ESL
SCIENCE
BUSINESS
BILINGUAL
PRESCHOOL
MATHEMATICS
LIBRARY MEDIA
SOCIAL STUDIES
WORLD LANGUAGES
GIFTED & TALENTED
TECHNOLOGY EDUCATION
ENGLISH LANGUAGE ARTS
FINE & PERFORMING ARTS
FAMILY & CONSUMER SCIENCE
HEALTH & PHYSICAL EDUCATION

RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Pre-Calculus

Grade Level: 11-12

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

Jeffrey Kurczeski,

Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

The Board acknowledges the following who contributed to the preparation of this curriculum.

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Dr. Tiffany A. Beer, Director of Curriculum and Instruction

Dr. Aleya Shoieb, Superintendent of Schools

Subject/Course Title:
Pre-Calculus
Grades 11-12

Date of Board Adoption:
August 27, 2024

RAHWAY PUBLIC SCHOOLS CURRICULUM

Pre-Calculus: Grades 11-12

PACING GUIDE

Unit	Title	Pacing
1	Functions and Their Graphs	8 weeks
2	Polynomial and Rational Functions	10 weeks
3	Exponential and Logarithmic Functions	10 weeks
4	Trigonometric Functions	12 weeks

ACCOMMODATIONS

504 Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Provide extra visual and verbal cues and prompts.
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Weekly home-school communication tools (notebook, daily log, phone calls or email messages).
- Provide study sheets and teacher outlines prior to assessments.
- Quiet corner or room to calm down and relax when anxious.
- Reduction of distractions.
- Permit answers to be dictated.
- Hands-on activities.
- Use of manipulatives.
- Assign preferential seating.
- No penalty for spelling errors or sloppy handwriting.
- Follow a routine/schedule.
- Provide student with rest breaks.
- Use verbal and visual cues regarding directions and staying on task.
- Assist in maintaining agenda book.

IEP Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Differentiate reading levels of texts (e.g., Newsela).
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide extra visual and verbal cues and prompts.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Provide students with additional information to supplement notes.
- Modify questioning techniques and provide a reduced number of questions or items on tests.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Provide study sheets and teacher outlines prior to assessments.
- Use of manipulatives.
- Have students work with partners or in groups for reading, presentations, assignments, and analyses.
- Assign appropriate roles in collaborative work.
- Assign preferential seating.
- Follow a routine/schedule.

Gifted and Talented Accommodations:

- Differentiate reading levels of texts (e.g., Newsela).
- Offer students additional texts with higher lexile levels.
- Provide more challenging and/or more supplemental readings and/or activities to deepen understanding.
- Allow for independent reading, research, and projects.
- Accelerate or compact the curriculum.
- Offer higher-level thinking questions for deeper analysis.
- Offer more rigorous materials/tasks/prompts.
- Increase number and complexity of sources.
- Assign group research and presentations to teach the class.
- Assign/allow for leadership roles during collaborative work and in other learning activities.

ML Accommodations:

- Provide extended time.
- Assign preferential seating.
- Assign peer buddy who the student can work with.
- Check for understanding frequently.
- Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc…).
- Have student repeat directions.
- Make vocabulary words available during classwork and exams.
- Use study guides/checklists to organize information.
- Repeat directions.
- Increase one-on-one conferencing.
- Allow student to listen to an audio version of the text.
- Give directions in small, distinct steps.
- Allow copying from paper/book.
- Give student a copy of the class notes.

