ESL
SCIENCE
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WORLD LANGUAGES
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TECHNOLOGY EDUCATION
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HEALTH & PHYSICAL EDUCATION

RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Accelerated Mathematics

Grade Level: 7

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

Jeffery Kurczeski,

Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

The Board acknowledges the following who contributed to the preparation of this curriculum.

Carlos Sabates, Mathematics Teacher

Dr. Tiffany A. Beer, Director of Curriculum and Instruction

Dr. Aleya Shoieb, Superintendent of Schools

Subject/Course Title:
Accelerated Mathematics
Grade 7

Date of Board Adoption:
August 27, 2024

RAHWAY PUBLIC SCHOOLS CURRICULUM

Accelerated Mathematics: Grade 7

PACING GUIDE

Unit	Title	Pacing
1	Rigid Transformations and Congruence	3 weeks
2	Scale Drawings, Similarity, and Slope	3 weeks
3	Writing and Solving Equations	6 weeks
4	Inequalities, Expressions, and Equations	6 weeks
5	Linear Relationships	5 weeks
6	Functions and Volume	7 weeks
7	System of Equations, Exponents, and Scientific Notation	5 weeks
8	Polynomial Equations, Quadratic Functions, Pythagorean Theorem, and Irrational Numbers	5 weeks

ACCOMMODATIONS

504 Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Provide extra visual and verbal cues and prompts.
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Weekly home-school communication tools (notebook, daily log, phone calls or email messages).
- Provide study sheets and teacher outlines prior to assessments.
- Quiet corner or room to calm down and relax when anxious.
- Reduction of distractions.
- Permit answers to be dictated.
- Hands-on activities.
- Use of manipulatives.
- Assign preferential seating.
- No penalty for spelling errors or sloppy handwriting.
- Follow a routine/schedule.
- Provide student with rest breaks.
- Use verbal and visual cues regarding directions and staying on task.
- Assist in maintaining agenda book.

IEP Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Differentiate reading levels of texts (e.g., Newsela).
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide extra visual and verbal cues and prompts.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Provide students with additional information to supplement notes.
- Modify questioning techniques and provide a reduced number of questions or items on tests.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Provide study sheets and teacher outlines prior to assessments.
- Use of manipulatives.
- Have students work with partners or in groups for reading, presentations, assignments, and analyses.
- Assign appropriate roles in collaborative work.
- Assign preferential seating.
- Follow a routine/schedule.

Gifted and Talented Accommodations:

- Differentiate reading levels of texts (e.g., Newsela).
- Offer students additional texts with higher lexile levels.
- Provide more challenging and/or more supplemental readings and/or activities to deepen understanding.
- Allow for independent reading, research, and projects.
- Accelerate or compact the curriculum.
- Offer higher-level thinking questions for deeper analysis.
- Offer more rigorous materials/tasks/prompts.
- Increase number and complexity of sources.
- Assign group research and presentations to teach the class.
- Assign/allow for leadership roles during collaborative work and in other learning activities.

ML Accommodations:

- Provide extended time.
- Assign preferential seating.
- Assign peer buddy who the student can work with.
- Check for understanding frequently.
- Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc…).
- Have student repeat directions.
- Make vocabulary words available during classwork and exams.
- Use study guides/checklists to organize information.
- Repeat directions.
- Increase one-on-one conferencing.
- Allow student to listen to an audio version of the text.
- Give directions in small, distinct steps.
- Allow copying from paper/book.
- Give student a copy of the class notes.

	● Provide written and oral instructions. ● Differentiate reading levels of texts (e.g., Newsela). ● Shorten assignments. ● Read directions aloud to student. ● Give oral clues or prompts. ● Record or type assignments. ● Adapt worksheets/packets. ● Create alternate assignments. ● Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted. ● Allow student to resubmit assignments. ● Use small group instruction. ● Simplify language. ● Provide scaffolded vocabulary and vocabulary lists. ● Demonstrate concepts possibly through the use of visuals. ● Use manipulatives. ● Emphasize critical information by highlighting it for the student. ● Use graphic organizers. ● Pre-teach or pre-view vocabulary. ● Provide student with a list of prompts or sentence starters that they can use when completing a written assignment. ● Provide audio versions of the textbooks. ● Highlight textbooks/study guides. ● Use supplementary materials. ● Give assistance in note taking ● Use adapted/modified textbooks. ● Allow use of computer/word processor. ● Allow student to answer orally, give extended time (time-and-a-half). ● Allow tests to be given in a separate location (with the ESL teacher). ● Allow additional time to complete assignments and/or assessments. ● Read question to student to clarify. ● Provide a definition or synonym for words on a test that do not impact the validity of the exam. ● Modify the format of assessments. ● Shorten test length or require only selected test items. ● Create alternative assessments. ● On an exam other than a spelling test, don’t take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Rigid Transformations and Congruence

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students learn to understand and use the terms "reflection," "rotation," and "translation," recognizing what determines each type of transformation. For example, two points determine a translation. They learn to understand and use the terms "transformation" and "rigid transformation." They identify and describe translations, rotations, reflections, and sequences of these, using the terms "corresponding sides" and "corresponding angles," and recognizing that lengths and angle measures are preserved. They draw images of figures under rigid transformations on and off square grids and the coordinate plane. They use rigid transformations to generate shapes and to reason about measurements of figures. They learn to understand congruence of plane figures in terms of rigid transformations. They recognize when one plane figure is congruent or not congruent to another. They use the definition of "congruent" and properties of congruent figures to justify claims of congruence or non-congruence. They investigate whether sets of angle and side length measurements determine unique triangles, multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms "complementary," "supplementary," "vertical angles," and "unique."

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:

7.G.A.2 Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations.

a. Lines are transformed to lines, and line segments to line segments of the same length.
b. Angles are transformed to angles of the same measure.
c. Parallel lines are transformed to parallel lines.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. Describe the effects of dilations, translations, rotations, and reflections using coordinates.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of

triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Science

MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.

MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:

Students will understand that…

- Utilizing the properties of rotation, reflection, and translation to model and relate pre-images to their resultant image through physical representations.
- Applying an effective sequence of transformations to prove that two figures are congruent.
- Recognizing a dilation as the reduction or an enlargement of a figure and determining the scale factor.
- Applying a sequence of transformations to determine that figures are similar when corresponding parts are proportional.
- Justifying facts about angles created by a set of parallel lines cut by a transversal.

- Justifying facts about the interior and exterior angles of a triangle, and the angle-angle relationship used to identify similar triangles.
- Exploring strategies to compare and contrast triangles within a set, identifying unique characteristics that differentiate triangle types, and applying this knowledge to geometric problem-solving.

Unit Essential Questions:

- If a figure is transformed on the coordinate plane, will the resulting figure always be congruent to the original?
- What relationships are formed when a set of parallel lines is cut by a transversal?
- What properties do the angles of a triangle possess?
- What strategies can students employ to compare triangles within a set, recognizing key characteristics that differentiate various types of triangles and their properties?

Knowledge and Skills:

Students will know...

- How to describe
  - movements of figures.
  - observations about transforming parallel lines.
  - transformations using corresponding points, line segments, and angles.
  - observations about angle measurements.
  - positioning and movement of side lengths and angles.
  - transformations found in tessellations and in designs with rotational symmetry.
- How to generalize
  - about categories for movement.
  - about rotating line segments.
  - about the relationship between vertical angles.
  - about corresponding segments and length.
  - about alternate interior angles.
  - about the sum of angles in a triangle.
  - about categories for unique triangles.
- How to justify
  - whether or not rigid transformations could produce an image.
  - whether or not shapes are congruent.
  - whether or not polygons are congruent.
  - whether or not triangles can be created from given angle measurements.
  - whether or not shapes are identical copies.
  - whether or not measurements determine unique triangles.
- Vocabulary: vertex, clockwise, counterclockwise, reflection, rotation, translation, image, sequence of transformations, coordinate plane, rigid transformation, corresponding, congruent, right angle, alternate interior angles, transversal, supplementary, complementary, vertical angles, straight angle, and tessellation.

Students will be able to...

- Determine coordinates that represent the image of a polygon or line segment in the coordinate plane after a transformation.
- Draw and label the image of figures that result from translations, rotations, and reflections on a square or isometric grid.
- Explain the sequence of transformations that takes one figure to its image.
- Draw and label rigid transformations of lines and parallel lines and explain the relationship between the original and its image under the transformation.

