ESL
SCIENCE
BUSINESS
BILINGUAL
PRESCHOOL
MATHEMATICS
LIBRARY MEDIA
SOCIAL STUDIES
WORLD LANGUAGES
GIFTED & TALENTED
TECHNOLOGY EDUCATION
ENGLISH LANGUAGE ARTS
FINE & PERFORMING ARTS
FAMILY & CONSUMER SCIENCE
HEALTH & PHYSICAL EDUCATION

RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Advanced Placement Statistics

Grade Level: 11-12

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

Jeffrey Kurczeski,

Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

The Board acknowledges the following who contributed to the preparation of this curriculum.

Leslie Breen, Mathematics Teacher

Dr. Tiffany A. Beer, Director of Curriculum and Instruction

Dr. Aleya Shoieb, Superintendent of Schools

Subject/Course Title:
Advanced Placement Statistics
Grades 11-12

Date of Board Adoption:
August 27, 2024

RAHWAY PUBLIC SCHOOLS CURRICULUM

Advanced Placement Statistics: Grades 11-12

PACING GUIDE

Unit	Title	Pacing
1	Exploring One-Variable Data	3 weeks
2	Exploring Two-Variable Data	2 weeks
3	Collecting Data	2 weeks
4	Probability, Random Variables and Probability Distributions	4 weeks
5	Sampling Distributions	3 weeks
6	Inference for Categorical Data: Proportions	4 weeks
7	Inference for Quantitative Data: Means	3 weeks
8	Inference for Categorical Data: Chi-Squared	2 weeks
9	Inference for Quantitative Data: Slope	2 weeks

ACCOMMODATIONS

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504 Accommodations:
- Provide scaffolded vocabulary and vocabulary lists.
- Provide extra visual and verbal cues and prompts.
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Weekly home-school communication tools (notebook, daily log, phone calls or email messages).
- Provide study sheets and teacher outlines prior to assessments.
- Quiet corner or room to calm down and relax when anxious.
- Reduction of distractions.
- Permit answers to be dictated.
- Hands-on activities.
- Use of manipulatives.
- Assign preferential seating.
- No penalty for spelling errors or sloppy handwriting.
- Follow a routine/schedule.
- Provide student with rest breaks.
- Use verbal and visual cues regarding directions and staying on task.
- Assist in maintaining agenda book.

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IEP Accommodations:
- Provide scaffolded vocabulary and vocabulary lists.
- Differentiate reading levels of texts (e.g., Newsela).
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide extra visual and verbal cues and prompts.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Provide students with additional information to supplement notes.
- Modify questioning techniques and provide a reduced number of questions or items on tests.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Provide study sheets and teacher outlines prior to assessments.
- Use of manipulatives.
- Have students work with partners or in groups for reading, presentations, assignments, and analyses.
- Assign appropriate roles in collaborative work.
- Assign preferential seating.
- Follow a routine/schedule.

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Gifted and Talented Accommodations:
- Differentiate reading levels of texts (e.g., Newsela).
- Offer students additional texts with higher lexile levels.
- Provide more challenging and/or more supplemental readings and/or activities to deepen understanding.
- Allow for independent reading, research, and projects.
- Accelerate or compact the curriculum.
- Offer higher-level thinking questions for deeper analysis.
- Offer more rigorous materials/tasks/prompts.
- Increase number and complexity of sources.
- Assign group research and presentations to teach the class.

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ML Accommodations:
- Provide extended time.
- Assign preferential seating.
- Assign peer buddy who the student can work with.
- Check for understanding frequently.
- Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc…).
- Have student repeat directions.
- Make vocabulary words available during classwork and exams.
- Use study guides/checklists to organize information.
- Repeat directions.
- Increase one-on-one conferencing.
- Allow student to listen to an audio version of the text.

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● Assign/allow for leadership roles during collaborative work and in other learning activities.	● Give directions in small, distinct steps. ● Allow copying from paper/book. ● Give student a copy of the class notes. ● Provide written and oral instructions. ● Differentiate reading levels of texts (e.g., Newsela). ● Shorten assignments. ● Read directions aloud to student. ● Give oral clues or prompts. ● Record or type assignments. ● Adapt worksheets/packets. ● Create alternate assignments. ● Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted. ● Allow student to resubmit assignments. ● Use small group instruction. ● Simplify language. ● Provide scaffolded vocabulary and vocabulary lists. ● Demonstrate concepts possibly through the use of visuals. ● Use manipulatives. ● Emphasize critical information by highlighting it for the student. ● Use graphic organizers. ● Pre-teach or pre-view vocabulary. ● Provide student with a list of prompts or sentence starters that they can use when completing a written assignment. ● Provide audio versions of the textbooks. ● Highlight textbooks/study guides. ● Use supplementary materials. ● Give assistance in note taking ● Use adapted/modified textbooks. ● Allow use of computer/word processor. ● Allow student to answer orally, give extended time (time-and-a-half). ● Allow tests to be given in a separate location (with the ESL teacher). ● Allow additional time to complete assignments and/or assessments. ● Read question to student to clarify. ● Provide a definition or synonym for words on a test that do not impact the validity of the exam. ● Modify the format of assessments. ● Shorten test length or require only selected test items. ● Create alternative assessments. ● On an exam other than a spelling test, don’t take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Exploring One-Variable Data

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Unit 1 introduces students to data and the vocabulary of statistics. Students also learn to talk about data in real-world contexts. Variability in data may seem to suggest certain conclusions about the data distribution, but not all variation is meaningful. Statistics allows us to develop shared understandings of uncertainty and variation. In this unit, students will define and represent categorical and quantitative variables, describe and compare distributions of one-variable data, and interpret statistical calculations to assess claims about individual data points or samples. Students will also begin to apply the normal distribution model as an introduction to how theoretical models for populations can be used to describe some distributions of sample data. Later units will more fully develop probabilistic modeling and inference.

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:
S.ID.A.1 Summarize, represent, and interpret data on a single count or measurement variable**.**
S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

College Board Standards:
VAR-1.A Identify questions to be answered, based on variation in one-variable data.
VAR-1.B Identify variables in a set of data.
VAR-1.C Classify types of variables.
UNC-1.A Represent categorical data using frequency or relative frequency tables.
UNC-1.B Describe categorical data represented in frequency or relative tables.
UNC-1.C Represent categorical data graphically.
UNC-1.D Describe categorical data represented graphically.
UNC-1.E Compare multiple sets of categorical data.
UNC-1.F Classify types of quantitative variables.
UNC-1.G Represent quantitative data graphically.
UNC-1.H Describe the characteristics of quantitative data distributions.
UNC-1.I Calculate measures of center and position for quantitative data.

UNC-1.J Calculate measures of variability for quantitative data.
UNC-1.K Explain the selection of a particular measure of center and/or variability for describing a set of quantitative data.
UNC-1.L Represent summary statistics for quantitative data graphically
UNC-1.M Describe summary statistics of quantitative data represented graphically.
UNC-1.N Compare graphical representations for multiple sets of quantitative data.
UNC-1.O Compare summary statistics for multiple sets of quantitative data.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…
- Given that variation may be random or not, conclusions are uncertain.
- Graphical representations and statistics allow us to identify and represent key features of data.
- The normal distribution can be used to represent some population distributions.

Unit Essential Questions:
- What is statistics?
- What can we learn from data?

Knowledge and Skills:
Students will know…
- A categorical variable takes on values that are category names or group labels.
- A quantitative variable is one that takes on numerical values for a measured or counted quantity.
- A frequency table gives the number of cases falling into each category. A relative frequency table gives the proportion of cases falling into each category.
- Percentages, relative frequencies, and rates all provide the same information as proportions.
- Counts and relative frequencies of categorical data reveal information that can be used to justify claims about the data in context.
- Bar charts (or bar graphs) are used to display frequencies (counts) or relative frequencies (proportions) for categorical data.
- The height or length of each bar in a bar graph corresponds to either the number or proportion of observations falling within each category.
- There are many additional ways to represent frequencies (counts) or relative frequencies (proportions) for categorical data.
- Graphical representations of a categorical variable reveal information that can be used to justify claims about the data in context.
- Frequency tables, bar graphs, or other representations can be used to compare two or more data sets in terms of the same categorical variable.
- A discrete variable can take on a countable number of values. The number of values may be finite or countably infinite, as with the counting numbers.
- A continuous variable can take on infinitely many values, but those values cannot be counted. No matter how small the interval between two values of a continuous variable, it is always possible to determine another value between them.
- In a histogram, the height of each bar shows the number or proportion of observations that fall within the interval corresponding to that bar. Altering the interval widths can change the appearance of the histogram.
- In a stem and leaf plot, each data value is split into a "stem" (the first digit or digits) and a "leaf" (usually the last digit).
- A dotplot represents each observation by a dot, with the position on the horizontal axis corresponding to the data value of that observation, with nearly identical values stacked on top of each other.
- A cumulative graph represents the number or proportion of a data set less than or equal to a given number.
- There are many additional ways to graphically represent distributions of quantitative data.
- Descriptions of the distribution of quantitative data include shape, center, and variability (spread), as well as any unusual features such as outliers, gaps, clusters, or multiple peaks.
- Outliers for one-variable data are data points that are unusually small or large relative to the rest of the data.

- A distribution is skewed to the right (positive skew) if the right tail is longer than the left. A distribution is skewed to the left (negative skew) if the left tail is longer than the right. A distribution is symmetric if the left half is the mirror image of the right half.
- Univariate graphs with one main peak are known as unimodal. Graphs with two prominent peaks are bimodal. A graph where each bar height is approximately the same (no prominent peaks) is approximately uniform
- A gap is a region of a distribution between two data values where there are no observed data.
- Clusters are concentrations of data usually separated by gaps.
- Descriptive statistics does not attribute properties of a data set to a larger population, but may provide the basis for conjectures for subsequent testing.
- A statistic is a numerical summary of sample data.
- The mean is the sum of all the data values divided by the number of values. For a sample, the mean is denoted by x-bar :$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where x_i represents the ith data point in the sample and *n* represents the number of data values in the sample.
- The median of a data set is the middle value when data are ordered. When the number of data points is even, the median can take on any value between the two middle values.
- The first quartile, Q1, is the median of the half of the ordered data set from the minimum to the position of the median. The third quartile, Q3, is the median of the half of the ordered data set from the position of the median to the maximum. Q1 and Q3 form the boundaries for the middle 50% of values in an ordered data set.
- The pth percentile is interpreted as the value that has p% of the data less than or equal to it.
- Three commonly used measures of variability (or spread) in a distribution are the range, interquartile range, and standard deviation.
- The range is defined as the difference between the maximum data value and the minimum data value. The interquartile range (IQR) is defined as the difference between the third and first quartiles: Q3 − Q1. Both the range and the interquartile range are possible ways of measuring variability of the distribution of a quantitative variable.
- Standard deviation is a way to measure variability of the distribution of a quantitative variable. For a sample, the standard deviation is denoted by s: $s_x = \sqrt{\frac{1}{n-1}\sum (x_1 - \overline{x})^2}$. The square of the sample standard s^2 , is called the sample variance.
- Changing units of measurement affects the values of the calculated statistics.
- There are many methods for determining outliers. Two methods frequently used in this course are:
  i. An outlier is a value greater than 1.5 × IQR above the third quartile or more than 1.5 × IQR below the first quartile.
  ii. An outlier is a value located 2 or more standard deviations above, or below, the mean.
- The mean, standard deviation, and range are considered nonresistant (or non-robust) because they are influenced by outliers. The median and IQR are considered resistant (or robust), because outliers do not greatly (if at all) affect their value.
- Taken together, the minimum data value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum data value make up the five-number summary.
- A boxplot is a graphical representation of the five-number summary (minimum, first quartile, median, third quartile, maximum). The box represents the middle 50% of data, with a line at the median and the ends of the box corresponding to the quartiles. Lines ("whiskers") extend from the quartiles to the most extreme point that is not an outlier, and outliers are indicated by their own symbol beyond this.

