring in centimeters. (MP6, MP8)
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.3	Measure angles with a protractor. (MP5) ☼
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.4	Determine the perimeter and area of two-dimensional figures and label with appropriate units. (MP2, MP6) ✣ ☼
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.5	Find the areas of geometric figures that can be decomposed into rectangular shapes using tools like dot or grid paper. Label area measurements using square units. (MP2)

Grade	Strand	Anchor Standard	Code	Benchmark
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.6	Explain why the area of a rectangle can be calculated by multiplying the length by the width and use the formula $A = l \text{ x } w$ to calculate the area of rectangles with whole number side lengths. (MP3)
4	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	4.2.3.7	Make change up to $20 with place values, using $ and ¢ symbols appropriately. (MP7) $
4	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	4.2.4.1	Draw points, line segments, rays, lines, angles and perpendicular and parallel lines. Identify these in two-dimensional figures. (MP5) ☼
4	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	4.2.4.2	Create representations of triangles given the relationships among the sides (scalene, isosceles, equilateral) and the angles (acute, right, obtuse). (MP1, MP6) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
4	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	4.2.4.3	Sort and classify quadrilaterals in a hierarchy, including squares, rectangles, trapezoids, rhombuses, parallelograms and kites. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. (MP1, MP6) ✣ μ ☼
4	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	4.2.4.4	Create a representation and describe the front, top and side views of three-dimensional figures composed of cubes and rectangular prisms. (MP5)
4	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	4.2.4.5	Draw the nets of cubes. Recognize nets that will and will not form cubes. (MP1)

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.1	Recognize that in a multi-digit whole number, a digit in one place represents 10 times what it represents in the place to its right. (MP7) $
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.2	Compare and order whole numbers from 0 to 1,000,000 with place value understanding, number lines and other tools using >, = and < symbols to record the results of comparisons. (MP4) $
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.3	Estimate sums and differences, within 1,000,000, using strategies based on place value, approximation and properties of operations. (MP3) ✣ $ ☼
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.4	Estimate products and quotients of multi-digit whole numbers by using simple multiplicative relationships, approximation and place value to assess the reasonableness of results. (MP1) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.5	Fluently multiply two numbers from 0 to 12 using flexible strategies based on the associative, commutative and distributive properties of multiplication. (MP6, MP7)
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.6	Use place value language to describe how to multiply a number by 10, 100 and 1,000. (MP7)
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.7	Flexibly decompose numbers into addends or factors to multiply two two-digit numbers with a one-digit number, by and up to a four-digit number. Justify the strategy using equations, rectangular arrays and area models. (MP2, MP7) µ
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.8	Solve contextual situations using division with dividends up to the thousands place and using one-digit divisors. Strategies may include using visual models, partial quotients, the commutative, associative and distributive properties and repeated subtraction. (MP7, MP8) $ µ

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.9	Solve multi-step contextual situations requiring the use of addition, subtraction and multiplication of multi-digit whole numbers. Use various strategies, including the relationship between operations, the use of technology and the context of the situation to assess the reasonableness of results. (MP4, MP7) ✣ $
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.10	Read, write, represent and plot on a number line fractional values between 0 and 3, including mixed numbers and fractions greater than 1 with denominators of 2, 3, 4, 5, 6, 8, 10 and 12. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. (MP6)
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.11	Explain why a fraction $\frac{a}{b}$ is equivalent to the product $a \times \frac{1}{b}$ using visual models and language. (MP2)

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.12	Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n\ x\ a}{n\ x\ b}$ by using visual models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (MP3)
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.13	Compare and order fractions between 0 and 3 and justify reasoning using pictures, position on a number line and selecting, when appropriate, among strategies such as using a common numerator, referencing a benchmark and using a common denominator. (MP3) µ
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.14	Recognize the relationship between decimals and fractions. Read and write decimals and fractions in both decimal and fraction notations using words, symbols and expanded form. Recall the fraction and decimal equivalent forms of one-half, one-quarter and three-quarters. (MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.15	Compare and order decimal values to the hundredths and justify using place value language, a number line and models such as dimes, pennies, 10 x 10 grids and base 10 blocks. Use place value concepts to name and model equivalent forms of decimal values. (MP4) $
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.16	Use visual models to add and subtract fractions with denominators of 2, 4 and 8 with results up to 2. (MP4)
4	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	4.3.5.17	Use the four operations to make financial decisions based on income, spending, saving, credit and charitable giving. (MP1) $
4	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	4.3.6.1	Use relational thinking to find a missing value in an open number sentence with multi-digit whole number multiplication and division expressions. Determine if the equation is true or false and justify your reasoning. (MP3) $

Grade	Strand	Anchor Standard	Code	Benchmark
4	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	4.3.6.2	Make conjectures and justifications using the distributive property to justify multi-digit multiplication with true/false and open number equations. (MP3) #
4	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	4.3.7.1	Develop an explicit rule that generalizes a visual pattern relating the figure number with the number of items in that figure. Use the rule to find the number of items in figure *n*. (MP7) µ
4	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	4.3.7.2	Use words to write a rule for multiplicative patterns and solve contextual situations. Compare and contrast patterns or rules that are additive and multiplicative, using a variety of strategies including tables, drawings and algebraic equations with a symbol for the unknown number to represent the situation. (MP7, MP8) $ ☼
4	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	4.3.7.3	Generate a number or shape pattern that follows a given descriptive rule. Identify and explain apparent features of the pattern that were not explicit in the rule itself. (MP1, MP3) $ ☼