	● Provide written and oral instructions. ● Differentiate reading levels of texts (e.g., Newsela). ● Shorten assignments. ● Read directions aloud to student. ● Give oral clues or prompts. ● Record or type assignments. ● Adapt worksheets/packets. ● Create alternate assignments. ● Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted. ● Allow student to resubmit assignments. ● Use small group instruction. ● Simplify language. ● Provide scaffolded vocabulary and vocabulary lists. ● Demonstrate concepts possibly through the use of visuals. ● Use manipulatives. ● Emphasize critical information by highlighting it for the student. ● Use graphic organizers. ● Pre-teach or pre-view vocabulary. ● Provide student with a list of prompts or sentence starters that they can use when completing a written assignment. ● Provide audio versions of the textbooks. ● Highlight textbooks/study guides. ● Use supplementary materials. ● Give assistance in note taking ● Use adapted/modified textbooks. ● Allow use of computer/word processor. ● Allow student to answer orally, give extended time (time-and-a-half). ● Allow tests to be given in a separate location (with the ESL teacher). ● Allow additional time to complete assignments and/or assessments. ● Read question to student to clarify. ● Provide a definition or synonym for words on a test that do not impact the validity of the exam. ● Modify the format of assessments. ● Shorten test length or require only selected test items. ● Create alternative assessments. ● On an exam other than a spelling test, don’t take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Functions and Their Graphs

Target Course/Grade Level: Pre-Calculus/Grades 11-12

Unit Summary: In this unit, students will review the parent functions of the elementary functions. Students will learn to evaluate, analyze, transform, and combine functions, with an emphasis on the graphs of these functions. The concepts of rate of change and inverse functions will be studied in more depth. The difference quotient will be introduced and developed.

Approximate Length of Unit: 8 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), f(x + k) and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.B.4 Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = $2x^3$ or $f(x) = \frac{(x+1)}{(x-1)}$ for x ≠ 1.

b. Verify by composition that one function is the inverse of another.

c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. Produce an invertible function from a non-invertible function by restricting the domain.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

f. Graph trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (.97)^t$, $y = (1.01)^{12t}$, $y = (1.01)^{t/10}$, and classify them as representing exponential growth or decay.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.

L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that…

- The slope of a line can be used to solve real-life problems.
- Functions can be used to model and solve real-world problems.
- New functions can be formed by adding, subtracting, multiplying or dividing two functions.
- Not all functions have an inverse function.

Unit Essential Questions:

- What is a function?
- What are the characteristics of different functions?
- What is the relationship between the equation of a function and its graph?
- How are functions used in the real world?

Knowledge and Skills:

Students will know…

- The slopes of parallel and perpendicular lines.
- If a relation is a function or not.
- The input and output values of a function vary in tandem according to the function rule, which can be expressed graphically, numerically, analytically, or verbally.
- A function is increasing over an interval of its domain if, as the input values increase, the output values always increase.
- A function is decreasing over an interval of its domain if, as the input values increase, the output values always decrease.
- The graph intersects the x-axis when the output value is zero. The corresponding input values are said to be zeros of the function.
- The characteristics of the parent functions for: linear, absolute value, square root, quadratic, cubic, and rational functions.
- The characteristics of the graphs of different functions.
- The process for forming a composition of two functions.

- How to find the domain for combinations of functions.
- The process for finding an inverse function algebraically, and verifying that functions are inverse functions algebraically and graphically.

Students will be able to...

- Determine the slope of a line.
- Write linear equations given the slope and a point on the line.
- Utilize and understand function notation.
- Evaluate functions and difference quotients.
- Evaluate the domain and range of functions.
- Graph piecewise-defined functions.
- Identify even and odd functions.
- Perform transformations on parent functions, identify these transformations, and use these transformations to sketch the graphs of functions.
- Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
- Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
- Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
- Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Application of concepts to real-world examples.
- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, Ron. (2020). Precalculus with Limits - A Graphing Approach. 8th Edition. Boston: Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
  - http://www.ixl.com
  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Polynomial and Rational Functions

Target Course/Grade Level: Pre-Calculus/Grades 11-12

Unit Summary: Students will further develop their skills by analyzing polynomial and rational functions. Students will continue to deepen their understanding of the connection between the equation of the function and its graphical representation. Students will strengthen their ability to develop mathematical models and use them to make predictions.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior.
f. Graph trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (.97)^t$, $y = (1.01)^{12t}$, $y = (1.01)^{t/10}$, and classify them as representing exponential growth or decay.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

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L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…

- Polynomial and rational functions can be used to model real-world situations.
- When finding the zeros of a polynomial function, the algebraic and graphical solutions must give the same values.