- Identify a rigid transformation using a drawing of a figure and its image.
- Identify side lengths and angles that have equivalent measurements in composite shapes and explain why they are equivalent.
- Compare and contrast side lengths, angle measures, and other features of shapes using rigid transformations to explain why a shape is or is not congruent to another.
- Justify that two polygons on a grid are congruent using the definition of congruence in terms of rigid transformations.
- Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.
- Generalize that the sum of angles in a triangle is 180 degrees using rigid transformations or the congruence of alternate interior angles of parallel lines cut by a transversal.
- Create tessellations and designs with rotational symmetry using rigid transformations.
- Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.
- Draw triangles with two given angle measures and one side length, one given angle and two side lengths, or three side lengths.
- Justify where 3 measures of angles or sides determine a unique triangle or more than one triangle, or if no triangle is possible.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Triangle Square Dance - The purpose of this activity is for students to begin to observe and describe translations and rotations. In groups of 2, they describe one of 3 possible dances, presented in cartoon form, and the partner identifies which dance is being described. Identify students who use specific and detailed language to describe the dance and select them to share during class discussion.
- Lesson 6: Which One? - The purpose of this activity is to decide if there is a sequence of translations, rotations, and reflections that take one figure to another and, if so, to produce one such sequence. Deciding whether or not such a sequence is possible uses the knowledge that translations, rotations, and reflections do not change side lengths or angle measures.
- Lesson 9: Triangle Plus One - The purpose of this task is to use rigid transformations to describe an important picture that students have seen previously when they developed the formula for the area of a triangle. They first found the area of a parallelogram to be base times height and then, to find one half base times height for the area of a triangle, they "composed" two copies of a triangle to make a parallelogram. The focus of this activity is on developing this precise language to describe a familiar geometric situation.

- Lesson 11: Area, Perimeter, and Congruence - Sometimes people characterize congruence as "same size, same shape." The problem with this is that it isn't clear what we mean by "same shape." All of the figures in this activity have the same shape because they are all rectangles, but they are not all congruent. Students examine a set of rectangles and classify them according to their area and perimeter. Then they identify which ones are congruent. Because congruent shapes have the same side lengths, congruent rectangles have the same perimeter. But rectangles with the same perimeter are not always congruent. Congruent shapes, including rectangles, also have the same area. But rectangles with the same area are not always congruent. Highlighting important features, like perimeter and area, which can be used to quickly establish that two shapes are not congruent develops MP7, identifying fundamental properties shared by any pair of congruent shapes.
- Lesson 13: Tear It Up - The students found that the sum of the angles of all the triangles on the cards was 180 degrees and questioned if all triangles have the same angle sum. In this activity, students experiment with the converse: If we know the measures of three angles sum to 180 degrees, can these three angles be the interior angles in a triangle?
- Lesson 17: Revisiting: How Many You Can You Draw? - Students continue to practice drawing triangles from given conditions and categorizing their results. This activity focuses on the inclusion of a single angle and two sides. Again, they do not need to memorize which conditions result in unique triangles, but should begin to notice how some conditions (such as the equal side lengths) result in certain requirements for the completed triangle.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.1 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.1 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Scale Drawings, Similarity, and Slope

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students learn to understand and use the terms "scaled copy," "to scale," "scale factor," "scale drawing," and "scale," and recognize when two pictures or plane figures are or are not scaled copies of each other. They use tables to reason about measurements in scaled copies. They make, interpret, and reason about scale drawings. These include maps and floor plans that have scales with and without units. Students then refine their thinking about scaled copies as they learn to understand and use the term "dilation" and to recognize that a dilation is determined by a point called the "center" and a number called the "scale factor." They learn that under a dilation, the image of a circle is a circle and the image of a line is a line parallel to the original. They draw images of figures under dilations on and off the coordinate plane. They describe correspondences between a figure and its dilated image, and recognize that angle measures are preserved, but lengths are multiplied by the scale factor. They learn to understand similarity of plane figures in terms of rigid transformations and dilations. They learn to recognize when one plane figure is similar or not similar to another. They use the definition of "similar" and properties of similar figures to justify claims of similarity or non-similarity. Students learn the terms "slope" and "slope triangle," and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope.

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. Describe the effects of dilations, translations, rotations, and reflections using coordinates.

8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Science

MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.

MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:

Students will understand that…

- The concept of scale drawings and how they represent real-world objects or geometric figures at a reduced or enlarged scale.
- Applying scale factors to accurately resize geometric figures in scale drawings while maintaining proportional relationships between corresponding sides and angles.
- Dilations are a new type of transformation and a way to create a scaled copy.
- Translating, reflecting, rotating, and dilating geometric figures can help determine if they are similar and that similar figures have lengths that have the same scale factor.
- Two distinct points on a line determine the same slope as the slope of the overall line.

Unit Essential Questions:

- How can critical thinking skills be applied when determining the scale factor in a scale drawing of a geometric figure?
- In what ways can problem-solving strategies help in accurately scaling down real-world objects onto paper in a scale drawing?
- If a figure is transformed on the coordinate plane, will the resulting figure always be congruent to the original?
- What relationships are formed when a set of parallel lines is cut by a transversal?
- What properties do the angles of a triangle possess?

Knowledge and Skills:

Students will know...

- How to explain
  - how to use scale drawings to find actual distances.
  - how to apply dilations to find specific images.
  - how to determine whether triangles are congruent, similar, or neither.
  - strategies for finding missing side lengths.
  - how to apply dilations to find specific images of points.
  - reasoning for a conjecture.
- How to describe
  - features of scaled copies.
  - observations about scaled rectangles.
  - observations about dilated points, circles, and polygons.
  - sequences of transformations.
  - observations about side lengths in similar triangles.
  - relevant features of a classroom with a scale drawing.
- How to justify
  - reasoning about scaled copies.
  - that polygons are similar.
  - that triangles are similar.
  - why the height of objects and the length of their shadows are approximately proportional.
- Vocabulary: scaled copy, scale factor, scale, scale drawing, dilation, center of dilation, similar, and slope.

Students will be able to...

- Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures.
- Draw a scaled copy of a figure using a given scale factor.
- Create a scale drawing given the actual measurements of the object or given another scale drawing at a different scale.
- Explain how to use scales and scale drawings to calculate actual distances and areas.
- Create a dilation of a figure given a scale factor and center of dilation.
- Describe a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points.
- Identify the center, scale factor, and image of a dilation.
- Calculate unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.
- Justify that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or by checking that two pairs of corresponding angles are congruent.
- Comprehend the term “slope” to mean a number that tells how steep a line is.

- Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line and use it to justify whether a point (x,y) is on the line by verifying that the values of x and y satisfy the equation.
- Calculate the unknown heights of objects by using proportional reasoning and explain (orally) the solution method.
- Justify (orally) why the relationship between the height of objects and the length of their shadows cast by the sun is approximately proportional.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Pairs of Scaled Polygons - In this activity, students hone their understanding of scaled copies by working with more complex figures. Students work with a partner to match pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.
- Lesson 4: Sizing Up a Basketball Court - In this introductory activity, students explore the meaning of scale. They begin to see that a scale communicates the relationship between lengths on a drawing and corresponding lengths in the objects they represent, and they learn some ways to express this relationship.
- Lesson 7: Apollo Lunar Module - In this activity, students use a scale drawing and a scale expressed without units to calculate actual lengths. Students will need to make a choice about which units to use, and some choices make the work easier than others.
- Lesson 8: Number Talk: Remembering Fraction Division - This Number Talk gives students an opportunity to recall strategies for computation problems that will arise in the lesson. While many strategies may emerge, the focus of these problems is for students to recall and rehearse a reliable way to divide a mixed number by a whole number.
- Lesson 10: Card Sort: Matching Dilations on a Coordinate Grid - This activity adds the structure of coordinates and this extra structure plays a key role, allowing students to name points. Students match figures with their dilated images, using coordinates to describe the center of dilation and the vertices. The same strategies that were used previously in dilating images, on a circular grid and with no grid, will be useful here.
- Lesson 12: Find Someone Similar - In the previous activity, students learned that in order to be similar, two figures must have congruent corresponding angles and proportional corresponding side lengths. In this activity, students apply this knowledge. Each student has a card with a figure on it and they identify someone with a similar (but not congruent) figure.