Grade 5 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
5	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	5.1.1.1	Notice and describe patterns in data-rich situations or given related data sets that are descriptive and comparative. Ask meaningful statistical questions that can be answered with data. (MP7) ✣
5	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	5.1.1.2	Compare and contrast between qualitative and quantitative data. (MP3, MP6) ✣
5	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	5.1.1.3	Collect and organize data to answer statistical questions and analyze measures of center (mean and median) and variability (range). Represent data in a variety of ways, including technology. (MP5) # µ
5	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	5.1.1.4	Critically analyze data visualizations using measures of center and variability, including but not limited to double-bar graphs, line graphs and line plots to support a claim and solve situations. (MP3, MP7) $ #

Grade	Strand	Anchor Standard	Code	Benchmark
5	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.	5.1.1.5	Compare and contrast different data displays to determine how the visualizations impact analysis and interpretation. (MP3, MP8) #
5	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	5.1.2.1	List outcomes from a probability experiment in a frequency table. (MP4) ☼
5	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	5.1.2.2	Use a frequency table to record results from an experiment to make predictions. Place predictions on a number line from 0 to 1. (MP4) # ☼

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
5	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	5.2.3.1	Develop, justify and use formulas to determine the area of parallelograms and triangles. Find the areas of polygons that can be decomposed into parallelograms and triangles. (MP3) # µ

Grade	Strand	Anchor Standard	Code	Benchmark
5	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	5.2.3.2	Estimate the area of two-dimensional shapes, both polygons and non-polygons, using tools such as dot or grid paper. (MP1, MP5)
5	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	5.2.3.3	Use unit cubes to measure volume. Describe a unit cube as a cube with side length 1 unit that is said to have “one cubic unit” of volume and can be used to measure volume. (MP5, MP6)
5	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	5.2.3.4	Use various strategies to measure the volume and surface area of three-dimensional shapes made of a collection of unit cubes. (MP5, MP6) ✣ ☼
5	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	5.2.3.5	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes. Show that the volume is the same by unit cubes as by multiplying the edge lengths (l x w x h) or by multiplying the height by the area of the base. (MP3, MP8) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
5	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	5.2.4.1	Classify and describe prisms and pyramids by their defining attributes and the number of edges, faces, vertices and bases. (MP1, MP6) ✣ ☼
5	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	5.2.4.2	Recognize, draw and compare different nets for prisms, pyramids, cylinders and cones. (MP1)

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.1	Multiply two multi-digit numbers using an efficient strategy. Strategies include decomposing factors into factors, decomposing factors into sums or using an area model. Justify the chosen strategy using properties of operations and place value. (MP1, MP7) µ

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.2	Divide multi-digit numbers by a one-digit or two-digit divisor using efficient and generalizable procedures based on knowledge of place value and the properties of operations that may include partial quotients and standard algorithms. Recognize that quotients can be represented in a variety of ways, including a whole number with a remainder, a fraction, a mixed number or a decimal. (MP7)
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.3	Consider the context of a problem involving division to select the most useful form of the quotient and the remainder. (MP2) $ ☼
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.4	Solve multi-step contextual situations requiring addition, subtraction, multiplication and division of multi-digit whole numbers. Use various strategies including the inverse relationships between operations, the use of technology and the context of the situation to assess the reasonableness of results. (MP4) ✜ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.5	Generate equivalent fractions of the forms $\frac{a}{b} = \frac{n\ x\ a}{n\ x\ b}$ and $\frac{a}{b} = \frac{a \div n}{b \div n}$ and justify relationships using visual models. (MP3) µ
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.6	Given a value, mentally find 0.1 more or 0.1 less, 0.01 more or 0.01 less and 0.001 more or 0.001 less than the number. Justify reasoning by referencing a visual model. (MP2, MP3) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.7	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. (MP2, MP7) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.8	Recognize and flexibly generate equivalences between fractions and decimals to the thousandths place and justify using visual models, place value language and symbols. (MP3, MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.9	Compare and order decimal values to the thousandths. Justify using place value language and visual models. (MP3, MP4) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.10	Estimate sums and differences of fractions and mixed numbers to the nearest half. Justify reasoning using benchmarks. (MP3)
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.11	Fluently add and subtract fractions with unlike denominators (including mixed numbers) and justify using equivalent fractions, visual models and the number line. (MP3) µ
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.12	Estimate sums and differences of decimals. (MP2) $

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.13	Solve contextual situations using addition and subtraction of positive rational numbers represented as fractions (including mixed numbers) or decimals using visual models, equations and properties of operations. (MP4) ✣ $ ☼
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.14	Represent multiplication of a whole number of fractional groups, $n \times \frac{a}{b}$, using visual models, including a number line, and explain how the picture shows the product. (MP3) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.15	Represent contextual multiplication situations of a fractional amount of a whole number amount, $\frac{a}{b}$ of a group of *n*, using visual models, including a number line, and explain how the picture shows the product. (MP3) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.16	Represent contextual measurement situations using division of the form $n \div \left(\frac{a}{b}\right)$ where *n* is the total and $\frac{a}{b}$ is the amount per group. Use a visual model and explain how the picture shows the number of groups. (MP3) $

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.17	Solve multi-step contextual situations using addition and subtraction of positive rational numbers. Use various strategies, including the inverse relationships between operations and the context of the situation, to assess the reasonableness of results. (MP4) ✣ $ ☼
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.18	Use the four operations to compare and contrast different ways of paying and receiving payments. Identify the advantages and disadvantages of each method of payment, including checks, credit cards, debit cards and electronic payments. (MP1, MP7) $
5	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	5.3.5.19	Use the four operations to create an individual or group budget based on wants and needs and explore examples of debt and manageability of debt and its long-term impact. (MP1, MP7) $
5	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	5.3.6.1	Use relational thinking to find a missing value in an open number sentence with addition and subtraction of fractions and decimal expressions. Determine if the equation is true or false and justify the reasoning. (MP3) $ #

