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RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Trigonometry

Grade Level: 11-12

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

Jeffery Kurczeski,

Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

The Board acknowledges the following who contributed to the preparation of this curriculum.

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Rachael Cane, Mathematics Teachers

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Dr. Aleya Shoieb, Superintendent of Schools

Subject/Course Title:
Trigonometry
Grades 11- 12

Date of Board Adoption:
August 27, 2024

RAHWAY PUBLIC SCHOOLS CURRICULUM

Trigonometry: Grades 11-12

PACING GUIDE

Unit	Title	Pacing
1	Trigonometry with Right Triangles	10 weeks
2	Circular Functions	10 weeks
3	Graphs of Trigonometric Functions	10 weeks
4	Trigonometric Identities and Equations Trigonometry with Oblique Triangles	10 weeks

ACCOMMODATIONS

<p>504 Accommodations:</p> <ul style="list-style-type: none"> ● Provide scaffolded vocabulary and vocabulary lists. ● Provide extra visual and verbal cues and prompts. ● Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials. ● Provide links to audio files and utilize video clips. ● Provide graphic organizers and/or checklists. ● Provide modified rubrics. ● Provide a copy of teaching notes, especially any key terms, in advance. ● Allow additional time to complete assignments and/or assessments. ● Provide shorter writing assignments. ● Provide sentence starters. ● Utilize small group instruction. ● Utilize Think-Pair-Share structure. ● Check for understanding frequently. ● Have student restate information. ● Support auditory presentations with visuals. ● Weekly home-school communication tools (notebook, daily log, phone calls or email messages). ● Provide study sheets and teacher outlines prior to assessments. ● Quiet corner or room to calm down and relax when anxious. ● Reduction of distractions. ● Permit answers to be dictated. ● Hands-on activities. ● Use of manipulatives. ● Assign preferential seating. ● No penalty for spelling errors or sloppy handwriting. ● Follow a routine/schedule. ● Provide student with rest breaks. ● Use verbal and visual cues regarding directions and staying on task. ● Assist in maintaining agenda book. 	<p>IEP Accommodations:</p> <ul style="list-style-type: none"> ● Provide scaffolded vocabulary and vocabulary lists. ● Differentiate reading levels of texts (e.g., Newsela). ● Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials. ● Provide extra visual and verbal cues and prompts. ● Provide links to audio files and utilize video clips. ● Provide graphic organizers and/or checklists. ● Provide modified rubrics. ● Provide a copy of teaching notes, especially any key terms, in advance. ● Provide students with additional information to supplement notes. ● Modify questioning techniques and provide a reduced number of questions or items on tests. ● Allow additional time to complete assignments and/or assessments. ● Provide shorter writing assignments. ● Provide sentence starters. ● Utilize small group instruction. ● Utilize Think-Pair-Share structure. ● Check for understanding frequently. ● Have student restate information. ● Support auditory presentations with visuals. ● Provide study sheets and teacher outlines prior to assessments. ● Use of manipulatives. ● Have students work with partners or in groups for reading, presentations, assignments, and analyses. ● Assign appropriate roles in collaborative work. ● Assign preferential seating. ● Follow a routine/schedule.
<p>Gifted and Talented Accommodations:</p> <ul style="list-style-type: none"> ● Differentiate reading levels of texts (e.g., Newsela). ● Offer students additional texts with higher lexile levels. ● Provide more challenging and/or more supplemental readings and/or activities to deepen understanding. ● Allow for independent reading, research, and projects. ● Accelerate or compact the curriculum. ● Offer higher-level thinking questions for deeper analysis. ● Offer more rigorous materials/tasks/prompts. ● Increase number and complexity of sources. ● Assign group research and presentations to teach the class. ● Assign/allow for leadership roles during collaborative work and in other learning activities. 	<p>ML Accommodations:</p> <ul style="list-style-type: none"> ● Provide extended time. ● Assign preferential seating. ● Assign peer buddy who the student can work with. ● Check for understanding frequently. ● Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc...). ● Have student repeat directions. ● Make vocabulary words available during classwork and exams. ● Use study guides/checklists to organize information. ● Repeat directions. ● Increase one-on-one conferencing. ● Allow student to listen to an audio version of the text. ● Give directions in small, distinct steps. ● Allow copying from paper/book. ● Give student a copy of the class notes.

