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RAHWAY PUBLIC SCHOOLS

CURRICULUM & INSTRUCTION

Course: Mathematics

Grade Level: 7

This curriculum is part of the Educational Program of Studies of the Rahway Public Schools.

ACKNOWLEDGMENTS

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Subject/Course Title:
Mathematics
Grade 7

Date of Board Adoption:
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RAHWAY PUBLIC SCHOOLS CURRICULUM

Mathematics: Grade 7

PACING GUIDE

Unit	Title	Pacing
1	Scale Drawings	4 weeks
2	Introducing Proportional Relationships	5 weeks
3	Measuring Circles	4 weeks
4	Proportional Relationships and Percentages	5 weeks
5	Rational Number Arithmetic	5 weeks
6	Expressions, Equations, and Inequalities	6 weeks
7	Angles, Triangles, and Prisms	5 weeks
8	Probability and Sampling	6 weeks

ACCOMMODATIONS

<p>504 Accommodations:</p> <ul style="list-style-type: none"> ● Provide scaffolded vocabulary and vocabulary lists. ● Provide extra visual and verbal cues and prompts. ● Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials. ● Provide links to audio files and utilize video clips. ● Provide graphic organizers and/or checklists. ● Provide modified rubrics. ● Provide a copy of teaching notes, especially any key terms, in advance. ● Allow additional time to complete assignments and/or assessments. ● Provide shorter writing assignments. ● Provide sentence starters. ● Utilize small group instruction. ● Utilize Think-Pair-Share structure. ● Check for understanding frequently. ● Have student restate information. ● Support auditory presentations with visuals. ● Weekly home-school communication tools (notebook, daily log, phone calls or email messages). ● Provide study sheets and teacher outlines prior to assessments. ● Quiet corner or room to calm down and relax when anxious. ● Reduction of distractions. ● Permit answers to be dictated. ● Hands-on activities. ● Use of manipulatives. ● Assign preferential seating. ● No penalty for spelling errors or sloppy handwriting. ● Follow a routine/schedule. ● Provide student with rest breaks. ● Use verbal and visual cues regarding directions and staying on task. ● Assist in maintaining agenda book. 	<p>IEP Accommodations:</p> <ul style="list-style-type: none"> ● Provide scaffolded vocabulary and vocabulary lists. ● Differentiate reading levels of texts (e.g., Newsela). ● Provide adapted/alternate/excerpted versions of the text and/or modified supplementary materials. ● Provide extra visual and verbal cues and prompts. ● Provide links to audio files and utilize video clips. ● Provide graphic organizers and/or checklists. ● Provide modified rubrics. ● Provide a copy of teaching notes, especially any key terms, in advance. ● Provide students with additional information to supplement notes. ● Modify questioning techniques and provide a reduced number of questions or items on tests. ● Allow additional time to complete assignments and/or assessments. ● Provide shorter writing assignments. ● Provide sentence starters. ● Utilize small group instruction. ● Utilize Think-Pair-Share structure. ● Check for understanding frequently. ● Have student restate information. ● Support auditory presentations with visuals. ● Provide study sheets and teacher outlines prior to assessments. ● Use of manipulatives. ● Have students work with partners or in groups for reading, presentations, assignments, and analyses. ● Assign appropriate roles in collaborative work. ● Assign preferential seating. ● Follow a routine/schedule.
<p>Gifted and Talented Accommodations:</p> <ul style="list-style-type: none"> ● Differentiate reading levels of texts (e.g., Newsela). ● Offer students additional texts with higher lexile levels. ● Provide more challenging and/or more supplemental readings and/or activities to deepen understanding. ● Allow for independent reading, research, and projects. ● Accelerate or compact the curriculum. ● Offer higher-level thinking questions for deeper analysis. ● Offer more rigorous materials/tasks/prompts. ● Increase number and complexity of sources. ● Assign group research and presentations to teach the class. ● Assign/allow for leadership roles during collaborative work and in other learning activities. 	<p>ML Accommodations:</p> <ul style="list-style-type: none"> ● Provide extended time. ● Assign preferential seating. ● Assign peer buddy who the student can work with. ● Check for understanding frequently. ● Provide language feedback often (such as grammar errors, tenses, subject-verb agreements, etc...). ● Have student repeat directions. ● Make vocabulary words available during classwork and exams. ● Use study guides/checklists to organize information. ● Repeat directions. ● Increase one-on-one conferencing. ● Allow student to listen to an audio version of the text. ● Give directions in small, distinct steps. ● Allow copying from paper/book. ● Give student a copy of the class notes.

- Provide written and oral instructions.
- Differentiate reading levels of texts (e.g., Newsela).
- Shorten assignments.
- Read directions aloud to student.
- Give oral clues or prompts.
- Record or type assignments.
- Adapt worksheets/packets.
- Create alternate assignments.
- Have student enter written assignments in criterion, where they can use the planning maps to help get them started and receive feedback after it is submitted.
- Allow student to resubmit assignments.
- Use small group instruction.
- Simplify language.
- Provide scaffolded vocabulary and vocabulary lists.
- Demonstrate concepts possibly through the use of visuals.
- Use manipulatives.
- Emphasize critical information by highlighting it for the student.
- Use graphic organizers.
- Pre-teach or pre-view vocabulary.
- Provide student with a list of prompts or sentence starters that they can use when completing a written assignment.
- Provide audio versions of the textbooks.
- Highlight textbooks/study guides.
- Use supplementary materials.
- Give assistance in note taking
- Use adapted/modified textbooks.
- Allow use of computer/word processor.
- Allow student to answer orally, give extended time (time-and-a-half).
- Allow tests to be given in a separate location (with the ESL teacher).
- Allow additional time to complete assignments and/or assessments.
- Read question to student to clarify.
- Provide a definition or synonym for words on a test that do not impact the validity of the exam.
- Modify the format of assessments.
- Shorten test length or require only selected test items.
- Create alternative assessments.
- On an exam other than a spelling test, don't take points off for spelling errors.

UNIT 1 OVERVIEW

Content Area: Mathematics

Unit Title: Scale Drawings

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor. Also, the angle measures in a scaled copy are the same as in the original figure. This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships later. Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures. Note that the study of scaled copies is limited to pairs of figures that have the same orientation — in other words, they are not rotations or reflections of each other. Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

Approximate Length of Unit: 4 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Career Readiness, Life Literacies, and Key Skills:

- 9.2.8.CAP.2** Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CR.7.1 Cite several pieces of textual evidence and make relevant connections to support analysis of what an informational text says explicitly as well as inferences drawn from the text.

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

W.IW.7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

W.WP.7.4 With some guidance and support from peers and adults, develop and strengthen writing as needed by planning; flexibly making editing and revision choices and sustaining effort to complete complex writing tasks; and focusing on how well purpose and audience have been addressed.

