ESL
SCIENCE
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HEALTH & PHYSICAL EDUCATION

RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Content Area: Mathematics

Course: Applied Calculus

Grade Level: 11-12

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

Jeffery Kurczeski,

Program Supervisor of 7-12 Math & Science and 9-12 Business & Technology Education

The Board acknowledges the following who contributed to the preparation of this curriculum.

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Dr. Tiffany A. Beer, Director of Curriculum and Instruction

Dr. Aleya Shoieb, Superintendent of Schools

Subject/Course Title:
Applied Calculus
Grades 11-12

Date of Board Adoption:
August 27, 2024

RAHWAY PUBLIC SCHOOLS CURRICULUM

Applied Calculus: Grades 11-12

PACING GUIDE

Unit	Title	Pacing
1	Limits and Continuity	10 weeks
2	Derivatives	10 weeks
3	Applications of Derivatives	10 weeks
4	Integration and Accumulation of Change	10 weeks

ACCOMMODATIONS

504 Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Provide extra visual and verbal cues and prompts.
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Weekly home-school communication tools (notebook, daily log, phone calls or email messages).
- Provide study sheets and teacher outlines prior to assessments.
- Quiet corner or room to calm down and relax when anxious.
- Reduction of distractions.
- Permit answers to be dictated.
- Hands-on activities.
- Use of manipulatives.
- Assign preferential seating.
- No penalty for spelling errors or sloppy handwriting.
- Follow a routine/schedule.
- Provide student with rest breaks.
- Use verbal and visual cues regarding directions and staying on task.
- Assist in maintaining agenda book.

IEP Accommodations:

- Provide scaffolded vocabulary and vocabulary lists.
- Differentiate reading levels of texts (e.g., Newsela).
- Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials.
- Provide extra visual and verbal cues and prompts.
- Provide links to audio files and utilize video clips.
- Provide graphic organizers and/or checklists.
- Provide modified rubrics.
- Provide a copy of teaching notes, especially any key terms, in advance.
- Provide students with additional information to supplement notes.
- Modify questioning techniques and provide a reduced number of questions or items on tests.
- Allow additional time to complete assignments and/or assessments.
- Provide shorter writing assignments.
- Provide sentence starters.
- Utilize small group instruction.
- Utilize Think-Pair-Share structure.
- Check for understanding frequently.
- Have student restate information.
- Support auditory presentations with visuals.
- Provide study sheets and teacher outlines prior to assessments.
- Use of manipulatives.
- Have students work with partners or in groups for reading, presentations, assignments, and analyses.
- Assign appropriate roles in collaborative work.
- Assign preferential seating.
- Follow a routine/schedule.

Gifted and Talented Accommodations:

- Differentiate reading levels of texts (e.g., Newsela).
- Offer students additional texts with higher lexile levels.
- Provide more challenging and/or more supplemental readings and/or activities to deepen understanding.
- Allow for independent reading, research, and projects.
- Accelerate or compact the curriculum.
- Offer higher-level thinking questions for deeper analysis.
- Offer more rigorous materials/tasks/prompts.
- Increase number and complexity of sources.
- Assign group research and presentations to teach the class.
- Assign/allow for leadership roles during collaborative work and in other learning activities.

ML Accommodations:

- Provide extended time.
- Assign preferential seating.
- Assign peer buddy who the student can work with.
- Check for understanding frequently.
- Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc…).
- Have student repeat directions.
- Make vocabulary words available during classwork and exams.
- Use study guides/checklists to organize information.
- Repeat directions.
- Increase one-on-one conferencing.
- Allow student to listen to an audio version of the text.
- Give directions in small, distinct steps.
- Allow copying from paper/book.
- Give student a copy of the class notes.