- Provide written and oral instructions.
- Differentiate reading levels of texts (e.g., Newsela).
- Shorten assignments.
- Read directions aloud to student.
- Give oral clues or prompts.
- Record or type assignments.
- Adapt worksheets/packets.
- Create alternate assignments.
- Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted.
- Allow student to resubmit assignments.
- Use small group instruction.
- Simplify language.
- Provide scaffolded vocabulary and vocabulary lists.
- Demonstrate concepts possibly through the use of visuals.
- Use manipulatives.
- Emphasize critical information by highlighting it for the student.
- Use graphic organizers.
- Pre-teach or pre-view vocabulary.
- Provide student with a list of prompts or sentence starters that they can use when completing a written assignment.
- Provide audio versions of the textbooks.
- Highlight textbooks/study guides.
- Use supplementary materials.
- Give assistance in note taking
- Use adapted/modified textbooks.
- Allow use of computer/word processor.
- Allow student to answer orally, give extended time (time-and-a-half).
- Allow tests to be given in a separate location (with the ESL teacher).
- Allow additional time to complete assignments and/or assessments.
- Read question to student to clarify.
- Provide a definition or synonym for words on a test that do not impact the validity of the exam.
- Modify the format of assessments.
- Shorten test length or require only selected test items.
- Create alternative assessments.
- On an exam other than a spelling test, don't take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Trigonometry with Right Triangles

Target Course/Grade Level: Trigonometry/Grades 11-12

Unit Summary: Students will be introduced to trigonometric ratios. Students will then be able to find missing side lengths and missing angle measures for triangles and apply this knowledge to solve practical problems.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

G.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G.SRT.D.10 Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

- L.VL.11–12.3** Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
- W.AW.11–12.1** Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
- W.IW.11–12.2** Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
- W.WR.11–12.5** Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
- W.RW.11–12.7** Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
- SL.PE.11–12.1** Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- SL.II.11–12.2** Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
- SL.PI.11–12.4** Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- SL.AS.11–12.6** Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

- HS-ETS1-2** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that...

- Similarity is an essential part of the right triangle definitions of the trigonometric functions.
- A missing side or angle of a triangle may be found using the right triangle definitions of the trigonometric ratios.
- It is possible to evaluate the trigonometric functions for special right triangles without a calculator.

Unit Essential Questions:

- How are the trigonometric ratios defined?
- When is it possible to find the missing side or angle of a triangle?
- How do you find the missing side or angle of a triangle?
- How can the knowledge of trigonometry be applied to real-life problem-solving?

Knowledge and Skills:

Students will know...

- The Angle-Angle Similarity Theorem.
- Triangle similarity theorems.
- Proportionality theorems.
- The Pythagorean Theorem.
- Special right triangles.
- Proportional relationships in right triangles.
- Right triangle trigonometric ratios: sine, cosine, and tangent.
- The sine of an angle equals the cosine of its complement.

- How to simplify radicals and radical expressions.

Students will be able to...

- Approximate a radical to one decimal place.
- Apply the Pythagorean Theorem to find the length of a missing side of a right triangle.
- Implement the Angle-Angle Similarity theorem, the Side-Side-Side Similarity theorem, and the Side-Angle-Side Similarity theorem to determine whether triangles are similar.
- Determine the length of a missing side of a right triangle, and round the answer to the nearest tenth.
- Determine the length of a missing side of a right triangle, and leave the answer in the simplest radical form.
- Determine if three given side lengths form an acute, obtuse, or right triangle.
- Use special right triangles to solve real-life problems.
- Apply characteristics of 30-60-90 triangles to find the length of a missing side of a right triangle.
- Utilize the length in one triangle to find a missing length in another adjoining triangle.
- Apply characteristics of 45-45-90 triangles to find the length of a missing side of a right triangle.
- Discover the value of trigonometric ratios given a right triangle with side measurements.
- Discover a missing length of a right triangle using trigonometric ratios.
- Discover the value of missing angles given a right triangle with side measurements.
- Solve real-world problems utilizing the trigonometric ratios of a right triangle.
- Select the appropriate methods to solve right triangles, including Pythagorean Theorem, special right triangles, and trigonometry.
- Implement methods for solving right triangles to solve real-life problems.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Application of concepts to real-world examples.
- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- Teacher developed worksheets and activities
- Useful Websites for Teachers to Explore:
 - www.illustrativemathematics.org
 - <http://www.ixl.com>
 - www.kutasoftware.com
 - <https://www.khanacademy.org/>
 - <https://learnzillion.com/>
 - <https://www.teachingchannel.org/>
 - <http://illuminations.nctm.org>