Science

MS-PS1-1 Develop models to describe the atomic composition of simple molecules and extended structures.

MS-PS2-2 Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object.

Social Studies

6.1.8.GeoSV.3.a Use maps and other geographic tools to construct an argument on the impact of geography on the developments and outcomes of the American Revolution including New Jersey's pivotal role.

6.1.8.EconNE.4.b Analyze how technological innovations affected the status and social class of different groups of people and explain the outcomes that resulted.

Unit Understandings:

Students will understand that...

- The concept of scale drawings and how they represent real-world objects or geometric figures at a reduced or enlarged scale.
- Applying scale factors to accurately resize geometric figures in scale drawings while maintaining proportional relationships between corresponding sides and angles.
- Developing problem-solving skills by applying scale drawing techniques to solve real-life scenarios, such as map scaling, architectural design, or model building.
- Through scale drawings, analyzing and interpreting proportional relationships between dimensions in geometric figures, understanding how changes in scale impact the overall structure.
- Effectively communicating their mathematical reasoning behind scale drawings, including justifying their scaling decisions, interpreting scale factors, and explaining their problem-solving strategies.

Unit Essential Questions:

- How can critical thinking skills be applied when determining the scale factor in a scale drawing of a geometric figure?

- In what ways can problem-solving strategies help in accurately scaling down real-world objects onto paper in a scale drawing?
- How does creativity play a role in visualizing and creating scale drawings of geometric figures?
- What are the benefits of collaborating with peers when working on scale drawing projects, and how can this collaboration enhance the final outcome?
- When working independently on a scale drawing task, how can you apply innovative approaches to address challenges and achieve precision in your work?

Knowledge and Skills:

Students will know...

- How to represent
 - a scaled copy for a given scale factor.
 - distances using different scales.
 - relevant features of a classroom with a scale drawing.
- How to generalize
 - about corresponding distances and angles in scaled copies.
 - about scale factors greater than, less than, and equal to 1.
 - about scale factors and area.
 - about scale factors with and without units.
- How to explain
 - how to use scale drawings to find actual distances.
 - how to use scale drawings to find actual distances, speed, and elapsed time.
 - how to use scale drawings to find actual areas.
- Vocabulary: scaled copy, corresponding scale factor, reciprocal, area, scale drawing, scale

Students will be able to...

- Determine whether a figure is a scaled copy of another figure, by examining corresponding side lengths and angle measures.
- Draw a scaled copy of a figure using a given scale factor.
- Create a scale drawing given the actual measurements of the object or give another scale drawing at a different scale.
- Explain how to use scales and scale drawings to calculate actual distances and areas.
- Compare, contrast, and critique (orally) scale drawings of the classroom.
- Generate an appropriate scale to represent an actual distance on a limited drawing size, and explain (orally) the reasoning.
- Make simplifying assumptions and determine what information is needed to create a scale drawing of the classroom.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets

- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Pairs of Scaled Polygons - In this activity, students hone their understanding of scaled copies by working with more complex figures. Students work with a partner to match pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.
- Lesson 5: Scaling A Puzzle - This activity gives students a chance to apply what they know about scale factors, lengths, and angles and create scaled copies without the support of a grid. Students work in groups of 3 to complete a jigsaw puzzle, each group member scaling 2 non-adjacent pieces of a 6-piece puzzle with a scale factor of $\frac{1}{2}$. The group then assembles the scaled pieces and examines the accuracy of their scaled puzzle. Consider having students use a color in place of the cross-hatching.
- Lesson 7: Sizing Up a Basketball Court - In this introductory activity, students explore the meaning of scale. They begin to see that a scale communicates the relationship between lengths on a drawing and corresponding lengths in the objects they represent, and they learn some ways to express this relationship.
- Lesson 11: Apollo Lunar Module - In this activity, students use a scale drawing and a scale expressed without units to calculate actual lengths. Students will need to make a choice about which units to use, and some choices make the work easier than others.
- Lesson 12: The World's Largest Flag: In this activity, students use a scale without units to find actual and scaled distances that involve a wider range of numbers, from 0.02 to 2,000. They also return to thinking about how the area of a scale drawing relates to the area of the actual thing.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.1
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.1 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 2 OVERVIEW

Content Area: Mathematics

Unit Title: Introducing Proportional Relationships

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on previous work with equivalent ratios and helps prepare students for the study of linear functions in future courses. Students begin by looking at tables. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality." Next, students use equations of the form to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship. Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through the origin. They interpret points on the graph, including the point. Here is an example of a graph, an equation, and a table that all represent the same proportional relationship. By the end of the unit, students should be comfortable working with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether or not a relationship is proportional. In a later unit, students will apply proportional reasoning to solve multi-step problems and calculate more complex rates.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $(\frac{1}{2})/(\frac{1}{4})$ miles per hour, equivalently 2 miles per hour.
- 7.RP.A.2** Recognize and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
 - Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.
 - Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1,r)$ where the unit rate is.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CI.7.3 Analyze the relationship between a central idea and supporting details.

RI.AA.7.7 Interpret the vocabulary in a text, analyze the context in which it appears, and demonstrate understanding of figurative language.

W.IW7.2 Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

SL.UM.7.5 Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

MS-PS3-1 Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.

MS-ESS3-4 Construct an argument supported by evidence for how increases in human population and per-capita consumption of natural resources impact Earth's systems.

Social Studies

6.2.8.HistoryCC.1.a Describe the influence of the agricultural revolution on population growth and the subsequent development of civilizations (e.g., the impact of food surplus from farming).

6.2.8.EconEM.3.a Analyze the impact of expanding land and sea trade routes as well as a uniform system of exchange in the Mediterranean World and Asia.

Unit Understandings:

Students will understand that...

- The concept of proportional relationships and constants of proportionality recognizes the direct correlation between two quantities and the consistent ratio they maintain.
- Strategies to determine whether a relationship is proportional by analyzing patterns, calculating ratios, and identifying the presence or absence of a constant ratio.
- Exploring how to compare and represent situations with varying constants of proportionality, discerning how changes in ratios impact the nature of the relationship.
- Through mathematical reasoning and evidence-based explanations, justify their conclusions on whether a relationship is proportional or non-proportional, applying critical thinking skills to evaluate the validity of their claims.

- Representing proportional and nonproportional relationships graphically, algebraically, and with tables, showcasing a comprehensive understanding of multiple ways to express and interpret mathematical relationships.

Unit Essential Questions:

- How can you identify a proportional relationship from a set of data, and what characteristics define such relationships?
- What methods can be used to determine whether a given relationship is proportional or non-proportional, and how do these methods differ?
- In what ways can you compare situations with different constants of proportionality, and how does this comparison impact the interpretation of the relationship?
- How do you justify mathematically whether a relationship is proportional or non-proportional, and what evidence is needed to support your reasoning?
- What are the various ways to represent proportional relationships graphically, algebraically, and with tables, and how do these representations provide insights into the nature of the relationship?