- Lesson 15: Similar Triangles on the Same Line - In this activity, students explain why certain triangles with one side along the same line are similar. This fact about the triangles will be used to define the slope of the line. Students may show that the triangles are similar by describing a sequence of transformations and dilations or by AA (or AAA). Alternatively, they may use the fact that grid lines are parallel and use what they know about the angles where a transverse meets a pair of parallel lines.
- Lesson 16: What We Mean by an Equation of a Line - Prior to this lesson, students have seen that right triangles with a horizontal side, a vertical side, and a long side along the same line are all similar. This activity exploits this structure to examine the coordinates of points lying on a particular line. The discussion then produces an equation for the line. In the case where the line goes through (0,0), the equation will be familiar from prior work with proportional relationships but in the next lesson, similar triangles will be essential.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.2 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.2 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Writing and Solving Equations

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students solve equations of the forms px + q = r and p(x + q) = r where p, q and r are rational numbers. They draw, interpret, and write equations in one variable for balanced "hanger diagrams," and write expressions for sequences of instructions. They use tape diagrams together with equations to represent situations with one unknown quantity. They learn algebraic methods for solving equations. Students will be presented with the foundational skills related to solving linear equations and the connected skills of solving absolute value equations and rewriting equations and formulas. Students will be familiar with the idea of equality from their prior learning and the notion of what is done on one side, must be done on the other side to keep the equation balanced. Students will be rewriting equations and formulas, which provides the students with insight into the structure of equations and the operations performed.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $p(x + q) < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per

sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Science

MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.

MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:

Students will understand that…

- Reasoning quantitatively and using units to solve problems.
- Creating equations that describe numbers or relationships.
- Solving equations is a process of reasoning and explaining the reasoning.
- Developing the ability to interpret solutions to equations, and understanding how to analyze and apply solutions in various mathematical contexts to solve problems effectively.
- Interpreting and representing non-proportional situations with constant rates of change, utilizing methods such as tape diagrams to visually depict relationships and patterns within the given scenarios.
- Through collaborative discussions, critiquing the reasoning of their peers about expressions and corresponding diagrams, evaluating the validity of arguments, identifying logical flaws, and providing constructive feedback to enhance understanding.
- General strategies for solving equations and determining when expressions are equivalent, applying critical thinking skills to analyze and reason about various mathematical concepts and problem-solving techniques.

Unit Essential Questions:

- How can real-world situations be represented as equations?
- How can you solve an equation for the variable of interest?
- How can we interpret solutions to equations effectively, and what insights can we gain from analyzing these solutions in various mathematical contexts?
- In what ways can non-proportional situations with constant rates of change be interpreted and represented using methods like tape diagrams, enhancing our understanding of relationships and patterns within the scenarios?
- How can we critically critique the reasoning of peers regarding expressions and corresponding diagrams, evaluate the validity of arguments, and provide constructive feedback to deepen understanding?
- Why is it important to analyze and critique reasoning about solving equations and equivalent expressions, and how does this process contribute to our ability to generalize problem-solving approaches and identify equivalent mathematical expressions effectively?

Knowledge and Skills:

Students will know...

- How to interpret
  - non-proportional situations with constant rates of change.
  - solutions to equations.
  - non-proportional situations with constant rates of change.
  - equations representing angle measurements.
- How to compare
  - stories with corresponding tape diagrams.
  - tape diagrams with corresponding equations.
  - hanger diagrams and equations.
  - solution pathways.
- How to explain

    - strategies for using hanger diagrams to solve equations using hanger diagrams.
    - strategies for solving equations.
    - reasoning about situations, tape diagrams, and equations.
    - how to find unknown angle measurements.
- Writing and solving
    - one-step linear equations.
    - multi-step linear equations.
    - equations with variables on both sides.
    - equations involving absolute value.
    - literal equations for given variables.
- Proportional reasoning and analyzing units when solving problems.
- Vocabulary: equivalent expressions, variables, one, two, and multi-step linear equations, solution, equivalent equations, ratios, proportions, rate, accuracy, and literal equation.

Students will be able to...

- Create diagrams and equations in the form of $px + q = r$ and $p(x + q) = r$ to represent situations.
- Interpret equations in the form $px + q = r$ and $p(x + q) = r$ that represent relationships in diagrams and situations.
- Solve equations of the form $px + q = r$ and $p(x + q) = r$, including those that involve fractions, decimals, and negative numbers, and explain the solution method.
- Solve real-world problems leading to equations of the form $px + q = r$ or $p(x + q) = r$.
- Write an equation to represent a relationship between angle measures and solve the equation to find unknown angle measures.
- Solve simple and multi-step equations.
- Describe how to solve equations.
- Analyze the measurements used to solve a problem and judge the level of accuracy appropriate for the solution.
- Apply properties of equality to produce equivalent equations for one-step and multi-step equations.
- Write linear equations that model real-life situations for one-step and multi-step equations.
- Utilize ratios and rates to solve real-life problems.
- Apply properties of equality using variable terms.
- Assess when an equation has zero, one, or infinitely many solutions.
- Evaluate the two linear equations related to a given absolute value equation.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 2: Every Picture Tells a Story - In this activity, students explain how a tape diagram represents a situation. They also use the tape diagram to reason about the value of the unknown quantity. Students are not expected to write and solve equations here; any method they can explain for finding values for x and y is acceptable. While some students might come up with equations to describe the diagram and solve for the unknown, there is no need to focus on developing those ideas at this time.
- Lesson 6: Categories of Equations Card Sort - The goal of this activity is for students to notice the structure of equations. Any way of sorting is fine, but the discussion should land on explaining how equations involving an expression like $p(x + q)$ are different from ones that have an expression like $px + q$. Monitor for different ways groups choose to categorize the equations, but especially for categories that distinguish between these two types of expressions. As students work, encourage them to refine their descriptions of equations using more precise language and mathematical terms.
- Lesson 9: Old and New Ways to Solve - These are all solvable by thinking "what value would make the equation true." So, it's straightforward to figure out what the solution would be, but these equations present an opportunity to demonstrate that "doing the same thing to each side" still works when there are negative numbers. Monitor for students who reason about what value would make the equation true and those who reason by doing the same thing to each side.
- Lesson 13: Calculate the Measure - This activity is a culmination of all the work students have done with angles in this unit. With less support than in previous activities, students come up with equations that represent the relationships between angles in a figure. Then, students solve their equation to find each unknown angle measure.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, R. and Boswell, L. (2022). Algebra 1. Erie, PA: Big Ideas Learning, LLC.
- Illustrative Math (IM) Unit 7.3 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.3 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Inequalities, Expressions and Equations

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students expand on their previous work writing and solving equations of the form $px + q = r$ and $p(x + q) = r$. They solve linear inequalities in one variable and represent their solutions on the number line, understanding that there may be infinitely many solutions, and show solutions symbolically and on the number line. They generate expressions that are equivalent to a given numerical or linear expression. These include equations in which the variable occurs on each side of the equal sign, and equations with no solutions, exactly one solution, and infinitely many solutions. They learn that any one such equation is false, true for one value of the variable, or true for all values of the variable. They interpret solutions in the contexts from which the equations arose. Students will be presented with the foundational skills related to solving linear inequalities and the connected skills of solving absolute value inequalities and rewriting equations and formulas. Students will be familiar with the idea of equality from their prior learning and the notion of what is done on one side, must be done on the other side to keep the equation balanced. As the unit progresses, the students start to focus on inequalities and begin to solve increasingly more complex inequalities, eventually leading to the solving of compound inequalities and absolute value inequalities.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3° C > -7° C to express the fact that -3° C is warmer than -7° C.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $p(x + q) < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form$x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables using the substitution method and estimate solutions by graphing the equations. Solve simple cases by inspection. For example: by inspection, conclude that 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. Solve 3x + y = 30 and y = 2x using the substitution method; Solve y = 3x + 1 and y = -2x + 7 using the substitution method.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.
N.Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Career Readiness, Life Literacies, and Key Skills:
9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.
9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.
9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:
ELA
L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.
RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.
RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.
W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.
SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Science
MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.
MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the

amplitude of a wave is related to the energy in a wave.