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	5.3.6.2	Make conjectures and justifications about numerical expressions involving parentheses and the four operations using the properties of operations, properties of algebra, decomposition and composition to generate equivalent numerical expressions. (MP3) $
5	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	5.3.7.1	Use a rule or table to represent ordered pairs of positive integers and graph these ordered pairs on a coordinate system. (MP4) ☼
5	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	5.3.7.2	Identify and explain apparent relationships between two patterns from given rules, using tables or ordered pairs on a coordinate system. (MP7, MP8)
5	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	5.3.7.3	Represent contextual situations by graphing whole and half number points in the first quadrant of the coordinate plane. Interpret coordinate values of points in the context of the situation. (MP4) ✜ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
5	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	5.3.7.4	Use ratio tables with whole numbers to solve situations with additive and multiplicative reasoning. Interpret multiplication as scaling. (MP2) ✣ $ μ ☼
5	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	5.3.7.5	Develop an explicit rule that generalizes a visual pattern relating the figure number with the number of items in that figure. Use the rule to find the number of items in figure *n*. (MP7, MP8)

Grade 6 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
6	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	6.1.1.1	Recognize a statistical question as one that anticipates variability in the data, compares differences between groups and collects categorical or numerical data related to the question, and accounts for it in the answer. (MP2) ✣
6	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	6.1.1.2	Design and conduct investigations and experiments to gather data, while considering cultural perspectives, to answer statistically investigative questions considering variability and justifying choice of variables. (MP3, MP6) ✣ $ # ☼
6	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	6.1.1.3	Identify, determine and interpret measures of center (mean and median) and measures of variability (range, interquartile range, mean-absolute deviation) to answer a statistically investigative question, summarizing the distribution of data using the measures of center and variability. (MP1, MP2) $ µ

Grade	Strand	Anchor Standard	Code	Benchmark
6	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	6.1.1.4	Create a visualization about a data set to describe patterns, highlight relationships or illustrate features of the distribution of the data to answer or help answer their statistically investigative question. Visualizations should represent the data in appropriate ways, including tables, dot plots, stem-and-leaf plots, histograms and box plots while incorporating any other relevant information that helps to tell a story about the data. (MP5, MP6) #
6	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	6.1.1.5	Compare and communicate competing explanations for data trends observed, considering cultural perspectives and reasonable alternatives given the variability in findings. (MP3, MP6) ☼
6	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	6.1.2.1	Determine the sample space (set of possible outcomes) for a given experiment. Sample space may be determined by the use of tree diagrams, tables or pictorial representations. (MP1) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
6	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	6.1.2.2	Determine the theoretical probability of an event using the ratio between the size of the event and the size of the sample space. Represent probabilities as percentages, fractions and decimals between 0 and 1 inclusive. (MP4)
6	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	6.1.2.3	Calculate experimental probabilities from experiments where the theoretical probability is known, recognizing that there may be differences between theoretical and experimental probability. Represent the probabilities as percentages, fractions and decimals between 0 and 1 inclusive. Use experimental probabilities to make predictions when actual probabilities are unknown. (MP4) # ☼

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
6	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	6.2.3.1	Calculate the measurements of the surface area of rectangular and triangular prisms using appropriate units. Justify the formulas used. Justification may involve decomposition, nets or other models. (MP1, MP3) ✣ # ☼
6	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	6.2.3.2	Calculate the measurement of the volume of prisms (including triangular and nonrectangular prisms) using appropriate units. Justify the formulas used. Justification may involve decomposition or other models. (MP1, MP7) ✣ # ☼
6	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	6.2.3.3	Solve situations in various contexts involving conversion of time, weights, capacities, lengths and area within measurement systems using appropriate units. (MP4, MP5) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
6	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	6.2.3.4	Estimate time, weights, capacities, lengths and dollar amounts using benchmarks in measurement systems with appropriate units. (MP1) ✣ $ ☼
6	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	6.2.3.5	Find the area of special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes. Apply these techniques in contextual mathematical situations. (MP2, MP5) ☼
6	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	6.2.4.1	Determine missing angle measures in a triangle using the fact that the sum of the interior angles of a triangle is 180^0. Use models to illustrate this fact. (MP8, MP2) ✣ ☼
6	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	6.2.4.2	Decompose polygons into triangles to investigate the sum of the interior angles of polygons. (MP1, MP7) #

Grade	Strand	Anchor Standard	Code	Benchmark
6	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	6.2.4.3	Draw polygons in the coordinate plane given coordinates for the vertices. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in contextual situations. (MP4, MP7) ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.1	Use positive and negative numbers to describe quantities having opposite directions or values, represent quantities in contexts and explain the meaning of 0 in situations including credits/debits, temperature above/below zero, elevation above/below sea level and positive/negative electric charge. (MP4, MP5) ✣ $ ☼
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.2	Locate positive and negative rational numbers on a number line. Plot ordered pairs of positive and negative rational numbers on a coordinate grid. (MP4, MP6) $

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.3	Interpret statements of inequality (<, >, =) as statements about the relative position of two numbers on a number line, including positive and negative rational numbers in various forms. (MP1) ☼ $
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.4	Factor whole numbers. Express a whole number as a product of prime factors with exponents. Identify a prime number as a whole number greater than one whose only factors are one and itself. (MP7, MP8)
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.5	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum or difference of two whole numbers, from 1 through 100, with a common factor as a multiple of a sum or difference of two whole numbers with no common factor. (MP2, MP8)
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.6	Understand the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value as magnitude for a positive or negative quantity in context. (MP4, MP7) $