Knowledge and Skills:

Students will know...

- How to compare
 - approaches to solving problems involving proportional relationships.
 - proportional relationships with nonproportional relationships.
 - tables, descriptions, and graphs representing the same situations.
 - graphs of proportional relationships.
- How to interpret
 - representations showing equivalent ratios.
 - tables showing equivalent ratios.
 - situations involving proportional relationships.
 - how a graph represents features of a situation.
- How to generalize
 - about proportional relationships.
 - about equations that represent proportional relationships.
 - about how a constant of proportionality is represented by graphs and tables.
- Vocabulary: equivalent ratios, constant of proportionality, proportional relationship, reciprocal, origin, and coordinate plane.

Students will be able to...

- Determine the constant of proportionality for a proportional relationship represented in a table, and interpret it in context.
- Explain how to calculate unknown values in a table that represents a proportional relationship.
- Use an equation to solve problems involving a proportional relationship.
- Write an equation of the form $y = kx$ to represent a proportional relationship, given a table or a description of the situation.
- Determine whether the values in a table could represent a proportional relationship.
- Use a table to determine whether an equation represents a proportional relationship.
- Determine whether a given graph represents a proportional relationship.
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret points on the graph of a proportional relationship.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Mystery Mixtures - The purpose of this activity is for students to articulate that the taste of the mixture depends both on the amount of water and the amount of drink mix used to make the mixture. Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyze contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can't, make strategic pairings of students for this task.
- Lesson 9: Info Gap: Biking and Rain - In this info gap activity, students write equations for several proportional relationships given in the contexts of a bike ride and steady rainfall. They use the equations to make predictions.
- Lesson 10: Matching Tables and Graphs - Students work in pairs to match tables to graphs and to practice articulating their reasoning. This task is intended to foster an understanding of correspondences between tables and graphs. Students sort the graphs and justify their sorting schemes. Then, they compare the way they sorted their graphs with a different group. The purpose of this activity is to illustrate the idea that the graph of a proportional relationship is a line through the origin.
- Lesson 14: One Scenario, Four Representations - In this activity, students choose from different lists of things to define their own proportional and nonproportional relationships. Some of the things on the list will be familiar and others will be unfamiliar. This is a significant change from previous activities where students were always given two quantities and they had to decide if they were proportional or not. This new step gives students the opportunity to think about what quantities are related to some of the items on the lists, which is an important step in modeling with mathematics.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.2
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.2 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 3 OVERVIEW

Content Area: Mathematics

Unit Title: Measuring Circles

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students learn to understand and use the term “circle” to mean the set of points that are equally distant from a point called the “center.” They gain an understanding of why the circumference of a circle is proportional to its diameter, with a constant of proportionality π . They see informal derivations of the fact that the area of a circle is equal to π times the square of its radius. Students use the relationships of circumference, radius, diameter, and area of a circle to find lengths and areas, expressing these in terms of π or using appropriate approximations of π to express them numerically.

Approximate Length of Unit: 4 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.A.2** Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.G.B.4.** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.RP.A.2** Recognize and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.
- 7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- 7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for

a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

RI.CI.7.3 Analyze the relationship between a central idea and supporting details.

W.IW.7.2: Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

SL.UM.7.5 Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

MS-PS1-2 Analyze and interpret data on the properties of substances before and after the substances interact to determine if a chemical reaction has occurred.

MS-PS3-1 Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.

MS-LS1-3 Use argument supported by evidence for how the body is a system of interacting subsystems composed of groups of cells.

Unit Understandings:

Students will understand that...

- Having a deep understanding of measuring circles, including concepts such as radius, diameter, circumference, and area, and how these measurements are interconnected.
- Analyzing and critiquing reasoning related to circle measurements, identifying and evaluating different approaches to solving circle-related problems, and assessing the validity of the presented arguments.
- Their reasoning behind circle measurements, including discussions on different approximations of pi, understanding the significance of pi in calculating circle properties, and justifying their choices of approximation methods.
- Through exploring circle graphs and deconstructed circles, they will describe the features of graphs representing circle measurements, such as how changes in radius or diameter affect the shape of the graph and the relationships between different circle measurements.

- Effectively communicating their understanding of circle measurements, reasoning processes, and critiques of circle-related arguments, utilizing mathematical language and evidence-based explanations to support their claims.

Unit Essential Questions:

- How do we measure different aspects of circles, such as radius, diameter, circumference, and area, and how are these measurements interconnected?
- What strategies can we use to critique reasoning about circle measurements, and how can we evaluate the validity of various approaches to solving circle-related problems?
- Why is it important to explain our reasoning behind circle measurements, including discussions on different approximations of pi, and how do these approximations impact our calculations?
- What features characterize graphs representing circle measurements, and how do changes in radius, diameter, or circumference affect the shape and properties of these graphs?
- How can we effectively describe and deconstruct circles to analyze their components and relationships, enhancing our understanding of circle measurements and geometric concepts?

Knowledge and Skills:

Students will know.

- How to generalize
 - about categories for sorting circles.
 - about the relationship between circumference and diameter.
 - about circumference and rotation.
 - about the relationship between the radius and the area of a circle.
- How to justify
 - reasoning about circumference and perimeter.
 - estimates for the areas of circles.
 - reasoning about areas of curved figures.
 - reasoning about the cost of stained glass windows.
- How to interpret
 - situations involving circles.
 - floor plans and maps.
 - situations involving circumference and area.
- Vocabulary: radius, diameter, circumference, circle, pi, area of a circle, and squared.

Students will be able to...

- Recognize that there are proportional relationships between the circumference, diameter, and radius of circles and express these relationships using equations.
- Solve problems involving circumference.
- Justify that the area of a circle can be calculated with the formula $A=\pi r^2$.
- Recognize that the area of a circle is not proportional to its diameter or radius.
- Solve problems involving the area of a circle.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Perimeter of a Square - In this activity, students examine the relationship between the length of the diagonal and the perimeter for squares of different sizes. This prepares students for examining the relationship between the diameter and circumference of circles in a future lesson.
- Lesson 2: Sorting Round Objects - The purpose of this activity is to build on students’ prior experience with circles and help them refine their definition of a circle. First, students sort pictures of round objects based on whether or not they are circular. Then, they compare the size of the circles to begin a discussion of what aspects of a circle can be measured. Lastly, the teacher introduces the terms diameter, center, radius, and circumference, so students can identify these measurements in the pictures of the circular objects. When students focus on what shapes have in common and describe their common features (and the deviations of non-examples) to build a definition, they express regularity in repeated reasoning.
- Lesson 7: Estimating Areas of Circles - In a previous lesson, students measured various circular objects and graphed the measurements to see that there appears to be a proportional relationship between the diameter and circumference of a circle. In this activity, students use a similar process to see that the relationship between the diameter and area of a circle is not proportional. This echoes the earlier exploration comparing the length of a diagonal of a square to the area of the square, which was also not proportional.
- Lesson 10: Card Sort: Circle Problems - The purpose of this activity is for students to think about how the circumference and area of circles apply to real-world situations. First, students sort slips based on whether the question is related to the circumference or area of a circle. Next, each group focuses on one of the questions.. They estimate appropriate measurements for the context and use these measurements to calculate a reasonable answer.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.3
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.3 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 4 OVERVIEW