Unit Understandings:
Students will understand that…

- Reasoning quantitatively and using units to solve problems.
- Creating equations and inequalities that describe numbers or relationships.
- Solving equations as a process of reasoning and explaining the reasoning.
- Solving equations and inequalities in one variable.
- Linear equations represent proportional relationships between variables and are connected to ratios and rates with lines and triangles.
- Writing and solving equations can be completed in a variety of ways and there are multiple approaches to reach the same solution.
- Given descriptions of real-world situations, they can write and solve linear equations in one variable, interpreting solutions in the contexts from which the equations arose.

Unit Essential Questions:

- How can real-world situations be represented as equations or inequalities?
- How can you solve an equation or inequality for the variable of interest?
- How do you use patterns to understand mathematics and model situations?
- How do algebraic representations relate and compare to one another?
- How can we communicate and generalize algebraic relationships?

Knowledge and Skills:
Students will know...

- How to justify
    - reasoning about solutions to inequalities.
    - reasoning about solutions to inequalities.
    - the need for specific information to write and solve inequalities.
    - reasoning about the distributive property.
    - predictions about solutions of linear equations.
- How to critique
    - the reasoning of others.
    - reasoning of peers about expressions and corresponding diagrams.
    - reasoning about equivalent expressions.
    - reasoning about maintaining balance in equations.
    - solutions of linear equations.
- How to generalize
    - about the relationships between shapes.
    - about when expressions are equivalent.
    - about the structures of equations that have infinite and no solutions.
    - about the structures of equations that have one, infinite, and no solutions.
- Writing and solving
    - compound inequalities.
    - multi-step inequalities.
    - inequalities involving absolute value.
- Formulate inequalities and represent solutions of inequalities on number lines.
- Proportional reasoning and analyzing units when solving problems.
- Vocabulary: One, two, and multi-step inequalities, solution, inequality, solution of an inequality, equivalent inequalities, compound inequalities, absolute value inequalities, term, factor (an expression), expand (an expression), constant term, and coefficient.

Students will be able to...

- Draw and label a graph on a number line that represents all the solutions to an inequality.
- Solve an inequality of the form $px + q > r$ or $px + q < r$ and interpret the solution.
- Write an inequality of the form $px + q > r$ or $px + q < r$ to represent a situation with a constraint.
- Apply properties of operations to write an expression with fewer terms that is equivalent to a given expression.
- Apply the distributive property to factor or expand an expression.
- Write equivalent equations and describe the moves that are used.
- Write equivalent equations to solve linear equations in one variable.
- Describe features of linear equations that have one solution, no solution, or many solutions.
- Interpret the solution of an equation in one variable in context.
- Compare and contrast solving inequalities with solving equations.
- Graph and interpret inequalities.
- Apply the Addition and Subtraction Properties of Inequality to produce equivalent inequalities.
- Apply the Multiplication and Division Properties of Inequality to produce equivalent inequalities.
- Produce multi-step inequalities to solve real-life problems.
- Solve and graph solutions of compound inequalities.
- Develop absolute value inequalities to solve real-life problems.
- Connect equation-solving techniques to solve real-life problems.
- Apply techniques for solving inequalities to solve real-life applications.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 4: Earning Money for Soccer Stuff - Previously in this unit, students wrote expressions and equations that are similar to the ones in this activity. Here, they are prompted in a scaffolded way to notice that they can express not just that an outcome can be equal to a value, but that an outcome can be at least as much as a value by using the new notation $\geq$.
- Lesson 6: Giving Advice - In this activity, students set up and solve inequalities that represent real-life situations. Students will think about how to interpret their mathematical solutions. For example, if they use w to represent width in centimeters and find $w < 25.5$, does that mean $w = -10$ is a solution to the inequality?
- Lesson 9: Making Sides Equal - In this activity, students use what they have learned so far to find a missing term that makes two expressions equivalent. They have many tools at their disposal to reason about the missing term. For example, for the first problem $6x + ? = 10x$, they might reason as in the last activity and write out the sum of 6 x's and ? on one side, and a string of 10 x's on the other side, and reason that 4 x's are needed to make the sides equivalent. Alternatively, they might

reason with the distributive property, and rewrite the left side as $x(6 + ?) = 10x$. These alternative ways of reasoning about equivalent expressions should be highlighted in the discussion.

- Lesson 13: Trading Moves - The goal of this activity is for students to build fluency in solving equations with variables on each side. Students describe each step in their solution process to a partner and justify how each of their changes maintains the equality of the two expressions.
- Lesson 15: Thinking About Solutions - Students who pause to think about the structure of a complex equation before taking steps to solve it can find the most efficient solution paths and, sometimes, notice that there is no single solution to be found. The goal of this lesson is to encourage students to make this pause part of their routine and to build their skill at understanding and manipulating the structure of equations through the study of two special types of equations: ones that are always true and ones that are never true.
- Lesson 16: Thinking About Solutions Some More - In this activity, students solve a variety of equation types; both in form and number of solutions. After solving the 10 equations, groups sort them into categories of their choosing. The goal of this activity is to encourage students to look at the structure of equations before solving them and to build fluency in solving complex equations. For example, students who notice that equation D, $3 - 4x + 5 = 2(8 - 2x)$, has the same number of x's on each side but a different constant know that there are no values of x that make the equation true. Similarly, equation J has the same number of x's on each side and the same constants on each side, meaning that all values of x make the equation true. These up-front observations allow students to avoid spending time working out the steps to re-write the equation into a simpler form where the number of solutions to the equations is easier to see.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, R. and Boswell, L. (2022). Algebra 1. Erie, PA: Big Ideas Learning, LLC.
- Illustrative Math (IM) Unit 7.4 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.4 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 5 OVERVIEW

Content Area: Mathematics

Unit Title: Linear Relationships

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students learn to understand and use the terms "rate of change," "linear relationship," and "vertical intercept." They deepen their understanding of slope, and they learn to recognize connections among rate of change, slope, and constant of proportionality, and between linear and proportional relationships. They represent linear relationships with tables, equations, and graphs that include lines with negative slopes or vertical intercepts, and horizontal and vertical lines. They learn to use the term "solution of an equation" when working with one linear equation in two variables, understand the graph of a linear equation as the set of its solutions, and what is meant by a solution for a system of linear equations. In particular, that the pair of values that satisfies a system of equations are coordinates of a point that lies on the graphs of all the equations in the system, and, conversely, a point that lies on the graphs of all the equations in the system has coordinates that satisfy all the equations in the system. Students learn to understand and use the terms "system of equations," "solution for the system of equations," "zero solutions," "no solution," "one solution," and "infinitely many solutions." Students conclude the unit working with bivariate data sets. They learn to understand and use the terms "scatter plot" and "association," and describe associations as "positive" or "negative" and "linear" or "non-linear." Students describe scatter plots, using a term previously used to describe univariate data "cluster," and the new term "outlier." They fit lines to scatter plots and informally assess their goodness of fit by judging the closeness of the data points to the lines, and compare predicted and actual values.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables using the substitution method and estimate solutions by graphing the equations. Solve simple cases by inspection. For example: by inspection, conclude that $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot

simultaneously be 5 and 6. Solve $3x + y = 30$ and $y = 2x$ using the substitution method; Solve $y = 3x + 1$ and $y = -2x + 7$ using the substitution method.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Science

MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.

MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:

Students will understand that…

- Utilizing linear relationships and their representations helps them make sense of and solve problems.
- The rate of change is a way to describe the rate per 1 in a linear relationship and is utilized to determine the numerical value of the slope of a line.
- There is a relationship between the coordinates of points on a graph and the solution of an equation.
- Constructing scatter plots for bivariate data and identifying and interpreting data patterns (clustering, outliers, positive or negative association, possible lines of best fit, and nonlinear association).
- Constructing frequency/relative frequency tables to analyze and describe possible associations between two variables.
- Bivariate data has two variables, and graphs such as scatter plots can be useful for displaying and analyzing this type of data. The conclusions made from the data depend on how it is represented and summarized.
- Tables and graphs of functions allow for conclusions to be drawn about their rate of change, intercepts, etc.