	● Provide written and oral instructions. ● Differentiate reading levels of texts (e.g., Newsela). ● Shorten assignments. ● Read directions aloud to student. ● Give oral clues or prompts. ● Record or type assignments. ● Adapt worksheets/packets. ● Create alternate assignments. ● Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted. ● Allow student to resubmit assignments. ● Use small group instruction. ● Simplify language. ● Provide scaffolded vocabulary and vocabulary lists. ● Demonstrate concepts possibly through the use of visuals. ● Use manipulatives. ● Emphasize critical information by highlighting it for the student. ● Use graphic organizers. ● Pre-teach or pre-view vocabulary. ● Provide student with a list of prompts or sentence starters that they can use when completing a written assignment. ● Provide audio versions of the textbooks. ● Highlight textbooks/study guides. ● Use supplementary materials. ● Give assistance in note taking ● Use adapted/modified textbooks. ● Allow use of computer/word processor. ● Allow student to answer orally, give extended time (time-and-a-half). ● Allow tests to be given in a separate location (with the ESL teacher). ● Allow additional time to complete assignments and/or assessments. ● Read question to student to clarify. ● Provide a definition or synonym for words on a test that do not impact the validity of the exam. ● Modify the format of assessments. ● Shorten test length or require only selected test items. ● Create alternative assessments. ● On an exam other than a spelling test, don’t take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Limits and Continuity

Target Course/Grade Level: Applied Calculus/Grades 11-12

Unit Summary: Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as x approaches a point. This distinction allows us to extend the understanding of asymptotes and holes in graphs with formal definitions of continuity. Consider reviewing rational functions when introducing limits, rather than beginning the year with a full review of precalculus topics. Limits are the foundation for differentiation and integration. They are the basis for important definitions and theorems that are used to solve realistic problems involving change and to justify conclusions.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:
ELA
RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…

- Calculus allows us to generalize knowledge about motion to diverse problems involving change.
- Reasoning with definitions, theorems, and properties can be used to justify claims about limits.
- Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.
- Existence theorems allow us to conclude a function's behavior on an interval without precisely locating that behavior.

Unit Essential Questions:

- Can change occur in an instant?
- How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?
- How do we close loopholes so that a conclusion about a function is always true?

Knowledge and Skills:

Students will know...

- Calculus uses limits to understand and model dynamic change.
- Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.
- The limit concept allows us to define the instantaneous rate of change in terms of average rates of change.
- Given a function f, the limit of f(x) as x approaches c is a real number R if f(x) can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is lim lim f (x) as $x \rightarrow c = R$.
- A limit can be expressed in multiple ways, including graphically, numerically, and analytically.
- The concept of a limit includes one-sided limits.
- Graphical information about a function can be used to estimate limits.
- Because of issues of scale, graphical representations of functions may miss important function behavior.
- A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- Numerical information can be used to estimate limits.
- One-sided limits can be determined analytically or graphically.
- Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.
- It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.
- Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- A function f is continuous at x = c provided that f(c) exists, lim f (x) as $x \rightarrow c$ exists, and lim f (x) as $x \rightarrow c = f(c)$.
- A function is continuous on an interval if the function is continuous at each point in the interval.
- Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.
- If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value of the limit of the function as x approaches that point.
- For a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.
- The concept of a limit can be extended to include infinite limits.
- Asymptotic and unbounded behavior of functions can be described and explained using limits.
- The concept of a limit can be extended to include limits at infinity.
- Limits at infinity describe end behavior.

- The difference quotients [f (a+ h) - f(a)] / h (x) and [f (x) - f(a)] / [x - a] express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at x=a can be expressed by lim [f (a+ h) - f(a)] / h (x) as h → 0 or lim [f (x) - f(a)] / [x - a] as x → a, provided the limit exists. These are equivalent forms of the derivative and are denoted f ′ (a).
- If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'.
- A continuous function may fail to be differentiable at a point in its domain

Students will be able to...

- Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.
- Represent limits analytically using correct notation.
- Interpret limits expressed in analytic notation, graphs of functions, and tables.
- Apply appropriate mathematical rules or procedures, with and without technology
- Determine the limits of functions using limit theorems (properties of limits).
- Determine the limits of functions using equivalent expressions for the function. Techniques include:
    - Factoring and dividing common factors of rational functions
    - Multiplying by an expression involving the conjugate of a sum or difference to simplify functions involving radicals
    - Using alternate forms of trigonometric functions
- Identify an appropriate mathematical rule or procedure based on the classification of a given expression.
- Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
- Connect multiple representations of limits
- Identify a re-expression of mathematical information presented in a given representation.
- Justify conclusions about continuity at a point using the definition.
- Identify an appropriate mathematical definition, theorem, or test to apply.
- Determine intervals over which a function is continuous.
- Determine values of x or solve for parameters that make discontinuous functions continuous (remove discontinuities), if possible.
- Interpret the behavior of functions using limits involving infinity
    - Connect infinite limits and vertical asymptotes.
    - Connect limits at infinity and horizontal asymptotes.
- Determine average rates of change using difference quotients
- Represent the derivative of a function as the limit of a difference quotient.
- Determine the equation of a line tangent to a curve at a given point.
- Estimate derivatives.
- Explain the relationship between differentiability and continuity.