Content Area: Mathematics

Unit Title: Proportional Relationships and Percentages

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students use ratios, scale factors, unit rates (also called constants of proportionality), and proportional relationships to solve multi-step, real-world problems that involve fractions and percentages. They use long division to write fractions presented in the form a/b as decimals, e.g., $11/30=0.36$ (6 repeating). They learn to understand and use the terms “repeating decimal,” “terminating decimal,” “percent increase,” “percent decrease,” “percent error,” and “measurement error.” They represent amounts and corresponding percent rates with double-number line diagrams and tables. They use these terms and representations in reasoning about situations involving sales taxes, tips, markdowns, markups, sales commissions, interest, depreciation, and scaling a picture. Students use equations to represent proportional relationships in which the constant of proportionality arises from a percentage, e.g., the relationship between the price paid and the amount of sales tax paid.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- 7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”
- 7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $(\frac{1}{2})/(\frac{1}{4})$ miles per hour, equivalently 2 miles per hour.
- 7.RP.A.2** Recognize and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t=pn$.
- d. Explain what a point on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1,r)$ where r is the unit rate.
- e. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

RI.CI.7.3 Analyze the relationship between a central idea and supporting details.

W.IW.7.2: Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

SL.UM.7.5 Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

MS-PS3-1 Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.

MS-LS1-3 Use argument supported by evidence for how the body is a system of interacting subsystems composed of groups of cells.

Unit Understandings:

Students will understand that...

- The ability to compare measurements using various units, understand scale factors in resizing objects or figures and interpret decimal and fraction representations in different contexts.
- Comparing and contrasting representations of increases or decreases of quantities using fractions and decimals, identifying patterns and relationships between the original and changed amounts.
- The concept of using constants of proportionality to solve problems efficiently, recognizing how these constants establish proportional relationships and enable streamlined problem-solving approaches.
- Through real-world scenarios, exploring relationships with percent increase and decrease, understanding how percentages reflect changes in quantities, and applying this knowledge to practical problem-solving situations.
- Justifying the specific information needed to solve percent change problems, and articulating why certain data points, percentages, or values are essential for accurately calculating and interpreting percent changes in various contexts.

Unit Essential Questions:

- How can measurements be effectively compared using different units, scale factors, and decimal or fraction representations, and what insights can be gained from these comparisons?
- What are the key differences between representations of an increase or decrease of an amount using fractions and decimals, and how do these representations impact our understanding of changes in quantities?
- In what ways can constants of proportionality be generalized to efficiently solve problems, and how do these constants establish proportional relationships that aid in problem-solving processes?
- How do relationships with percent increase and decrease influence our interpretation of changes in quantities, and how can percentages be utilized to analyze and solve practical problems effectively?
- Why is it important to justify the specific information required to solve percent change problems, and what considerations should be taken into account when determining the necessary data for accurate percentage calculations in various contexts?

Knowledge and Skills:

Students will know...

- How to interpret
 - situations involving constant speed.
 - concrete problems involving percent increase and decrease.
 - problems involving sales tax and tips.
 - concrete situations involving percent error.
- How to explain
 - how to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount.
 - how to solve percent change problems.
 - strategies for solving percent problems with fractional percentages.
 - how to measure lengths and interpret measurement errors.
 - strategies for solving percent error problems.
- How to represent
 - situations involving percent increase and decrease.
 - situations with percent error.
 - situations from the news involving percent change.
- Vocabulary: percentage, unit rate, tape diagram, repeating decimal, long division, percent increases, percent decrease, measurement error, and percent error.

Students will be able to...

- Create algebraic expressions that represent a situation involving adding or subtracting a fraction of the initial value.
- Solve problems about proportional relationships with fractional quantities.
- Use long division to generate a decimal representation of a fraction.
- Create algebraic expressions or equations that represent a situation involving percent increase or decrease.
- Use diagrams to solve problems involving percent increase or decrease.
- Calculate measurement error, and express it as a percentage of the actual value.
- Solve problems involving tax, tip, simple interest, markup, markdown, or commission.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 4: Walking Half as Much Again - In this activity, students find patterns in situations to connect to the distributive property. These patterns build understanding of the equations $x + 0.5x = (1 + 0.5)x = 1.5x$ or $x + \frac{1}{2}x = (1 + \frac{1}{2})x = 1\frac{1}{2}x$. Students should see that the expressions are all representations of the same thing. Students learn that multiplying a number by $\frac{1}{2}$ and adding that product to the original numbers is the same as multiplying by 1.5. This idea is extended to percentages in the following activity.
- Lesson 5: More and Less with Decimals - This activity continues work done in the previous lesson that connected various representations of proportional relationships including images, equations, and descriptions. In this activity, students match diagrams, descriptions, and equations that represent a proportional relationship, involving variables x and y . Students then create their own diagram to represent an equation.
- Lesson 11: Card Sort: Percentage Situations - This activity gives students an opportunity to practice various vocabulary terms that come along with percentages. Students are asked to sort scenarios to different descriptors using the images, sentences or questions found on the scenario cards. The questions found on the scenario cards are intended to help students figure out which descriptor the scenario card belongs under.
- Lesson 12: Info Gap: Sporting Goods - The purpose of this info gap activity is for students to identify the essential information needed to determine the total savings after various discounts are applied to different items. The info gap structure requires students to make sense of problems by determining what information is necessary and then to ask for the information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

- Lesson 13: Measuring to the Nearest - The purpose of this task is to notice how differences in recorded measurements can result from the level of precision of your measuring device. Students use rulers that have varying levels of accuracy to measure the same lines. This warm-up gets the conversation started around measurement errors that will continue in the following activities.
- Lesson 15: Info Gap: Quality Control - This info gap activity uses a quality control situation working with percent error. It is very common that products in a factory are checked to make sure that they meet certain specifications. In this case, the odometer of a car is tested and the amount of liquid in a bottle that is automatically filled is checked. It makes sense that the decision should be based on percent error and not on absolute error.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.4
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.4 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 5 OVERVIEW