- Summary statistics of quantitative data, or of sets of quantitative data, can be used to justify claims about the data in context.
- If a distribution is relatively symmetric, then the mean and median are relatively close to one another. If a distribution is skewed right, then the mean is usually to the right of the median. If the distribution is skewed left, then the mean is usually to the left of the median.
- Any of the graphical representations, e.g., histograms, side-by-side boxplots, etc., can be used to compare two or more independent samples on center, variability, clusters, gaps, outliers, and other features.
- Any of the numerical summaries (e.g., mean, standard deviation, relative frequency, etc.) can be used to compare two or more independent samples.
- A parameter is a numerical summary of a population.
- Some sets of data may be described as approximately normally distributed. A normal curve is mound-shaped and symmetric. The parameters of a normal distribution are the population mean, μ, and the population standard deviation, σ.
- For a normal distribution, approximately 68% of the observations are within 1 standard deviation of the mean, approximately 95% of observations are within 2 standard deviations of the mean, and approximately 99.7% of observations are within 3 standard deviations of the mean. This is called the empirical rule.
- Many variables can be modeled by a normal distribution.
- A standardized score for a particular data value is calculated as (data value – mean)/(standard deviation), and measures the number of standard deviations a data value falls above or below the mean.
- One example of a standardized score is a z-score, which is calculated as z -score = $(\frac{x_i - \mu}{\sigma})$.
  A z-score measures how many standard deviations a data value is from the mean.
- Technology, such as a calculator, a standard normal table, or computer-generated output, can be used to find the proportion of data values located on a given interval of a normally distributed random variable.
- Given the area of a region under the graph of the normal distribution curve, it is possible to use technology, such as a calculator, a standard normal table, or computer-generated output, to estimate parameters for some populations.
- Percentiles and z-scores may be used to compare relative positions of points within a data set or between data sets.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Describe data presented numerically or graphically.
- Construct numerical or graphical representations of distributions.
- Compare distributions or relative positions of points within a distribution.
- Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Gallery Walk:** Have students work in groups of four to construct a dotplot, a stem-and-leaf plot, a histogram, or a boxplot for a set of student-generated data (e.g., time in minutes to get to school). After the gallery walk, discuss what information can be seen more easily in each graph (e.g., boxplots can easily show the IQR).
    - **Notice and Wonder:** Display just the graphs from 2018 FRQ 5. Have students think individually for one minute about how the graphs compare. Then ask them, "What do you notice? What do you wonder? What questions could be answered with these graphs?" Have students share their ideas with a partner then debrief the ideas as a class.
    - **Reversing Interpretations:** Give pairs of students four pictures of normal distributions with various parts shaded. Have students create the question that could have resulted in the picture shown (e.g., if a value of 15 is labeled and the distribution is shaded to the right of 15, students could write "What is the probability that a value is more than 15?").

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Exploring Two-Variable Data

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Building on Unit 1, students will explore relationships in two-variable categorical or quantitative data sets. They will use graphical and numerical methods to investigate an association between two categorical variables. Skills learned while working with two-way tables will transfer to calculating probabilities in Unit 4. Students will describe form, direction, strength, and unusual features for an association between two quantitative variables. They will assess correlation and, if appropriate, use a linear model to predict values of the response variable from values of the explanatory variable. Students will interpret the least-squares regression line in context, analyze prediction errors (residuals), and explore departures from a linear pattern.

Approximate Length of Unit: 2 weeks

LEARNING TARGETS

NJ Student Learning Standards:
S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

College Board Standards:
VAR-1.D Identify questions to be answered about possible relationships in data.
UNC-1.P Compare numerical and graphical representations for two categorical variables.
UNC-1.Q Calculate statistics for two categorical variables.
UNC-1.R Compare statistics for two categorical variables.
UNC-1.S Represent bivariate quantitative data using scatterplots.
DAT-1.A Describe the characteristics of a scatter plot.
DAT-1.B Determine the correlation for a linear relationship.
DAT-1.C Interpret the correlation for a linear relationship.
DAT-1.D Calculate a predicted response value using a linear regression model.
DAT-1.E Represent differences between measured and predicted responses using residual plots.
DAT-1.F Describe the form of association of bivariate data using residual plots.
DAT-1.G Estimate parameters for the least-squares regression line mode.
DAT-1.H Interpret coefficients for the least-squares regression line model.
DAT-1.I Identify influential points in regression.
DAT-1.J Calculate a predicted response using a least squares regression line for a transformed data set.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:

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RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- Graphical representations and statistics allow us to identify and represent key features of data.
- Regression models may allow us to predict responses to changes in an explanatory variable.
- Interpret coefficients for the least-squares regression line model.

- Calculate a predicted response using a least squares regression line for a transformed data set.

Unit Essential Question:
- Are variables related?

Knowledge and Skills:
Students will know...
- Apparent patterns and associations in data may be random or not.
- Side-by-side bar graphs, segmented bar graphs, and mosaic plots are examples of bar graphs for one categorical variable, broken down by categories of another categorical variable.
- Graphical representations of two categorical variables can be used to compare distributions and/or determine if variables are associated.
- A two-way table, also called a contingency table, is used to summarize two categorical variables. The entries in the cells can be frequency counts or relative frequencies.
- A joint relative frequency is a cell frequency divided by the total for the entire table.
- The marginal relative frequencies are the row and column totals in a two-way table divided by the total for the entire table.
- A conditional relative frequency is a relative frequency for a specific part of the contingency table (e.g., cell frequencies in a row divided by the total for that row).
- Summary statistics for two categorical variables can be used to compare distributions and/or determine if variables are associated.
- A bivariate quantitative data set consists of observations of two different quantitative variables made on individuals in a sample or population.
- A scatterplot shows two numeric values for each observation, one corresponding to the value on the x-axis and one corresponding to the value on the y-axis.
- An explanatory variable is a variable whose values are used to explain or predict corresponding values for the response variable.
- A description of a scatter plot includes form, direction, strength, and unusual features.
- The direction of the association shown in a scatterplot, if any, can be described as positive or negative.
- A positive association means that as values of one variable increase, the values of the other variable tend to increase. A negative association means that as values of one variable increase, values of the other variable tend to decrease.
- The form of the association shown in a scatterplot, if any, can be described as linear or non-linear to varying degrees.
- The strength of the association is how closely the individual points follow a specific pattern, e.g., linear, and can be shown in a scatterplot. Strength can be described as strong, moderate, or weak.
- Unusual features of a scatter plot include clusters of points or points with relatively large discrepancies between the value of the response variable and a predicted value for the response variable.
- The correlation, r, gives the direction and quantifies the strength of the linear association between two quantitative variables.
- The correlation coefficient can be calculated by: $r = \frac{1}{n-1}\sum(\frac{x_i - \bar{x}}{s_x})(\frac{y_i - \bar{y}}{s_y})$. However, the most common way to determine r is by using technology.
- A correlation coefficient close to 1 or −1 does not necessarily mean that a linear model is appropriate.
- The correlation, r, is unit-free, and always between −1 and 1, inclusive. A value of r 0 indicates that there is no linear association. A value of r 1 or r 1 indicates that there is a perfect linear association.

- A perceived or real relationship between two variables does not mean that changes in one variable cause changes in the other. That is, correlation does not necessarily imply causation.
- A simple linear regression model is an equation that uses an explanatory variable, *x,* to predict the response variable, *y*.
- The predicted response value, denoted by $\hat{y}$, is calculated as $\hat{y}$ = +a bx, where a is the y-intercept and b is the slope of the regression line, and x is the value of the explanatory variable.
- Extrapolation is predicting a response value using a value for the explanatory variable that is beyond the interval of x-values used to determine the regression line. The predicted value is less reliable as an estimate the further we extrapolate.
- The residual is the difference between the actual value and the predicted value: residual = $y - \hat{y}$.
- A residual plot is a plot of residuals versus explanatory variable values or predicted response values.
- Apparent randomness in a residual plot for a linear model is evidence of a linear form to the association between the variables.
- Residual plots can be used to investigate the appropriateness of a selected model.
- The least-squares regression model minimizes the sum of the squares of the residuals and contains the point $(\bar{x},\bar{y})$.
- The slope, b, of the regression line can be calculated as $b = r(\frac{s_y}{s_x})$ where *r* is the correlation between x and y, s_y is the sample standard deviation of the response variable, *y*, and s_x is the sample standard deviation of the explanatory variable, *x*.
- Sometimes, the y-intercept of the line does not have a logical interpretation in context.
- In simple linear regression, r^2 is the square of the correlation, r. It is also called the coefficient of determination. r^2 is the proportion of variation in the response variable that is explained by the explanatory variable in the model.
- The coefficients of the least-squares regression model are the estimated slope and y-intercept.
- The slope is the amount that the predicted *y*-value changes for every unit increase in *x*.
- The y-intercept value is the predicted value of the response variable when the explanatory variable is equal to 0. The formula for the y-intercept, a, is $a = \bar{y} - b\bar{x}$.
- An outlier in regression is a point that does not follow the general trend shown in the rest of the data and has a large residual when the Least Squares Regression Line (LSRL) is calculated.
- A high-leverage point in regression has a substantially larger or smaller x-value than the other observations have.
- An influential point in regression is any point that, if removed, changes the relationship substantially. Examples include much different slopes, y-intercept, and/or correlation. Outliers and high leverage points are often influential.
- Transformations of variables, such as evaluating the natural logarithm of each value of the response variable or squaring each value of the explanatory variable, can be used to create transformed data sets, which may be more linear in form than the untransformed data.
- Increased randomness in residual plots after transformation of data and/or movement of r^2 to a value closer to 1 offers evidence that the least-squares regression line for the transformed data is a more appropriate model to use to predict responses to the explanatory variable than the regression line for the untransformed data.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Compare distributions or relative positions of points within a distribution.

- Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response.
- Construct numerical or graphical representations of distributions.
- Describe data presented numerically or graphically.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Quickwrite:** Give students a scatterplot and its associated computer output. Have them identify and describe the meaning of the following values in the context of the problem: slope, y-intercept, coefficient of determination, and standard error of the residuals. Also have them calculate the correlation and explain how they found it. Have students compare their write-ups in groups of three to four.
    - **Reversing Interpretations:** Instead of asking students to interpret a residual, give them the residual and the equation of the least-squares regression line and ask them to make a prediction for a particular observation (e.g., "One wolf in the pack had a length of 1.4 m and a residual of −9.87. What does that −9.87 tell us about that particular wolf?")
    - **Build the Model Solution:** Provide students with strips of paper containing portions of the model solution for 2018 FRQ 1 and have them work to assemble the phrases into a solution for the FRQ. Words can be grouped for part a, as follows: [The estimate of the intercept is] [72.95]. [It is] [estimated that] [the average time to] [finish checkout] [if there are no other customers in line] [is 72.95 seconds]. Additional numbers or phrases for part a could include [174.40], [is 174.50 seconds], and [the time to].
    - **Predict and Confirm:** Have students toss a handful of M&Ms and record how many land M side up. This is trial 1. Then have them remove the ones that were M side up. For trial 2, have students toss the remaining candies (the ones left over after removing the ones that landed M side up) and record how many land M side up on the second toss. Ask

students to think about the trend and make a prediction: Will it be linear? A scatterplot of many trials should show a nonlinear relationship.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Collecting Data

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Depending on how data are collected, we may or may not be able to generalize findings or establish evidence of causal relationships. For example, if random selection is not used to obtain a sample from a population, bias may result and statistics from the sample cannot be assumed to generalize to the population. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect. Students learn important principles of sampling and experimental design in this unit; they will learn about statistical inference in Units 6–9.

Approximate Length of Unit: 2 weeks

LEARNING TARGETS

NJ Student Learning Standards:
S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
S.ID.C.9 Distinguish between correlation and causation.
S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
S.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling
S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S.IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed).

College Board Standards:
VAR-1.E Identify questions to be answered about data collection methods.
DAT-2.A Identify the type of a study.
DAT-2.C Identify a sampling method, given a description of a study.
DAT-2.E Identify potential sources of bias in sampling methods.

VAR-3.A Identify the components of an experiment.
VAR-3.B Describe elements of a well-designed experiment.
VAR-3.C Compare experimental designs and methods.
VAR-3.D Explain why a particular experimental design is appropriate.
VAR-3.E Interpret the results of a well-designed experiment.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

HS-ETS1-4 Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- The way we collect data influences what we can and cannot say about a population.
- Well-designed experiments can establish evidence of causal relationships.

Unit Essential Questions:

- What do our data values tell us?
- Why might the data we collected not be valid for drawing conclusions about an entire population?

Knowledge and Skills:

Students will know...

- Methods for data collection that do not rely on chance result in untrustworthy conclusions.
- A population consists of all items or subjects of interest.
- A sample selected for study is a subset of the population.
- In an observational study, treatments are not imposed. Investigators examine data for a sample of individuals (retrospective) or follow a sample of individuals into the future collecting data (prospective) in order to investigate a topic of interest about the population. A sample survey is a type of observational study that collects data from a sample in an attempt to learn about the population from which the sample was taken.
- In an experiment, different conditions (treatments) are assigned to experimental units (participants or subjects).
- It is only appropriate to make generalizations about a population based on samples that are randomly selected or otherwise representative of that population.
- A sample is only generalizable to the population from which the sample was selected.
- It is not possible to determine causal relationships between variables using data collected in an observational study.
- When an item from a population can be selected only once, this is called sampling without replacement. When an item from the population can be selected more than once, this is called sampling with replacement.
- A simple random sample (SRS) is a sample in which every group of a given size has an equal chance of being chosen. This method is the basis for many types of sampling mechanisms. A few examples of mechanisms used to obtain SRSs include numbering individuals and using a random number generator to select which ones to include in the sample, ignoring repeats, using a table of random numbers, or drawing a card from a deck without replacement.
- A stratified random sample involves the division of a population into separate groups, called strata, based on shared attributes or characteristics (homogeneous grouping). Within each stratum a simple random sample is selected, and the selected units are combined to form the sample.
- A cluster sample involves the division of a population into smaller groups, called clusters. Ideally, there is heterogeneity within each cluster, and clusters are similar to one another in their composition. A simple random sample of clusters is selected from the population to form the sample of clusters. Data are collected from all observations in the selected clusters.
- A systematic random sample is a method in which sample members from a population are selected according to a random starting point and a fixed, periodic interval.
- A census selects all items/subjects in a population.

- There are advantages and disadvantages for each sampling method depending upon the question that is to be answered and the population from which the sample will be drawn.
- Bias occurs when certain responses are systematically favored over others.
- When a sample is comprised entirely of volunteers or people who choose to participate, the sample will typically not be representative of the population (voluntary response bias).
- When part of the population has a reduced chance of being included in the sample, the sample will typically not be representative of the population (undercoverage bias).
- Individuals chosen for the sample for whom data cannot be obtained (or who refuse to respond) may differ from those for whom data can be obtained (nonresponse bias).
- Problems in the data-gathering instrument or process result in response bias. Examples include questions that are confusing or leading (question wording bias) and self-reported responses.
- Non-random sampling methods (for example, samples chosen by convenience or voluntary response) introduce potential for bias because they do not use chance to select the individuals.
- The experimental units are the individuals (which may be people or other objects of study) that are assigned treatments. When experimental units consist of people, they are sometimes referred to as participants or subjects.
- An explanatory variable (or factor) in an experiment is a variable whose levels are manipulated intentionally. The levels or combination of levels of the explanatory variable(s) are called treatments.
- A response variable in an experiment is an outcome from the experimental units that is measured after the treatments have been administered.
- A confounding variable in an experiment is a variable that is related to the explanatory variable and influences the response variable and may create a false perception of association between the two.
- A well-designed experiment should include the following:
  a. Comparisons of at least two treatment groups, one of which could be a control group.
  b. Random assignment/allocation of treatments to experimental units.
  c. Replication (more than one experimental unit in each treatment group).
  d. Control of potential confounding variables where appropriate.
- In a completely randomized design, treatments are assigned to experimental units completely at random. Random assignment tends to balance the effects of uncontrolled (confounding) variables so that differences in responses can be attributed to the treatments
- Methods for randomly assigning treatments to experimental units in a completely randomized design include using a random number generator, a table of random values, drawing chips without replacement, etc.
- In a single-blind experiment, subjects do not know which treatment they are receiving, but members of the research team do, or vice versa.
- In a double-blind experiment neither the subjects nor the members of the research team who interact with them know which treatment a subject is receiving.
- A control group is a collection of experimental units either not given a treatment of interest or given a treatment with an inactive substance (placebo) in order to determine if the treatment of interest has an effect.
- The placebo effect occurs when experimental units have a response to a placebo.
- For randomized complete block designs, treatments are assigned completely at random within each block.
- Blocking ensures that at the beginning of the experiment the units within each block are similar to each other with respect to at least one blocking variable. A randomized block design helps to separate natural variability from differences due to the blocking variable.
- A matched pairs design is a special case of a randomized block design. Using a blocking variable, subjects (whether they are people or not) are arranged in pairs matched on relevant factors. Matched pairs may be formed naturally or by the experimenter. Every pair receives both

treatments by randomly assigning one treatment to one member of the pair and subsequently assigning the remaining treatment to the second member of the pair. Alternatively, each subject may get both treatments.
- There are advantages and disadvantages for each experimental design depending on the question of interest, the resources available, and the nature of the experimental units.
- Statistical inference attributes conclusions based on data to the distribution from which the data were collected.
- Random assignment of treatments to experimental units allows researchers to conclude that some observed changes are so large as to be unlikely to have occurred by chance. Such changes are said to be statistically significant.
- Statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.
- If the experimental units used in an experiment are representative of some larger group of units, the results of an experiment can be generalized to the larger group. Random selection of experimental units gives a better chance that the units will be representative.

Students will be able to...
- Identify the question to be answered or the problem to be solved.
- Describe an appropriate method for gathering and representing data.
- Make an appropriate claim or draw an appropriate conclusion.
- Identify key and relevant information to answer a question or solve a problem.
- Interpret statistical calculations and findings to assign meaning or assess a claim.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?
- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?
- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
  - **Graphic Organizer:** Provide students with a table listing all possible combinations of whether a study involves random sampling (yes or no) and random assignment (yes or no). Ask them to fill in each cell with both the type of conclusion that is appropriate (association or causation) and the generalizability of the results (to the population or to only those similar to the study participants).

    - **Odd One Out:** After modeling an odd one out example, have students form groups of four and give each of them a description of a statistical study. Explain that three of the studies are of the same type (observational or experimental) and one is different. Have students work together in their groups to determine which study is the odd one out and explain why.
    - **Password-Style Games:** After completing the lessons on sampling and surveying, use the following 10 terms in a password-style game: census, simple random sample, stratified random sample, cluster sample, systematic random sample, bias, voluntary response bias, undercoverage, nonresponse bias, and response bias. The winner is the pair whose partner guesses the most terms correctly from the descriptions given.
    - **Think-Pair-Share:** Provide students with a description of a well-designed experiment (e.g., 2010 FRQ 1) and ask them to individually identify the type of design, the experimental units, the treatments, and how the study addresses the principles of a well-designed experiment (including random assignment, control, blinding, and replication). Then have students share their thoughts with their neighbor.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Probability, Random Variables and Probability Distributions

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and to make statistical inferences. Simulations and concrete examples can help students to understand the abstract definitions and calculations of probability. This unit builds on an understanding of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables. Interpretations of probabilities and parameters associated with a probability distribution should use appropriate units.

Approximate Length of Unit: 4 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.A.3 Understand the conditional probability of A given B as, $\frac{P(A \text{ and } B)}{P(B)}$ and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

S.CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S.CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

S.CP.B.8 Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$ and interpret the answer in terms of the model.