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Sample Activities:
    - **Notation Read Aloud:** Begin by writing a limit expression in analytical form (e.g., lim x as x → 0−), and then read the expression aloud to the class: "The limit of x cubed as x approaches 0 from the left." Do the same for 1–2 additional examples that use a variety of limit notations (e.g., the symbol for infinity). Then have students pair up and take turns reading aloud different limit expressions to one another.
    - **Create Representations:** Present students with a limit expression in analytical form (e.g., lim (x^2 -9) /(x-3) as x→3), and then have them translate that expression into a variety of representations: constructing a graph, creating a table of values, and writing it as a verbal expression. Then have students check their graphs and tables using technology.
    - **Work Backward:** Present students with a set of limit problems. Rather than determining the given limits, have them make a list of the various strategies that would be used to determine the limits (e.g., factoring, multiplying by conjugate, and simplifying using trigonometric identities). After confirming their list is complete, have students work in pairs to create and write limit problems, each requiring one of the listed strategies. Then have them swap problems with another pair of students to complete each other's problems.
    - **Discussion Groups:** Give each group of students a piecewise-defined function, a graph paper, and a list of x-values. Have them graph the function, then discuss whether the function is continuous or discontinuous at each x-value, and explain why. Ask students to take turns recording the group's conclusion for each x-value. If continuous, have students discuss and show that all three continuity conditions are satisfied. If discontinuous, have students state which condition was not satisfied.

RESOURCES

Teacher Resources:

- **Textbook:** *Brief Calculus: An Applied Approach*, *9th edition,* Larson, R., (2013), Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
  - http://www.ixl.com
  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Derivatives

Target Course/Grade Level: Applied Calculus/Grades 11-12

Unit Summary: Derivatives allow us to determine instantaneous rates of change. To develop an understanding of how the definition of the derivative applies limits to average rates of change, create opportunities for students to explore average rates of change over increasingly small intervals. Graphing calculator explorations of how various operations affect slopes of tangent lines help students to make sense of basic rules and properties of differentiation. Encourage students to apply the order of operations as they select differentiation rules. Developing differentiation skills will allow students to model realistic instantaneous rates of change and to analyze graphs.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

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W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
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SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
Science
HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:
Students will understand that…

- Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.
- Recognizing that a function's derivative may also be a function allows us to develop knowledge
- about the related behaviors of both.
- Recognizing opportunities to apply derivative rules can simplify differentiation.
- Reasoning with definitions, theorems, and properties can be used to determine a limit.

Unit Essential Questions:

- What is a derivative and how does it differ in various situations?
- What can you predict about *f* given *f'*?
- When there are multiple approaches, how should you choose the best method?
- What are the advantages of having different ways to represent a derivative?
- Why do mathematical properties and rules for simplifying and evaluating limits apply to differentiation?

Knowledge and Skills:

Students will know...

- Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form f(x) = x^r.
- Sums, differences, and constant multiples of functions can be differentiated using derivative rules.
- The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.
- In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit.
- Derivatives of products of differentiable functions can be found using the product rule
- Derivatives of quotients of differentiable functions can be found using the quotient rule.
- The chain rule provides a way to differentiate composite functions.
- The chain rule is the basis for implicit differentiation.
- The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists.
- The chain rule applied with the definition of an inverse function, or the formula for the derivative of an inverse function, can be used to find the derivatives of inverse trigonometric functions.
- Differentiating f ′ produces the second derivative f ″, provided the derivative of f ′ exists; repeating this process produces higher order derivatives of f.
- Higher-order derivatives are represented with a variety of notations. For y = f(x), notations for the second derivative include d^2y/dx^2, f ″(x), and y ″. Higher-order derivatives can be denoted d^(n)y/dx^n or f ^(n)(x).
- The derivative of a function can be interpreted as the instantaneous rate of change concerning its independent variable.
- The derivative can be used to express information about rates of change in applied contexts.
- The unit for f '(x) is the unit for f divided by the unit for x.
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
- The derivative can be used to solve problems involving rates of change in applied contexts.
- The chain rule is the basis for differentiating variables in a related rates problem concerning the same independent variable.
- Other differentiation rules, such as the product rule and the quotient rule, may also be necessary to differentiate all variables concerning the same independent variable.
- The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.