Content Area: Mathematics

Unit Title: Rational Number Arithmetic

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students interpret signed numbers in contexts (e.g., temperature, elevation, deposit and withdrawal, position, direction, speed and velocity, percent change) together with their sums, differences, products, and quotients. (“Signed numbers” include all rational numbers, written as decimals or in the form ab .) Students use tables and number line diagrams to represent sums and differences of signed numbers or changes in quantities represented by signed numbers such as temperature or elevation, becoming more fluent in writing different numerical addition and subtraction equations that express the same relationship. They compute sums and differences of signed numbers. They plot points in the plane with signed number coordinates, representing and interpreting sums and differences of coordinates. They view situations in which objects are traveling at a constant speed (familiar from previous units) as proportional relationships. For these situations, students use multiplication equations to represent changes in position on number line diagrams or distance traveled, and interpret positive and negative velocities in context. They become more fluent in writing different numerical multiplication and division equations for the same relationship. Students extend their use of the “next to” notation to include negative numbers and products of numbers, e.g., writing $-5x$ and $(-5)(-10)$ rather than $(-5) \cdot (x)$ and $(-5) \cdot (-10)$. They extend their use of the fraction bar to include variables as well as numbers, writing $-8.5 \div x$ as well as $-8.5/x$.

Approximate Length of Unit: 6 weeks

LEARNING TARGETS

NJ Student Learning Standards:

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9 \frac{3}{4}$ inches long in the center of a door that is $27 \frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. For example, in the first round of a game, Maria scored 20 points. In the second round of the same game, she lost 20 points. What is her score at the end of the second round?

b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

(Clarification: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

RI.CI.7.3 Analyze the relationship between a central idea and supporting details.

W.IW.7.2: Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

SL.UM.7.5 Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

MS-PS3-1 Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.

MS-LS1-3 Use argument supported by evidence for how the body is a system of interacting subsystems composed of groups of cells.

Unit Understandings:

Students will understand that...

- Justifying reasoning about distances on a number line, understanding the significance of negative numbers and their application in various contexts, such as measuring distances, and understanding account balances, and debt.
- Exploring the process of determining changes in temperature using signed numbers, examining how positive and negative values represent temperature shifts and applying this knowledge to real-life temperature scenarios.
- Utilizing inverse operations to find information within the realm of signed numbers, recognizing the role of inverse operations in solving equations, uncovering hidden data, and simplifying mathematical processes.
- Through practical applications, modeling situations involving signed numbers to represent transactions, movements, or relationships, enhances their ability to interpret and solve problems requiring an understanding of positive and negative values.

Unit Essential Questions:

- How can we justify reasoning about distances on a number line and negative numbers, including their applications in understanding account balances, debt, and real-world scenarios?
- What strategies can be employed to explain the process of determining changes in temperature using signed numbers and how can this knowledge be applied to practical situations?
- How do we find information using inverses in the context of signed numbers, and what role do inverse operations play in solving equations and uncovering hidden data?
- In what ways can we model situations involving signed numbers to represent transactions, movements, or mathematical relationships, and how does modeling aid in understanding and solving complex problems?

Knowledge and Skills:

Students will know...

- How to interpret
 - situations involving signed numbers.
 - tables with signed numbers.
 - bank statements with signed numbers.

- How to represent
 - the addition of signed numbers on a number line.
 - situations involving signed numbers.
 - elevation changes.
 - position, speed, and direction.
- How to generalize
 - about subtracting and adding signed numbers.
 - about differences and magnitude.
 - about multiplying negative numbers.
 - about additive and multiplicative inverses.
- Vocabulary: absolute value, positive number, negative number, deposit, withdrawal, solution (to an equation), rational number, and variable.

Students will be able to...

- Apply addition and subtraction of signed numbers to represent situations and solve problems.
- Calculate the sum or difference of two rational numbers.
- Apply multiplication and division of signed numbers to represent situations and solve problems.
- Calculate the product or quotient of two rational numbers.
- Apply the four operations with rational numbers to solve problems.
- Solve an equation of the form $x + p = q$ or $px = q$, where p , q , and x are rational numbers.
- Write an equation of the form $x + p = q$ or $px = q$ (where p , q , and x are rational numbers) to represent a situation.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 1: Card Sort: Rational Numbers - This activity reviews ordering integers first, and then rational numbers second. Many of the numbers also have their additive inverse in the set, which can help students use the structure of the number line to order the numbers.
- Lesson 9: Rational Numbers Multiplication Grid - The students revisit the representation of a multiplication chart, which may be familiar from previous grades; however, in this activity, the multiplication chart is extended to include negative numbers. Students identify and continue patterns to complete the chart and see that it fits the patterns in the chart for the product of two negative numbers to be a positive number.
- Lesson 10: Card Sort: Matching Expressions - This activity reminds students of the links between positive fractions and multiplication and prepares them to think about division as multiplication by the reciprocal; this will be important for dividing negative numbers. Students will use earlier

work and their work in previous lessons in this unit to extend what they know about the division of positive rationals to all rational numbers.

- Lesson 13: Card Sort: The Same But Different - In this activity students continue to build fluency operating with signed numbers as they match different expressions that have the same value. Students look for and use the relationship between inverse operations.
- Lesson 15: Trip to the Mountains - In this activity, students interpret equations that represent situations. The purpose is for students to see that equations of the form $x + p = q$ can be solved by adding the opposite of p to the equation, regardless of whether p is positive or negative. Students also see that equations of the form $px = q$ can be solved by multiplying the equation by the reciprocal of p . Through this work, students see that the structure of equations can be used to reason about a path to a solution even when negative values are included or when a variable can represent a negative number.
- Lesson 17: Your Own Stock Portfolio - This task introduces the concept of a stock portfolio being a selection of stocks an investor might own to try to make money—students examine the change in the portfolio and evaluate the value. They must use both positive and negative change and percentage change. Students then calculate their own portfolio using sample “Stock Prices”.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.5
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.5 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 6 OVERVIEW

Content Area: Mathematics

Unit Title: Expressions, Equations and Inequalities

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students solve equations of the forms $px+q=r$ and $p(x+q)=r$ where p , q , and r are rational numbers. They draw, interpret, and write equations in one variable for balanced “hanger diagrams,” and write expressions for sequences of instructions, e.g., “number puzzles.” They use tape diagrams together with equations to represent situations with one unknown quantity. They learn algebraic methods for solving equations. Students solve linear inequalities in one variable and represent their solutions on the number line. They understand and use the terms “less than or equal to” and “greater than or equal to,” and the corresponding symbols. They generate expressions that are equivalent to a given numerical or linear expression. Students formulate and solve linear equations and inequalities that represent real-world situations.

Approximate Length of Unit: 6 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
 - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

- 7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- Describe situations in which opposite quantities combine to make 0. For example, in the first round of a game, Maria scored 20 points. In the second round of the same game, she lost 20 points. What is her score at the end of the second round?
 - Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
 - Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
 - Apply properties of operations as strategies to add and subtract rational numbers.

