Mathematics Learning *and* Teaching

at *the International School of Beijing*

Guiding Questions

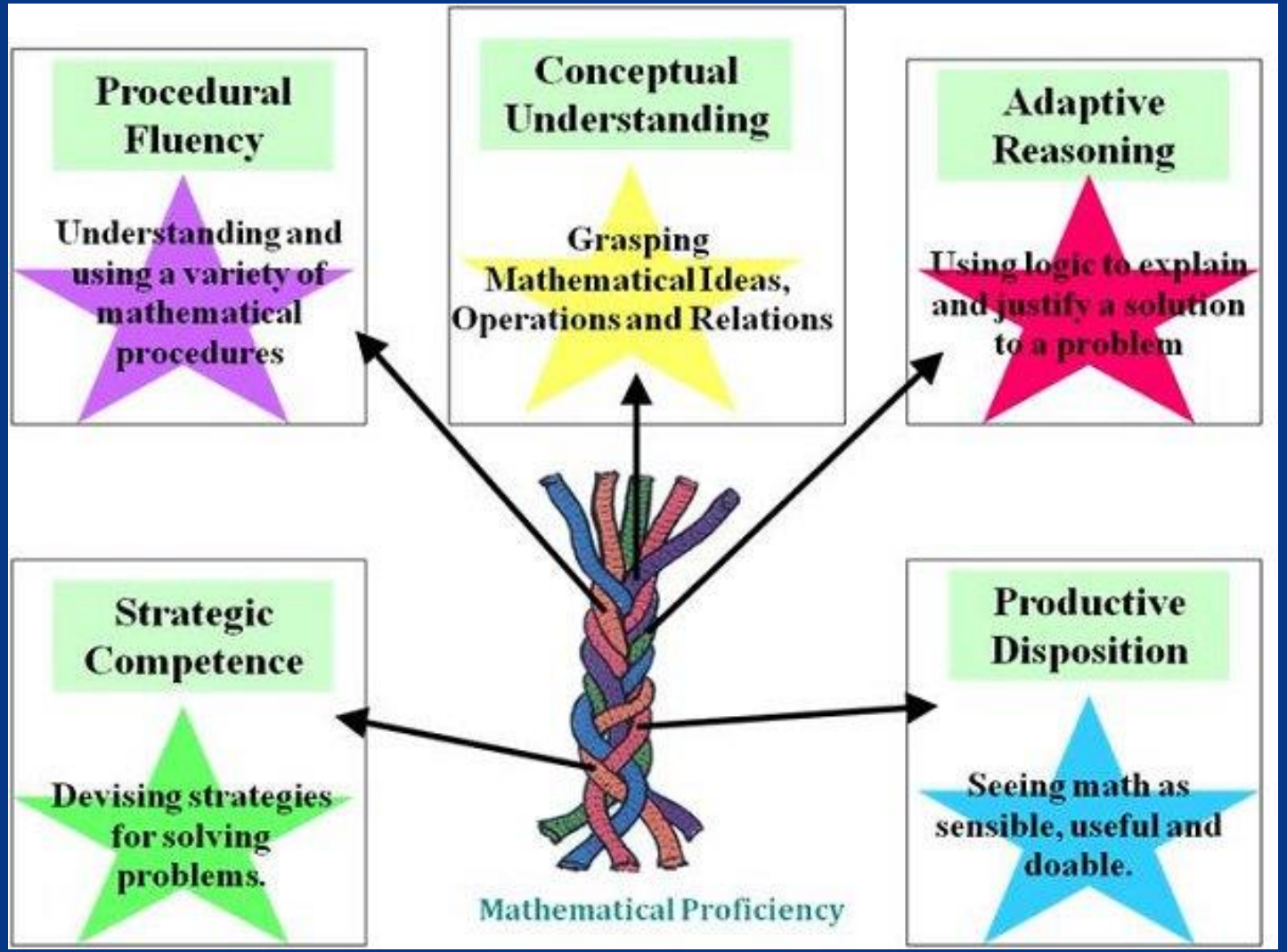
What Does Math Learning Look Like? Sound Like?

Why Are We Teaching & Learning in This Way?

How Are We Making Math Rigorous for All Learners?

Check-In

What does it mean to educate the *whole child* in mathematics?


Procedural Fluency
Understanding and using a variety of mathematical procedures
Conceptual Understanding
Grasping Mathematical Ideas, Operations and Relations
Adaptive Reasoning
Using logic to explain and justify a solution to a problem
Strategic Competence
Devising strategies for solving problems.
Productive Disposition
Seeing math as sensible, useful and doable.
Mathematical Proficiency

Conceptual Understanding: comprehension of mathematical concepts, operations, and relations

Procedural Fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

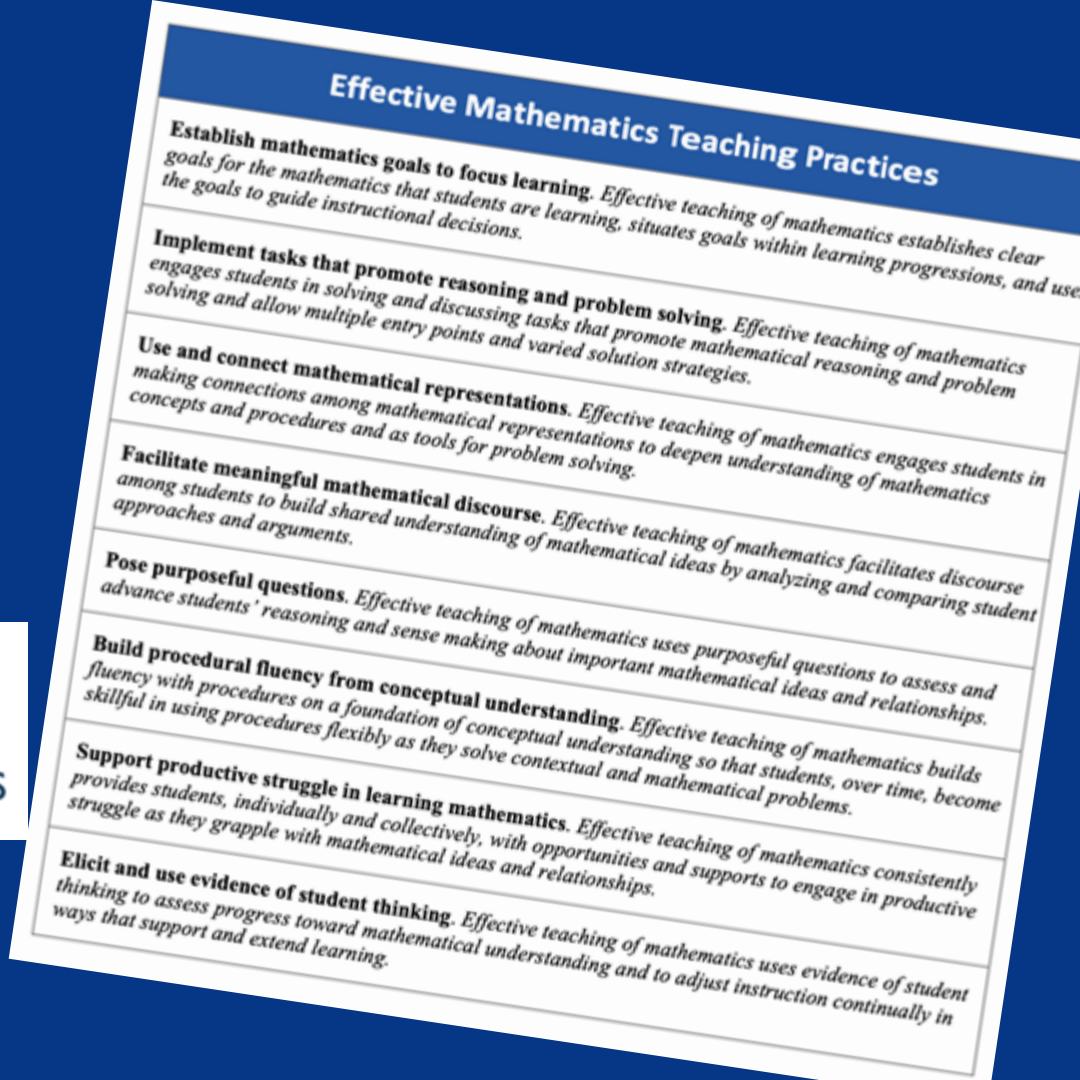
Strategic Competence: ability to formulate, represent, and solve mathematical problems