Unit Essential Questions:

- What do the variables m and b in the linear equation $y = mx + b$ represent?
- How can the unit rate of a proportional relationship be identified from the graph of the equation?
- Why do we graph linear equations, and what does the line represent?
- What is a real-life situation you can apply using the slope-intercept formula?
- What is bivariate data and how can displaying this type of data be useful?
- How can an equation with variables on both sides be used to solve real-world problems?
- How do we use the graph of systems of equations to determine the solution(s)?
- How can real-world problems be solved by using a system of equations?
- How can scatter plots and two-way tables be used to describe the relationship between bivariate data?

Knowledge and Skills:

Students will know…

- How to represent
  - situations involving proportional relationships.
  - constants of proportionality in different ways.
  - slope using expressions.
  - linear relationships using graphs, tables, equations, and verbal descriptions.
  - situations using negative slopes and slopes of zero.
  - situations by graphing lines and writing equations.
  - situations involving systems of linear equations.
  - data in organized ways.

    - data using two-way tables, bar graphs, and segmented bar graphs.
    - situations involving linear relationships.
- How to interpret
    - situations involving proportional relationships.
    - graphs using different scales.
    - slopes and intercepts of linear graphs.
    - situations using negative slopes and slopes of zero.
    - situations involving systems of linear equations.
    - tables and scatterplots of bivariate data.
    - tables, scatterplots, equations, and situations involving bivariate data.
    - situations involving linear relationships.
- How to explain
    - how to use a graph to determine information about a linear situation.
    - how to determine slope from a graph.
    - how slope relates to changes in a situation.
    - how to estimate using available data.
    - how to use tables and scatterplots to make estimates and predictions.
    - the meaning of slope for a situation.
    - how to use lines to show associations, identify outliers, and answer questions.
    - how to answer questions about systems of equations.
- Vocabulary: constant of proportionality, rate of change, linear relationship, slope, vertical intercept, the solution to an equation with two variables, scatter plot, outlier, positive association, negative association, independent variable, dependent variable, segmented bar graph, relative frequency, and two-way (frequency) table.

Students will be able to...

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Interpret multiple representations of a proportional relationship in context.
- Create and compare graphs that represent linear relationships with the same rate of change but different initial values.
- Create an equation that represents a linear relationship.
- Interpret the slope and y-intercept of the graph of a line in context.
- Create multiple representations of a linear relationship, including a graph, equation, and table.
- Interpret the slope of a non-increasing line in context.
- Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.
- Categorize systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety
- of strategies.
- Comprehend that solving a system of equations means finding values of the variables that make both equations true at the same time.
- Create a system of equations that represents a situation and interpret the solution in context.
- Create a scatter plot from a table of data, and describe the trend of the data.
- Interpret a point on a scatter plot in context.
- Describe the relationship between two variables using a line fit to data on a scatter plot.
- Draw a linear model to fit data in a scatter plot, and describe features of a line that fits data well.
- Interpret features of data on a scatter plot including linear and non-linear associations, outliers, slope of a linear model, and clustering.
- Calculate relative frequencies, and describe associations between variables using a relative frequency table.

- Create a two-way table and a segmented bar graph that represent relative frequencies, and interpret the frequencies in context.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 2: Card Sort: Proportional Relationships - The purpose of this activity is for students to identify the same proportional relationship graphed using different scales. Students will first sort the cards based on what proportional relationship they represent and then write an equation representing each relationship. Identify and select groups using different strategies to match graphs to share during the Activity Synthesis. For example, some groups may identify the unit rate for each graph in order to match while others may choose to write equations first and use those to match their graphs.
- Lesson 5: Slopes, Vertical, Intercepts, and Graphs - This task focuses on interpreting the slope of a graph and where it crosses the y-axis in context. Students are given cards describing situations with a given rate of change and cards with graphs. Students match each graph with a situation it could represent, and then use the context to interpret the meaning of the slope. They find where the line crosses the vertical axis, i.e., the vertical intercept, and interpret its meaning in each situation. They also decide if the two quantities in each situation are in a proportional relationship.
- Lesson 7: Translating a Line - This activity further examines parallel lines, including situations where the y-intercept is negative. In addition, students match lines represented in many different ways including graphically, verbal description, table of values, and equations.
- Lesson 9: Making Designs - The goal of this activity is for students to recognize information that determines the location of a line in the coordinate plane, and to practice distinguishing between positive and negative slopes. In this activity, one partner has a design that they verbally describe to their partner, who then tries to draw it. The purpose of this activity is to provide an environment where students have to describe or interpret the slope and locations of several lines. (Students are not expected to communicate by saying the equations of the lines, though there is nothing stopping them from doing so.) Students take turns describing and interpreting by doing this two times with two different designs.
- Lesson 11: I'll Take and X, Please - Students are given linear equations—some of which represent proportional relationships—in various forms, and are also given solutions to their partner's equations in the form of coordinates of a point. The student with the equation decides which quantity they would like to know, x or y, and requests this information from their partner. They then solve for the other quantity. The activity reinforces the concept that solutions to equations with two variables are a pair of numbers, and that knowing one can give you the other by using

the value you know and solving the equation. Students also have a chance to think about the most efficient way to find solutions for equations in different forms.

- Lesson 14: Different Types of Systems - While students have encountered equations with different numbers of solutions in earlier activities, this is the first activity where students connect systems of equations with their previous thinking about equations that have no solution, one solution, or infinitely many solutions. The purpose of this activity is for students to connect the features of the graph of the equations of a system to the number of solutions of a system. While students are not asked to solve the systems of equations, they may choose to rewrite the equations in equivalent forms as they work to graph the lines.
- Lesson 20: Fitting Lines - In this activity, students draw their own linear model to fit the data in a scatter plot. In one scatter plot, the data points are nearly linear, and in another, there is much more variation in the data. A discussion follows about what makes some lines a better fit than others.
- Lesson 23: Card Sort; Matching Representations - In this activity students become familiar with two-way tables, clustered bar graphs, and segmented bar graphs by matching different situations. They label the diagrams to match the data given and create a table to match the data shown in one of the bar graphs.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.5 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.5 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 6 OVERVIEW

Content Area: Mathematics

Unit Title: Functions and Volume

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students are introduced to the concept of a function. They learn to understand and use the terms "input," "output," and "function"—for example, "temperature is a function of time." They describe functions as increasing or decreasing between specific numerical inputs, and they consider the inputs of a function to be values of its independent variable and its outputs to be values of its dependent variable. They use tables, equations, and graphs to represent functions, and describe information presented in tables, equations, or graphs in terms of functions. In working with linear functions, students coordinate and synthesize their understanding of "constant of proportionality", "rate of change", "slope", and increasing and decreasing. Function notation and characteristics of functions are also studied, often in the context of graphing linear functions. The unit begins with a focus on function notation, discrete and continuous functions, and evaluating functions. The students will be introduced to two forms of linear equations - standard and slope-intercept. Students will also create equations in two variables to represent relationships between quantities. Students then turn to focus on features of 3-dimensional shapes and consider volume formulas as examples of functions. They analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism, and solve problems involving area, surface area, and volume. Students perceive similarities in structure between pairs of volume formulas: for a rectangular prism and a cylinder; and for a cylinder and a cone. Students rearrange these formulas to show functional relationships and use them to reason about how the volume of a figure changes as another measurement changes—for example, the height of a cylinder is proportional to its volume; if the radius of a cylinder triples, its volume becomes nine times larger.

Approximate Length of Unit: 7 weeks

LEARNING TARGETS

NJ Student Learning Standards:

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.A.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = (1) = 1, f(n + 1) = f(n) + f(n-1) for $n \geq 1$.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
F.BF.A.1 Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.C.9 Distinguish between correlation and causation.

Career Readiness, Life Literacies, and Key Skills:
9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.
9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.
9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:
ELA
L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.
RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.
RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.
W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.
SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Science
MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.
MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:
Students will understand that…

- A function is a relationship between inputs and outputs, meaning that each input results in exactly one output.
- Functions can be represented in a real-world situation in several ways, including identifying tables, equations, and graphs.
- Linear functions model relationships between quantities and provide information about graphs and other representations.
- Finding the volume of a three-dimensional figure can help them solve real-world problems.
- Articulating the positioning and movement of side lengths and angles in geometric figures, enabling them to analyze shapes based on their spatial relationships and properties.
- Describing cross sections of prisms and pyramids, identifying commonalities and distinctions in resulting shapes to deepen their understanding of three-dimensional geometry and shape classification.