Career Readiness, Life Literacies, and Key Skills:

- 9.2.8.CAP.2** Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.12** Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
- 9.4.8.CI.1** Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.
- 9.4.8.CT.2** Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3** Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
- 9.4.8.TL.2** Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

- RI.CI.7.2** Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.
- RI.CI.7.3** Analyze the relationship between a central idea and supporting details.
- W.IW.7.2:** Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.
- SL.PE.7.1** Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.
- SL.UM.7.5** Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

- MS-PS3-1** Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.
- MS-LS1-3** Use argument supported by evidence for how the body is a system of interacting subsystems composed of groups of cells.

Unit Understandings:

Students will understand that...

- Developing the ability to interpret solutions to equations, and understanding how to analyze and apply solutions in various mathematical contexts to solve problems effectively.

- Interpreting and representing non-proportional situations with constant rates of change, utilizing methods such as tape diagrams to visually depict relationships and patterns within the given scenarios.
- Describing the structure of equations and tape diagrams, recognizing the connections between mathematical expressions, visual representations, and problem-solving strategies.
- Through collaborative discussions, critiquing the reasoning of their peers about expressions and corresponding diagrams, evaluating the validity of arguments, identifying logical flaws, and providing constructive feedback to enhance understanding.
- General strategies for solving equations and determining when expressions are equivalent, applying critical thinking skills to analyze and reason about various mathematical concepts and problem-solving techniques.

Unit Essential Questions:

- How can we interpret solutions to equations effectively, and what insights can we gain from analyzing these solutions in various mathematical contexts?
- In what ways can non-proportional situations with constant rates of change be interpreted and represented using methods like tape diagrams, enhancing our understanding of relationships and patterns within the scenarios?
- What key elements define the structure of equations and tape diagrams, and how do these components contribute to problem-solving strategies and mathematical reasoning?
- How can we critically critique the reasoning of peers regarding expressions and corresponding diagrams, evaluating the validity of arguments and providing constructive feedback to deepen understanding?
- Why is it important to analyze and critique reasoning about solving equations and equivalent expressions, and how does this process contribute to our ability to generalize problem-solving approaches and identify equivalent mathematical expressions effectively?

Knowledge and Skills:

Students will know...

- How to compare
 - stories with corresponding tape diagrams.
 - tape diagrams with corresponding equations.
 - hanger diagrams and equations.
 - solution pathways.
 - descriptions of situations with corresponding inequalities.
- How to explain
 - strategies for using hanger diagrams to solve equations.
 - different strategies for solving equations (Lesson 9) and inequalities.
 - reasoning about situations, tape diagrams, and equations.
 - strategies for identifying and writing equivalent expressions.
- How to justify
 - reasoning about inequalities.
 - reasoning about solutions to inequalities.
 - the need for specific information in order to write and solve inequalities.
 - reasoning about the distributive property.
 - whether different sequences of calculations give the same result.
- Vocabulary: equivalent expressions, variable, solution to an inequality, term, factor, and expand.

Students will be able to...

- Create diagrams and equations in the form $px + q = r$ and $p(x + q) = r$ to represent situations.
- Interpret equations in the form $px + q = r$ and $p(x + q) = r$ that represent relationships in diagrams and situations.
- Solve equations of the form $px + q = r$ and $p(x + q) = r$, including those that involve fractions, decimals, and negative numbers, and explain the solution method.
- Solve real-world problems leading to equations of the form $px + q = r$ or $p(x + q) = r$.
- Draw and label a graph on a number line that represents all the solutions to an inequality.
- Solve an inequality of the form $px + q > r$ or $px + q < r$ and interpret the solution.
- Write an inequality of the form $px + q > r$ or $px + q < r$ to represent a situation with a constraint.
- Apply properties of operations to write an expression with fewer terms that are equivalent to a given expression.
- Apply the distributive property to factor or expand an expression.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 2: Every Picture Tells a Story - In this activity, students explain how a tape diagram represents a situation. They also use the tape diagram to reason about the value of the unknown quantity. Students are not expected to write and solve equations here; any method they can explain for finding values for x and y is acceptable. While some students might come up with equations to describe the diagram and solve for the unknown, there is no need to focus on developing those ideas at this time.
- Lesson 6: Categories of Equations Card Sort - The goal of this activity is for students to notice the structure of equations. Any way of sorting is fine, but the discussion should land on explaining how equations involving an expression like $p(x + q)$ are different from ones that have an expression like $px + q$. Monitor for different ways groups choose to categorize the equations, but especially for categories that distinguish between these two types of expressions. As students work, encourage them to refine their descriptions of equations using more precise language and mathematical terms.
- Lesson 9: Old and New Ways to Solve - These are all solvable by thinking “what value would make the equation true.” So, it’s straightforward to figure out what the solution would be, but these equations present an opportunity to demonstrate that “doing the same thing to each side” still works when there are negative numbers. Monitor for students who reason about what value would make the equation true and those who reason by doing the same thing to each side.
- Lesson 14: Earning Money for Soccer Stuff - Previously in this unit, students wrote expressions and equations that are similar to the ones in this activity. Here, they are prompted in a scaffolded

way to notice that they can express not just that an outcome can be equal to a value, but that an outcome can be at least as much as a value by using the new notation \geq .

- Lesson 17: Giving Advice - In this activity, students set up and solve inequalities that represent real-life situations. Students will think about how to interpret their mathematical solutions. For example, if they use w to represent width in centimeters and find $w < 25.5$, does that mean $w = -10$ is a solution to the inequality?

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.6
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.6 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 7 OVERVIEW

Content Area: Mathematics

Unit Title: Angles, Triangles and Prisms

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms “complementary,” “supplementary,” “vertical angles,” and “unique.” The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism, and solve problems involving area, surface area, and volume.

Approximate Length of Unit: 5 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.G.A.2** Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- 7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- 7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
 - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

- 7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- Describe situations in which opposite quantities combine to make 0. For example, in the first round of a game, Maria scored 20 points. In the second round of the same game, she lost 20 points. What is her score at the end of the second round?
 - Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
 - Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
 - Apply properties of operations as strategies to add and subtract rational numbers.

Career Readiness, Life Literacies, and Key Skills:

- 9.2.8.CAP.2** Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.12** Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
- 9.4.8.CI.1** Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.
- 9.4.8.CT.2** Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3** Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
- 9.4.8.TL.2** Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

- RI.CI.7.2** Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.
- RI.CI.7.3** Analyze the relationship between a central idea and supporting details.
- W.IW.7.2:** Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.
- SL.PE.7.1** Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.
- SL.UM.7.5** Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

- MS-PS3-1** Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.
- MS-LS1-6** Construct a scientific explanation based on evidence for the role of photosynthesis in the cycling of matter and flow of energy into and out of organisms.

