

# FOLSOM CORDOVA UNIFIED SCHOOL DISTRICT



## Integrated Math II Foundations

<b>Board Approval Date: October 21, 2021</b>	<b>Course Length: 2 Semesters</b>
<b>Grading: A-F</b>	<b>Credits: 5 Credits per Semester</b>
<b>Proposed Grade Level(s): 10, 11, 12</b>	<b>Subject Area: Mathematics</b> <b>Elective Area (if applicable):</b> N/A
<b>Prerequisite(s):</b> <b>“D” or better in Integrated Math I</b>	<b>Corequisite(s):</b> N/A
<b>CTE Sector/Pathway: N/A</b>	
<b>Intent to Pursue ‘A-G’ College Prep Status: No</b>	
<b>A-G Course Identifier: N/A</b>	
<b>Graduation Requirement: Yes</b>	
<b>Course Intent: District Course</b> <b>Program (if applicable): N/A</b>	
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### COURSE DESCRIPTION (Online Course)

This is a non-college preparatory course designed for students who have passed Integrated Math 1 but may not be ready for the rigor of Integrated Math II. The course is structured around problems and investigations that build spatial visualization skills, conceptual understanding of geometry topics, a few Algebra 1 topics not covered in Integrated Math I, and an awareness of connections between different ideas. Students are encouraged to investigate and conjecture to develop their reasoning skills. Lessons are structured for students to collaborate actively by working collaboratively with peers. The course will focus on the foundational geometry standards that students will learn in Integrated Math II, without

engaging in formal proofs. The big ideas of the course are presented in an integrated Algebra/geometry context.

**DETAILED UNITS OF INSTRUCTION:**

Unit Number/Title	Unit Essential Questions	Examples of Formative Assessments	Examples of Summative Assessment
<b>1. Exponential Functions</b>	<p>How do exponential functions model real-world problems and their solutions?</p> <p>How do the characteristics of exponential functions affect the graph?</p> <p>How do you use exponential growth and decay to model real-life situations?</p>	*Journal: Graphs of Exponential Functions	<p>*Modeling: Exponential Functions</p> <p>*Unit Test</p>
<b>2. Polynomials</b>	<p>How do exponential functions model real-world problems and their solutions?</p> <p>How do the characteristics of exponential functions affect the graph?</p> <p>How do you use exponential growth and decay to model real-life situations?</p>	*Journal: Multiplying Polynomials	<p>*Modeling: Multiplying Binomials</p> <p>*Unit Test</p>
<b>3. Factoring Polynomials</b>	<p>For a polynomial function, how are factors, zeros and x-intercepts related?</p> <p>For a polynomial function, how are factors and roots related?</p> <p>How can simplifying and factoring polynomials help solve real-world problems?</p>	*Journal: Factoring and Graphing	<p>*Modeling: Factoring <math>ax^2 + bx + c</math></p> <p>*Unit Test</p>
<b>4. Quadratic Equations and Functions</b>	<p>How are quadratic functions used to model, analyze, and interpret mathematical relationships?</p> <p>What are the advantages of a quadratic function in vertex form? In standard form?</p> <p>How is any quadratic</p>	*Journal: Completing the Square	<p>*Modeling: Linear, Quadratic, and Exponential Functions</p> <p>*Unit Test</p>

	function related to the parent quadratic function? How are the real solutions of a quadratic equation related to the graph of the related quadratic function?		
<b>5. Undoing Functions and Moving them Around</b>	How does understanding how to find the vertex of a quadratic function help in making decisions in real-life applications? What are the advantages of a quadratic function in vertex form? In standard form? How is any quadratic function related to the parent quadratic function $f(x)=x^2$ ?	*Journal: Shifting Functions	*Modeling: Stretching and Compressing Functions *Unit Test
<b>6. Triangles</b>	How can you describe the relationships among the angles of a triangle? How can you use angles to tell whether triangles are congruent? How can you use angles to tell whether triangles are similar? How can you use similarity and triangle theorems to prove congruency?	*Journal: Proofs of Congruence	*Modeling: Similarity Theorems *Unit Test
<b>7. Right Triangles and Trigonometry</b>	How do you find a side length or angle measure in a right triangle? How do trigonometric ratios relate to similar right triangles? How can you use trigonometry to solve real world problems?	*Journal: Proving the Pythagorean Theorem	*Modeling: Leaning Towers *Unit Test
<b>8. Conic Sections</b>	What determines the type of conic section you will be using?	*Practice Problems	*Unit Test