Unit Essential Questions:

- What do the variables m and b in the linear equation y = mx + b represent?
- How can the unit rate of a proportional relationship be identified from the graph of the equation?
- Why do we graph linear equations, and what does the line represent?
- What is a real-life situation you can apply using the slope-intercept formula?
- What is a linear function?
- What insights can be gained from describing the positioning and movement of side lengths and angles in geometric figures, and how does this knowledge contribute to analyzing and classifying shapes based on their attributes?
- How can students effectively describe cross sections of prisms and pyramids, identifying commonalities and differences in the resulting shapes to deepen their understanding of three-dimensional geometry?

Knowledge and Skills:
Students will know...

- How to compare
    - different representations of functions.
    - features of graphs, equations, and situations.
    - features of a situation with features of a graph.
    - temperatures are shown on a graph with different temperatures given in a table.
    - cross sections of figures.
    - the volumes of cones with the volumes of cylinders.
    - methods for finding and approximating the volume of a sphere as a function of its radius.
    - how to determine the characteristics of triangles and prisms.
- How to explain
    - how the height and volume of cylinders are related.
    - how to find the area and volume of prisms.
    - how to find the surface area of prisms.
    - reasoning about finding the volume of a cylinder.
    - reasoning about the relationship between volumes of hemispheres and volumes of boxes, cylinders, and cones.
    - how to determine the characteristics of triangles and prisms.
- How to describe
    - quantities in a situation.
    - approximately linear relationships.
    - relationships that are approximately piecewise linear.

    - cross sections of prisms and pyramids.
    - volume measurements and features of three-dimensional figures.
    - the effects of varying dimensions of rectangular prisms and cones on their volumes.
    - how to determine the characteristics of triangles and prisms.
- The concepts of functions.
- Describing the characteristics of a function.
- Identifying and graphing linear functions.
- Using function notation.
- Graphing and interpreting equations written in standard form.
- Finding the slope of a line.
- Using the slope-intercept form.
- Graphing transformations of linear functions.
- Graphing absolute value functions.
- Writing equations of lines in slope-intercept form.
- Writing equations of lines in point-slope form.
- Recognizing and writing equations of parallel and perpendicular lines.
- Using scatter plots and lines of fit to describe relationships between data.
- Analyzing lines of fit and finding lines of best fit.
- The concept of arithmetic sequences.
- Graphing and writing piecewise functions.
- Vocabulary: relation, function, independent variable, dependent variable, x-intercept, y-intercept, domain and range, slope, horizontal and vertical lines, parallel, perpendicular, slope-intercept form, standard form, point-slope form, transformations, scatter plots, best-fit lines, arithmetic sequence, volume, function, cross-section, base (of a prism or pyramid), cylinder, cone, sphere, surface area, and radius.

Students will be able to...

- Comprehend the structure of a function as having one and only one output for each allowable input.
- Draw the graph of a function that represents a content, and explain which quantity is a function of which.
- Interpret multiple representations of functions, including graphs, tables, and equations, and explain how to find information in each type of representation.
- Calculate the different rates of change of a piecewise linear function using a graph, and interpret the rates of change in context.
- Comprehend that any linear function can be represented by an equation in the form $y = mx + b$, where m and b are the rate of change and initial value of the function, respectively.
- Identify the graph of a linear function.
- Graph linear functions written in different forms.
- Describe the characteristics of a function.
- Explain how a transformation affects the graph of a linear function.
- Discover the domain and range of a function.
- Approximate when a function is positive, negative, increasing, or decreasing.
- Sketch a graph of a function from a verbal description.
- Create real-life problems that correspond to discrete or continuous data.
- Evaluate functions using function notation.
- Solve real-life problems using linear equations in standard form.
- Predict the slope-intercept form of a linear equation.
- Explain how translation, reflections, stretches, and shrinks affect graphs of functions.
- Determine the slope given ordered pairs, a graph, or a context.

- Write the equation of a line in different forms.
- Interpret scatter plots and analyze lines of fit.
- Write a function that represents an arithmetic sequence to solve a real-life problem.
- Discover the slope and y-intercept of a line.
- Solve real-life problems using equations in slope-intercept form.
- Utilize a point and the slope, or two points on a line to write the equation of a line.
- Write equations of parallel and perpendicular lines.
- Predict correlations between sets of data.
- Graph, identify, and write arithmetic sequences.
- Evaluate piecewise functions.
- Calculate the value of one dimension of a cylinder or cone, and explain the reasoning.
- Calculate the volume of a cylinder or cone.
- Calculate the volume of a sphere.
- Solve problems involving cones, cylinders, and spheres.
- Calculate the surface area and volume of a prism.
- Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Guess My Rule - The purpose of this activity is to introduce the idea of input-output rules. One student chooses inputs to tell a partner who uses a rule written on a card only they can see to respond with the corresponding output. The first student then guesses the rule on the card once they think they have enough input-output pairs to know what it is. Partners then reverse roles.
- Lesson 3: Dimes and Quarters - The purpose of this activity is to introduce the idea of input-output rules. One student chooses inputs to tell a partner who uses a rule written on a card only they can see to respond with the corresponding output. The first student then guesses the rule on the card once they think they have enough input-output pairs to know what it is. Partners then reverse roles.
- Lesson 6: Sketching a Story about a Boy and a Bike - The purpose of this task is for students to sketch a graph from a story. In order to make the sketch, students must select two quantities from the story to graph, decide which is the independent variable and which is the dependent variable, and create and label their axes based on their decisions.

- Lesson 8: Is it Filling Up or Draining Out? - The purpose of this activity is to connect features of an equation representing a function to what that means in a context. Students start with two functions that represent a tank being filled up and another being drained out and are asked to determine which equations represent which situation. This gives students the opportunity to connect initial value and slope, which they learned about in a previous unit, to the general form of the linear equation and to the fact that linear relationships are functions.
- Lesson 13: What's Your Estimate? - The purpose of this activity is for students to practice using precise language to describe how they estimated volumes of objects. Starting from an object of known volume, students must consider the difference in dimensions between the two objects. The focus here is on strategies to estimate the volume and units of measure used, not on exact answers or calculating volume using a formula.
- Lesson 20: Number Talk: Thirds - The purpose of this number talk is to elicit understandings and review strategies students have for finding the unknown value in an equation that involves the fraction ⅓. These understandings will be helpful later in this lesson when students are solving for the unknown length of the radius or height of a cone given its volume.
- Lesson 25: Info Gap: Unknown Dimensions - In this info gap activity, students determine and request the information needed to answer questions related to volume equations of cylinders, cones, and spheres.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, R. and Boswell, L. (2022). Algebra 1. Erie, PA: Big Ideas Learning, LLC.
- Illustrative Math (IM) Unit 7.6 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.6 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 7 OVERVIEW

Content Area: Mathematics

Unit Title: System of Equations, Exponents, and Scientific Notation

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: The first section of the unit begins with a lesson that reviews exponential expressions, including work with exponential expressions with bases 2 and ½. Students examine powers of 10. After working with these powers of 10, they consider what the value of 10^0 should be and define 10^0 to be 1. The students expand their work to numerical bases other than 10, using exponent rules with products of exponentials with the same base and contrasting it with products of exponentials with different bases. They note numerical instances of $a^n * b^n = (a \cdot b)^n$. The next section of the unit returns to powers of 10 as a prelude to the introduction of scientific notation. Students consider differences in magnitude of powers of 10 and use powers of 10 and multiples of powers of 10 to describe magnitudes of quantities. Students will be able to write and solve systems of linear equations and inequalities. The big understanding for students is that a solution of a system must satisfy every equation or inequality in the system. Many of the applications in this unit require students to write a system to represent a context, which is an essential skill for modeling with mathematics.Students will also be able to understand exponential functions and sequences. Students will extend the properties of integer exponents, introduced in middle school, to rational exponents. This leads to an introduction of exponential functions and then making a connection between exponential functions and geometric sequences.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.C.6 Solve systems of linear equations algebraically (include using the elimination method) and graphically, focusing on pairs of linear equations in two variables.

A.REI.D.11 Explain why the *x*-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x) ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1), f(n + 1) = f(n) + f(n - 1) for $n \geq 1$.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior**.**

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential function.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S.ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
c. Fit a linear function for a scatter plot that suggests a linear association.