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator
- Ruler
- Protractor

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Circular Functions

Target Course/Grade Level: Trigonometry/Grades 11-12

Unit Summary: Students will extend the definition of the trigonometric functions using the unit circle. They will be introduced to radian measure and determine trigonometric functions of angles using reference angles.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$ and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x , where x is any real number.

F.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:

ELA

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L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

- W.AW.11–12.1** Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
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- W.WR.11–12.5** Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
- W.RW.11–12.7** Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
- SL.PE.11–12.1** Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
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- SL.AS.11–12.6** Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

- HS-ETS1-2** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that...

- Similarity is an essential part of the right triangle definitions of the trigonometric functions.
- The sine of an angle equals the cosine of its complement.
- It is possible to evaluate the trigonometric functions for special right triangles without a calculator.

Unit Essential Questions:

- How are the basic trigonometric functions related to the unit circle?
- What are the units for angle measure and how are they related?
- How is the unit circle used to create the graphs of the trigonometric functions?

Knowledge and Skills:

Students will know...

- The six trigonometric functions.
- Drawing angles in standard position.
- Radian measure.
- Trigonometric functions of any angle.
- The radian measure of an angle is related to the arc length.
- To use reference angles to find the trigonometric values of special angles.
- That angles can be negative and greater than 360° .
- The basic trigonometric values can be found by drawing right triangles within the unit circle.
- That the trigonometric functions are periodic and derived from the unit circle.

Students will be able to...

- Define right triangle trigonometric functions.

- Evaluate trigonometric functions of any angle.
- Utilize trigonometric functions to find side lengths of right triangles.
- Draw angles in standard position.
- Justify the use and importance of radian measure.
- Convert between degrees and radians.
- Utilize radian measure to find arc lengths and the area of a sector.
- Calculate reference angles.
- Evaluate trigonometric functions given a point on an angle.
- Calculate coterminal angles.
- Evaluate trigonometric functions using the unit circle.
- Discover reference angles to evaluate trigonometric functions.
- Solve real-life problems involving projectiles.
- Measure quadrantal angles.
- Identify functions as positive or negative determined by the quadrants they terminate.
- Assess the six trigonometric functions when given a point on the terminal side of an angle.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Application of concepts to real-world examples.
- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- Teacher developed worksheets and activities
- Useful Websites for Teachers to Explore:
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- <http://www.ixl.com>
- www.kutasoftware.com
- <https://www.khanacademy.org/>
- <https://learnzillion.com/>
- <https://www.teachingchannel.org/>
- <http://illuminations.nctm.org>

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator
- Ruler
- Protractor

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Graphs of Trigonometric Functions

Target Course/Grade Level: Trigonometry/Grades 11-12

Unit Summary: Students will graph the basic trigonometric functions and the inverse functions. Students will be able to identify the various transformations from the equation and represent these transformations graphically. They will also apply these concepts to real-world examples.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, $f(x + k)$ and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

f. Graph trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (.97)^t$, $y = (1.01)^{12t}$, $y = (1.01)^{t/10}$, and classify them as representing exponential growth or decay.

Career Readiness, Life Literacies, and Key Skills:

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9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

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SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that...

- The unit circle is used to create the graphs of the basic trigonometric functions.
- The transformations applied to the parent function to sketch a new graph apply also to trigonometric functions.
- The graph of a sine function is a horizontal translation of the cosine function.
- Real-life behavior that is periodic may be modeled by a trigonometric function.

Unit Essential Questions:

- How is the unit circle related to the graphs of the basic trigonometric functions?
- What does it mean for a function to be periodic?
- How are the transformations of a trigonometric function shown algebraically and graphically?
- How can we use this knowledge to model real-life scenarios?

Knowledge and Skills:

Students will know...