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.7	Estimate solutions to situations with whole numbers, fractions and decimals and use the estimates to assess the reasonableness of the results in the context of the situation. (MP2) ✣ $ ☼
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.8	Multiply and divide fractions and mixed numbers using visual models to represent the situation leading towards generalizable algorithms. (MP2)
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.9	Solve mathematical situations requiring arithmetic, including multiplication and division, with decimals, fractions and mixed numbers, explaining the solution pathway. Interpret quotients in the context of the situation. (MP3, MP4) $ µ ☼
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.10	Solve situations using the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$ in context, including constant speed and unit pricing to make purchase decisions. (MP4) ✣ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	6.3.5.11	Solve percent situations using visual models including tables of equivalent ratios, tape diagrams or double number lines. Apply concepts of percentage including discounts, markups, tips and commission. Situations can include identifying the part given a whole and the percentage, and identifying the percentage given the part and the whole. (MP4, MP7) ✣ µ ☼
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.1	Generate equivalent numerical expressions involving positive rational numbers and justify why expressions are equivalent. (MP1, MP3)
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.2	Determine equivalences among fractions, decimals and percentages involving rational numbers. Convert between equivalent representations. (MP7) $
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.3	Represent mathematical situations using expressions, equations and inequalities involving variables and positive rational numbers. (MP4) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.4	Solve one-step equations, including equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all positive rational numbers. Use number sense, properties of arithmetic and the idea of maintaining equality on both sides of the equation. Interpret a solution in the original context and assess the reasonableness of results. (MP3, MP4)
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.5	Identify and use ratios to compare quantities. Understand that comparing quantities using ratios is not the same as comparing quantities using subtraction. (MP1) $ ☼
6	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	6.3.6.6	Solve ratio and rate situations, including mixtures and concentrations, by modeling with tables of equivalent ratios, tape diagrams, double number lines or equations. (MP4, MP7) ✣ µ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
6	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	6.3.7.1	Use variables to represent two quantities in a situation that change in relationship to one another. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. (MP2, MP8) ✣ $ µ

Grade 7 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.1	Formulate statistical investigative questions with data collection, keeping cultural perspectives in mind, which compare differences between groups, require a sample of a population and use categorical and/or numerical data. (MP3, MP6) ✜ $
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.2	Describe how statistics can be used to gain information about a population by examining a sample of the population. Explain how generalizations about a population are valid only when the sample is representative of that population and that random sampling tends to produce representative samples and support valid inferences. (MP2) ✜ ☼
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.3	Make inferences using statistics about population parameters based on a random sample from that population. (MP3)

Grade	Strand	Anchor Standard	Code	Benchmark
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.4	Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, variability and overall shape. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. Justify the choice of measures of center and variability, the shape of the data distribution and the context in which the data were gathered. (MP1, MP7)
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.5	Create a visualization about a data set, organizing and presenting the data in appropriate ways, including in tables, circle graphs and histograms, and incorporating any other relevant information that helps to tell a story about the data. (MP5, MP6) # μ
7	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	7.1.1.6	Compare and communicate multiple explanations for data trends observed, considering reasonable alternatives given the variability in findings and sampling methods and cultural perspectives. (MP3, MP6)

Grade	Strand	Anchor Standard	Code	Benchmark
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.1	Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as fractions, decimals and percentages. (MP2)
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.2	Approximate the probability of a chance event, where the theoretical probability is unknown, by collecting data and observing its long-run frequency. Represent the probabilities as percentages, fractions and decimals between 0 and 1 inclusive. Use approximate probabilities to make predictions when actual probabilities are unknown. (MP4) # ☼
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.3	Recognize that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space where the compound event occurs. (MP1)
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.4	Represent sample spaces for compound events by decomposing the events using methods such as organized lists, tables and/or tree diagrams. For an event described in everyday language, identify the outcomes in the sample space that compose the event. (MP5) #

Grade	Strand	Anchor Standard	Code	Benchmark
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.5	Design and use a simulation within a computational tool to generate frequencies for compound events. #
7	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	7.1.2.6	Find probabilities of compound events using organized lists, tables, tree diagrams and/or simulation via a computational tool. (MP5) # μ ☼

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
7	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	7.2.3.1	Demonstrate an understanding of the proportional relationship between the diameter and circumference of a circle and that the unit rate (constant of proportionality) is π. (MP1, MP2) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
7	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	7.2.3.2	Calculate the circumference and area of circles to solve situations in various contexts. (MP4, MP6) ✣ μ ☼
7	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	7.2.3.3	Calculate the arc length and area of sectors of circles (given the central angle) to solve situations in various contexts. (MP4, MP7) ✣ ☼
7	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	7.2.3.4	Calculate the surface area and volume of cylinders. Justify the formulas used. (MP3, MP8) ✣ ☼
7	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	7.2.4.1	Graph and describe translations and reflections of figures on a coordinate grid. Determine the coordinates of the vertices of the figure after the transformation. Describe the properties of congruency when performing translations and reflections. (MP8)