Adaptive Reasoning: capacity for logical thought, reflection, explanation, and justification

Productive Disposition: habitual inclination to see mathematics as sensible, useful,and worthwhile, coupled with a belief in diligence and one's own efficacy.

Mathematics Teaching *and* Learning

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS



Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*

Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*

Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*

Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*

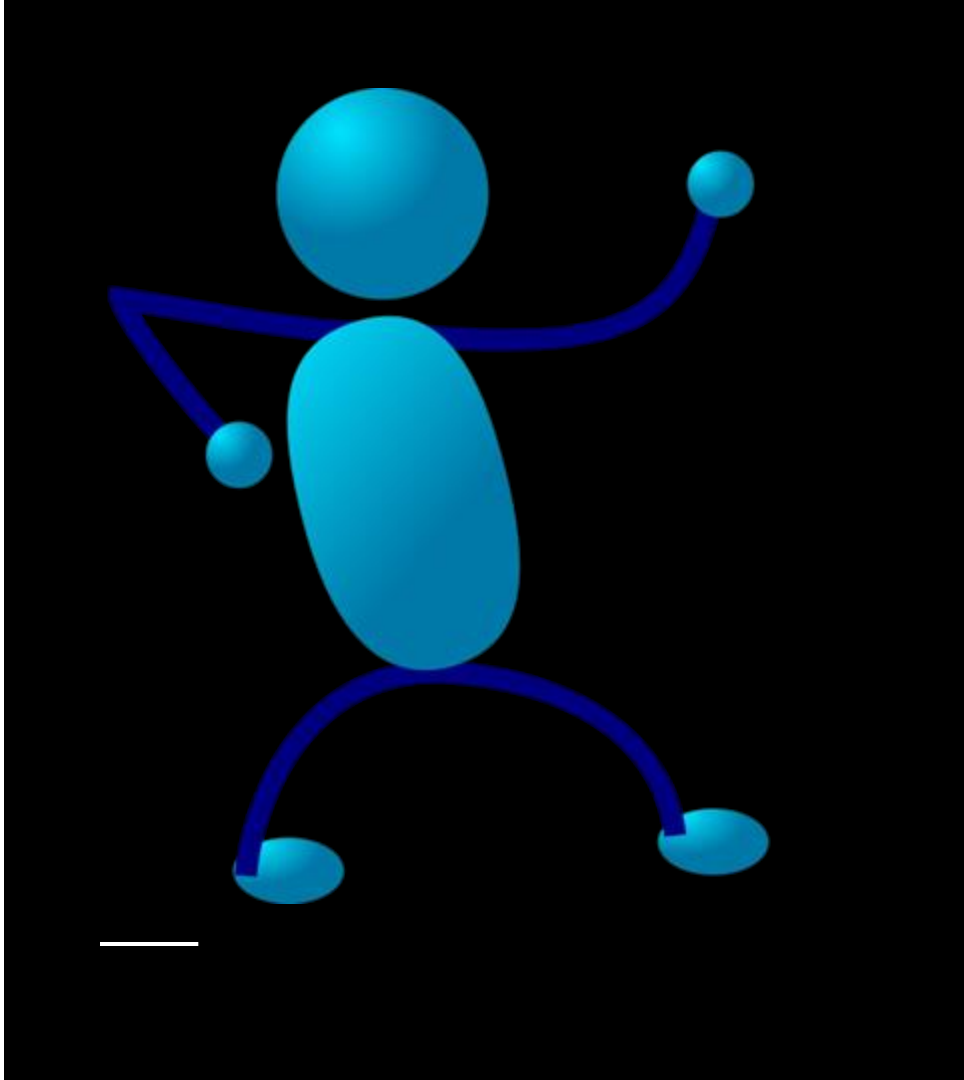
Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*

Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*

Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*

Let's Do Some Math!



Sample Lesson Design

→ *inviting all students into mathematics*

- ❑ Warm Up
- ❑ *Instructional Activities*
- ❑ *Lesson Synthesis*
- ❑ *Cool-Down*

Number Talk: Familiar Numbers

Warm-Up

- Number Talk

Find the value of each expression mentally.

- **10 × 6**

Find the value of each expression mentally.

- **10 × 6**
- **10 × 12**

Find the value of each expression mentally.

- **10 × 6**
- **10 × 12**
- **10 × 24**

Find the value of each expression mentally.

- **10 × 6**
- **10 × 12**
- **10 × 24**
- **5 × 24**

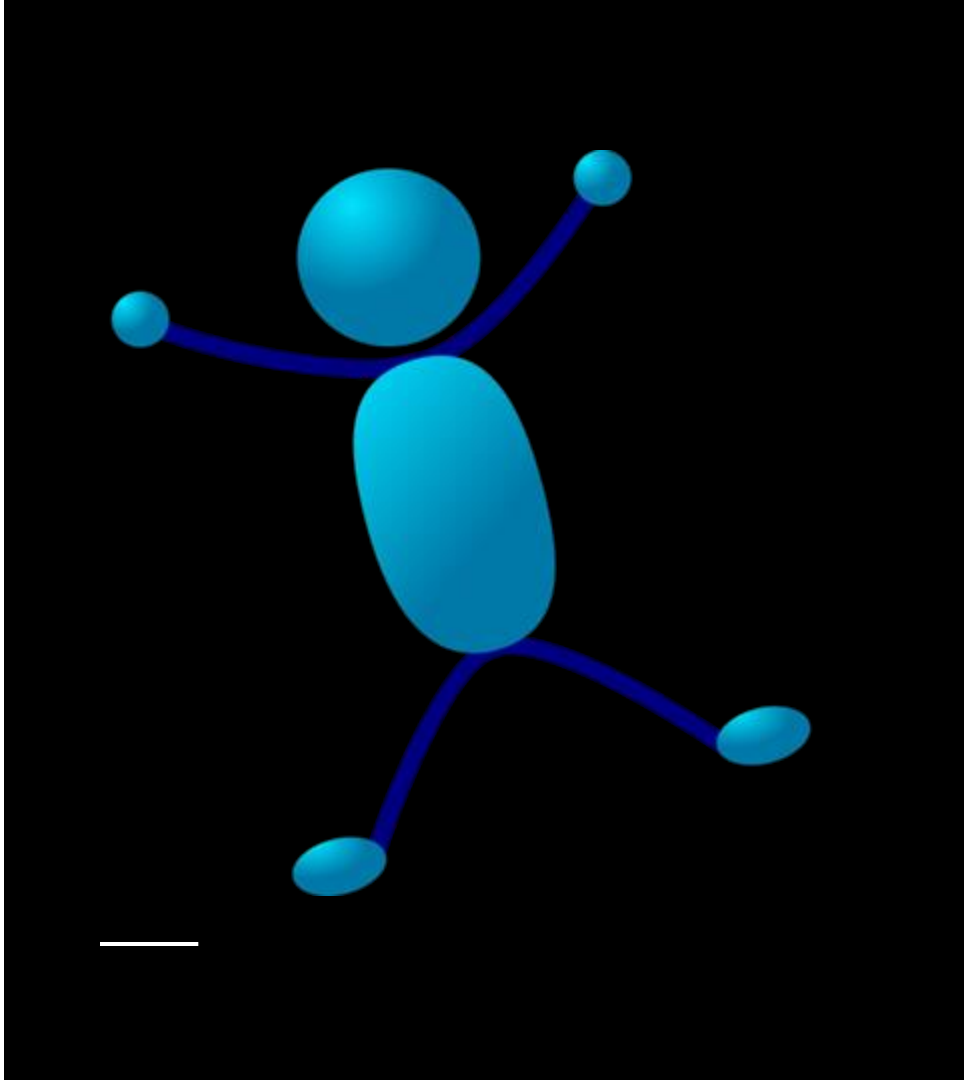
Let’s Talk About It!