Students will be able to...

- Estimate derivatives.
- Explain the relationship between differentiability and continuity.
- Calculate derivatives of familiar functions.
    - Apply the power, constant, sum, difference, and constant multiple rules.
- Interpret a limit as a definition of a derivative.

- Calculate derivatives of products and quotients of differentiable functions, compositions of differentiable functions, and implicitly defined functions.
- Use derivatives to find rates of change, velocity, and marginal business applications
- Simplify the derivative of a function using algebra
- Classify a higher-order derivative of a function
- Distinguish between functions written in implicit form and explicit form
- Solve related rate problems using implicit differentiation

EVIDENCE OF LEARNING

Assessment:
What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:
What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Sample Activities:
    - **Graph and Switch:** Present students with two or three functions and the graph of each function. Have each student choose a random derivative question and one function. Questions could include: Find the average rate of change on an interval, instantaneous rate of change at a point, derivative as a function, derivative value at a point, or equations for tangent or normal lines at a point. Have students answer their questions and place their answers onto the function's graph. Then have students share their solutions with each other to give and receive feedback.
    - **Match Mine:** Create cards containing graph images of functions with various continuous, discontinuous, differentiable, and non differentiable points or intervals. Provide each student in a pair with the same nine cards. Student A arranges their graphs in a 3 ë 3 grid, which is not visible to Student B. Student A describes each of their graph's positions using information about continuity and differentiability to describe the graph. Based on the descriptions, Student B attempts to arrange their cards to match the grid of Student A
    - **Error Analysis:** Assign a function to each student. Ask them to find the function's derivative using one or more derivative rules. Allow them to check their answers. Ask half of the class to redo their work to include an error, thus having the wrong answer. Ask students to record their correct or incorrect work on a card. Mix up the cards and redistribute them, having students determine if the answer is correct or incorrect. If incorrect, they should explain what error was made, and find the correct answer.

- **Graphic Organizer:** Provide students with colored paper, pens, and markers. Ask them to create a chart, a foldable card, or other creative methods to organize all the derivative rules. For each rule, have them include the mathematical definition, examples, pictures, and helpful hints to understand and remember the rule.
- **Round Table:** Provide each student with the same worksheet containing four functions that require the product rule or quotient rule when finding the derivative. Then have students sit in groups of four. Each student determines the derivative of function No. 1, and then they pass their papers clockwise to the next student. Each student checks the first problem and, if necessary, discusses any mistakes with the previous student. Each student now completes function No. 2 on the paper, and the process continues until each student has their original paperback.

RESOURCES

Teacher Resources:

- **Textbook:** *Brief Calculus: An Applied Approach*, *9th edition,* Larson, R., (2013), Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
  - http://www.ixl.com
  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Applications of Derivatives

Target Course/Grade Level: Applied Calculus/Grades 11-12

Unit Summary: In this unit, the superficial details of contextual applications of differentiation are stripped away to focus on abstract structures and formal conclusions. Reasoning with definitions and theorems establishes that answers and conclusions are more than conjectures; they have been analytically determined. When students showed supporting work for answers in previous units, students will learn to present justifications for their conclusions about the behavior of functions over certain intervals or the locations of extreme values or points of inflection. The unit concludes this study of differentiation by applying abstract reasoning skills to justify solutions for realistic optimization problems.

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.
9.4.12.TL.1 Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

Interdisciplinary Connections and Standards:

ELA

RI.CR.11–12.1 Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
L.SS.11–12.1 Demonstrate command of the system and structure of the English language when writing or speaking.
L.VL.11–12.3 Determine or clarify the meaning of unknown and multiple-meaning words an phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
W.AW.11–12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.
SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that…

- Existence theorems allow us to conclude a function's behavior on an interval without precisely locating that behavior.
- A function's derivative can be used to understand some behaviors of the function.

Unit Essential Questions:

- How does one determine a function's maximum or minimum value?
- How does one determine relative extrema and intervals on which a function is increasing or decreasing?
- What additional information is included in a sound mathematical argument about optimization that a simple description of an equivalent answer lacks?

Knowledge and Skills:

Students will know...