Career Readiness, Life Literacies, and Key Skills:
9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.
9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.
9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:
ELA
L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.
RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.
RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.
W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.
SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Science
MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.
MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:
Students will understand that…
- Integer exponents can be used to represent very small or very large numerical values.
- Extremely small and large numbers can be displayed using integer exponents or conceptually scientific notation.
- Applying properties of integer exponents to simplify and write equivalent numerical expressions.

- Utilizing scientific notation to estimate and express the values of very large or very small numbers and compare their values.
- Performing operations with numbers written in scientific notation.
- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.
- Construct and compare linear and exponential models and solve problems.

Unit Essential Questions:

- When can you apply integer exponents to real-world situations?
- Why is it useful to use scientific notation?
- What is the result of a base having a positive, a negative, or a zero exponent?
- How can you solve a system of linear equations?
- How can you use substitution and elimination to solve a system of linear equations?
- How can you use a system of linear equations to solve an equation with variables on both sides?
- How can you solve an exponential equation graphically?
- How are a function and its inverse related?

Knowledge and Skills:

Students will know...

- How to critique
  - reasoning about powers of powers.
  - reasoning about zero exponents.
  - applications of exponent rules.
  - reasoning about scientific notation.
- How to represent
  - situations using exponents.
  - large and small numbers using number lines, exponents, and decimals.
  - situations comparing quantities expressed in scientific notation.
- How to justify
  - reasoning about multiplying powers of 10.
  - reasoning about powers of powers.
  - reasoning about dividing powers of 10.
  - whether or not expressions are equivalent to exponential expressions.
  - reasoning about situations comparing powers of 10.
- Solving linear equations by multiple methods (graphing, substitution, or elimination).
- Solving linear equations with a different number of solutions.
- Graphing linear inequalities in two variables.
- Graphing and writing systems of linear inequalities.
- Writing equivalent expressions involving powers.
- Writing and evaluating the nth root of a number.
- Graphing and writing exponential functions.
- Writing and graph exponential growth and decay functions.
- Solving exponential equations.
- Identifying, extending, and graphing geometric sequences.
- Writing terms of recursively defined sequences and writing recursive rules for sequences.
- Vocabulary: exponent, base (of an exponent), reciprocal, scientific notation, system of linear equations, linear inequality in two variables, half-planes, system of linear inequalities, exponential function, exponential growth, exponential decay, exponential equation, nth roots, extraneous solutions, rational exponents, geometric sequences, square root function, radical function, cube root function, radical equation, inverse relation, and inverse function.

Students will be able to...

- Use exponent rules to generate equivalent numerical expressions for powers of 10.
- Use exponent rules to generate equivalent numerical expressions for expressions with different bases and bases other than 10.
- Compare very large or very small quantities expressed as a multiple of a power of 10.
- Use exponent rules and power of 10 to solve problems in context.
- Calculate with numbers in scientific notation and interpret them in context.
- Identify numbers written in scientific notation, including scientific notation that has been generated by technology.
- Identify a system of linear equations.
- Describe different methods for solving systems of linear equations.
- Analyze systems of linear equations and decide what solution method is most efficient.
- Predict whether a system of linear equations has one solution, no solution, or infinitely many solutions.
- Approximate the solution of a linear system using a graph.
- Solve real-life system problems using substitution and elimination.
- Compare and contrast solving systems of linear equations using substitution and elimination.
- Solve an absolute value equation by graphing.
- Assess solutions of a linear inequality in two variables in a real-life situation.
- Discover whether an ordered pair is a solution to a system of linear inequalities.
- Identify and use properties of exponents.
- Describe exponential functions.
- Analyze data, a graph, or a context to determine whether it represents exponential growth or decay.
- Model using an exponential function or a geometric sequence.
- Simplify expressions using properties of exponents.
- Evaluate and solve expressions involving rational exponents.
- Model real-life problems using exponential functions.
- Write exponential growth functions and exponential decay functions to solve real-life situations.
- Differentiate whether a sequence is arithmetic, geometric, or neither.
- Translate between recursive rules and explicit rules.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Broken Coin - The broken coin prompts students to think about repeated division, laying the foundation for later work on negative exponents. Understanding repeated division by 2 as being equivalent to repeated multiplication by ½ will later allow students to make sense of negative exponents.
- Lesson 3: Big Cube - The purpose of this warm-up is to introduce students to the idea of raising a value with an exponent to another power. Computing the volume of a cube whose side lengths are themselves powers of 10 introduces the basic structure of a power to a power, which will lead to a general exponent rule during later activities.
- Lesson 4: Zero Exponent - Students extend exponent rules to discover why it makes sense to define 10^0 as 1. Students create viable arguments and critique the reasoning of others when discussing the argument that 10^0 should be 0.
- Lesson 7: Exponent Rule Practice - This activity develops procedural fluency with exponent rules and encourages students to think about their own learning. Students choose 6 of 12 possible problems to solve, thereby identifying problems that they consider more difficult versus less difficult. Notice which problems students choose more than others, and which problems are skipped more than others. The first set of problems checks whether students can apply the exponent rules procedurally. The next set of problems checks whether students understand what negative exponents mean. The last set of problems asks students to evaluate exponents to check whether they understand the meaning of the zero exponent and the definition of exponents as repeated multiplication (by the base, or by the reciprocal of the base in the case of negative exponents).
- Lesson 10: The Speed of Light - This activity guides students to thinking in terms of scientific notation while investigating the properties of light. A number line that shows a power of 10 partitioned into 10 equal intervals is again used to illustrate the base-ten structure. Plotting numbers along it gives a clearer meaning to expressions that are a product of a single digit and a power of 10.
- Lesson 12: Meter Sticks to the Moon - The large quantities involved in these questions lend themselves to arithmetic with powers of 10, giving students the opportunity to make use of scientific notation before it is formally introduced. This activity was designed so students could practice modeling skills such as identifying essential features of the problem and gathering the required information. Students use powers of 10 and the number line as tools to make it easier to calculate and interpret results.
- Lesson 13: Info Gap: Distances in the Solar System - In this info gap activity, students continue to use scientific notation as a tool for working with small and large numbers—to describe quantities, make estimates, and make comparisons (e.g., to express how many times as much one is as the other).

RESOURCES

Teacher Resources:

- **Textbook:** Larson, R. and Boswell, L. (2022). Algebra 1. Erie, PA: Big Ideas Learning, LLC.
- Illustrative Math (IM) Unit 7.7 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.7 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 8 OVERVIEW

Content Area: Mathematics

Unit Title: Polynomial Equations, Quadratic Functions, Pythagorean Theorem, and Irrational Numbers

Target Course/Grade Level: Accelerated Mathematics/Grade 7

Unit Summary: In this unit, students work with geometric and symbolic representations of square and cube roots. They understand and use notation such as $\sqrt{2}$ and $\sqrt[3]{5}$ for square and cube roots. They understand the terms "rational number" and "irrational number," using long division to express fractions as decimals. They use their understanding of fractions to plot rational numbers on the number line and their understanding of approximation of irrationals by rationals to approximate the number-line location of a given irrational. Students learn (without proof) that $\sqrt{2}$ is irrational. They understand two proofs of the Pythagorean Theorem—an algebraic proof that involves manipulation of two expressions for the same area and a geometric proof that involves decomposing and rearranging two squares. They use the Pythagorean Theorem in two and three dimensions, e.g., to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane. Students will perform basic operations with polynomials and factoring polynomials to solve equations and reveal roots of polynomials. Students will be able to graph quadratic functions. Students will analyze different forms of quadratic functions to identify characteristics. Standard form and vertex form are developed by transforming the parent function f(x) = x^2. Intercept form is developed from identifying the x-intercepts of the graph. Students will be able to solve quadratic equations using a variety of methods: graphing, using square roots, completing the square, and using the Quadratic Formula. Students will consider which method is most efficient as they learn new methods. Students will also solve nonlinear systems of equations.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

8.NS.A.1 Know that there are numbers that are not rational, and approximate them by rational numbers. 1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.

a. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

b. Simplify numerical radicals, limiting to square roots (i.e. non perfect squares). For example, simplify $\sqrt{8}$ to $2\sqrt{2}$.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N.RN.A.3 Simplify radicals, including algebraic radicals (e.g. $\sqrt[3]{54} = 3\sqrt[3]{2}$, simplify $\sqrt{32x^2}$).