Unit Essential Questions:

- How are the characteristics of polynomial and rational functions shown algebraically and graphically?
- How are polynomial and rational functions used in the real world?

Knowledge and Skills:
Students will know...

- The characteristics of the graphs of polynomial functions.
- The characteristics of the graphs of rational functions.
- The Remainder and Factor Theorems.
- The Fundamental Theorem of Algebra.

Students will be able to...

- Utilize transformations to sketch graphs of polynomial functions.
- Write an equation of a parabola given its vertex and a point on the graph.
- Determine the zeros of polynomial functions.
- Solve equations by dividing polynomials by other polynomials.
- Determine the complex solutions of a quadratic equation.
- Discover the domains of rational functions.
- Observe vertical and horizontal asymptotes of graphs of rational functions.
- Detects the intercepts of rational functions.
- Determine the holes of rational functions.
- Analyze and sketch graphs of rational functions.
- Perform transformations on parent functions, identify these transformations, and use these transformations to sketch the graphs of functions.
- Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
- Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
- Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
- Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Application of concepts to real-world examples.
- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, Ron. (2020). Precalculus with Limits - A Graphing Approach. 8th Edition. Boston: Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
  - http://www.ixl.com
  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Exponential and Logarithmic Functions

Target Course/Grade Level: Pre-Calculus/Grades 11-12

Unit Summary: In this unit, students will study common and natural logarithms and the inverse relationship between exponential and logarithmic functions. Students will learn to solve exponential and logarithmic equations and use exponential and logarithmic models to describe real-world situations.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), f(x + k) and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.BF.B.4 Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.

b. Verify by composition that one function is the inverse of another.

c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. Produce an invertible function from a non-invertible function by restricting the domain.

F.BF-B.5 Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its

domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

f. Graph trigonometric functions, showing period, midline, and amplitude.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (.97)^t$, $y = (1.01)^{12t}$, $y = (1.01)^{t/10}$, and classify them as representing exponential growth or decay.

F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F. LE.A.4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
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Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…

- Exponential and logarithmic functions are inverse functions.
- The five most common types of models involve exponential or logarithmic functions.
- The graphs of exponential and logarithmic functions reflect the inverse nature of their relationship.
- Exponential and logarithmic graphs can be transformed in the same manner as any other parent function.

Unit Essential Questions:

- What are the characteristics of exponential and logarithmic functions?
- What is the relationship between the domain and range of exponential and logarithmic functions?

- How are the laws and properties of logarithms used to rewrite expressions and solve equations?
- How are exponential and logarithmic functions used in the real world?

Knowledge and Skills:

Students will know...

- The properties of the natural logarithmic function.
- The properties of logarithms (product, quotient, and power property).
- The characteristics of the graphs of exponential and logarithmic functions.
- Exponential functions model growth patterns where successive output values over equal-length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.
- $f(x) = log_b x$ and $g(x) = b^x$, where b > 0 and b b ≠ 1, are inverse functions. That is, g(f(x)) = f(g(x)) = x.

Students will be able to...

- Recognize, evaluate, and graph exponential and logarithmic functions.
- Utilize exponential and logarithmic functions to model and solve real-world problems.
- Rewrite logarithms with different bases.
- Use the properties of logarithms to expand and condense logarithmic expressions.
- Solve exponential and logarithmic equations.
- Perform transformations on parent functions, identify these transformations, and use these transformations to sketch the graphs of functions.
- Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
- Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
- Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
- Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
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- Application of concepts to real-world examples.

- Visuals of concepts that are introduced.
- Hands-on activities where applicable.