S.CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
S.MD.A.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
S.MD.A.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
S.MD.B.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
S.MD.B.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.B.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

College Board Standards:
VAR-1.F Identify questions suggested by patterns in data.
UNC-2.A Estimate probabilities using simulation
VAR-4.A Calculate probabilities for events and their complements.
VAR-4.B Interpret probabilities for events.
VAR-4.C Explain why two events are (or are not) mutually exclusive.
VAR-4.D Calculate conditional probabilities.
VAR-4.E Calculate probabilities for independent events and for the union of two events.
VAR-5.A Represent the probability distribution for a discrete random variable.
VAR-5.B Interpret a probability distribution.
VAR-5.C Calculate parameters for a discrete random variable.
VAR-5.D: Interpret parameters for a discrete random variable.
VAR-5.E Calculate parameters for linear combinations of random variables.
VAR-5.F Describe the effects of linear transformations of parameters of random variables.
UNC-3.A Estimate probabilities of binomial random variables using data from a simulation.
UNC-3.B Calculate probabilities for a binomial distribution.
UNC-3.C Calculate parameters for a binomial distribution.
UNC-3.D Interpret probabilities and parameters for a binomial distribution.
UNC-3.E Calculate probabilities for geometric random variables.
UNC-3.F Calculate parameters of a geometric distribution.
UNC-3.G Interpret probabilities and parameters for a geometric distribution.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2: Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3: Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- Simulation allows us to anticipate patterns in data.
- The likelihood of a random event can be quantified.
- Probability distributions may be used to model variation in populations.
- Probabilistic reasoning allows us to anticipate patterns in data.

Unit Essential Questions:

- How can an event be both random and predictable?
- What is randomness?
- What is the law of large numbers?

Knowledge and Skills:

Students will know...

- Patterns in data do not necessarily mean that variation is not random.
- A random process generates results that are determined by chance.
- An outcome is the result of a trial of a random process.
- An event is a collection of outcomes.
- Simulation is a way to model random events, such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance. Record the counts of simulated outcomes and the count total.

- The relative frequency of an outcome or event in simulated or empirical data can be used to estimate the probability of that outcome or event.
- The law of large numbers states that simulated (empirical) probabilities tend to get closer to the true probability as the number of trials increases
- The sample space of a random process is the set of all possible non-overlapping outcomes.
- If all outcomes in the sample space are equally likely, then the probability an event E will occur is defined as the fraction: $\frac{number\ of\ outcomes\ in\ event\ E}{total\ number\ of\ outcomes\ in\ sample\ space}$
- The probability of an event is a number between 0 and 1, inclusive.
- The probability of the complement of an event E, E′ or E^c , (i.e., not E) is equal to $1 - P(E)$.
- Probabilities of events in repeatable situations can be interpreted as the relative frequency with which the event will occur in the long run.
- The probability that events A and B both will occur, sometimes called the joint probability, is the probability of the intersection of A and B, denoted $P(A \cap B)$
- Two events are mutually exclusive or disjoint if they cannot occur at the same time. So $P(A \cap B) = 0$.
- The probability that event A will occur given that event B has occurred is called a conditional probability and denoted $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.
- The multiplication rule states that the probability that events A and B both will occur is equal to the probability that event A will occur multiplied by the probability that event B will occur, given that A has occurred. This is denoted $P(A \cap B) = P(A) \cdot P(B \mid A)$.
- Events A and B are independent if, and only if, knowing whether event A has occurred (or will occur) does not change the probability that event B will occur.
- If, and only if, events A and B are independent, then $P(A \mid B) = P(A)$, $P(B \mid A) = P(B)$, and
- $P(A \cap B) = P(A) \cdot P(B)$.
- The probability that event A or event B (or both) will occur is the probability of the union of A and B, denoted $P(A \cup B)$.
- The addition rule states that the probability that event A or event B or both will occur is equal to the probability that event A will occur plus the probability that event B will occur minus the probability that both events A and B will occur. This is denoted $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- The values of a random variable are the numerical outcomes of random behavior.
- A discrete random variable is a variable that can only take a countable number of values. Each value has a probability associated with it. The sum of the probabilities over all of the possible values must be 1.
- A probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.
- A cumulative probability distribution can be represented as a table or function showing the probability of being less than or equal to each value of the random variable.
- An interpretation of a probability distribution provides information about the shape, center, and spread of a population and allows one to make conclusions about the population of interest.
- A numerical value measuring a characteristic of a population or the distribution of a random variable is known as a parameter, which is a single, fixed value.

- The mean, or expected value, for a discrete random variable X is $\mu_x = \sum x_i \cdot P(x_i)$.

- The standard deviation for a discrete random variable X is $\sigma_x \sqrt{\sum (x_i - \mu_i)^2 \cdot P(x_i)}$.

- Parameters for a discrete random variable should be interpreted using appropriate units and within the context of a specific population.

- For random variables X and Y and real numbers a and b, the mean of the mean of aX + bY is $a\mu_x + b\mu_y$, and the variance of aX + bY is $a^2\sigma_x^2 + b^2\sigma_y^2$.
- For Y= a + bX , the probability distribution of the transformed random variable, Y, has the same shape as the probability distribution for X, so long as a > 0 and b > 0. The mean of Y is $\mu_y = a + b\mu_x$. The standard deviation of Y is $\sigma_y = |b|\sigma_x$.
- A probability distribution can be constructed using the rules of probability or estimated with a simulation using random number generators.
- A binomial random variable, X, counts the number of successes in n repeated independent trials, each trial having two possible outcomes (success or failure), with the probability of success p and the probability of failure 1 – p.
- The binomial probability function.
- If a random variable is binomial, its mean, μ_x, is np and its standard deviation, σ_x, is $\sqrt{np(1-p)}$.
- Probabilities and parameters for a binomial distribution should be interpreted using appropriate units and within the context of a specific population or situation.
- For a sequence of independent trials, a geometric random variable, X, gives the number of the trials on which the first success occurs. Each trial has two possible outcomes (success or failure) with the probability of success p and the probability of failure 1 – p.
- If a random variable is geometric, its mean, μ_x, is $\frac{1}{p}$ and its standard deviation, σ_x, is $\frac{\sqrt{1-p}}{p}$.
- The probability that the first success for repeated independent trials with probability of success p occurs on trial x is calculated as $P(X = x) = (1 - p)^{x-1}p,\ x = 1, 2, 3,...$ This is the geometric probability function.

Students will be able to...

- Identify the question to be answered or the problem to be solved.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Construct numerical or graphical representations of distributions.
- Determine parameters for probability distributions.
- Describe probability distributions.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Error Analysis:** Using 2015 FRQ 3, provide students with several answers containing errors for each part. Provide some responses with incorrect notation, incorrect work, missing work, work that shows calculator commands only, an incorrect formula or approach, and an incorrect final answer. Then ask students to identify the errors.
    - **Think-Pair-Share:** Provide students with a set of five probability questions: one for the complement rule, the conditional probability formula, the general multiplication rule, the multiplication rule for independent events, and the general addition rule. Ask students to individually identify the formula needed to solve each problem, without doing the final calculations. Then have them share their thoughts with a partner.
    - **Odd One Out:** After modeling an odd one out example, have students form groups of four and give each of them a description of either a binomial or a geometric random variable. Explain that three of their variables follow the same probability distribution and one is different. Have students work together in their groups to determine whose is the odd one out and explain why
    - **Predict and Confirm:** Ask students to consider couples who plan to continue having children until they have one girl and predict how many children they think these couples will have, on average. Then ask each student to perform 10 trials using a coin toss where Heads = Girl and Tails = Boy. A trial is finished once one girl is observed and the number of total children is recorded. Combine the class results and calculate the average. Confirm with the geometric mean formula once it is discussed.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 5 OVERVIEW

Content Area: Mathematics

Unit Title: Sampling Distributions

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: This unit applies probabilistic reasoning to sampling, introducing students to sampling distributions of statistics they will use when performing inference in Units 6 and 7. Students should understand that sample statistics can be used to estimate corresponding population parameters and that measures of center (mean) and variability (standard deviation) for these sampling distributions can be determined directly from the population parameters when certain sampling criteria are met. For large enough samples from any population, these sampling distributions can be approximated by a normal distribution. Simulating sampling distributions helps students to understand how the values of statistics vary in repeated random sampling from populations with known parameters.

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.A.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S.MD.A.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

S.MD.A.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

College Board Standards:

VAR-1.G Identify questions suggested by variation in statistics for samples collected from the same population.

VAR-6.A Calculate the probability that a particular value lies in a given interval of a normal distribution.

VAR-6.B Determine the interval associated with a given area in a normal distribution.

VAR-6.C Determine the appropriateness of using the normal distribution to approximate probabilities for unknown distributions.

UNC-3.H Estimate sampling distributions using simulation.

UNC-3.I Explain why an estimator is or is not unbiased.

UNC-3.J Calculate estimates for a population parameter.

UNC-3.K Determine parameters of a sampling distribution for sample proportions.

UNC-3.L Determine whether a sampling distribution for a sample proportion can be described as approximately normal.
UNC-3.M Interpret probabilities and parameters for a sampling distribution for a sample proportion
UNC-3.N Determine parameters of a sampling distribution for a difference in sample proportions.
UNC-3.O Determine whether a sampling distribution for a difference of sample proportions can be described as approximately normal.
UNC-3.P Interpret probabilities and parameters for a sampling distribution for a difference in proportions.
UNC-3.Q Determine parameters for a sampling distribution for sample means.
UNC-3.R Determine whether a sampling distribution of a sample mean can be described as approximately normal.
UNC-3.S Interpret probabilities and parameters for a sampling distribution for a sample mean.
UNC-3.T Determine parameters of a sampling distribution for a difference in sample means.
UNC-3.U Determine whether a sampling distribution of a difference in sample means can be described as approximately normal.
UNC-3.V Interpret probabilities and parameters for a sampling distribution for a difference in sample means

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- The normal distribution may be used to model variation.
- Probabilistic reasoning allows us to anticipate patterns in data.

Unit Essential Questions:

- How likely is it to get a value this large just by chance?
- How can we anticipate patterns in the values of a statistic from one sample to another?
- What is the Central Limit Theorem?

Knowledge and Skills:

Students will know...

- Variation in statistics for samples taken from the same population may be random or not..
- A continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.
- A continuous random variable with a normal distribution is commonly used to describe populations. The distribution of a normal random variable can be described by a normal, or "bell-shaped," curve.
- The area under a normal curve over a given interval represents the probability that a particular value lies in that interval.
- The boundaries of an interval associated with a given area in a normal distribution can be determined using z-scores or technology, such as a calculator, a standard normal table, or computer-generated output.
- Intervals associated with a given area in a normal distribution can be determined by assigning appropriate inequalities to the boundaries of the intervals:

  a.) $P(X < x_a) = \frac{p}{100}$ means that the lowest p% of values lie to the left of a x_a .

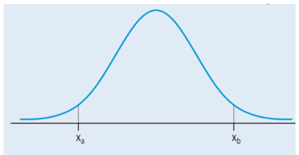
  b.) $P(x_a < X < x_b) = \frac{p}{100}$ means that the lowest p% of values between $x_a \; and \; x_b$.

  c.) $P(X > x_b) = \frac{p}{100}$ means that the lowest p% of values lie to the right of a x_a .

  d.) To determine the most extreme p% of values requires dividing the area associated with p% into two equal areas on either extreme of the distribution:

  $P(X < x_a) = \frac{1}{2}\frac{p}{100}$ and $P(X > x_b) = \frac{1}{2}\frac{p}{100}$

  means that half of the p% most extreme values lie to the left of x_a and half of the p% most extreme values lie to the right of x_b .