Unit Understandings:

Students will understand that...

- The use of precise language to compare angle measurements accurately, developing a deep understanding of geometric properties and enhancing communication skills in describing angles.
- Exploring strategies to compare and contrast triangles within a set, identifying unique characteristics that differentiate triangle types, and applying this knowledge to geometric problem-solving.
- Investigating and describing cross sections of various figures, gaining insights into the three-dimensional structure of shapes and enhancing their spatial visualization skills.
- Through exploration, describing the characteristics of pattern blocks, recognizing patterns, symmetries, and transformations within different configurations to build spatial reasoning abilities.
- Articulating the positioning and movement of side lengths and angles in geometric figures, enabling them to analyze shapes based on their spatial relationships and properties.
- Describing cross sections of prisms and pyramids, identifying commonalities and distinctions in resulting shapes to deepen their understanding of three-dimensional geometry and shape classification.

Unit Essential Questions:

- How can language be effectively utilized to compare angle measurements, and what role does precise terminology play in enhancing our understanding of geometric properties?
- What strategies can students employ to compare triangles within a set, recognizing key characteristics that differentiate various types of triangles and their properties?
- In what ways can cross sections of figures be compared and described, and how do these comparisons provide insights into the three-dimensional structure of geometric shapes?
- How do the characteristics of pattern blocks influence our understanding of spatial relationships, symmetry, and geometric transformations within different configurations?
- What insights can be gained from describing the positioning and movement of side lengths and angles in geometric figures, and how does this knowledge contribute to analyzing and classifying shapes based on their attributes?
- How can students effectively describe cross sections of prisms and pyramids, identifying commonalities and differences in the resulting shapes to deepen their understanding of three-dimensional geometry?

Knowledge and Skills:

Students will know...

- How to critique
 - reasoning about measuring angles.
 - reasoning about the decomposition of prisms.
 - reasoning about the surface area of prisms.
- How to explain
 - how to measure angles.
 - how to find unknown angle measurements.
 - how to find the volume of prisms.
 - how to find the surface area of prisms.
- How to interpret
 - situations involving intersecting lines in order to form a conjecture.
 - which information is relevant to answer questions.
 - equations representing angle measurements.
 - situations involving volume and surface area.
- How to justify

- whether or not shapes are identical copies.
- whether or not measurements determine identical copies.
- whether or not measurements determine unique triangles.
- Vocabulary: straight angle, adjacent angle, right angle, supplementary, complementary, measurement error, vertical angles, cross section, base (of a prism or pyramid), prism, pyramid, volume, and surface area.

Students will be able to...

- Solve multi-step problems involving complementary, supplementary, and vertical angles.
- Write an equation to represent the relationship between angles in a given diagram.
- Draw triangles with two given angle measures and one side length, one given angle and two side lengths, or three side lengths.
- Justify where 3 measures of angles or sides determine a unique triangle or more than one triangle, or if no triangle is possible.
- Calculate the surface area and volume of a prism.
- Decide whether to calculate the surface area or volume of a prism to solve a problem in a real-world situation.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 4: Info Gap: Angle Finding - The purpose of this info gap activity is for students to see how they can use different pieces of information to solve for an unknown angle measure in a multi-step problem. During the whole class discussion, students are introduced to writing and solving equations to represent the relationships between angles.
- Lesson 6: What Can You Build? - The purpose of this activity is for students to explore a physical representation of polygons and make observations about triangles and quadrilaterals. This introductory activity serves to familiarize students with the tools and definitions they will use in future activities. Students may notice that some sets of 3 strips cannot make a triangle, but formalizing rules about what lengths can and cannot be used to form a triangle is not the goal of this lesson.
- Lesson 10: Revisiting: How Many You Can You Draw? - Students continue to practice drawing triangles from given conditions and categorizing their results. This activity focuses on the inclusion of a single angle and two sides. Again, they do not need to memorize which conditions result in unique triangles, but should begin to notice how some conditions (such as the equal side lengths) result in certain requirements for the completed triangle.

- Lesson 12: Finding Volume with Cubes - In this activity, students extend their understanding to see that even when the base is not a rectangle, they can still calculate the volume of a prism by multiplying the area of the base times the height of the prism.
- Lesson 14: Multifaceted - The purpose of this warm-up is for students to recognize important parts of solids in anticipation of computing volume and surface area. The figure used in the next activity is introduced in this warm-up as a way for students to start thinking about parts of solids and how we use them to compute surface area or volume.
- Lesson 15: Card Sort: Area or Volume - The purpose of this activity is for students to sort cards with questions that have a context referring to either volume or surface area of a prism. In previous lessons, students focused on determining volume or surface area and the two concepts were never presented side by side. Here, students are asked to sort questions with a context to determine if it makes more sense to think about surface area or volume when answering the question. After sorting, students think about what information they need to answer a question and estimate reasonable measurements to calculate the answer to their question.
- Lesson 17: Making the Prism - In this activity, students take the triangle they selected in the previous activity and use it as the base of their triangular prism. After students have drawn their net and before they cut it out and assemble it, make sure they have correctly positioned their bases, opposite from each other on the top and bottom of the rectangle and reflected. It will also make assembling the net easier for students if they draw lines subdividing the large rectangle into the individual rectangular faces and draw tabs where the faces will be glued or taped together.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.7
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.7 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.

UNIT 8 OVERVIEW

Content Area: Mathematics

Unit Title: Probability and Sampling

Target Course/Grade Level: Mathematics/Grade 7

Unit Summary: In this unit, students understand and use the terms “event,” “sample space,” “outcome,” “chance experiment,” “probability,” “simulation,” “random,” “sample,” “random sample,” “representative sample,” “overrepresented,” “underrepresented,” “population,” and “proportion.” They design and use simulations to estimate probabilities of outcomes of chance experiments and understand the probability of an outcome as its long-run relative frequency. They represent sample spaces (that is, all possible outcomes of a chance experiment) in tables and tree diagrams and as lists. They calculate the number of outcomes in a given sample space to find the probability of a given event. They consider the strengths and weaknesses of different methods for obtaining a representative sample from a given population. They generate samples from a given population, e.g., by drawing numbered papers from a bag and recording the numbers, and examine the distributions of the samples, comparing these to the distribution of the population. They compare two populations by comparing samples from each population.

Approximate Length of Unit: 6 weeks

LEARNING TARGETS

NJ Student Learning Standards:

- 7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
- 7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- 7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0

indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Career Readiness, Life Literacies, and Key Skills:

9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.

9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

9.4.8.CI.1 Assess data gathered on varying perspectives on causes of climate change (e.g., crosscultural, gender-specific, generational), and determine how the data can best be used to design multiple potential solutions.

9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Interdisciplinary Connections and Standards:

ELA

RI.CI.7.2 Determine a central idea in an informational text and explain how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.