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for x^2=49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a+bi for real numbers a and b.

A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where and/or are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

L.SS.7.1 Demonstrate command of the system and structure of the English language when writing or speaking.

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

Science

MS-PS1-5 Develop and use a model to describe how the total number of atoms does not change in a chemical reaction and thus mass is conserved.

MS-PS4-1 Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

Unit Understandings:

Students will understand that…

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve quadratic equations and inequalities in one variable.
- Build a function that models a relationship between two quantities.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.
- Construct and compare polynomial models and solve problems.
- Rational numbers can be written as a ratio, whereas irrational numbers cannot be written as a ratio.
- Calculating square roots of perfect squares and identifying them as rational.
- The Pythagorean Theorem is a formula that can determine a missing side of a right triangle if the other two side measurements are known.
- Volume formulas calculate the amount of space enclosed by three-dimensional figures and can be applied to real-life situations.

Unit Essential Questions:

- How can you solve a polynomial equation?
- How can you recognize and factor special products?
- How can you compare the growth rates of linear, exponential, and quadratic functions?
- How can you multiply and divide square roots?
- How can you use a graph to solve a quadratic equation in one variable?
- How can you derive a formula that can be used to write the solutions of any quadratic equation in standard form?
- What is the difference between rational and irrational numbers?
- Where and when can the Pythagorean Theorem be applied?

Knowledge and Skills:

Students will know...

- How to explain
  - strategies for finding area.
  - strategies for approximating and finding square roots.
  - strategies for finding triangle side lengths.
  - predictions about situations involving right triangles and strategies to verify.
  - strategies for finding distances between points on a coordinate plane.
  - strategies for approximating the value of cube roots.
- How to represent
  - square roots on a number line.
  - relationships between side lengths and areas.
  - cube edge lengths and volumes using cube roots.
  - cube roots on a number line.
  - rational numbers as ratios, decimal expansions, and on a number line.

- How to justify
    - which squares have side lengths in a given range.
    - ordering of irrational numbers.
    - ordering of hypotenuse lengths.
- Graphing and describing square root functions.
- Graphing and describing cube root functions.
- Solving radical equations and identifying any extraneous solutions.
- Understanding the relationship between inverse functions.
- Adding, subtracting, multiplying, and dividing polynomials.
- Utilizing patterns to find products of polynomials.
- Solving polynomial equations in factored form.
- Factoring polynomial expressions using the Greatest Common Factor.
- Factoring trinomial expressions when a is one and when a is not 1.
- Recognizing and factoring special products (difference of two squares and perfect square trinomials).
- Factoring a polynomial by grouping.
- Recognizing when a polynomial is factored completely.
- Graphing and describing quadratic functions in any form (monomial, binomial, or trinomial quadratics; vertex form; intercept form).
- Comparing the characteristics of linear, exponential, and quadratic functions.
- Utilize properties of radicals to write equivalent expressions.
- Utilize graphs to solve quadratic equations and find zeros of functions.
- Solving quadratic equations using square roots.
- Solving quadratic equations and finding maximum and minimum values of quadratic functions by completing the square.
- Utilizing the Quadratic Formula and its discriminant to solve and analyze quadratic equations.
- Solving nonlinear systems graphically and algebraically.
- Vocabulary: square root, irrational number, rational number, Pythagorean Theorem, hypotenuse, legs, cube root, repeating decimal, monomial, binomial, trinomial, leading coefficient, zeros of a function, vertex, axis of symmetry, maximum value, minimum value, average rate of change, quadratic functions, quadratic formula, radical expression, rationalizing the denominator, like radicals, and system of nonlinear equations.

Students will be able to...

- Identify domains and ranges of radical functions.
- Graph square root and cube root functions.
- Solve radical equations.
- Find inverses of relations and functions.
- Graph and describe transformations of square root and cube root functions.
- Investigate square root and cube root functions to solve real-life problems.
- Conclude whether the inverse of a function is also a function.
- Comprehend the term “irrational number” to mean a number that is not rational and that is an example of an irrational number.
- Comprehend the term “square root of” and the notation to mean the side length of a square whose area is square units.
- Use the square root symbol to represent solutions to equations of the form and represent the square root as a point on the number line.
- Calculate the distance between two points in the coordinate plane by using the Pythagorean Theorem.
- Explain an area-based algebraic proof of the Pythagorean Theorem.

- Use the Pythagorean Theorem to calculate unknown side lengths of right triangles and to solve problems within a context.
- Coordinate representations of a cube root, including cube root notation, decimal representation, the edge length of a cube or given volume, and a point on the number line.
- Represent rational numbers as equivalent decimals and fractions.
- Identify a system of linear equations.
- Describe different methods for solving systems of linear equations.
- Analyze systems of linear equations and decide what solution method is most efficient.
- Predict whether a system of linear equations has one solution, no solution, or infinitely many solutions.
- Approximate the solution of a linear system using a graph.
- Solve real-life system problems using substitution and elimination.
- Compare and contrast solving systems of linear equations using substitution and elimination.
- Solve an absolute value equation by graphing.
- Assess solutions of a linear inequality in two variables in a real-life situation.
- Discover whether an ordered pair is a solution to a system of linear inequalities.
- Identify and use properties of exponents.
- Describe exponential functions.
- Analyze data, a graph, or a context to determine whether it represents exponential growth or decay.
- Model using an exponential function or a geometric sequence.
- Simplify expressions using properties of exponents.
- Evaluate and solve expressions involving rational exponents.
- Model real-life problems using exponential functions.
- Write exponential growth functions and exponential decay functions to solve real-life situations.
- Differentiate whether a sequence is arithmetic, geometric, or neither.
- Translate between recursive rules and explicit rules.
- Identify domains and ranges of radical functions.
- Graph square root and cube root functions.
- Solve radical equations.
- Find inverses of relations and functions.
- Graph and describe transformations of square root and cube root functions.
- Investigate square root and cube root functions to solve real-life problems.
- Conclude whether the inverse of a function is also a function.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 2: One Square - The purpose of this activity is for students to estimate the side length of a square via a geometric construction that relates the side length of the square to a point on the number line, and verify their estimate using techniques from the previous lesson. Once students connect the side length to a point on the number line, they learn that this number has a name and a special notation to denote it: square root and the square root symbol. While this is students' first formal introduction to square roots, they will have many opportunities to deepen their understanding of square roots and practice using square root notation in later activities and lessons.
- Lesson 4: Solutions on a Number Line - The purpose of this activity is for students to use rational approximations of irrational numbers to place both rational and irrational numbers on a number line and to reinforce the definition of a square root as a solution to the equation of the form $x^2 = a$. This is also the first time that students have thought about negative square roots.
- Lesson 6: Adding Up Areas - The purpose of this activity is for students to work through an area-based algebraic proof of the Pythagorean Theorem. One of the figures used in this particular proof was first encountered by students at the start of the year during a unit on transformations and again in a recent lesson where they reasoned about finding the area of the triangles.
- Lesson 8: Cutting Corners - The purpose of this activity is for students to use the Pythagorean Theorem to reason about distances and speeds to figure out who will win a race. Students must translate between the context and the geometric representation of the context and back. Identify students whose work is clearly labeled and organized to share during the whole-class discussion.
- Lesson 10: Cube Roots Values - The purpose of this activity is for students to think about cube roots in relation to the two whole number values they are closest to. Students are encouraged to use the fact that $\sqrt[3]{a}$ is a solution to the equation $x^3 = a$. Students can draw a number line if that helps them reason about the magnitude of the given cube roots, but this is not required. However students reason, they need to explain their thinking.
- Lesson 11: Recalculating Rational Numbers - The purpose of this task is for students to rewrite rational numbers with terminating decimal expansions in fraction form and fractions with terminating decimal expansions as decimals. This activity is the first of a series of three in which students rewrite numbers in different ways, supporting their understanding of what rational and irrational numbers are and how they can be represented.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, R. and Boswell, L. (2022). Algebra 1. Erie, PA: Big Ideas Learning, LLC.
- Illustrative Math (IM) Unit 7.8 Accelerated
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.8 Accelerated Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper

- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.