- A point on a function where the first derivative equals zero or fails to exist is a critical point of the function.
- All local (relative) extrema occur at critical points of a function, though not all critical points are local extrema.
- The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing.
- The first derivative of a function can determine the location of the relative (local) extrema of the function.
- Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints.
- The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval.
- The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity.
- The second derivative of a function may be used to locate points of inflection for the graph of the original function.
- The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum.
- When a continuous function has only one critical point on an interval on its domain and the critical point corresponds to a relative (local) extremum of the function on the interval, then that critical point also corresponds to the absolute (global) extremum of the function on the interval.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Graphical, numerical, and analytical information from f' and f'' can be used to predict and explain the behavior of f.
- Key features of the graphs of f, f', and f'' are related to one another.
- The derivative can be used to solve optimization problems; that is, finding a minimum or maximum value of a function on a given interval.
- Minimum and maximum values of a function take on specific meanings in applied contexts.
- A point on an implicit relation where the first derivative equals zero or does not exist is a critical point of the function.
- Applications of derivatives can be extended to implicitly defined functions.
- Second derivatives involving implicit differentiation may be relations of x, y, and dy/dx.

Students will be able to...

- Justify conclusions about the behavior of a function based on the behavior of its derivatives.
- Calculate minimum and maximum values in applied contexts or analysis of functions.
- Interpret minimum and maximum values calculated in applied contexts.
- Determine critical points of implicit relations.
- Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Sample Activities:
    - **Critique Reasoning:** Arrange students in groups of four to six, provide them with a function's derivative (e.g., $g'(x) = 5x + 3$), and ask them to determine if g(x) is increasing or decreasing at a specific x-value, for example, $x = -3$. Ask students to share the reasoning for their conclusion with classmates in their group. Members of the group can then provide feedback and suggestions.
    - **Think-Pair-Share:** Provide students with a graph of f′ and a graph of f′′. Ask them to identify relative extrema and practice writing justifications for relative extrema using the first or second derivative test. Once they've written their justification, ask them to pair with a partner and share their justifications. Students can then discuss similarities or differences in their justification wording.
    - **Create a Plan:** Provide students with a function represented analytically on a closed interval. Ask them to discuss and write x-values that are viable candidates for absolute extrema. Once they have established the viable candidates, ask them to design a method for analyzing
    - **Predict and Confirm:** Provide students with the graph of a differentiable function, for example, f(x)=x^3-4x^2+4x+1 , but do not provide the rule for the function. Ask students to sketch a graph of the derivative of the function. Once students are done, reveal the rule for f(x). Ask students to calculate f′(x), and use technology to graph f′(x) and compare it to their sketched graph.

RESOURCES

Teacher Resources:

- **Textbook:** *Brief Calculus: An Applied Approach*, *9th edition,* Larson, R., (2013), Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
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  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Integration and Accumulation of Change

Target Course/Grade Level: Applied Calculus/Grades 11-12

Unit Summary: This unit establishes the relationship between differentiation and integration using the Fundamental Theorem of Calculus. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines the accumulation of change over an interval, just as differentiation determines the instantaneous rate of change at a point. Students should understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes).

Approximate Length of Unit: 10 weeks

LEARNING TARGETS

NJ Student Learning Standards:

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

F.BF.A.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Career Readiness, Life Literacies, and Key Skills:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1 Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2 Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

9.4.12.IML.2 Evaluate digital sources for timeliness, accuracy, perspective, credibility of the source, and relevance of information, in media, data, or other resources.

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Interdisciplinary Connections and Standards:

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W.IW.11–12.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

W.WR.11–12.5 Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

W.RW.11–12.7 Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes.

SL.PE.11–12.1 Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2 Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.AS.11–12.6 Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

Science

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Understandings:

Students will understand that…

- Definite integrals allow us to solve problems involving the accumulation of change over an interval.
- Definite integrals can be approximated using geometric and numerical methods.
- The Fundamental Theorem of Calculus connects differentiation and integration.
- Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.
- Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration.

Unit Essential Questions:

- How does the Fundamental Theorem of Calculus connect integral and differential calculus?
- How does estimating finite sums lay the foundation for finding the value of a definite integral?
- How is integrating to find areas related to differentiating to find slopes?

Knowledge and Skills:

Students will know...