RI.CI.7.3 Analyze the relationship between a central idea and supporting details.

W.IW.7.2: Write informative/explanatory texts (including the narration of historical events, scientific procedures/experiments, or technical processes) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.

SL.PE.7.1 Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

SL.UM.7.5 Include multimedia components (e.g., graphics, sound) and visual displays in presentations to clarify concepts or findings.

Science

MS-LS1-3 Use argument supported by evidence for how the body is a system of interacting subsystems composed of groups of cells.

MS-ESS2-4 Develop a model to describe the cycling of water through Earth's systems driven by energy from the sun and the force of gravity.

Unit Understandings:

Students will understand that...

- Developing the ability to critique predictions about the mean of random samples, evaluating the accuracy of forecasts, and enhancing their understanding of statistical measures in data analysis.
- General concepts related to sample spaces, predictions, sampling methods, and fairness, applying critical thinking skills to analyze and interpret various scenarios within the realm of probability and statistics.
- Through language usage, representing data obtained from repeated experiments, effectively communicating findings and patterns to convey meaningful insights from statistical investigations.
- Representing probabilities and sample spaces visually, utilizing diagrams and models to illustrate the likelihood of outcomes and deepen their understanding of probability concepts.
- By interpreting situations involving sample spaces, probability calculations, and populations, learners will enhance their ability to analyze real-world scenarios, make informed decisions, and draw conclusions based on statistical data.

Unit Essential Questions:

- How can students effectively critique predictions about the mean of random samples, and what strategies can be employed to evaluate the accuracy of statistical forecasts in data analysis?
- In what ways can students generalize concepts related to sample spaces, predictions, sampling methods, and fairness, and how does this generalization enhance their understanding of probability and statistical reasoning?
- What role does language play in representing data from repeated experiments, and how can effective communication through language aid in conveying insights and patterns discovered through statistical analyses?
- How can visual representations of probabilities and sample spaces enhance students' understanding of likelihood and outcomes, and what insights can be gained from interpreting these visual models in probabilistic scenarios?
- How do students interpret and analyze situations involving sample spaces, probability calculations, and population data to make informed decisions, draw conclusions, and apply statistical concepts to real-world contexts?

Knowledge and Skills:

Students will know...

- How to describe
 - observations and predictions during a game.
 - patterns observed in repeated experiments.
 - chance experiments to model situations.
 - a simulation used to model a situation.
 - observations about data sets.

- How to explain
 - predictions.
 - how to determine which events are more likely.
 - possible differences in experimental and theoretical probability.
 - how to use simulations to estimate probability.
 - how to use a simulation to answer questions about the situation.
- How to justify
 - whether situations are surprising and possible.
 - which samples are or are not representative of a larger population.
 - which samples correspond with each show, which show is most appropriate for a commercial, and whether a movie is eligible for an award.
 - reasoning about samples and populations.
 - whether or not differences between samples are meaningful.
- How to compare
 - sample spaces and probability of outcomes for different spinners.
 - methods for writing sample spaces.
 - heights of two groups.
 - measures of center with samples.
 - sampling methods.
 - populations based on samples.
- Vocabulary: event, chance experiment, outcome, probability, random, sample space, simulation, mean absolute deviation (MAD), mean, median, population, sample, representative sample, interquartile range (IQR), and proportion.

Students will be able to...

- Use the results from a repeated experiment to estimate the probability of an event, and justify the estimate.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.
- Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify that it represents a situation.
- Interpret or create a list, table, or tree diagram that represents the sample space of a compound event, and use the sample space to write the probability for an event.
- Describe methods to obtain a random sample from a population, and explain why it is representative of the population.
- Explain why samples are necessary and describe a sample and population for a given statistical questions
- Determine whether two populations are likely to be meaningfully different by reasoning about center and spread.
- Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain the reasoning.

EVIDENCE OF LEARNING

Assessment:

What evidence will be collected and deemed acceptable to show that students truly “understand”?

- End of Unit Common Assessment - See folder for assessment links.
- Warm-Ups
- Cool Downs
- Section Checkpoints
- Practice Sets
- Renaissance Star Math Diagnostic Assessment – Fall, Winter, Spring

Learning Activities:

What differentiated learning experiences and instruction will enable all students to achieve the desired results?

- Lesson 2: Card Sort: Likelihood - In this lesson, the students move one more step closer to quantifying the likelihood of scenarios by ordering them individually rather than into groups. Some of the scenarios have a numerical probability expressed as a percentage, some in decimal form, some as a fraction, and some do not have a numerical probability given. This gives students the opportunity to work with probabilities expressed as percentages, decimals, and fractions.
- Lesson 5: Making My Head Spin - In this activity, students return to calculating probabilities using the sample space, and they compare the calculated probabilities to the outcomes of their actual trials. Students have a chance to construct arguments about why probability estimates based on carrying out the experiment many times might differ from the expected probability. Students use a spinner in this activity, which will be helpful when designing simulations in upcoming lessons.
- Lesson 6: Diego's Walk - In this activity, students estimate the probability of a real-world event by simulating the experience with a chance experiment. Students see that multiple simulation methods can result in similar estimates for the probability of the actual event.
- Lesson 10: Designing Simulations - In this activity, each group is assigned a situation for which they will design and perform a simulation to estimate the probability. Students will give a short presentation on the methods and results of their simulation for the class after they have designed and run the simulation. Students will need to attend to precision as well as present arguments for the simulation method they chose. At this stage, students have experienced a large number of simulation methods and should be able to design their own to represent the situations using the appropriate tools.
- Lesson 16: Reactions Times - In previous lessons, students examined the estimation of the mean and median for populations using data from a sample. In this activity, students apply similar reasoning to estimating the proportion of a population that matches certain characteristics. Students collect a sample of 20 reaction times and compute the fraction of responses in their sample that are in a given range. Then, in the discussion, students compare their estimations to the known population proportion and use the class's proportions to gauge the accuracy of their estimate.
- Lesson 19: Info Gap: Comparing Populations - In this info gap activity, students work together to compare two populations from information about samples from each of the populations. Students must pay attention to the information they need in order to solve the problem and the types of questions they could ask to get to the answer.

RESOURCES

Teacher Resources:

- Illustrative Math (IM) Unit 7.8
- IM Student Workbook
- IM Blackline Masters
- Khan Academy IM Unit 7.8 Companion

Equipment Needed:

- Manipulatives
- IM Student Workbooks
- Student Whiteboards
- Chart Paper
- Dry Erase Markers
- Online Approved Digital Resources
- Chromebooks
- Bags, resealable plastic, or envelopes.
- Geometry Toolkit: rulers, graph paper, pattern blocks, straightedges, colored pencils, index cards, patty paper or tracing paper, pencil boxes, and scissors.