How did the first three expressions help you find the value of the last one?

- **10 × 6**
- **10 × 12**
- **10 × 24**
- **5 × 24**

Mathematics Teaching & Learning

zoom-out



ISB Vision: EMPOWERED WITH PURPOSE AND COMPASSION

ISB Mission:

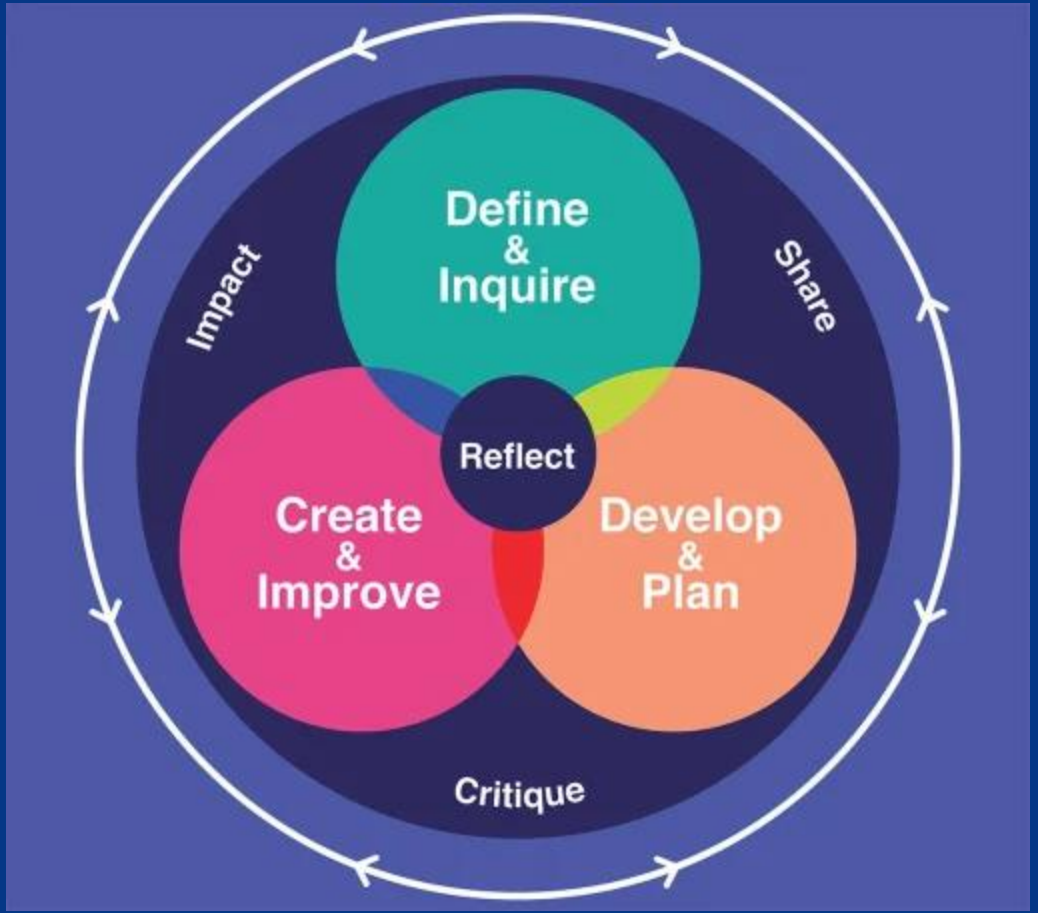
We are an inspiring international community in Beijing, where thinkers and leaders find their place in the world and serve others. So that our learning is at its best, we build strong relationships and set high expectations together. We are committed to challenging and joyful learning with the freedom to explore.

Real-world and Impact

Research-based

Student-Centered
Driven by the Learner

Design and STEM



At ISB we instill an appreciation of mathematics by providing a challenging program that is differentiated and appropriately rigorous for each student. In a nurturing and motivating environment, students explore and question, construct conceptual understandings and procedural fluency.

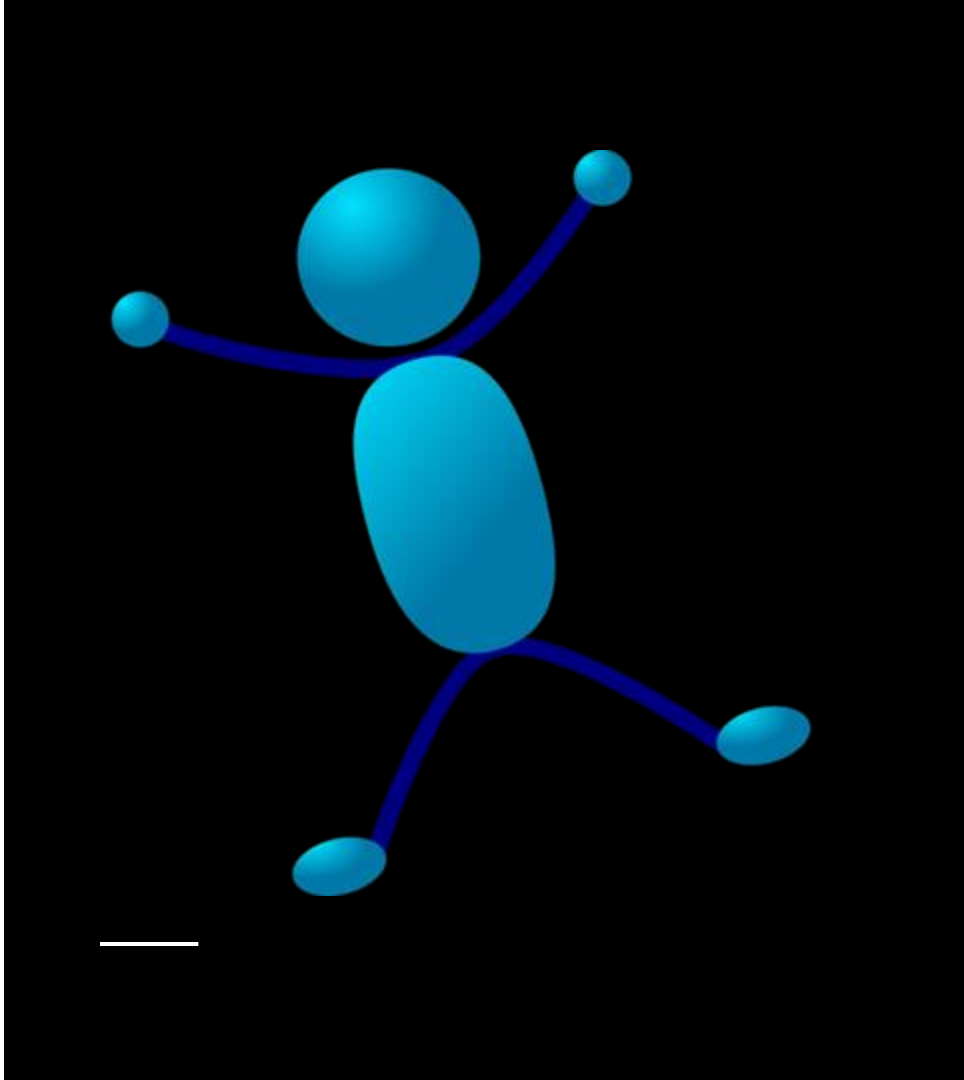
The Mathematics program values:

- the use of various models, tools, manipulatives, and technology
- flexibility in solving problems
- engaging and meaningful contexts
- communication of reasoning

We believe that students learn Mathematics best when they:

- are encouraged to take risks and see mistakes as learning opportunities
- productively struggle with problems and persevere in solving them
- Experience success in an open and collaborative learning environment

Rigor, Depth, and Coherence



Common Core State Standards – Mathematics

Standards Progressions

<table>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS</th></tr>
<tr><td colspan="1">Counting and Cardinality</td><td colspan="8"></td><td rowspan="3">Number and Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="6">Geometry</td><td colspan="3">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Grade 4 » Introduction

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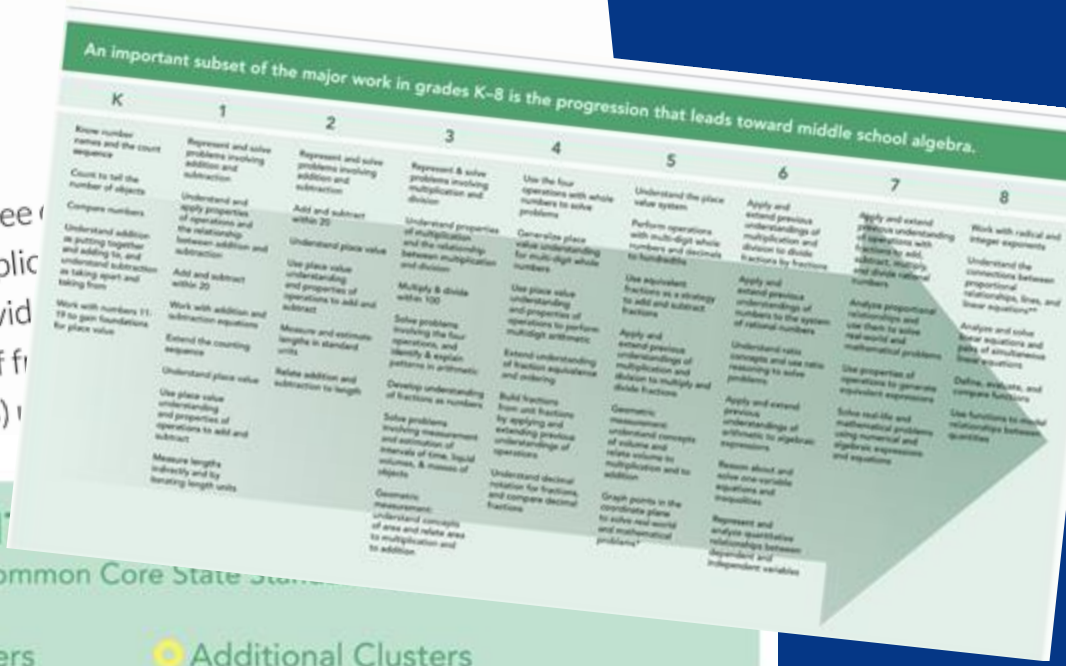
In Grade 4, instructional time should focus on three
understanding and fluency with multi-digit multiplic
dividing to find quotients involving multi-digit divid
fraction equivalence, addition and subtraction of f
whole numbers; (3)



MAJOR, SUPPORTING, AND ADDI

Emphases are given at the cluster level. Refer to the Common Core State
specific standards that fall within each cluster.

Key: Major Clusters Supporting Clusters Additional Clusters

4.OA.A	Use the four operations with whole numbers to solve problems.
4.OA.B	Gain familiarity with factors and multiples.
4.OA.C	Generate and analyze patterns.

Open Middle

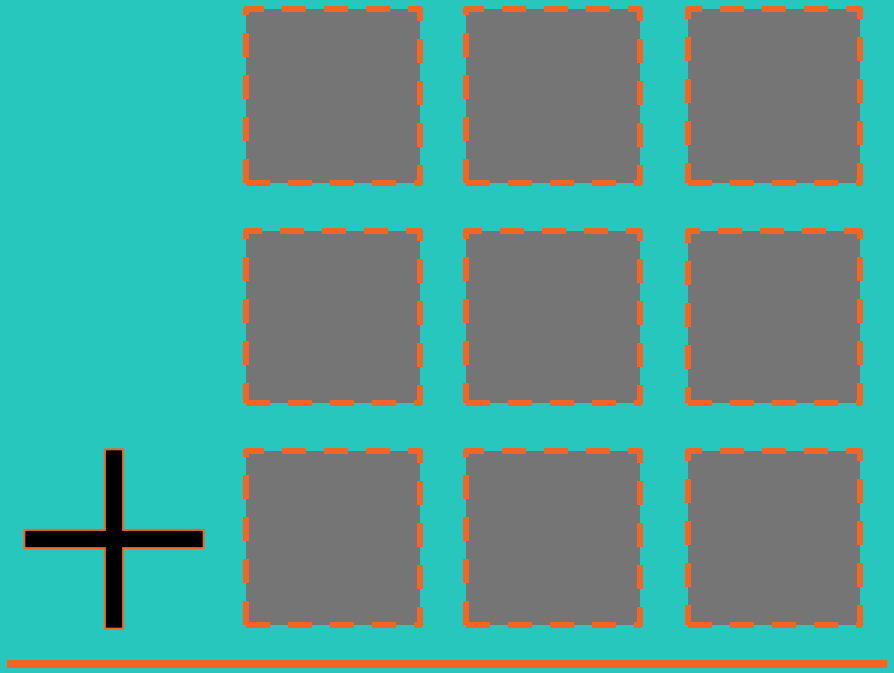
SUBTRACTION TO GET THE SMALLEST DIFFERENCE

Directions: Place any digit, 1 through 9, in the boxes below to create the smallest possible difference. Each digit can only be used once.

CLOSE TO 1000

Arrange the digits 1-9, using each digit only once, into three 3-digit whole numbers.

Make the sum as close to 1000 as possible.



Using all of the digits 0 to 9
(each digit only once),
find the maximum possible value of the
following expression.

$$\left(\square.\square\square\square \times 10^{\square}\right) \times \left(\square.\square\square\square \times 10^{\square}\right)$$

Given the equation or graph of a function f. Using the digits 0 to 9 only once, find the values of $a, ..., h$ so that the values of the sum of the following three expressions is as close to 0 as possible.