RESOURCES

Teacher Resources:

- **Textbook:** Larson, Ron. (2020). Precalculus with Limits - A Graphing Approach. 8th Edition. Boston: Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
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  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Trigonometric Functions

Target Course/Grade Level: Pre-Calculus/Grades 11-12

Unit Summary: In this unit, students will build upon their trigonometry concepts that were studied in geometry. The right triangle definition of the trigonometric functions will be reviewed and expanded to develop the unit circle. From the unit circle, the graphs of the sine and cosine functions will be studied. Students will be able to recognize and write the fundamental trigonometric identities. They will also be able to evaluate, verify, simplify, and rewrite trigonometric expressions. Students will apply these identities and their knowledge of algebra to solve trigonometric equations.

Approximate Length of Unit: 12 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$ use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$ and 2 $\pi - x$ in terms of their values for x, where x is any real number.

F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F.TF.C.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2\theta = 1$ and use it to find $sin(\theta)$, $cos(\theta)$ or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$ or $tan(\theta)$ and the quadrant of the angle.

F.TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…
- Radians and degrees are used to measure angles.
- Reference angles are used to evaluate trigonometric functions of any angle.
- The unit circle may be used to develop the graphs of the parent function of the sine and cosine functions.

Unit Essential Questions:
- How are the right triangle definitions of trigonometric functions and the trigonometric functions based on the unit circle related?
- How are the trigonometric functions of angles and their reference angles related?
- How are the graphs of the sine and cosine functions related to the unit circle?

- How can trigonometric identities be used to solve equations?

Knowledge and Skills:

Students will know...

- The right triangle definitions of trigonometric functions.
- The unit circle and the process of evaluating trigonometric functions of any angle using the unit circle.
- The radian measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin subtended by the angle to the radius of that same circle. For a unit circle that has radius 1, the radian measure is the same as the length of the subtended arc.
- The fundamental trigonometric identities, and use and apply these identities.
- The characteristics of the graphs of the sine and cosine functions.
- Given an angle of measure θ in standard position and a circle with radius r centered at the origin, there is a point, P, where the terminal ray intersects the circle. The coordinates of point P are $(r\cos\theta, r\sin\theta)$.
- That the sine and cosine functions are periodic.
- The graph of a periodic relationship can be constructed from the graph of a single cycle of the relationship.
- For inverse trigonometric functions, the input and output values are switched from their corresponding trigonometric functions, so the output value of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function.
- The sum and difference formulas.
- The double-angle formulas.
- Because trigonometric functions are periodic, there are often infinitely many solutions to trigonometric equations.

Students will be able to...

- Describe and sketch angles.
- Utilize degree and radian measures.
- Convert between degrees and radians.
- Use radian measure to solve problems involving arc length or angular velocity.
- Discover co-terminal angles.
- Evaluate trigonometric functions.
- Calculate the exact value of a trigonometric function.
- Develop the unit circle with special right triangles.
- Use trigonometry to solve a right triangle (including application problems).
- Discover the reference angles.
- Utilize the unit circle to develop the graphs of the parent function of the sine and cosine functions.
- Describe the domain and range of sine, cosine, and tangent functions.
- Perform transformations on the parent functions, identify these transformations and use these transformations to sketch graphs.
- Apply sine and cosine functions to model real-world situations.
- Recognize and write the fundamental trigonometric identities
- Use the fundamental trigonometric identities to evaluate trigonometric functions, and simplify and rewrite trigonometric expressions
- Verify trigonometric identities.
- Solve trigonometric equations of quadratic type.
- Use the sum and difference formulas to evaluate trigonometric functions, verify trigonometric identities and solve trigonometric equations.

- Solve equations and inequalities represented analytically, with and without technology.
- Apply numerical results in a given mathematical or applied context.
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  - https://learnzillion.com/
  - https://www.teachingchannel.org/

- http://illuminations.nctm.org

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