- The area of the region between the graph of a rate of change function and the x axis gives the accumulation of change.
- In some cases, the accumulation of change can be evaluated by using geometry.
- If a rate of change is positive (negative) over an interval, then the accumulated change is positive (negative).
- The unit for the area of a region defined by the rate of change is the unit for the rate of change multiplied by the unit for the independent variable.
- Definite integrals can be approximated for functions that are represented graphically, numerically, analytically, and verbally.
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- Definite integrals can be approximated using numerical methods, with or without technology.
- Depending on the behavior of a function, it may be possible to determine whether an approximation for a definite integral is an underestimate or overestimate of the value of the definite integral.
- The limit of an approximating Riemann sum can be interpreted as a definite integral.
- A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- The definite integral of a continuous function f over the interval [a, b] is the limit of Riemann sums as the widths of the subintervals approach 0.
- A definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
- The definite integral can be used to define new functions.
- If f is a continuous function on an interval containing a, then $\frac{d}{dx}\int_a^x f(t)dt = f(x)$, where x is in the interval.
- Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, the reversal of limits of integration, and the integral of a function over adjacent intervals.
- The definition of the definite integral may be extended to functions with removable or jump discontinuities.
- An antiderivative of a function f is a function g whose derivative is f.
- If a function f is continuous on an interval containing a, the function defined by $F(x) = \int_a^x f(t)dt = f(x)$ is an antiderivative of f for x in the interval.

- If f is continuous on the interval [a, b] and F is an antiderivative of f, then $\int_a^b f(x)dx = F(b) - F(a)$.
- $\int f(x)dx$ is an indefinite integral of the function f and can be expressed as $\int f(x)dx$=F(x)+C,

  where $F'(x) = f(x)$ and C is any constant.
- Differentiation rules provide the foundation for finding antiderivatives.
- Many functions do not have closed-form antiderivatives.
- Substitution of variables is a technique for finding antiderivatives.
- For a definite integral, the substitution of variables requires corresponding changes to the limits of integration.

Students will be able to...

- Approximate a definite integral using geometric and numerical methods.
- Interpret the limiting case of the Riemann sum as a definite integral.
- Represent the limiting case of the Riemann sum as a definite integral.
- Represent accumulation functions using definite integrals.
- Represent accumulation functions using definite integrals.
- Calculate a definite integral using areas and properties of definite integrals
- Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.
- Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.
- For integrands requiring substitution or rearrangements into equivalent forms:
  - (a) Determine indefinite integrals.
  - (b) Evaluate definite integrals

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly "understand"?

- End of Unit Common Assessment - See folder for assessment links.
- Formative: warm-up activities, exploratory activities, class discussions, student participation, homework, and exit tickets.
- Summative: quizzes, tests, projects, and benchmark assessments.
- Open-ended problems that involve written responses with justification of answers.

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Interactive Platforms: Desmos, Kahoot, Delta Math, Formative, Quizizz, Quizlet, Google Forms, Mathspace, PearDeck, Freckle, Geogebra, Gimkit, and Khan Academy.
- Group Work Suggestion: quiz trade, circuits, limit war, matching card games, jeopardy, relay review, and speed dating.
- Sample Activities:
  - **Quickwrite:** Present the class with several examples of definite integrals set equal to Riemann sums in summation notation. Ask students to take five minutes to identify and write about all common elements between the two expressions and why they think the

two expressions are equivalent. After finishing the five minutes, ask students to share their observations with the class

- **Look For a Pattern:** Present students with several indefinite integrals and proposed, yet incorrect, antiderivatives. Ask them to check the antiderivatives by differentiating each and comparing to the original integrands. As students see that each antiderivative is incorrect, ask them to identify a pattern within the errors. Identifying this pattern will establish the foundation for integrating using substitution.
- **Odd One Out:** To help students select a strategy, form groups of four, presenting each student an indefinite integral whose integrand is rational. For each group, include one integrand that requires long division or completing the square. Ask students to decide if their example fits with the group. Identifying the odd one out will help students connect the integrand form to the appropriate strategy.

RESOURCES

Teacher Resources:

- **Textbook:** *Brief Calculus: An Applied Approach*, *9th edition,* Larson, R., (2013), Cengage Learning.
- Useful Websites for Teachers to Explore:
  - www.illustrativemathematics.org
  - http://www.ixl.com
  - www.kutasoftware.com
  - https://www.khanacademy.org/
  - https://learnzillion.com/
  - https://www.teachingchannel.org/
  - http://illuminations.nctm.org

Equipment Needed:

- Projector, Computer/Laptop, Chromebooks, Document Camera, Graphing Calculator