1. $\lim_{x \to a} f(x) + \lim_{x \to b} f(x)$

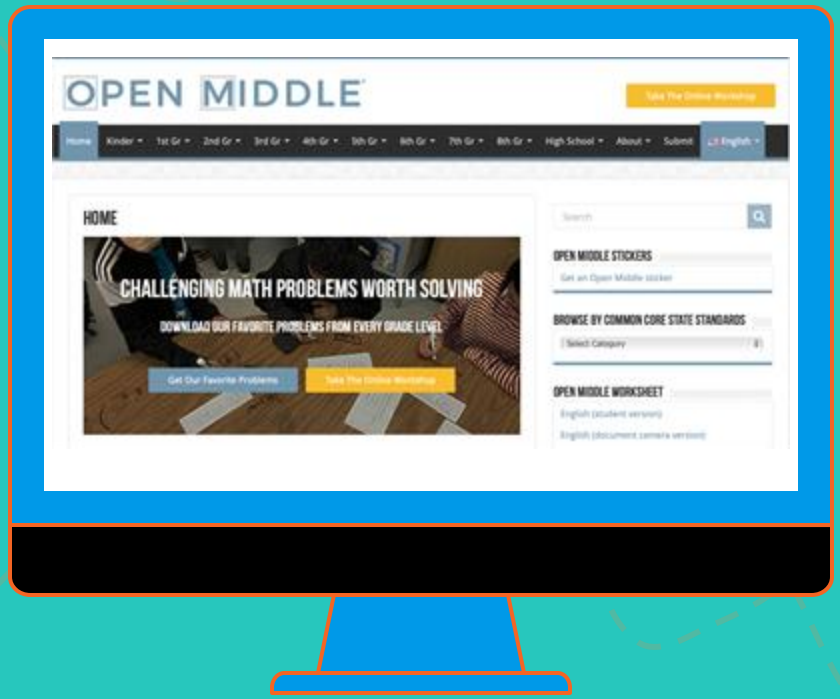
2. $f'(c) + f'(d)$

3. $\int_e^f f(x)dx + \int_g^h f(x)dx$

Note: The potential difficulty of the problem depends highly on the type of function. I would start at piecewise linear.

OPEN MIDDLE

How might these problems value thinking over performing?



What is Rigor?

- Rigor requires that **students construct meaning** and impose structure on situations rather than expect to find them already apparent. (Resnick, 1987)
- Rather than reproduce knowledge, students **manipulate what they know** in order to learn something new from the process.
- Students need **deep, authentic command** of mathematical concepts.

Myths and Truths About Rigor / Common Core State Standards

What is Rigor?

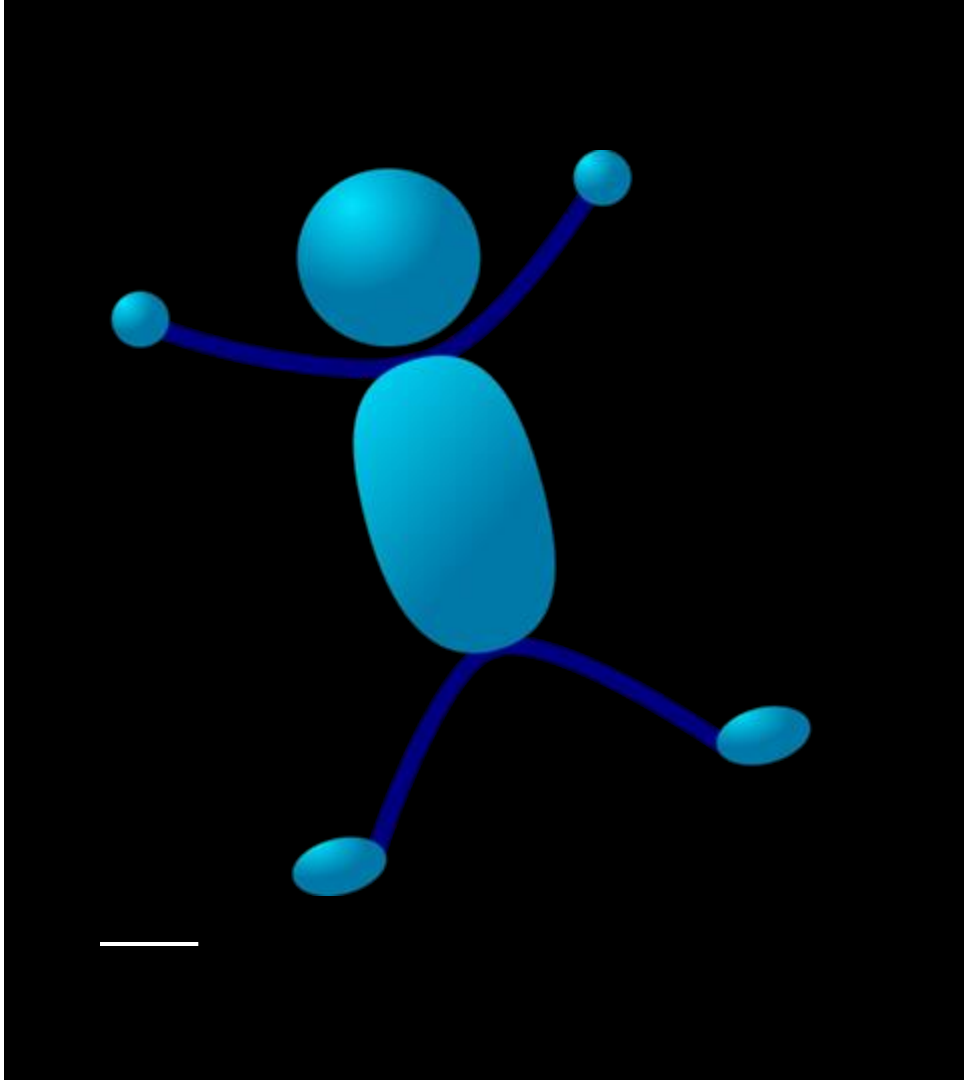
Educators pursue three aspects of rigor in the major work of the grade level:

- Conceptual Understanding
- Procedural Skills and Fluency
- Application *(Reasoning, Problem-Solving)*

→ It is ***not*** making math "harder" or introducing topics at earlier grades.