	How do you use the midpoint and distance formulas? Why are circles, ellipses, hyperbolas, and parabolas called “conic sections”?		
<b>9. Three Dimensional Solids</b>	How can you identify, describe, and classify three-dimensional solids? What are the characteristics of 3-dimensional figures?	*Journal: Volume	*Modeling: Solids *Unit Test

**ESSENTIAL STANDARDS:**

STANDARDS

Content Standards

Math Practice Standards

Key Advances from Mathematics I

Students extend their previous work with linear and exponential expressions, equations, and systems of equations and inequalities to quadratic relationships.

A parallel extension occurs from linear and exponential functions to quadratic functions: students begin to analyze functions in terms of transformations.

Building on their work with transformations, students produce increasingly formal arguments about geometric relationships, particularly around notions of similarity.

Critical Areas of Instruction

Extend the laws of exponents to rational exponents

Compare key characteristics of quadratic functions with those of linear and exponential functions

Create and solving equations and inequalities involving linear, exponential, and quadratic expressions

Extend work with probability

Establish criteria for similarity of triangles based on dilations and proportional reasoning

NUMBER and QUANTITY

Major Clusters

The Complex Number System

N-CN-1 Know there is a complex number such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

N-CN-2 Use the relation  $i^2 = -1$ , and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN-7 Solve quadratic equations with real coefficients that have complex solutions.

Additional/Supporting Clusters:

The Real Number System

N-RN-1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N-RN-2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-RN-3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a non-zero rational number and an irrational number is irrational.

N-RN-8 Extend polynomial identities to the complex numbers. For example, rewrite as  $x^2 + 4 = (x + 2i)(x - 2i)$ .

N-RN-9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## ALGEBRA

### Seeing Structure in Expressions

#### Major Clusters

A-SSE-1a Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE-2 Use the structure of an expression to identify ways to rewrite it. For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .

A-SSE-3a Factor a quadratic expression to reveal the zeros of the function it defines.

A-SSE-3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

### Perform Arithmetic Operations on Polynomials

A-APR-1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### Create Equations that Describe Numbers or Relationships

A-CED-4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [Include formulas involving quadratic terms.]

### Solve Equations and Inequalities in One Variable

A-REI-4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a + bi$  for real numbers  $a$  and  $b$ .

A-REI-7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .

### Additional/Supporting Clusters:

#### The Real Number System

A-SSE-1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret  $P(1 + r)^n$  as the product of  $P$  and a factor not depending on  $P$ .

A-SSE-3c Use the properties of exponents to transform expressions for exponential functions

#### Create Equations that Describe Numbers or Relationships

A-CED-1 Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

#### Solve Equations and Inequalities in One Variable

A-REI-4a. Use the method of completing the square to transform any quadratic equation into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

## FUNCTIONS

## Major Clusters

Interpret functions that arise in applications in terms of the context.

F-IF-4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF-5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF-6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

F-IF-7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF-8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Build new functions from existing functions.

F-BF-3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x+k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$ , given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Interpret expressions for functions in terms of the situation they model.

F-LE-3 Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

Additional/Supporting Clusters:

Analyze functions using different representations.

F-IF-7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF-8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ , and  $y = 1.2^t/10$ , and classify them as representing exponential growth or decay.

F-IF-9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Build a function that models a relationship between two quantities.

F-BF-1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations.

Build new functions from existing functions.

F-BF-4 Find inverse functions.

a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$ .

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE-3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Trigonometric Functions

F-TF-8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

## GEOMETRY

### Major Clusters

#### Prove Geometric Theorems

G-CO-10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Prove theorems involving similarity.

G-SRT-4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G-SRT-5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

G-SRT-6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT-8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Understand and apply theorems about circles.

G-C-2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Translate between the geometric description and the equation for a conic section.