- How to describe and graph sine, cosine, tangent, cotangent, secant, and cosecant functions.
- The basic shapes and behaviors of the parent graphs of the six trigonometric functions.
- Writing trigonometric functions.
- Evaluating trigonometric functions.
- Transformations also apply to trigonometric functions.
- The relationship between trigonometric functions and their inverses.
- Asymptotes occur in some of the trigonometric graphs.
- Trigonometric functions are used to model periodic behavior.

Students will be able to...

- Evaluate characteristics of the trigonometric function graphs (sine, cosine, tangent, cotangent, secant, and cosecant).
- Graph trigonometric parent functions.
- Write and graph trigonometric functions using frequency.
- Discover a trigonometric model for a set of data using technology.
- Write trigonometric functions for a given graph.
- Identify period, amplitude, and phase shift for a given equation or graph.
- Graph the inverse sine, cosine, and tangent functions.
- Utilize $y = a \sin bx$ and $y = a \cos bx$ to determine amplitudes, periods, and frequency and graph results.
- Assess the domain and range of the trigonometric functions.
- Use the general equations $y = a \sin(bx+c)+d$ and $y = a \cos (bx+c)+d$, to find phase shift/vertical shift to graph given equations.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.

- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Application of concepts to real-world examples.
- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- Teacher developed worksheets and activities
- Useful Websites for Teachers to Explore:
 - www.illustrativemathematics.org
 - <http://www.ixl.com>
 - www.kutasoftware.com
 - <https://www.khanacademy.org/>
 - <https://learnzillion.com/>
 - <https://www.teachingchannel.org/>
 - <http://illuminations.nctm.org>

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator
- Ruler
- Protractor

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Trigonometric Identities and Equations/Trigonometry with Oblique Triangles

Target Course/Grade Level: Trigonometry/Grades 11-12

Unit Summary: Students will be able to recognize and write the fundamental trigonometric identities. They will also be able to evaluate, verify, simplify, and rewrite trigonometric expressions. Students will apply these identities and their knowledge of algebra to solve trigonometric equations. To connect this final unit to Unit 1, students will prove and apply the Laws of Sines and Cosines.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

G.SRT.C.10 Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.C.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles

F.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2 \theta = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$ or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$ or $\tan(\theta)$ and the quadrant of the angle.

F.TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

- L.VL.11–12.3** Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
- W.AW.11–12.1** Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
- W.IW.11–12.2** Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
- W.WR.11–12.5** Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
- W.RW.11–12.7** Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
- SL.PE.11–12.1** Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- SL.II.11–12.2** Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
- SL.PI.11–12.4** Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- SL.AS.11–12.6** Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

- HS-ETS1-2** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that...

- Identities may be proven.
- Identities can be used to simplify trigonometric expressions and equations.
- If a triangle has two known side lengths and one known angle that is not the included angle, then there may be one solution, two solutions, or no solution.

Unit Essential Questions:

- What are the different trigonometric identities?
- How can trigonometric identities be used to solve equations?
- How can algebra skills be applied to solving trigonometric equations and formulas?

Knowledge and Skills:

Students will know...

- The basic trigonometric identities, negative identities, and Pythagorean identities.
- How to use trigonometric identities to evaluate trigonometric functions and simplify trigonometric functions.
- The Law of Sines.
- The Law of Cosines.
- Sum and Difference Formulas.
- The Double-Angle and Half-Angle Formulas.
- How to find unknown side lengths and angle measures of acute and obtuse triangles.

Students will be able to...

- Evaluate trigonometric functions using the trigonometric identities.
- Simplify trigonometric expressions using the trigonometric identities.
- Assess and verify the trigonometric identities.
- Solve trigonometric equations using the basic trigonometric identities.
- Prove the Law of Sines and the Law of Cosines.
- Discover areas of triangles using formulas that involve sine.
- Utilize the Law of Sines and the Law of Cosines to find unknown parts of a triangle.
- Solve real-life problems using the Law of Sines and the Law of Cosines.
- Evaluate trigonometric expressions using sum and difference formulas.
- Simplify trigonometric expressions using sum and difference formulas.
- Solve trigonometric equations using sum and difference formulas.
- Compare using the trigonometric identities and the sum and difference formulas for evaluating, simplifying, and solving trigonometry problems.

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