Grade	Strand	Anchor Standard	Code	Benchmark
7	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	7.2.4.2	Describe the properties of similarity, distinguishing between similarity and congruency, compare geometric figures for similarity and determine scale factors. (MP1, MP6) ✣ ☼
7	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	7.2.4.3	Apply scale factors, length ratios and area ratios to determine side lengths and areas of similar geometric figures. (MP5)
7	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	7.2.4.4	Use proportional reasoning and ratios to solve situations involving scale drawings and conversions of measurement units. (MP2, MP4) ✣ $ ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.1	Know that every rational number can be written as the ratio of two integers and as a terminating or repeating decimal. Recognize that π is not rational but that it can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14. (MP8)
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.2	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. Recognize that the opposite of the opposite of a number is the number itself, such as –(–3) = 3. Recognize that 0 is its own opposite. (MP7)
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.3	Compare positive and negative rational numbers expressed in various forms including the symbols <, >, =, ≤ and ≥. (MP4) $
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.4	Recognize subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in contexts. (MP7, MP8) ✣ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.5	Create a budget using positive and negative rational numbers for an event and calculate what percentage of the total budget each category comprises. Justify choices of the allocation of the available resources. (MP1, MP4, MP5) ✣ $ µ ☼
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.6	Solve mathematical situations involving adding, subtracting, multiplying and dividing positive and negative rational numbers that are integers, fractions and terminating decimals. Use efficient and generalizable procedures. Raise positive rational numbers to whole-number exponents. (MP4) $ ☼
7	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	7.3.5.7	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. (MP1, MP4) $ ☼
7	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	7.3.6.1	Use properties of algebra to generate equivalent numerical and algebraic expressions containing rational numbers, grouping symbols and whole number exponents. Properties of algebra include associative, commutative and distributive laws. (MP1)

Grade	Strand	Anchor Standard	Code	Benchmark
7	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	7.3.6.2	Represent and solve contextual situations resulting in two-step equations and inequalities involving variables and positive/negative rational numbers. Graph the solution set of the inequality and interpret the solution in context. (MP4, MP5) $ ☼
7	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	7.3.6.3	Evaluate algebraic expressions, including expressions containing rational numbers, absolute value and whole number exponents, by applying computational hierarchy of operations at specified values of their variables. (MP7) #
7	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	7.3.6.4	Solve multi-step situations involving proportional relationships in numerous contexts using models, such as tables of equivalent ratios involving scaling up and down, tape diagrams, double number lines or equations. (MP4, MP7) ✣ $ µ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
7	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	7.3.6.5	Solve multi-step financial literacy situations including simple interest, tax, markups and markdowns, tips and commissions, fees, percent increase and decrease and percent error. Distinguish between sales tax and income tax for earned wages. Situations include finding the whole, the part and the percent. (MP7, MP8) $ µ ☼
7	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	7.3.7.1	Represent proportional relationships with tables, verbal descriptions, symbols, equations and graphs. Translate from one representation to another. Determine the unit rate (constant of proportionality or rate of change) given any of these representations. (MP2, MP8) ✣ $ µ
7	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	7.3.7.2	Express a relationship between two variables, the independent (*x*) and the dependent (*y*), as proportional if it can be written in the form $\frac{y}{x} = k$ or $y = kx$. Distinguish proportional relationships from non-proportional relationships, including relationships where the *x* and *y* value are not 0. (MP8) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
7	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	7.3.7.3	Determine whether two quantities are in a proportional relationship by testing for equivalent ratios in a table or graphing on a coordinate plane and by observing whether the graph is a straight line through the origin. Use graphing technology to examine the relationship between the unit rate and the line for a given situation. (MP1) ✣ μ ☼

Grade 8 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.1	Formulate statistical investigative questions, such as questions about variation, the differences between groups and associations between two numerical variables. (MP3) ✣
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.2	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association. (MP5, MP8)
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.3	Identify when to use straight lines to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and assess the model fit by judging the closeness of the data points to the line. (MP5, MP6)

Grade	Strand	Anchor Standard	Code	Benchmark
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.4	Use the equation of a linear model to solve situations in the context of bivariate measurement data. Interpret the slope and intercepts in context of the variables. (MP4, MP6) $ #
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.5	Create data visualizations about a data set. Organize and present the data in appropriate ways, including in tables and scatter plots, and incorporate other relevant information that helps to tell a story and support a claim about the data. (MP1, MP6) ✜ $ # µ
8	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	8.1.1.6	Compare and communicate competing explanations for data trends observed, considering the reasonableness of the model's predictions and correlation versus causation. (MP3, MP6) $ #

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
8	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	8.2.3.1	Informally justify the Pythagorean Theorem and its converse by using measurements, diagrams or computer software. (MP2, MP3) ✣ ☼
8	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	8.2.3.2	Apply the Pythagorean Theorem to solve multi-step contextual situations by determining unknown side lengths in right triangles in both two- and three-dimensional shapes. (MP4) µ ☼
8	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	8.2.3.3	Determine the distance between two points on a horizontal or vertical line in a coordinate system. Use the Pythagorean Theorem to find the distance between any two points in a coordinate system. (MP7) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
8	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	8.2.4.1	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (MP8)
8	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	8.2.4.2	Given a line on a coordinate plane and the coordinates of a point not on the line, find lines through that point that are parallel and perpendicular to the given line, using graphing technology or hand drawn graphs. (MP7) ✣ #
8	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	8.2.4.3	Identify the different types of solutions possible for a system of linear equations (no solution, one solution, infinitely many solutions). Using slope, compare the number of solutions to the graphical representation of pairs of lines that are intersecting, parallel or identical. (MP2) $