Math Learning & Teaching

zoom-in



Problem-Based Learning

Purposeful Representations

Coherent Progression

All Students are Capable Learners of Mathematics

Learning Mathematics by Doing Mathematics

Balancing Rigor

Instructional Routines

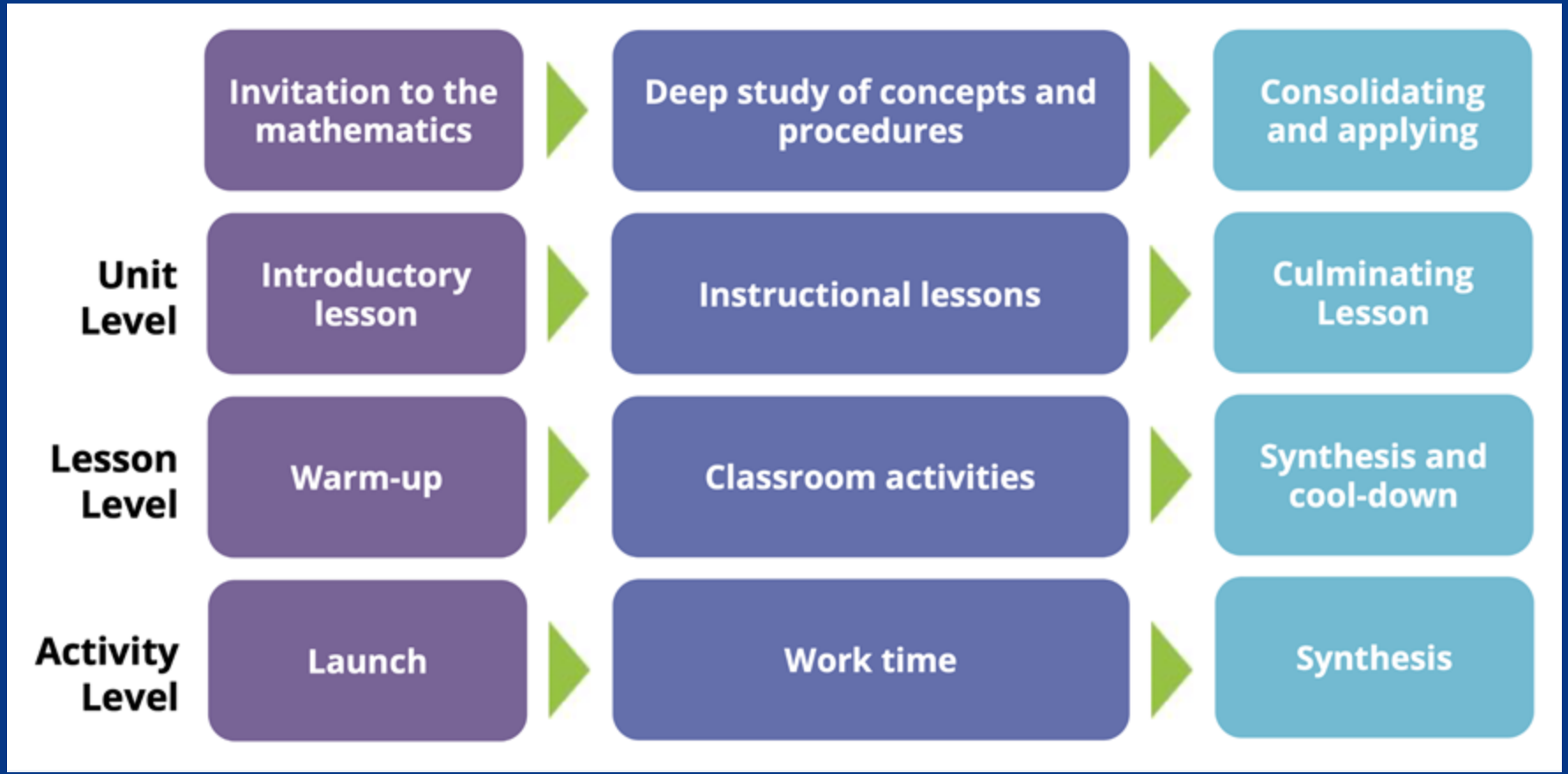
Using the 5 Practices for Orchestrating Productive Discussions

Community Building

Model with Mathematics K–5

Teacher Learning Through Curriculum Materials

Task Complexity


Invitation to the mathematics
Deep study of concepts and procedures
Consolidating and applying
Unit Level
Introductory lesson
Instructional lessons
Culminating Lesson
Lesson Level
Warm-up
Classroom activities
Synthesis and cool-down
Activity Level
Launch
Work time
Synthesis

Unit 1 ▸
Introduction to Multiplication

Students represent and solve multiplication problems through the context of picture and bar graphs that represent categorical data.

Unit 2 ▸
Area and Multiplication

Students learn about area concepts and relate area to multiplication and to addition.

Unit 3 ▸
Wrapping Up Addition and Subtraction Within 1,000

Students use place value understanding to round whole numbers and add and subtract within 1,000. They also represent and solve two-step word problems using addition, subtraction, and multiplication and assess the reasonableness of answers.

Unit 4 ▸
Relating Multiplication to Division

Students learn about and use the relationship between multiplication and division, place value understanding, and the properties of operations to multiply and divide whole numbers within 100. They also represent and solve two-step word problems using the four operations.

Unit 5 ▸
Fractions as Numbers

Students develop an understanding of fractions as numbers and of fraction equivalence by representing fractions on diagrams and number lines, generating equivalent fractions, and comparing fractions.

Unit 6 ▸
Measuring Length, Time, Liquid Volume, and Weight

Students generate and represent length measurement data in halves and fourths of an inch on line plots. They learn about and estimate relative units of measure including weight, liquid volume, and time, and use the four operations to solve problems involving measurement.

Unit 7 ▸
Two-Dimensional Shapes and Perimeter

Students reason about shapes and their attributes, with a focus on quadrilaterals. They solve problems involving the perimeter and area of shapes.

Unit 8 ▸
Putting It All Together

Students consolidate and solidify their understanding of various concepts and skills related to major work of the grade. They also continue to work toward fluency goals of the grade.

Section A ▸
Size and Location of Fractions

Lesson 1 ▸
Lesson 2 ▸
Lesson 3 ▸
Lesson 4 ▸
Lesson 5 ▸
Lesson 6 ▸

Practice Problems ▸

Section B ▸
Equivalent Fractions

Lesson 7 ▸
Lesson 8 ▸
Lesson 9 ▸
Lesson 10 ▸
Lesson 11 ▸

Practice Problems ▸

Section C ▸
Fraction Comparison

Lesson 12 ▸
Lesson 13 ▸
Lesson 14 ▸
Lesson 15 ▸
Lesson 16 ▸
Lesson 17 ▸

Practice Problems ▸

Lesson 7 ▸

Equivalent Fractions

Generate equivalent fractions using a representation that makes sense to students.

Lesson 8 ▸

Equivalent Fractions on the Number Line

Reason about and generate equivalent fractions on the number line.

Lesson 9 ▸

Explain Equivalence

Determine if given fractions are equivalent in a way that makes sense to them.

Given a pair of equivalent fractions, explain why they are equivalent.

Lesson 10 ▸

Use Multiples to Find Equivalent Fractions

Make sense of a way to generate equivalent fractions by using multiples of the numerator and denominator.

Lesson 11 ▸

Use Factors to Find Equivalent Fractions

Generate equivalent fractions by using factors of the numerator and denominator.

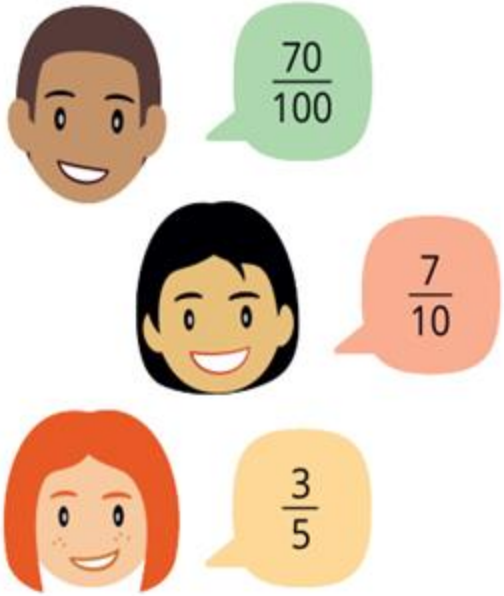
Reason about fraction equivalence numerically, by using multiples or factors of the numerator and denominator.