- Normal distributions are symmetrical and "bell-shaped." As a result, normal distributions can be used to approximate distributions with similar characteristics.
- A sampling distribution of a statistic is the distribution of values for the statistic for all possible samples of a given size from a given population.
- The central limit theorem (CLT) states that when the sample size is sufficiently large, a sampling distribution of the mean of a random variable will be approximately normally distributed.
- The central limit theorem requires that the sample values are independent of each other and that *n* is sufficiently large.
- A randomization distribution is a collection of statistics generated by simulation assuming known values for the parameters. For a randomized experiment, this means repeatedly randomly reallocating/reassigning the response values to treatment groups.
- The sampling distribution of a statistic can be simulated by generating repeated random samples from a population.
- When estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator is equal to the population parameter.
- When estimating a population parameter, an estimator exhibits variability that can be modeled using probability.
- A sample statistic is a point estimator of the corresponding population parameter.
- For independent samples (sampling with replacement) of a categorical variable from a population with population proportion, p, the sampling distribution of the sample proportion, $\hat{p}$, has a mean, $\mu_{\hat{p}}$ = p and a standard deviation, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- If sampling without replacement, the standard deviation of the sample proportion is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
- For a categorical variable, the sampling distribution of the sample proportion, pˆ, will have an approximate normal distribution, provided the sample size is large enough: np ≥10 and n (1-p) ≥10.
- Probabilities and parameters for a sampling distribution for a sample proportion should be interpreted using appropriate units and within the context of a specific population.
- For a categorical variable, when randomly sampling with replacement from two independent populations with population proportions p_1 and p_2, the sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ has a mean $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$ and a standard deviation $\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
- If sampling without replacement, the standard deviation of the difference in sample proportions is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
- The sampling distribution of the difference in sample proportions $\hat{p}_1 - \hat{p}_2$ will have an approximate normal distribution provided the sample sizes are large enough.
- Parameters for a sampling distribution for a difference of proportions should be interpreted using appropriate units and within the context of a specific population.

- For a numerical variable, when random sampling with replacement from a population with mean μ and standard deviation, σ, the sampling distribution of the sample mean has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
- If sampling without replacement, the standard deviation of the sample mean is smaller than what is given by the formula above. If the sample size is less than 10% of the population size, the difference is negligible.
- For a numerical variable, if the population distribution can be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled with a normal distribution.
- For a numerical variable, if the population distribution cannot be modeled with a normal distribution, the sampling distribution of the sample mean, $\bar{x}$, can be modeled approximately by a normal distribution, provided the sample size is large enough, e.g., greater than or equal to 30.
- Probabilities and parameters for a sampling distribution for a sample mean should be interpreted using appropriate units and within the context of a specific population.
- For a numerical variable, when random sampling with replacement from two independent populations with population means μ_1 and μ_2 and population standard deviations σ_1 and σ_2 the sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ has a mean $\mu_{(\bar{x}_1 - \bar{x}_2)} = \mu_1 - \mu_2$ and standard deviation, $\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$.
- If sampling without replacement, the standard deviation of the difference in sample means is smaller than what is given by the formula above. If the sample sizes are less than 10% of the population sizes, the difference is negligible.
- The sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ can be modeled with a normal distribution if the two population distributions can be modeled with a normal distribution.
- The sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$ can be modeled approximately by a normal distribution if the two population distributions cannot be modeled with a normal distribution but both sample sizes are greater than or equal to 30.
- Probabilities and parameters for a sampling distribution for a difference of sample means should be interpreted using appropriate units and within the context of a specific population.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Construct numerical or graphical representations of distributions.
- Determine parameters for probability distributions.
- Describe probability distributions.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.

- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Think Aloud:** Group students into pairs within a larger group of four. Have each student individually read 2014 FRQ 3 and think aloud with their partner, brainstorming ways to begin each part of the question. Each student then independently completes all parts. Have the pairs compare answers within their groups, improving their individual responses as necessary. Groups can then compare their responses with other groups. Finally, have students score their responses according to the rubric.
    - **Use Manipulatives:** From a large container of pennies, have each student take two random samples of size 5, two of size 10, and two of size 25, and record the dates on those pennies. Have students calculate the mean of the dates in each sample and then construct four “dotplots” on the floor: one using the pennies, one using nickels placed at the mean of the student’s sample size 5, one using dimes placed at the mean of the sample size 10, and one using quarters placed at the mean of the sample size 25.
    - **Password-Style Games:** Have partners sit facing opposite sides of the room. Display vocabulary terms from the unit on the classroom screen. Have the students facing the screen describe the terms to their partner who then tries to guess the terms described. After half of the terms have been used, have students switch roles. Terms to include: parameter, statistic, sampling distribution, distribution of sample data, sample distribution, unbiased estimator, sampling variability of a statistic, bias, sample proportion, sample mean, μ_p, σ_p, $\mu_{\bar{x}}$, $\sigma_{\bar{x}}$, and central limit theorem.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 6 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Categorical Data: Proportions

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: This unit introduces statistical inference, which will continue through the end of the course. Students will analyze categorical data to make inferences about binomial population proportions. Provided conditions are met, students will use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They will also interpret the two types of errors that can be made in a significance test, their probabilities, and possible consequences in context.

Approximate Length of Unit: 4 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:

VAR-1.G: Identify questions suggested by variation in statistics for samples collected from the same population.

VAR-6.A Calculate the probability that a particular value lies in a given interval of a normal distribution.

VAR-6.B Determine the interval associated with a given area in a normal distribution.

VAR-6.C Determine the appropriateness of using the normal distribution to approximate probabilities for unknown distributions.

UNC-3.H Estimate sampling distributions using simulation.

UNC-3.I Explain why an estimator is or is not unbiased.

UNC-3.J Calculate estimates for a population parameter.

UNC-3.K Determine parameters of a sampling distribution for sample proportions.

UNC-3.L Determine whether a sampling distribution for a sample proportion can be described as approximately normal.

UNC-3.M Interpret probabilities and parameters for a sampling distribution for a sample proportion

UNC-3.N Determine parameters of a sampling distribution for a difference in sample proportions.

UNC-3.O Determine whether a sampling distribution for a difference of sample proportions can be described as approximately normal.

UNC-3.P Interpret probabilities and parameters for a sampling distribution for a difference in proportions.
UNC-3.Q Determine parameters for a sampling distribution for sample means.
UNC-3.R Determine whether a sampling distribution of a sample mean can be described as approximately normal.
UNC-3.S Interpret probabilities and parameters for a sampling distribution for a sample mean.
UNC-3.T Determine parameters of a sampling distribution for a difference in sample means.
UNC-3.U Determine whether a sampling distribution of a difference in sample means can be described as approximately normal.
UNC-3.V Interpret probabilities and parameters for a sampling distribution for a difference in sample means

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:
Students will understand that…
- Given that variation may be random or not, conclusions are uncertain..
- An interval of values should be used to estimate parameters, in order to account for uncertainty.
- The normal distribution may be used to model variation.
- Significance testing allows us to make decisions about hypotheses within a particular context.
- Probabilities of Type I and Type II errors influence inference.

Unit Essential Questions:
- When can we use a normal distribution to perform inference calculations involving population proportions?
- How can we narrow the width of a confidence interval?
- What is the difference between a confidence interval and a test statistic?

Knowledge and Skills:
Students will know...
- Variation in shapes of data distributions may be random or not.
- The appropriate confidence interval procedure for a one-sample proportion for one categorical variable is a one sample z-interval for a proportion.
- In order to make assumptions necessary for inference on population proportions, means, and slopes, we must check for independence in data collection methods and for selection of the appropriate sampling distribution.
- In order to calculate a confidence interval to estimate a population proportion, p, we must check for independence and that the sampling distribution is approximately normal.
- Based on sample data, the standard error of a statistic is an estimate for the standard deviation for the statistic. The standard error of $\hat{p}$ is $SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- A margin of error gives how much a value of a sample statistic is likely to vary from the value of the corresponding population parameter.
- For categorical variables, the margin of error is the critical value (z*) times the standard error (SE) of the relevant statistic.
- The formula for margin of error can be rearranged to solve for n, the minimum sample size needed to achieve a given margin of error. For this purpose, use a guess for $\hat{p}$ or use $\hat{p} = 0.5$ in order to find an upper bound for the sample size that will result in a given margin of error.
- In general, an interval estimate can be constructed as a point estimate ± (margin of error).
- Critical values represent the boundaries encompassing the middle C% of the standard normal distribution, where C% is an approximate confidence level for a proportion.
- Confidence intervals for population proportions can be used to calculate interval estimates with specified units.
- A confidence interval for a population proportion either contains the population proportion or it does not, because each interval is based on random sample data, which varies from sample to sample.
- We are C% confident that the confidence interval for a population proportion captures the population proportion.

- In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the population proportion.
- Interpreting a confidence interval for a one sample proportion should include a reference to the sample taken and details about the population it represents.
- A confidence interval for a population proportion provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- When all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a population proportion, the width of the interval is proportional to $\frac{1}{\sqrt{n}}$.
- For a given sample, the width of the confidence interval for a population proportion increases as the confidence level increases.
- The width of a confidence interval for a population proportion is exactly twice the margin of error.
- The null hypothesis is the situation that is assumed to be correct unless evidence suggests otherwise, and the alternative hypothesis is the situation for which evidence is being collected.
- For hypotheses about parameters, the null hypothesis contains an equality reference (=, ≥, or ≤), while the alternative hypothesis contains a strict inequality (<, >, or ≠). The type of inequality in the alternative hypothesis is based on the question of interest. Alternative hypotheses with < or > are called one-sided, and alternative hypotheses with ≠ are called two sided. Although the null hypothesis for a one sided test may include an inequality symbol, it is still tested at the boundary of equality.
- The null hypothesis for a population proportion is: $H_o : p = p_o$, where p_o is the null hypothesized value for the population proportion.
- A one-sided alternative hypothesis for a proportion is either $H_a : p < p_o$ or $H_a : p > p_o$
  A two-sided alternative hypothesis is $H_a : p \neq p_o$
- For a one-sample z-test for a population proportion, the null hypothesis specifies a value for the population proportion, usually one indicating no difference or effect.
- For a single categorical variable, the appropriate testing method for a population proportion is a one-sample z-test for a population proportion.
- In order to make statistical inferences when testing a population proportion, we must check for independence and that the sampling distribution is approximately normal.
- The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or when a probability model is assumed to be true, a theoretical distribution (z).
- When using a z-test, the standardized test statistic can be written:
  test statistic = $\frac{sample\ statistic - null\ value\ of\ parameter}{standard\ deviation\ of\ statistic}$. This is called a z-statistic for proportions.
- A p-value is the probability of obtaining a test statistic as extreme or more extreme than the observed test statistic when the null hypothesis and probability model are assumed to be true. The significance level may be given or determined by the researcher.
- The p-value is the proportion of values for the null distribution that are as extreme or more extreme than the observed value of the test statistic.
- An interpretation of the p-value of a significance test for a one-sample proportion should recognize that the p-value is computed by assuming that the probability model and null hypothesis are true, i.e., by assuming that the true population proportion is equal to the particular value stated in the null hypothesis.
- The significance level, α, is the predetermined probability of rejecting the null hypothesis given that it is true.
- A formal decision explicitly compares the p-value to the significance level, α. If the p-value $\leq \alpha$, reject the null hypothesis. If the p-value $> \alpha$, fail to reject the null hypothesis.