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.1	Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. (MP1, MP7) ☼
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.2	Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line. Estimate the value of expressions involving irrational numbers. (MP2, MP6)
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.3	Know and apply the properties of positive and negative integer exponents to generate equivalent numerical expressions. (MP1)
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.4	Express approximations of very large and very small numbers using scientific notation. Understand how technology displays numbers in scientific notation. Compare numbers expressed in scientific notation using the symbols <, >, =, ≤ and ≥. (MP4, MP5) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.5	Multiply and divide numbers expressed in scientific notation. Express answers in scientific notation. (MP5) $ ☼
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.6	Solve situations in various contexts involving calculating and comparing simple and compound interest. Compound interest situations are limited to compounded annually. (MP7, MP8) $ ☼
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.7	Solve multi-step contextual situations comparing how interest rate and loan length affect the cost of credit. Calculate the total cost of repaying a loan under various rates of interest and over different periods of time. (MP4, MP5) ✜ $ µ ☼
8	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	8.3.5.8	Compare and contrast employment opportunities and their payment methods, including per hour, with or without tips, salary, per diem and piecework pay. Justify financial decisions with representations, including linear relationships. (MP1, MP4) ✜ $ µ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.1	Justify steps in generating equivalent algebraic expressions and identify the properties used. Properties include the associative, commutative, distributive, identity and inverse. (MP1, MP3)
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.2	Evaluate algebraic expressions, including expressions containing radicals and absolute values, by applying computational hierarchy of operations at specified values of their variables. (MP7)
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.3	Solve multi-step equations in one variable, including equivalent linear expressions. Solve for one variable in a multivariable equation in terms of the other variables. Justify the steps by identifying the properties of the equality used. (MP3, MP4)
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.4	Use the relationship between square roots and squares of a number to solve situations. (MP2, MP6) ☼
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.5	Represent linear relationships in point-slope and standard form and convert to slope-intercept form. (MP7)

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.6	Reason abstractly, involving variables as a point, slope or intercept, to compare general forms of linear relationships, including point-point, point-slope, standard form and slope-intercept form. (MP7) µ
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.7	Represent relationships in various contexts using multi-step linear inequalities. Solve linear inequalities using properties of inequalities. Graph the solutions on a number line and interpret the solutions in context. (MP4) $ ☼
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.8	Represent relationships in various contexts with equations and inequalities involving the absolute value of a linear expression. Solve such equations and inequalities and graph the solutions on a number line. (MP4) $ ☼
8	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	8.3.6.9	Represent relationships in various contextual situations using systems of linear equations. Solve systems of linear equations in two variables, symbolically and graphically, understanding that the solution corresponds to the point of intersection of their graphs. (MP4, MP5) $ # ☼

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.1	Compare graphical properties of proportional and non-proportional linear relationships, including slope. (MP7) $ ☼
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.2	Analyze visual patterns to distinguish between linear and non-linear patterns. For linear patterns, describe how a pattern is changing, name the n^{th} term and write an equation to generalize the n^{th} term. (MP1, MP7) ✣ # ☼
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.3	Recognize that a function is a rule that assigns each input to exactly one output. Use the function to represent the relationship in which changing the input (independent) variable, by an amount, leads to a change in the output (dependent) variable, a constant multiplied by that amount. Recognize that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Use functional notation, such as $f(x)$, to represent such relationships. (MP4, MP7) µ

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.4	Represent linear functions with tables, verbal descriptions, symbols, equations and graphs. Translate from one representation to another. (MP5) $ ☼
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.5	Explain how changes to the values *m* or *b* in the linear function $f(x) = mx + b$ affect the graph of the function. Use graphing technology to examine these effects. Recognize that the graph of the linear equation $y = mx + b$ comes from *b* units translation of $y = mx$ graph. (MP1, MP5) µ # ☼
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.6	Identify graphical properties of linear functions in the form $f(x) = mx + b$, including slope, y-intercept and x-intercept. Know that the graph is a straight line, the slope (*m*) equals the rate of change, the y-intercept (*b*) is the value of the function at $x = 0$ and the x-intercept is the value of the function at $f(x) = 0$. (MP1, MP5) $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.7	Recognize that an arithmetic sequence is a linear function that can be expressed in the form $f(x) = mx + b$, where $x = 0, 1, 2, 3, \ldots$ (MP1, MP8)
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.8	Recognize that a geometric sequence is a non-linear function that can be expressed in the form $f(x) = a(b)^x$, where $x = 0, 1, 2, 3, \ldots$ (MP1, MP8)
8	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	8.3.7.9	Represent arithmetic and geometric sequences using equations, tables, graphs and verbal descriptions and use them to solve situations. (MP4)

Grades 9–11 Standards

Strand 1: Data and Probability

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.1	Formulate statistical investigative questions and pose hypotheses. These include questions about variation or the differences between groups, associations between quantitative and categorical variables or pairing together multiple analyses. (MP1, MP3) ✜ $ µ ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.2	Explain how choices concerning data collection methods can affect the quality, size, speed, accessibility and cost of the data. (MP3) $ ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.3	Analyze issues of bias by considering data collection methods and cultural perspectives. (MP3) ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.4	Explain the purposes of and differences among sample surveys, experiments and observational studies. Explain whether randomization in each allows for conclusions of causation and/or generalization of a population. (MP3) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.5	Analyze and explain when arguments based on data confuse correlation and causation. (MP3)
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.6	Compute using technology or estimate the correlation coefficient of a linear model. Interpret the linear model in the context of the data. (MP5, MP6) $
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.7	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Identify data sets for which such a procedure is not appropriate. Tools can include calculators, spreadsheets, apps or tables. (MP5, MP6) $ ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.8	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples or simulated samples of the same size to gauge the variation in estimates or predictions. (MP1, MP4) # ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.9	Use statistics appropriate to the shape of the data distribution to compare the center and spread of two or more data sets. (MP4)
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.10	Create and analyze data displays, including scatter plots, histograms and boxplots using technology. (MP1) ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.11	Identify, create and compare statistical models with linear and exponential functions, including linear regression. Assess the reasonableness of model fit using residuals and correlation coefficients. (MP4) # µ
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.12	Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports. (MP3, MP6) $ ☼