Sample Lesson Design

→ *inviting all students into mathematics*

- ❑ *Warm Up*
- ❑ Instructional Activities
- ❑ Lesson Synthesis
- ❑ *Cool-Down*

Andre, Lin, and Clare are representing $\frac{70}{100}$ on a number line.



- Andre said, "Oh, no! We'll need to partition the line into 100 equal parts and count 70 parts just to mark one point!"
- Lin said, "What if we mark $\frac{7}{10}$ instead? We could partition the line into just 10 parts and count 7 parts."
- Clare said, "What if we partition the line into 5 parts and mark $\frac{3}{5}$?"

Activity Purpose

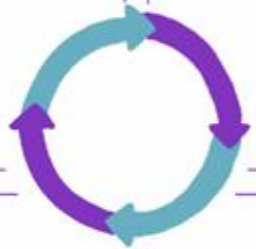
→ In this activity, students look closely at the **relationships** of fractions with denominator 5, 10, and 100. They use their **observations and understanding to identify** equivalent fractions and to explain why two fractions are or are not equivalent.

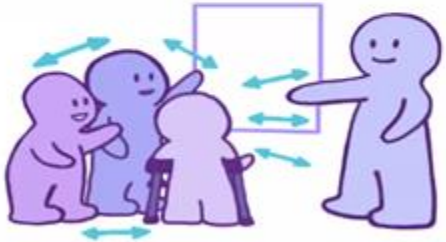
→ When students **analyze and criticize the reasoning** presented in the activity statements and when they discuss their work with classmates, they are **critiquing the reasoning of others and improving their arguments** (MP3).

Students learn math by *doing* math

1. Teacher ensures students understand the question.

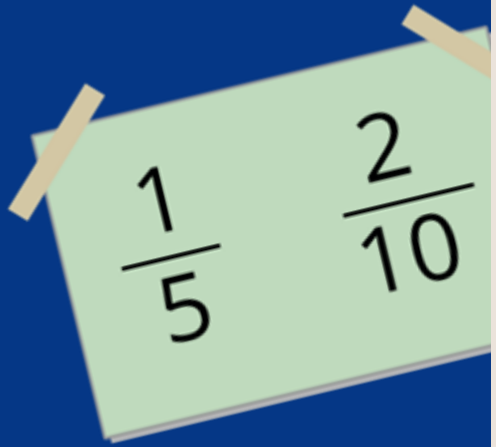
2. Students work individually. Teacher monitors, listens, questions.





4. Teacher helps students synthesize their learning.

3. Students work in groups. Teacher monitors, listens, and asks questions to understand students' thinking.



For the set of 2 fractions:

Explain or show how you know the fractions are equivalent.

Write a new equivalent fraction on a sticky note and add it to the poster. Think of a fraction that hasn't already been written by someone else.

We visited poster _____ , which shows_____ and _____.

New equivalent fraction: _____.

For the set of 3 fractions:

Identify 2 fractions that are equivalent. Explain your reasoning.

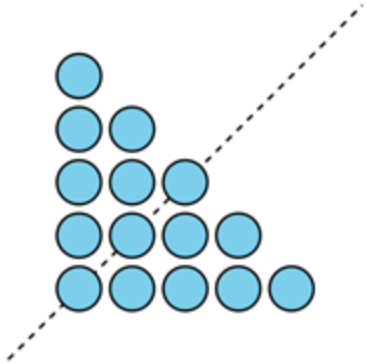
We visited poster _____ , which shows_____ and _____.

Contents

Problem 6 (Exploration)

Andre says that there are an odd number of circles in this picture.

Do you agree with Andre? Explain or show your reasoning.



Activity Purpose

→ This activity gives students opportunities to **practice explaining or showing** whether two fractions are equivalent.

→ Students may do so using a **visual representation, by reasoning** about the number and size of the fractional parts in each fraction, or by **thinking about multiplicative relationships** between the numbers in the given fractions.

Sample Lesson Design

→ *inviting all students into mathematics*

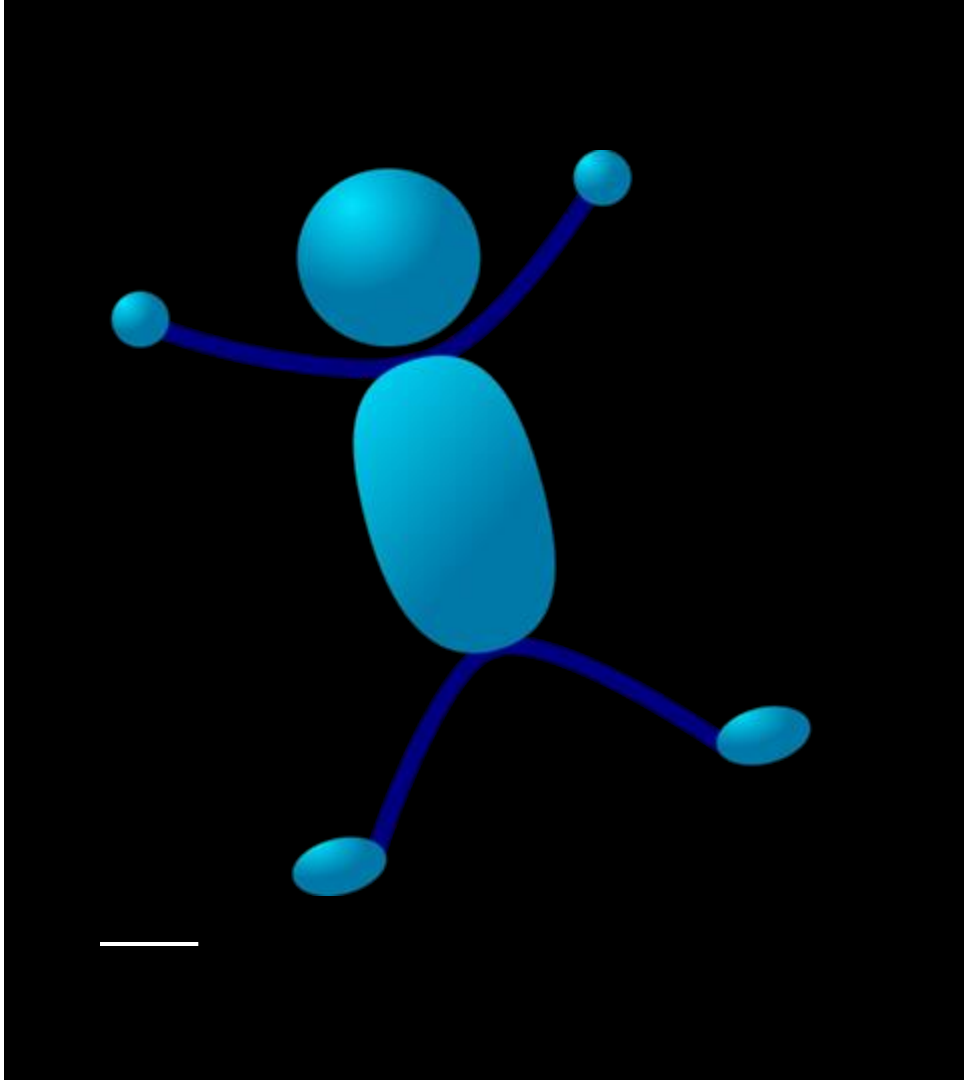
- ❑ *Warm Up*
- ❑ *Instructional Activities*
- ❑ *Lesson Synthesis*
- ❑ Cool-Down

1. Explain or show why this statement is true: $\frac{5}{4}$ is equivalent to $\frac{15}{12}$. Use a number line, if it helps.