- Rejecting the null hypothesis means there is sufficient statistical evidence to support the alternative hypothesis. Failing to reject the null means there is insufficient statistical evidence to support the alternative hypothesis.
- The conclusion about the alternative hypothesis must be stated in context.
- A significance test can lead to rejecting or not rejecting the null hypothesis, but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.
- Small p-values indicate that the observed value of the test statistic would be unusual if the null hypothesis and probability model were true, and so provide evidence for the alternative. The lower the p-value, the more convincing the statistical evidence for the alternative hypothesis.
- p-values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis and probability model were true, so do not provide convincing statistical evidence for the alternative hypothesis nor do they provide evidence that the null hypothesis is true.
- A formal decision explicitly compares the p-value to the significanceα . If the p-value $\leq \alpha$, then reject the null hypothesis, $H_o : p = p_o$. If the p-value $>\alpha$, then fail to reject the null hypothesis.
- The results of a significance test for a population proportion can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- A Type I error occurs when the null hypothesis is true and is rejected (false positive).
- A Type II error occurs when the null hypothesis is false and is not rejected (false negative).
- The significance level is the probability of making a Type I error, if the null hypothesis is true.
- The power of a test is the probability that a test will correctly reject a false null hypothesis.
- The probability of making a Type II error = 1 - power.
- The probability of a Type II error decreases when any of the following occurs, provided the others do not change: i. Sample size(s) increases. ii. Significance level of a test increases. iii. Standard error decreases. iv. True parameter value is farther from the null.
- Whether a Type I or a Type II error is more consequential depends upon the situation.
- Since the significance level,α , is the probability of a Type I error, the consequences of a Type I error influence decisions about a significance level.
- The appropriate confidence interval procedure for a two-sample comparison of proportions for one categorical variable is a two-sample z-interval for a difference between population proportions.
- In order to calculate confidence intervals to estimate a difference between proportions, we must check for independence and that the sampling distribution is approximately normal.
- Confidence intervals for a difference in proportions can be used to calculate interval estimates with specified units.
- In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference in population proportions.
- Interpreting a confidence interval for difference between population proportions should include a reference to the sample taken and details about the population it represents.
- A confidence interval for difference in population proportions provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- For a two-sample test for a difference of two proportions, the null hypothesis specifies a value of 0 for the difference in population proportions, indicating no difference or effect.
- The null hypothesis for a difference in proportions is:$H_o : p_1 = p_2$ or $H_o : p_1 - p_2 = 0$.
- A one-sided alternative hypothesis for a difference in proportions is $H_a : p_1 < p_2$ or $H_a : p_1 > p_2$. A two-sided alternative hypothesis for a difference of proportions is $H_a : p_1 \neq p_2$.
- For a single categorical variable, the appropriate testing method for the difference of two population proportions is a two-sample z-test for a difference between two population proportions.

- In order to make statistical inferences when testing a difference between population proportions, we must check for independence and that the sampling distribution is approximately normal.
- An interpretation of the p-value of a significance test for a difference of two population proportions should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population proportions are equal to each other.
- A formal decision explicitly compares the p-value to the significance α. If the p-value $\leq \alpha$, then reject the null hypothesis. . If the p-value $> \alpha$, then fail to reject the null hypothesis.
- The results of a significance test for a difference of two population proportions can serve as the statistical reasoning to support the answer to a research question about the two populations that were sampled.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Identify key and relevant information to answer a question or solve a problem.
- Identify an appropriate inference method for confidence intervals.
- Verify that inference procedures apply in a given situation.
- Construct a confidence interval, provided conditions for inference are met.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Justify a claim based on a confidence interval.
- Make an appropriate claim or draw an appropriate conclusion.
- Identify null and alternative hypotheses.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Interpret statistical calculations and findings to assign meaning or assess a claim
- Justify a claim using a decision based on significance tests.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Error Analysis:** Give student pairs a worksheet with 20 sets of hypotheses (including hypotheses for a population proportion and for the difference of two proportions), each

with a common student mistake. Have students circle the incorrect part, write why the circled component is incorrect, and then write the correct hypotheses. Include errors such as using statistics instead of parameters, and interchanging the = and > in the two hypotheses.

- **Sentence Starters:** For a given question, provide students with a set of hypotheses, p-value, significance level, and context. Have them compare the p-value to the significance level to determine whether or not to reject the null hypothesis. Using a given sentence starter with blanks to fill in, have students write a sentence in context explaining if they have enough evidence to "reject H_o ", or if they will "fail to reject H_o ." Make sure students avoid the common mistake of implying that evidence supports an "accept H_o " conclusion or a "reject Ha " conclusion.
- **The Scribe and the Calculator:** Have students work with a partner to construct and interpret a confidence interval for a population proportion. Only one partner is allowed to use the calculator, and only the other partner is allowed to write. When a calculation needs to be made, the scribe can only describe to the calculator operator which buttons to push; when writing needs to be done, the calculator operator can only describe to the scribe what needs to be written. Have students switch roles when constructing and interpreting a confidence interval for the difference of two population proportions.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 7 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Quantitative Data: Means

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: In this unit, students will analyze quantitative data to make inferences about population means. Students should understand that $t*$ and t-tests are used for inference with means when the population standard deviation is not known. Using s for in the formula for z gives a slightly different value, t, whose distribution, which depends on sample size, has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a t-distribution tend to be further from the mean than for a normal distribution. Students should understand how and why conditions for inference with proportions and means are similar and different.

Approximate Length of Unit: 3 weeks

LEARNING TARGETS

NJ Student Learning Standards:

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:

VAR-1.I Identify questions suggested by probabilities of errors in statistical inference.

VAR-7.A Describe t-distributions.

UNC-4.O Identify an appropriate confidence interval procedure for a population mean, including the mean difference between values in matched pairs.

UNC-4.P Verify the conditions for calculating confidence intervals for a population mean, including the mean difference between values in matched pairs.

UNC-4.Q Determine the margin of error for a given sample size for a one-sample t-interval.

UNC-4.R Calculate an appropriate confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.S Interpret a confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.T Justify a claim based on a confidence interval for a population mean, including the mean difference between values in matched pairs.

UNC-4.U Identify the relationships between sample size, width of a confidence interval, confidence level, and margin of error for a population mean.

VAR-7.B Identify an appropriate testing method for a population mean with unknown σ , including the mean difference between values in matched pairs.

VAR-7.C Identify the null and alternative hypotheses for a population mean with unknown σ , including the mean difference between values in matched pairs.
VAR-7.D Verify the conditions for the test for a population mean, including the mean difference between values in matched pairs.
VAR-7.E Calculate an appropriate test statistic for a population mean, including the mean difference between values in matched pairs.
DAT-3.E Interpret the p-value of a significance test for a population mean, including the mean difference between values in matched pairs.
DAT-3.F Justify a claim about the population based on the results of a significance test for a population mean.
UNC-4.W Verify the conditions to calculate confidence intervals for the difference of two population means.
UNC-4.X Determine the margin of error for the difference of two population means.
UNC-4.Y Calculate an appropriate confidence interval for a difference of two population means.
UNC-4.Z Interpret a confidence interval for a difference of population means.
UNC-4.AA A Justify a claim based on a confidence interval for a difference of population means.
UNC-4.AB Identify the effects of sample size on the width of a confidence interval for the difference of two means.
VAR-7.F Identify an appropriate selection of a testing method for a difference of two population means.
VAR-7.G Identify the null and alternative hypotheses for a difference of two population means.
VAR-7.H Verify the conditions for the significance test for the difference of two population means.
VAR-7.I Calculate an appropriate test statistic for a difference of two means.
DAT-3.G Interpret the p-value of a significance test for a difference of population means.
DAT-3.H Justify a claim about the population based on the results of a significance test for a difference of two population means in context.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:
Students will understand that…
- Given that variation may be random or not, conclusions are uncertain.
- The t-distribution may be used to model variation.
- An interval of values should be used to estimate parameters, in order to account for uncertainty.
- Significance testing allows us to make decisions about hypotheses within a particular context.

Unit Essential Questions:
- How do we know whether to use a t-test or a z-test for inference with means?
- How can we make sure that samples are independent?
- Why is it inappropriate to accept a hypothesis as true based on the results of statistical inference testing?

Knowledge and Skills:
Students will know...
- Random variation may result in errors in statistical inference.
- When s is used instead of to calculate a test statistic, the corresponding distribution, known as the t-distribution, varies from the normal distribution in shape, in that more of the area is allocated to the tails of the density curve than in a normal distribution.
- As the degrees of freedom increase, the area in the tails of a t-distribution decreases.
- Because σ is typically not known for distributions of quantitative variables, the appropriate confidence interval procedure for estimating the population mean of one quantitative variable for one sample is a one-sample t-interval for a mean.
- For one quantitative variable, X, that is normally distributed, the distribution of $t = \frac{(\bar{x} - \mu)}{\frac{s}{\sqrt{n}}}$ is a t-distribution with n −1 degrees of freedom.
- Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for confidence intervals proceeds as for a population mean.
- In order to calculate confidence intervals to estimate a population mean, we must check for independence and that the sampling distribution is approximately normal.
- The critical value t* with n −1 degrees of freedom can be found using a table or computer-generated output.