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9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.13	Analyze and interpret data using various measures, such as difference in shapes, center and spread to draw conclusions, identify trends and describe relationships, accounting for possible effects of extreme data points (outliers). (MP1) $
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.14	Evaluate reports based on published data by identifying the source of the data, the design of the study and the way the data are analyzed and displayed. (MP3) $ ☼
9–11	Data and Probability	Data Sciences: Identify, formulate and investigate statistical questions by collecting data, considering cultural perspectives, analyzing and interpreting data and communicating the results.	9.1.1.15	Identify and explain misleading uses of data along with how to use spreadsheets, tables or graphing technology to recognize and analyze distortions in data displays. Use interactive data visualizations to support and influence different points of view. (MP3) $ #
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.1	Select and apply counting procedures such as the multiplication and addition principles, permutations, combinations and tree diagrams to determine the elements and size of a sample space. (MP1, MP7)

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.2	Describe events as subsets of a sample space using characteristics of the outcomes. Use Venn diagrams, tables or lists to depict events and compute event sizes. Create representations to find unions, intersections and complements of events. (MP1) #
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.3	Apply probability concepts, such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve situations. (MP4) # ☼
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.4	Calculate experimental probabilities by repeatedly performing simulations or experiments involving a probability model and by using relative frequencies of outcomes. Recognize that as the number of trials increases, the closer the experimental probability approaches the theoretical probability. (MP4)
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.5	Construct and interpret two-way frequency tables when two categories are associated with each object being classified. Use the two-way table to decide if events are independent and to calculate conditional probabilities. (MP4, MP5) ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.6	Compare the recursion to create the n^{th} row of Pascal's triangle, the recursion to expand $(x + y)^n$, the recursion to generate all sequences of heads and tails on *n* coin flips and the recursion to generate all subsets of a set with *n* elements. Establish connections among the various representations. (MP8)
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.7	Calculate and make decisions based on expected values using theoretical or experimental probabilities in scenarios, including finance or games of chance. (MP1, MP6) $ μ ☼
9–11	Data and Probability	Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.	9.1.2.8	Calculate and make decisions to obtain a desired outcome (maximum or minimum value or maximum or minimum probability) based on expected value and theoretical or experimental probabilities in scenarios such as games of chance or finance. (MP6, MP7) $ ☼

Strand 2: Spatial Reasoning

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.1	Apply the Pythagorean Theorem and distance formula to compute perimeters of polygons and areas of right triangles and rectangles on coordinate planes. (MP1) ☼
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.2	Apply properties of 45°-45°-90° and 30°-60°-90° triangles to solve situations and logically justify results. (MP1, MP4) ☼
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.3	Use trigonometric ratios and the Pythagorean Theorem and its converse to solve contextual situations, including right triangles or polygons that can be decomposed into triangles. (MP4, MP7) ☼

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9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.4	Develop, justify and use decomposition to determine the formula for surface area and volume of various three-dimensional figures. (MP7, MP8) ✣ # ☼
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.5	Use the formulas for surface area and volume of various three-dimensional figures to solve multi-step contextual situations. (MP4, MP5) ✣ ☼
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.6	Use units of measure and dimensional analysis to solve multi-step situations. Interpret units consistently in formulas. Interpret the scale and the origin in graphs and data displays. (MP1, MP4) ✣ $ ☼
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.7	Compute the unit cost and total cost to cover different areas and volumes in applications, such as painting, carpeting and gardening. Optimize costs for various models, including linear and quadratic. Use technology, including spreadsheets. (MP4, MP5) $ µ ☼

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9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.8	Use similarity to determine the side ratios in right triangles as properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (MP8)
9–11	Spatial Reasoning	Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.	9.2.3.9	Apply dilations of scale factor k on length, area and volume and recognize the effect is multiplication by k, k^2, k^3 respectively. (MP2) μ ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.1	Apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve situations and logically justify results. (MP1, MP3)
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.2	Apply properties of equilateral, isosceles and scalene triangles to solve situations and logically justify results. (MP1, MP3) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.3	Apply properties such as interior and exterior angles and parallel and perpendicular sides to define, classify and solve situations involving regular and irregular polygons. (MP1) ✣ ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.4	Apply properties of congruent figures to solve situations and logically justify results. (MP1, MP3) ✣ ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.5	Accurately interpret and use words and phrases such as “if ... then,” “if and only if,” “all” and “not.” Recognize the logical relationships between an “if ... then” statement and its inverse, converse and contrapositive. (MP6, MP7) ✣ # ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.6	Analyze the validity of a logical argument and give counterexamples to disprove a statement. (MP1, MP3) ✣ # ☼

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9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.7	Construct logical arguments using axioms, definitions, theorems and postulates that clearly justify the reasoning. (MP3, MP6) #
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.8	Identify, describe and solve situations using relationships among inscribed angles, circumscribed angles, radii and chords in circles. (MP5) ✣ ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.9	Describe and solve situations using the relationships of angles formed outside the circle for both secants and tangent lines. (MP7) ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.10	Apply properties of similar figures to solve situations and logically justify results. (MP3) ✣

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.11	Show that two triangles are similar using the AA, SAS and SSS similarity criteria. (MP7)
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.12	Make geometric constructions with a variety of tools, including dynamic geometric technology, to examine theorems, make and test conjectures, represent transformations and develop mathematical reasoning skills in multi-step situations. (MP5) ☼
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.13	Create and compare rigid and non-rigid transformations, and connect the preimage and image to congruence. (MP2)
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.14	Describe the steps used to show various sequences of transformations, applying reflections, rotations, translations and/or dilations of geometric figures that map one figure onto the other. (MP8) ✜ #

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.15	Apply geometric methods to solve design situations, including designing an object or structure to satisfy physical constraints or optimize resources. (MP4) $ µ
9–11	Spatial Reasoning	Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.	9.2.4.16	Apply concepts of density based on area and volume in modeling contextual situations, including population density, BTUs per cubic foot and data storage. (MP4) ☼