2. Diego wrote $\frac{11}{5}$ and $\frac{55}{10}$ as equivalent fractions. Are those fractions equivalent? Explain or show how you know. Use a number line, if it helps.

Are You Ready For More?

Helping at Home



What do you Notice?

Wonder?

UNIT 2

Fraction Equivalence and Comparison

FAMILY MATERIALS

120 Number Board

Base Ten Blocks

Ten Frames

Two-Color Counters

Unifix Cubes

Color Tiles

Number Lines

Fraction Tiles & Number Line

Pattern Blocks

Number Cubes

Spinners

Geoboard

Algebra Tiles

Prime Factor Tiles

Math Balance

Place Value Disks

Help students **rely more on themselves** to determine whether something is mathematically correct

19 Is this a **reasonable answer**?

20 Does that make **sense**?

21 **Why** do you think that? Why is that true?

22 Can you **draw a picture or make a model** to show that?

23 **How** did you reach that conclusion?

48 **What would happen if ___**?

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

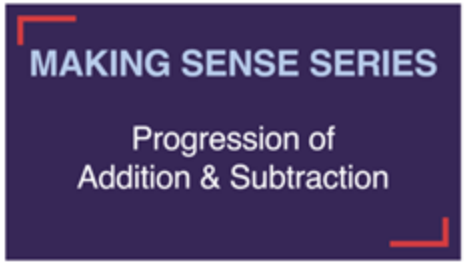
89 Have you tried making a **guess**?

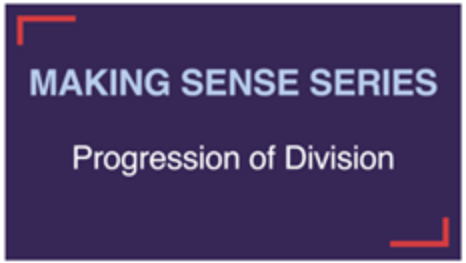
90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

Are You Ready For More?



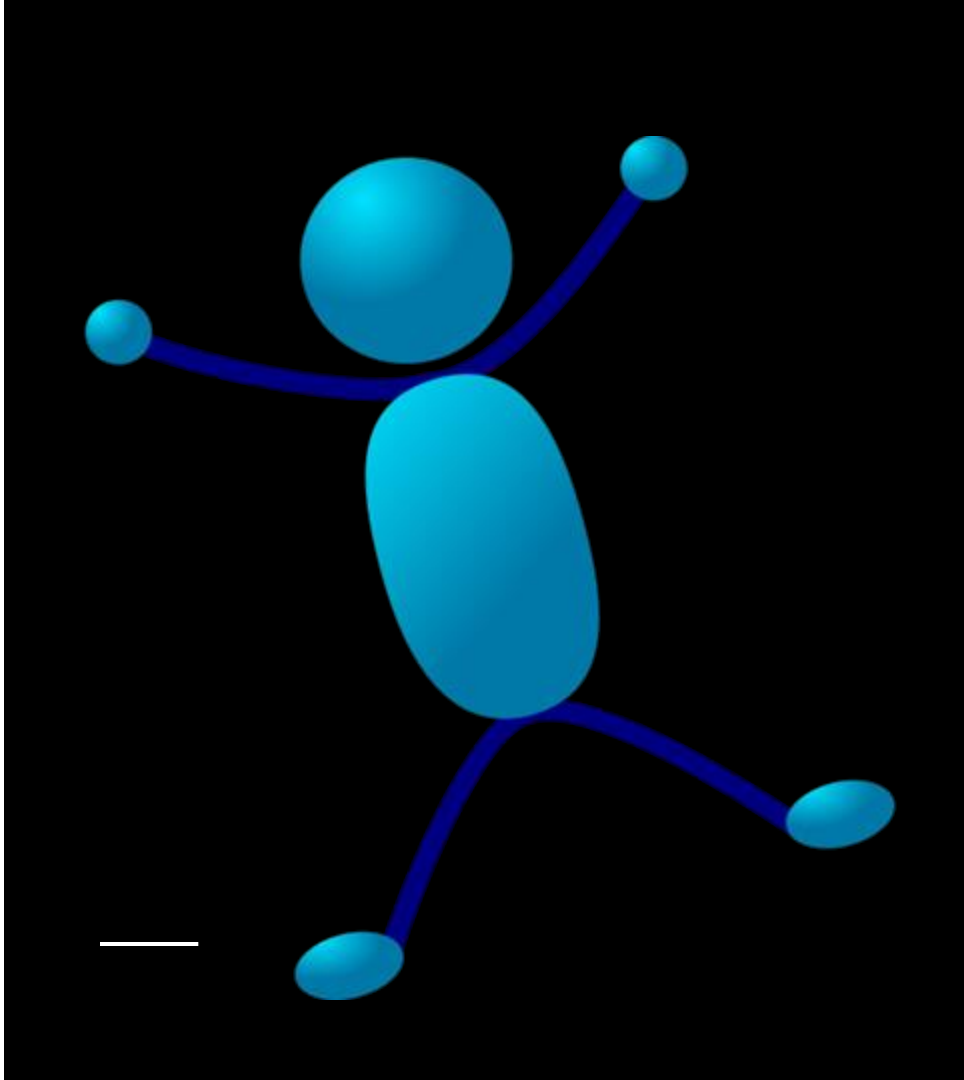



Rummi
CLASSIC


ACM
trusted voice


MONOPOLY
GO

Why is This Important Now?



Graduating Class of 2037!

What might be important skills for future-ready graduates?

WORLD ECONOMIC FORUM

Top 10 skills of 2025

Type of skill

- Problem-solving
- Self-management
- Working with people
- Technology use and development

Source: Future of Jobs Report 2020, World Economic Forum.

Mathematical Reasoning

Content Knowledge

Contexts

21st Century Skills

What is Mathematical Literacy?

Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It helps individuals know the role that mathematics plays in the world and make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st Century citizens.

What's new in PISA 2022

PISA 2022 aims to consider mathematics in a rapidly changing world driven by new technologies and trends in which citizens are creative and engaged, making non-routine judgments for themselves and the society in which they live. This brings into focus the ability to reason mathematically, which has always been a part of the PISA framework. This technology change is also creating the need for students to understand those computational thinking concepts that are part of mathematical literacy. Finally, the framework recognizes that improved computer-based assessment is available to most students within PISA.

Organizational Freedom

How Students...

In silence, alone	Talk	In groups, talking as they work
Cold Calling	Share	No hands up
Write down the teacher's dictated words	Record	Take notes when they want
Open Book and work when teacher says	Spend Time	Work on problems for as long as is helpful
Sitting at desks	Move	Get up to access resources

Mathematical Thinking

How Students...

Same answers & methods	Interpret	Different interpretations, methods, answers
No or same way of seeing	See	Seeing maths differently
Tell methods, with no need	Learn New Ideas	Be curious, discover need
Mistakes are punished	Think About Mistakes	Mistakes are encouraged, valued
Answer textbook questions	Inquire, create	Pose questions, pursue inquiries, be creative.

Guiding Questions

What Does Math Learning Look Like? Sound LIke?

Why Are We Teaching & Learning in This Way?

How Are We Making Math Rigorous for All Learners?

Check-Out

- I used to think...
- Now I think...
- I am still wondering...