G-GPE-1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

#### Explain Volume Formulas and Use Them to Solve Problems

G-GMD-3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G-GMD-5 Know that the effect of a scale factor  $k$  greater than zero on length, area, and volume is to multiply each by  $k$ ,  $k^2$ ,  $k^3$  and , respectively; determine length, area, and volume measures using scale factors

G-GMD-6 Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems.

#### Additional/Supporting Clusters

Prove geometric theorems.

G-CO-9 Prove theorems about lines and angles. Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO-11 Prove theorems about parallelograms. Theorems include opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Understand similarity in terms of similarity transformations.

G-SRT-1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT-2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality

of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT-3 Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

Similarity, Right Triangles, and Trigonometry

G-SRT-8.1 Derive and use the trigonometric ratios for special right triangles ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$  and  $45^\circ$ ,  $45^\circ$ ,  $90^\circ$ ).

Understand and apply theorems about circles.

G-C-1 Prove that all circles are similar.

G-C-3 Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

G-C-4 Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G-C-5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians.

Translate between the geometric description and the equation for a conic section.

G-GPE-2 Derive the equation of a parabola given a focus and directrix.

Use coordinates to prove simple geometric theorems algebraically.

G-GPE-4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove, that a figure defined by four given points in the coordinate plane, is a rectangle; prove or disprove that the point (1,

3), lies on the circle centered at the origin and containing the point (0,2) [include simple circle theorems].

G-GPE-6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Explain volume formulas and use them to solve problems.

G-GMD-1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

## STATISTICS and PROBABILITY

### Major Clusters

Understand independence and conditional probability and use them to interpret data.

S-CP-1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP-2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

S-CP-3 Understand the conditional probability of A given B as the  $P(A \text{ and } B) / P(B)$ , and interpret independence of A and B, as saying the conditional probability of A given B, is the same as the probability given A, and the conditional probability of B given A, is the same as the probability of B.

S-CP-4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

S-CP-5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP-6 Find the conditional probability of A given B as the fraction of 's outcomes that also belong to and interpret the answer in terms of the model.



S-CP-7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

S-CP-8 Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$  and interpret the answer in terms of the model.

S-CP-9 Use permutations and combinations to compute probabilities of compound events and solve problems.

#### Additional Supporting Clusters

Use probability to evaluate outcomes of decisions.

S-MD-6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S-MD-7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

### **RELEVANT STANDARDS AND FRAMEWORKS, CONTENT/PROGRAM SPECIFIC STANDARDS:**

#### **Link to Common Core Standards (if applicable):**

Educational standards describe what students should know and be able to do in each subject in each grade. In California, the State Board of Education decides on the standards for all students, from kindergarten through high school.

<https://www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf>

#### **Link to Framework (if applicable):**

Curriculum frameworks provide guidance for implementing the content standards adopted by the State Board of Education (SBE). Frameworks are developed by the Instructional Quality Commission, formerly known as the Curriculum Development and Supplemental Materials Commission, which also reviews and recommends textbooks and other instructional materials to be adopted by the SBE.

<https://www.cde.ca.gov/ci/ma/cf/documents/mathfwmathematics2jl.pdf>

#### **Link to Subject Area Content Standards (if applicable):**

Content standards were designed to encourage the highest achievement of every student, by defining the knowledge, concepts, and skills that students should acquire at each grade level.

#### **Link to Program Content Area Standards (if applicable):**

Program Content Area Standards apply to programs such as International Baccalaureate, Advanced Placement, Career and Technical Education, etc.

**TEXTBOOKS AND RESOURCE MATERIALS:**

**Textbooks**

<b>Board Approved</b>	<b>Pilot Completion Date (If applicable)</b>	<b>Textbook Title</b>	<b>Author(s)</b>	<b>Publisher</b>	<b>Edition</b>	<b>Date</b>
<i>Yes</i>		<i>APEX: Mathematics II</i>		APEX		<i>2019</i>

**Other Resource Materials**

N/A

**Supplemental Materials**

Board approved supplemental materials (Including but not limited to: Film Clips, Digital Resources, Supplemental texts, DVDs, Programs (Pebble Creek, DBQ, etc.):

N/A