- The standard error for a sample mean is given by $SE = \frac{s}{\sqrt{n}}$ where s is the sample standard deviation.
- For a one-sample t-interval for a mean, the margin of error is the critical value ( ) t*times the standard error (SE).
- The point estimate for a population mean is the sample mean, $\bar{x}$.
- For the population mean for one sample with unknown population standard deviation, the confidence interval is $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$.
- A confidence interval for a population mean either contains the population mean or it does not, because each interval is based on data from a random sample, which varies from sample to sample.
- We are C% confident that the confidence interval for a population mean captures the population mean.
- An interpretation of a confidence interval for a population mean includes a reference to the sample taken and details about the population it represents.
- A confidence interval for a population mean provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- When all other things remain the same, the width of a confidence interval for a population mean tends to decrease as the sample size increases.
- For a single mean, the width of the interval is proportional to $\frac{1}{\sqrt{n}}$.
- For a given sample, the width of the confidence interval for a population mean increases as the confidence level increases
- The appropriate test for a population mean with unknown is a one-sample t-test for a population mean.
- Matched pairs can be thought of as one sample of pairs. Once differences between pairs of values are found, inference for significance testing proceeds as for a population mean.
- The null hypothesis for a one-sample t-test for a population mean is $H_o: \mu = u_o$ where u_o0 is the hypothesized value. Depending upon the situation, the alternative hypothesis is $H_a: \mu < u_o$, $H_a: \mu > u_o$ or $H_a: \mu \neq u_o$.
- When finding the mean difference, μ_d, between values in a matched pair, it is important to define the order of subtraction.
- In order to make statistical inferences when testing a population mean, we must check for independence and that the sampling distribution is approximately normal.
- For a single quantitative variable when random sampling with replacement from a population that can be modeled with a normal distribution with mean μ and standard deviation σ, the sampling distribution of $t = \frac{(\bar{x} - \mu)}{\frac{s}{\sqrt{n}}}$ has a t-distribution with n – 1 degrees of freedom.
- An interpretation of the p-value of a significance test for a population mean should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population mean is equal to the particular value stated in the null hypothesis.
- A formal decision explicitly compares the p-value to the significance . If the p-value $\leq \alpha$, then reject the null hypothesis, $H_a: \mu = u_o$. If the p-value $> \alpha$, then fail to reject the null hypothesis.
- The results of a significance test for a population mean can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- Consider a simple random sample from population 1 of size n_1, mean μ_1, and standard deviation σ_1 and a second simple random sample from population 2 of size n_2, mean μ_2, and

standard deviation σ_2 . If the distributions of populations 1 and 2 are normal or if both n_1 and n_2 are greater than 30, then the sampling distribution of the difference of means, $\bar{x}_1 - \bar{x}_2$ is also normal. The mean for the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$.

- The appropriate confidence interval procedure for one quantitative variable for two independent samples is a two-sample t-interval for a difference between population means.
- In order to calculate confidence intervals to estimate a difference of population means, we must check for independence and that the sampling distribution is approximately normal.
- For the difference of two sample means, the margin of error is the critical value (t *) times the standard error (SE) of the difference of two means.
- The standard error for the difference in two sample means with sample standard deviations, s_1 and s_2, $\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$.
- The point estimate for the difference of two population means is the difference in sample means, $\bar{x}_1 - \bar{x}_2$.
- For a difference of two population means where the population standard deviations are not known, the confidence interval is $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$ where $\pm t^*$ are the critical values for the central C% of a t-distribution with appropriate degrees of freedom that can be found using technology.
- In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the difference of population means.
- An interpretation for a confidence interval for the difference of two population means should include a reference to the samples taken and details about the populations they represent.
- A confidence interval for a difference of population means provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- When all other things remain the same, the width of the confidence interval for the difference of two means tends to decrease as the sample sizes increase.
- For a quantitative variable, the appropriate test for a difference of two population means is a two-sample t-test for a difference of two population means.
- The null hypothesis for a two-sample t-test for a difference of two population means, μ_1 and μ_2 is $H_o: \mu_1 - \mu_2 = 0$ or $H_o: \mu_1 = \mu_2$. The alternative hypothesis is $H_a: \mu_1 < \mu_2$, $H_a: \mu_1 > \mu_2$ or $H_a: \mu_1 \neq \mu_2$.
- In order to make statistical inferences when testing a difference between population means, we must check for independence and that the sampling distribution is approximately normal.
- For a single quantitative variable, data collected using independent random samples or a randomized experiment from two populations, each of which can be modeled with a normal distribution, the sampling distribution of $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}}$ is an approximate t-distribution with degrees of freedom that can be found using technology.
- An interpretation of the p-value of a significance test for a two-sample difference of population means should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population means are equal to each other.
- A formal decision explicitly compares the p-value to the significance α . If the p-value $\leq \alpha$, then reject the null hypothesis $H_o: \mu_1 - \mu_2 = 0$ or $H_o: \mu_1 = \mu_2$. If the p-value $> \alpha$, then fail to reject the null hypothesis.

- The results of a significance test for a two-sample test for a difference between two population means can serve as the statistical reasoning to support the answer to a research question about the populations that were sampled.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Describe probability distributions.
- Construct a confidence interval, provided conditions for inference are met.
- Identify an appropriate inference method for confidence intervals.
- Verify that inference procedures apply in a given situation.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Justify a claim based on a confidence interval.
- Make an appropriate claim or draw an appropriate conclusion.
- Identify an appropriate inference method for significance tests.
- Identify null and alternative hypotheses.
- Verify that inference procedures apply in a given situation.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Justify a claim using a decision based on significance tests.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
    - **Team Challenge:** Give each team of three to four students a copy of 2004 FRQ 6, which focuses on the connection between a one-sample t-interval, a one-sample t-test, and the unfamiliar concept of a one-sided confidence interval. Challenge teams to collaboratively produce a model solution in 30 minutes.
    - **Discussion Groups:** Ask each group of three to four students to identify the conditions for performing a test about a population mean. For each condition, have them explain why the condition is required and what would go wrong with the test if the condition were violated. Have groups pair up and compare answers.

- **Team FRQ:** Give each team of four students copies of a free-response question that involves performing a two-sample t-test (e.g., 2011 FRQ 4). Have each team member take responsibility for writing one part of the model solution (hypotheses, procedure and conditions, calculations, conclusion) with group input.
- **Graphic Organizer:** Have students work in teams of two to three to develop a flowchart for determining which inference procedure from Units 6 and 7 to use in a given setting.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 8 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Categorical Data: Chi-Squared

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Unit 6 introduced inference for proportions of categorical data. Unit 8 introduces chi square tests, which can be used when there are two or more categories. Students need to understand how to select from the following tests: the chi-square test for goodness of fit (for a distribution of proportions of one categorical variable in a population), the chi-square test for independence (for associations between categorical variables within a single population), or the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding, teachers can make connections between frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between observed and expected counts relative to expected counts.

Approximate Length of Unit: 2 weeks

LEARNING TARGETS

NJ Student Learning Standards:
S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:
VAR-1.J Identify questions suggested by variation between observed and expected counts in categorical data.
VAR-8.A Describe chi-square distributions.
VAR-8.B Identify the null and alternative hypotheses in a test for a distribution of proportions in a set of categorical data.
VAR-8.C Identify an appropriate testing method for a distribution of proportions in a set of categorical data.
VAR-8.D Calculate expected counts for the chi-square test for goodness of fit.
VAR-8.E Verify the conditions for making statistical inferences when testing goodness of fit for a chi-square distribution.
VAR-8.F Calculate the appropriate statistic for the chi-square test for goodness of fit.
VAR-8.G Determine the p-value for chi-square test for goodness of fit significance test.
DAT-3.I Interpret the p-value for the chi-square test for goodness of fit.
DAT-3.J Justify a claim about the population based on the results of a chi-square test for goodness of fit.
VAR-8.H Calculate expected counts for two-way tables of categorical data.
VAR-8.I Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence.
VAR-8.J Identify an appropriate testing method for comparing distributions in two-way tables of categorical data.

VAR-8.K Verify the conditions for making statistical inferences when testing a chi-square distribution for independence or homogeneity.
VAR-8.L Calculate the appropriate statistic for a chi-square test for homogeneity or independence.
VAR-8.M Determine the p-value for a chi-square significance test for independence or homogeneity.
DAT-3.K Interpret the p-value for the chi-square test for homogeneity or independence.
DAT-3.L Justify a claim about the population based on the results of a chi-square test for homogeneity or independence.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:

Students will understand that…

- Given that variation may be random or not, conclusions are uncertain.
- The chi-square distribution may be used to model variation.
- Significance testing allows us to make decisions about hypotheses within a particular context.

Unit Essential Questions:

- How does increasing the degrees of freedom influence the shape of the chi-square distribution?
- Why is it inappropriate to use statistical inference to justify a claim that there is no association between variables?
- Are my results unexpected?

Knowledge and Skills:

Students will know...

- Variation between what we find and what we expect to find may be random or not.
- Expected counts of categorical data are counts consistent with the null hypothesis. In general, an expected count is a sample size times a probability
- The chi-square statistic measures the distance between observed and expected counts relative to expected counts.
- Chi-square distributions have positive values and are skewed right. Within a family of density curves, the skew becomes less pronounced with increasing degrees of freedom.
- For a chi-square goodness-of-fit test, the null hypothesis specifies null proportions for each category, and the alternative hypothesis is that at least one of these proportions is not as specified in the null hypothesis.
- When considering a distribution of proportions for one categorical variable, the appropriate test is the chi-square test for goodness of fit.
- Expected counts for a chi-square goodness-of-fit test are (sample size) (null proportion).
- In order to make statistical inferences for a chi-square test for goodness of fit we must check the following: a. To check for independence: i. Data should be collected using a random sample or randomized experiment. ii. When sampling without replacement, check that $n \leq 10\%N$. b. The chi-square test for goodness of fit becomes more accurate with more observations, so large counts should be used (shape). i. A conservative check for large counts is that all expected counts should be greater than 5.
- The test statistic for the chi-square test for goodness of fit is:

  $\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$ with degrees of freedom number of categories = −1.
- The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be either a randomization distribution or, when a probability model is assumed to be true, a theoretical distribution (chi-square).
- The p-value for a chi-square test for goodness of fit for a number of degrees of freedom is found using the appropriate table or computer generated output.
- An interpretation of the p-value for the chi-square test for goodness of fit is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
- A decision to either reject or fail to reject the null hypothesis is based on comparison of the p-value to the significance level, α.

- The results of a chi-square test for goodness of fit can serve as the statistical reasoning to support the answer to a research question about the population that was sampled.
- The expected count in a particular cell of a two-way table of categorical data can be calculated using the formula: $expected\ count = \frac{(row\ total)(column\ total)}{table\ total}$
- The appropriate hypotheses for a chi-square test for homogeneity are: H_o: There is no difference in distributions of a categorical variable across populations or treatments. H_a: There is a difference in distributions of a categorical variable across populations or treatments.
- The appropriate hypotheses for a chi-square test for independence are: H_o: There is no association between two categorical variables in a given population or the two categorical variables are independent. H_a: Two categorical variables in a population are associated or dependent.
- When comparing distributions to determine whether proportions in each category for categorical data collected from different populations are the same, the appropriate test is the chi-square test for homogeneity.
- To determine whether row and column variables in a two-way table of categorical data might be associated in the population from which the data were sampled, the appropriate test is the chi-square test for independence.
- In order to make statistical inferences for a chi-square test for two-way tables (homogeneity or independence), we must verify the following: a. To check for independence: i. For a test for independence: Data should be collected using a simple random sample. ii. For a test for homogeneity: Data should be collected using a stratified random sample or randomized experiment. iii. When sampling without replacement, check that $n \le 10\% N$. b. The chi-square tests for independence and homogeneity become more accurate with more observations, so large counts should be used (shape). i. A conservative check for large counts is that all expected counts should be greater than 5.
- The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic: $\chi^2 = \sum \frac{(Observed\ count - Expected\ count)^2}{Expected\ count}$

  with degrees of freedom equal to: (number of rows - 1)(number of columns - 1)
- The p-value for a chi-square test for independence or homogeneity for a number of degrees of freedom is found using the appropriate table or technology.
- For a test of independence or homogeneity for a two-way table, the p-value is the proportion of values in a chi-square distribution with appropriate degrees of freedom that are equal to or larger than the test statistic.
- An interpretation of the p-value for the chi-square test for homogeneity or independence is the probability, given the null hypothesis and probability model are true, of obtaining a test statistic as, or more, extreme than the observed value.
- A decision to either reject or fail to reject the null hypothesis for a chi-square test for homogeneity or independence is based on comparison of the p-value to the significance level, α.
- The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to a research question about the population that was sampled (independence) or the populations that were sampled (homogeneity).