Strand 3: Patterns and Relationships

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.1	Add, subtract, multiply and divide numbers in scientific notation. (MP6, MP7) ✣ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.2	Compare the definition of rational exponents and properties of radicals. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents. (MP6)
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.3	Apply knowledge of number systems extending from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers and from real numbers to complex numbers to solve equations. (MP2, MP7)
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.4	Use matrices to represent and manipulate data and interpret the results in context. Add, subtract and multiply matrices of appropriate dimensions. Multiply matrices by scalars. (MP4, MP5) #
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.5	Estimate and verify the cost of an item, including multiple discounts and taxes. Show an understanding of the order of operations. (MP6, MP8) $

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.6	Assess the reasonableness of a solution in its given context, including financial literacy applications. Compare the solution to appropriate graphical or numerical estimates. Interpret a solution in the original context. (MP3) $ ☼
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.7	Use the structure of an expression, equation and/or formula to create an equivalent form that is more helpful given the situation. Rearrange formulas to highlight a quantity of interest, using the same reasoning in solving equations. (MP6) $
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.8	Use the structure of an expression to write it in multiple ways. (MP7)
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.9	Find and evaluate the composition of multiple functions. (MP7)

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.10	Compute the time it takes to pay off a loan given the interest rate, loan amount and monthly payment. Determine the change in cost by the variation in loan amount, fixed or variable interest rates and payment amounts. Use technology, including spreadsheets. (MP1, MP5) ✣ $ ☼
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.11	Compute the total payments to pay off a loan given the interest rate, loan amount and monthly payment. Compare the total payments for various monthly payments and various interest rates. Use technology, including spreadsheets. (MP1, MP5) ✣ $ ☼
9–11	Patterns and Relationships	Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.	9.3.5.12	Compare different types of retirement plans, considering after tax or pretax contributions, employer match contributions, the benefit of starting contributions early and the consequences of use prior to retirement. (MP1) $
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.1	Add, subtract and multiply polynomials. (MP1, MP7)

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.2	Reason abstractly to compare general forms of quadratics, including vertex form, general form, factored form and the graph. Develop procedures to convert from one form to another. (MP2, MP7) μ
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.3	Choose and produce an equivalent form of a quadratic function, using symbolic and graphical methods, to identify the vertex, line of symmetry and intercepts of the parabola. (MP8)
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.4	Factor polynomials including common monomial factors from polynomials, quadratic polynomials and the difference of two squares. (MP7, MP8)
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.5	Solve quadratic equations by appropriate methods using factoring, completing the square, graphing or the quadratic formula. Find non-real complex roots when they exist. (MP2, MP7)
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.6	Compare the equation of a circle, the Pythagorean Theorem and the Distance Formula. Complete the square to find the center and radius of a circle given an equation. (MP2, MP8)

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.7	Solve situations involving relationships which are inversely proportional in various contexts, including rates. (MP4) ✣ $ ☼
9–11	Patterns and Relationships	Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.	9.3.6.8	Apply the properties of rational exponents and radicals to generate equivalent algebraic expressions. (MP2, MP7)
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.1	Represent and solve situations in various contexts, including financial literacy, using systems of linear equations, systems of linear inequalities and exponential and quadratic functions. (MP4) $ # ☼
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.2	Translate between graphs of quadratic, exponential and other functions (including absolute value, rational and polynomial), tables and symbolic representations. Sketch graphs and use graphing technology to graph functions. (MP5) $

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.3	Determine how vertical/ horizontal reflecting, translating and scaling affect the symbolic and graphical forms of a function. Use graphing technology to examine transformations. (MP3)
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.4	Express the terms in an arithmetic or geometric sequence recursively and by giving an explicit formula. (MP8) #
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.5	Express recursive patterns using recursive formulas. Calculate sequences defined by recursive formulas. (MP8) $
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.6	Find the domain and range of functions defined symbolically, graphically or in a context, including piecewise and step functions. Express solutions and recognize that some answers obtained may not be valid, including cases where the function inputs are discrete instead of continuous. (MP4) ✣ $ ☼

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.7	Describe the graph of a function using key features such as intercepts, maxima/minima, intervals of increase and decrease and end behavior. Draw conclusions from graphs of functions and other relations. (MP3) $ ☼
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.8	Define the compounding of interest *n* times per year according to a recursive formula. Compare the recursive definition of interest to the recursive definition of a geometric sequence $t(n) = r(t(n-1))$. Compare the interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to the general form of an exponential function $y = a(b)^x$. Explain the purpose of each part of the interest formula. (MP4, MP5) $ # µ
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.9	Find the inverse of a given function and justify the results using tables, graphs or algebra. (MP4, MP6)

Grade	Strand	Anchor Standard	Code	Benchmark
9–11	Patterns and Relationships	Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.	9.3.7.10	Use the concept of a function as a connection between inputs and outputs to find function values and use function notation. (MP2)

Appendix 1

Standards for Mathematical Practice

Math Practice 1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice 2: Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize (to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents) and the ability to contextualize (to pause as needed during the manipulation process in order to probe into the referents for the symbols involved). Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice 3: Construct viable arguments and appreciate and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. While communicating their own mathematical ideas is important, students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17–18)[3]. Together, students make sense of the mathematics by asking helpful questions that clarify or deepen everyone's understanding.

Math Practice 4: Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships

[3] Gutiérrez, Rochelle. 2017. "Living Mathematx: Towards a Vision for the Future." Philosophy of Mathematics Education Journal. November: 2-26. https://eric.ed.gov/?id=ED581384

mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice 5: Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice 6: Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice 7: Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in

a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice 8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.