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Describe probability distributions.
- Identify null and alternative hypotheses.
- Identify an appropriate inference method for significance tests.
- Determine relative frequencies, proportions, or probabilities using simulation or calculations.
- Verify that inference procedures apply in a given situation.

- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Justify a claim using a decision based on significance tests.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: Warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: Quizzes, tests, projects, and benchmark assessments.
- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activities:
  - **Discussion Groups:** Give each group of three to four students an example of a chi-square test involving a two-way table. Have students work together to state appropriate hypotheses, describe a Type 1 and Type 2 error in context, and give a possible consequence of each of those errors
  - **Graphic Organizer:** Have students work in teams of two to three to develop a chart that summarizes the three types of chi-square tests, including when each is appropriate, as well as the hypotheses, conditions, and degrees of freedom.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 9 OVERVIEW

Content Area: Mathematics

Unit Title: Inference for Quantitative Data: Slope

Target Course/Grade Level: Advanced Placement Statistics/Grades 11-12

Unit Summary: Students may be surprised to learn that there is variability in slope. In their experience in previous courses, the slope of the line of best fit does not vary for a particular set of bivariate quantitative data. However, suppose that every student in a university physics course collects data on spring length for 10 different hanging masses and calculates the least-squares regression line for their sample data. The students' slopes would likely vary as part of an approximately normal sampling distribution centered at the (true) slope of the population regression line relating spring length to hanging mass. In this unit, students will learn how to construct confidence intervals for and perform significance tests about the slope of a population regression line when appropriate conditions are met.

Approximate Length of Unit: 2 weeks

LEARNING TARGETS

NJ Student Learning Standards:
S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S.MD.A.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

College Board Standards:
VAR-1.K Identify questions suggested by variation in scatter plots.
UNC-4.AC Identify an appropriate confidence interval procedure for a slope of a regression model.
UNC-4.AD Verify the conditions to calculate confidence intervals for the slope of a regression model.
UNC-4.AE Determine the given margin of error for the slope of a regression model.
UNC-4.AF Calculate an appropriate confidence interval for the slope of a regression model.
UNC-4.AG Interpret a confidence interval for the slope of a regression model.
UNC-4.AH Justify a claim based on a confidence interval for the slope of a regression model.
UNC-4.AI Identify the effects of sample size on the width of a confidence interval for the slope of a regression model.
VAR-7.J Identify the appropriate selection of a testing method for a slope of a regression model.
VAR-7.K Identify appropriate null and alternative hypotheses for a slope of a regression model.
VAR-7.L Verify the conditions for the significance test for the slope of a regression model.
VAR-7.M Calculate an appropriate test statistic for the slope of a regression model.
DAT-3.M Interpret the p-value of a significance test for the slope of a regression model.
DAT-3.N Justify a claim about the population based on the results of a significance test for the slope of a regression model.

Career Readiness, Life Literacies, and Key Skills:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
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HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3 Evaluate a solution to a complex real-world problem, based on scientific knowledge, student generated sources of evidence, prioritized criteria, and trade off considerations.

Unit Understandings:
Students will understand that…
- Given that variation may be random or not, conclusions are uncertain.
- The chi-square distribution may be used to model variation.
- Significance testing allows us to make decisions about hypotheses within a particular context.

Unit Essential Questions:

- When is it appropriate to perform inference about the slope of a population regression line based on sample data?
- Why do we not conclude that there is no correlation between two variables based on the results of a statistical inference for slopes?

Knowledge and Skills:

Students will know...

- Variation in points' positions relative to a theoretical line may be random or non-random.
- Consider a response variable, y, that is linearly related to an explanatory variable, x. For a simple random sample of n observations, the sample regression line, $\hat{y} = a + bx$ is an estimate of the population regression line $\mu_y = \alpha + \beta x$ For a particular observation, (x_i , y_i), the residual from the sample regression line, $y_i - \bar{y}_i = y_i - (a + bx_i)$, is an estimate of $y_i - (\alpha + \beta x_i)$, the deviation of the response variable from the population regression line. For all points (x, y) in the population, the standard deviation of all of the deviations of the response variable from the population regression line, σ, can be estimated by the standard deviation of the residuals from the sample regression line $s = \sqrt{\frac{\Sigma(y_i - \bar{y}_i)^2}{n-2}}$.
- For a simple random sample of *n* observations, let *b* represent the slope of a sample regression line. Then the mean of the sampling distribution for *b* equals the population slope: $\mu_b = \beta$. The standard deviation of the sampling distribution for *b* is $\sigma_b = \frac{\sigma}{\sigma_x \sqrt{n}}$, where $\sigma_x = \sqrt{\frac{\Sigma(x_i - \hat{x}_i)^2}{n}}$.
- The appropriate confidence interval for the slope of a regression model is a t-interval for the slope.
- In order to calculate a confidence interval to estimate the slope of a regression line, we must check the following: a. The true relationship between *x* and *y* is linear. Analysis of residuals may be used to verify linearity. b. The standard deviation for *y*, σ_y , does not vary with *x*. Analysis of residuals may be used to check for approximately equal standard deviations for all *x*. c. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that $n \leq 10\%N$. d. For a particular value of *x*, the responses (*y*-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality. i. If the observed distribution is skewed, n should be greater than 30.
- For the slope of a regression line, the margin of error is the critical value (t^*) times the standard error (SE) of the slope.
- The standard error for the slope of a regression line with sample standard deviation, *s*, is $SE = \frac{s}{s_x \sqrt{n-1}}$,, where *s* is the estimate of σ and s_x is the sample standard deviation of the *x* values.
- The point estimate for the slope of a regression model is the slope of the line of best fit, *b*.
- For the slope of a regression model, the interval estimate is $b \pm t^*(SE_b)$.
- In repeated random sampling with the same sample size, approximately C% of confidence intervals created will capture the slope of the regression model, i.e., the true slope of the population regression model.

- An interpretation for a confidence interval for the slope of a regression line should include a reference to the sample taken and details about the population it represents.
- A confidence interval for the slope of a regression model provides an interval of values that may provide sufficient evidence to support a particular claim in context.
- When all other things remain the same, the width of the confidence interval for the slope of a regression model tends to decrease as the sample size increases.
- The appropriate test for the slope of a regression model is a t-test for a slope.
- The null hypothesis for a *t*-test for a slope is: H_o: $\beta = \beta_{o,}$ where $\beta_{o,}$ is the hypothesized value from the null hypothesis. The alternative hypothesis is H_a: $\beta < \beta_{o,}$ H_a: $\beta > \beta_{o,}$ or H_a: $\beta \neq \beta_o$.
- In order to make statistical inferences when testing for the slope of a regression model, we must check the following: a. The true relationship between *x* and *y* is linear. Analysis of residuals may be used to verify linearity. b. The standard deviation for *y*, σ_y, does not vary with *x*. Analysis of residuals may be used to check for approximately equal standard deviations for all *x*. c. To check for independence: i. Data should be collected using a random sample or a randomized experiment. ii. When sampling without replacement, check that $n \leq 10\%N$. d. For a particular value of *x,* the responses (*y*-values) are approximately normally distributed. Analysis of graphical representations of residuals may be used to check for normality. i. If the observed distribution is skewed, n should be greater than 30. ii. If the sample size is less than 30, the distribution of the sample data should be free from strong skewness and outliers.
- The distribution of the slope of a regression model assuming all conditions are satisfied and the null hypothesis is true (null distribution) is a t-distribution.
- For simple linear regression when random sampling from a population for the response that can be modeled with a normal distribution for each value of the explanatory variable, the sampling distribution of $t = \frac{b - \beta}{SE_b}$ has a t-distribution with degrees of freedom equal to n- 2. When testing the slope in a simple linear regression model with one parameter, the slope, the test for the slope has df = n - 1.
- An interpretation of the p-value of a significance test for the slope of a regression model should recognize that the p-value is computed by assuming that the null hypothesis is true, i.e., by assuming that the true population slope is equal to the particular value stated in the null hypothesis.
- A formal decision explicitly compares the p-value to the significance α . If the p-value $\leq \alpha$, then reject the null hypothesis, H_o: $\beta = \beta_o$. . If the p-value $> \alpha$, then fail to reject the null hypothesis.
- The results of a significance test for the slope of a regression model can serve as the statistical reasoning to support the answer to a research question about that sample.

Students will be able to...

- Identify the question to be answered or problem to be solved.
- Identify an appropriate inference method for confidence intervals.
- Verify that inference procedures apply in a given situation.
- Construct a confidence interval, provided conditions for inference are met.
- Interpret statistical calculations and findings to assign meaning or assess a claim.
- Justify a claim based on a confidence interval.
- Make an appropriate claim or draw an appropriate conclusion.
- Identify null and alternative hypotheses.
- Calculate a test statistic and find a p-value, provided conditions for inference are met.
- Justify a claim using a decision based on significance tests.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
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- FRQ (free response questions), daily observation of student work, and student/group presentations.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- AP Classroom and Khan Academy assignments.
- Sample Activity:
    - **Notation Read Aloud:** Have students read AP Exam questions aloud (e.g., 2011 FRQ 5, 2010 Form B FRQ 6, 2005 Form B FRQ 5, and 2001 FRQ 6), including the given notation. Remind students that the computer output provides the two-sided p-value, and that there are two different p-values in the chart: The top p-value is for the intercept, and the bottom p-value is for the slope. Then have students discuss each of the values in the computer output and carry out a test for the slope of a regression model.

RESOURCES

Teacher Resources:

- **Textbook**: Bock, David E., Paul F. Velleman and Richard D. DeVeaux. (2015). Stats: Modeling the World. 4th Edition. Boston: Pearson Education Inc.
- Khan Academy
- AP Classroom

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator