GEORGIA’S K-12 MATHEMATICS STANDARDS

Mathematics Teaching and Learning Resources

KINDERGARTEN

Comprehensive Grade Level Overview

MATHEMATICS

Georgia Department of Education

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SECTION 1: Grade-Level Description

Kindergarten Mathematics Overview

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in kindergarten should be devoted to numbers than to other topics. Being intentional with ongoing exposure to mathematical concepts such as counting and shape identification will support student efficacy and competence.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. *(Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.)* Students choose, combine, and apply effective strategies for answering quantitative and statistical questions, including quickly recognizing the quantity of a small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
2. Students describe their physical environment using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes (squares, circles, triangles, rectangles, hexagons, and octagons) presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes (cubes, cones, cylinders, and spheres). They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Numerical Reasoning

Numerical reasoning begins with students developing early foundational numeracy through reasoning about numbers by building relationships between number and quantity.

K.NR.1: Demonstrate and explain the relationship between numbers and quantities up to 20; connect counting to cardinality (the last number counted represents the total quantity in a set).
• **K.NR.1.1:** Count up to 20 objects in a variety of structured arrangements and up to 10 objects in a scattered arrangement • **K.NR.1.2:** When counting objects, explain that the last number counted represents the total quantity in a set (cardinality), regardless of the arrangement and order • **K.NR.1.3:** Given a number from 1-20, identify the number that is one more or one less. • **K.NR.1.4:** Identify pennies, nickels, and dimes and know their name and value.
K.NR.2: Use count sequences within 100 to count forward and backward in sequence
• **K.NR.2.1:** Count forward to 100 by tens and ones and backward from 20 by ones.

- **K.NR.2.2:** Count forward beginning from any number within 100 and count backward from any number within 20.

K.NR.3: Use place value understanding to compose and decompose numbers from 11–19

- **K.NR.3.1:** Describe numbers from 11 to 19 by composing (putting together) and decomposing (breaking apart) the numbers into ten ones and some more ones.

K.NR.4: Identify, write, represent, and compare numbers up to 20.

- **K.NR.4.1:** Identify written numerals 0-20 and represent a number of object with a written numeral 0-20 (with 0 representing a count of no objects).
- **K.NR.4.2:** Compare two sets of up to 10 objects and identify whether the number of objects in one group is more or less than the other group, using the words "greater than," "less than," or "the same as**".**

K.NR.5: Explain the concepts of addition, subtraction, and equality and use these concepts to solve real-life problems within 10.

- **K.NR.5.1:** Compose (put together) and decompose (break apart) numbers up to 10 using objects and drawings.
- **K.NR.5.2:** Represent addition and subtraction within 10 from a given authentic situation using a variety of representations and strategies.
- **K.NR.5.3:** Use a variety of strategies to solve addition and subtraction problems within 10.
- **K.NR.5.4:** Fluently add and subtract within 5 using a variety of strategies to solve practical, mathematical problems

Patterning and Algebraic Reasoning

Students begin sense-making with investigations of patterns. They use their work with repeating patterns to capitalize on their patterning and sense-making intuition. Students also begin to form generalizations from experiences with numbers and computation and identify mathematical structures.

K.PAR.6: Explain, extend, and create repeating patterns with a repetition, not exceeding 4 and describe patterns involving the passage of time

- **K.PAR.6.1:** Create, extend, and describe repeating patterns with numbers and shapes, and explain the rationale for the pattern.
- **K.PAR.6.2:** Describe patterns involving the passage of time using words and phrases related to actual events.

Measurement & Data Reasoning

Students use measurement reasoning to first discover measurable attributes of physical objects. They compare these measurable attributes indirectly at first, then more directly, using non-standard units (first grade). This leads to measuring with standard units beginning in second grade. Additionally, students create statistical investigative questions, determine strategies for gathering data and interpret this data to answer the statistical investigative question created. The goal is to use mathematics to make sense of the world around them by asking questions that they care about and by collecting, gathering, and interpreting data to answer those questions.

K.MDR.7: Observe, describe, and compare the physical and measurable attributes of objects and analyze graphical displays of data.
● **K.MDR.7.1:** Directly compare, describe, and order common objects, using measurable attributes (length, height, width, or weight) and describe the difference. ● **K.MDR.7.2:** Classify and sort up to ten objects into categories by an attribute; count the number of objects in each category and sort the categories by count. ● **K.MDR.7.3:** Ask questions and answer them based on gathered information, observations, and appropriate graphical displays to solve problems relevant to everyday life

Geometric and Spatial Reasoning

Students identify, sort and classify shapes based on attributes. Students also begin to add more formal geometric language as they identify and classify shapes through their properties.

K.GSR.8: Identify, describe, and compare basic shapes encountered in the environment, and form two-dimensional shapes and three-dimensional figures.
● **K.GSR.8.1:** Identify, sort, classify, analyze, and compare two-dimensional shapes and three-dimensional figures, in different sizes and orientations, using informal language to describe their similarities, differences, number of sides and vertices, and other attributes ● **K.GSR.8.2:** Describe the relative location of an object using positional words ● **K.GSR.8.3:** Use basic shapes to represent specific shapes found in the environment by creating models and drawings ● **K.GSR.8.4**: Use two or more basic shapes to form larger shapes.

SECTION 2: Curriculum Map & Pacing

Interactive Curriculum Map

Georgia's K-12 Mathematics Standards - Kindergarten							
Semester 1				Semester 2			
Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
Numerical Reasoning: Wondering About My World and Investigating to Find Answers Interdisciplinary Connection	Geometric & Spatial Reasoning: 2-D Shapes in My World Interdisciplinary Connection	Numerical Reasoning: How Many? (Numbers Up to 20) Interdisciplinary Connection	Numerical Reasoning: Understanding and Using Addition and Subtraction in My Life Interdisciplinary Connection	Numerical Reasoning: Using Numbers within 20 Interdisciplinary Connection	Geometric & Spatial Reasoning: 3-D Shapes in My World Interdisciplinary Connection	Measurement & Data Reasoning: Using Numbers and Data to Make Sense of My World Interdisciplinary Connection	Culminating Capstone Unit
4 - 5 weeks	**3 - 4 weeks**	**5 - 6 weeks**	**7 - 8 weeks**	**5 - 6 weeks**	**3 - 4 weeks**	**4 - 5 weeks**	**2 - 3 weeks**
K.NR.1 K.NR.2 K.NR.4 K.MDR.7 K.MP.1-8	K.GSR.8 K.MDR.7 K.PAR.6 K.MP.1-8	K.NR.1 K.NR.2 K.NR.3 K.NR.4 K.MDR.7 K.MP.1-8	K.NR.5 K.PAR.6 K.MDR.7 K.MP.1-8	K.NR.1 K.NR.2 K.NR.3 K.NR.4 K.MDR.7 K.MP.1-8	K.GSR.8 K.MDR.7 K.MP.1-8	K.NR.3 K.NR.5 K.PAR.6 K.MDR.7 K.MP.1-8	ALL STANDARDS K.MP.1-8
Ongoing interdisciplinary learning to impact the community and to explain real-life phenomena							
The concepts in each unit are presented based on a logical, mathematical progression. Each unique unit in sequence builds upon the previous unit.							
The Framework for Statistical Reasoning, Mathematical Modeling Framework, and the K-12 Mathematical Practices should be taught throughout the units.							

Mathematical Practices (K.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

Key for Course Standards: NR: Numerical Reasoning, **PAR:** Patterning & Algebraic Reasoning, **GSR:** Geometric & Spatial Reasoning, **MDR:** Measurement & Data Reasoning

Click here to access the entire Kindergarten Curriculum Map.

SECTION 3: Background Research & Support

Ensuring Access to a Well-Rounded Mathematics Education

Georgia's Plan for Ensuring Access to a Well-Rounded Mathematics Education was developed based on collaborative work among various district leadership teams and university partners. The theory of action leaders used to develop this action plan includes actions for district leaders, school leaders, teachers, and students in order to ensure that each student entrusted in our educational care is a prepared learner for life. The specific goals outlined in the plan are:

G O A L 1: Prepare, hire, develop and support high-quality mathematics teachers for all students.

G O A L 2: Create mathematics learning environments with established supports, structures and mindsets to honor each student's uniqueness, experiences, background knowledge, and interests in order to advance their learning.

G O A L 3: Provide a prevention and intervention infrastructure to ensure that each learner is provided with the necessary support to succeed.

G O A L 4: Recognize, understand and value each learner's individual uniqueness.

For a detailed description of each goal and the desired student outcome of accomplishing each goal, please refer to **Whole Child Mathematics Toolkit.**

Mathematical Mindsets

Growth mindset was pioneered by Carol Dweck, Lewis and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability (from With Math I Can).

You can learn how to use the power of growth mindset for yourself and your students in the resources below:

- Fostering Positive Mathematical Mindsets
- Youcubed: Inspire ALL Students with Open, Creative, Mindset Mathematics

Mathematical Reasoning

Mathematical reasoning is:

- making sense of mathematical ideas
- a critical skill that enables students to analyze and solve a given problem
- the process of converting information into mathematical language and then apply logic to determine desired results
- an argument made to justify one's process, procedure, or conjecture, to create strong conceptual foundations and connections, in order for students to be able to process new information.
- a way of thinking that moves students beyond memorizing facts, towards thinking beyond the rules and procedures to forming their own questions and conjectures.
- teaching that requires planning well-developed tasks and pre-planned questions, to allow students to communicate at higher levels of comprehension that mathematical reasoning requires.
- much more than asking, "Why?"

In classrooms that promote mathematical reasoning, students are engaged and building confidence as they collaborate and think creatively and critically to solve problems about the world around them. Students are presented with real-life phenomena, encouraged to ask mathematical questions to build logical sense of that phenomena, and engage with mathematical concepts while exploring those real-life phenomena within the key competencies/course standards for each grade level/course. Through this approach to teaching and learning, the mathematical concepts are addressed within a cluster of conceptual ideas rather than isolated skills.

Engaging students in the process of mathematical modeling and the Framework for Statistical Reasoning promotes mathematical reasoning. These two frameworks build on students' natural curiosity and offer them the opportunity to make sense of the world around them, make predictions, and/or make informed decisions. These skills prepare students for their chosen career and life in a fast-paced, ever-changing world.

Developing Mathematical Thinking

In mathematics teaching and learning, we encourage students to see mathematics in the world around them, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Students can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others." By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students (Catherine Twomey Fosnot and Maarten Dolk, Young Mathematicians at Work). While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for students to grapple with the unfamiliar content and engage in academic discourse; this perseverance leads to a deeper understanding and confidence that comes leading to student agency.

Computational Strategies

In mathematics, the emphasis is on the reasoning and thinking about the quantities within mathematical contexts. Specific mathematics strategies for teaching and learning are not mandated by the Georgia Department of Education or assessed on state or federally mandated tests. Students may solve problems in different ways and have the flexibility to choose a mathematical strategy that allows them to make sense of and strategically solve problems using efficient methods that are most comfortable for and makes sense to them. It is critical that teachers and parents remain partners to help each child grow to become a mathematically literate citizen. These standards preserve and affirm local control and flexibility. Area models, tape diagrams (bar models), and number line representations are a few examples of ways that students communicate their strategic thinking in a written form.

Effective Mathematics Teaching Practices

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically. The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using instructional practices that are effective in developing mathematics learning for all students. The eight Mathematics Teaching Practices below describe the essential teaching skills derived from the research-based learning principles, as well as other knowledge of mathematics teaching that has emerged over the last two decades. (NCTM, 2014)

1. Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

7. Support productive efforts of learning in mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in meaningful learning tasks as they grapple with mathematical ideas and relationships.
8. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

A deeper explanation of each of the 8 Effective Mathematics Teaching Practices can be found below and in Principles to Actions from NCTM.

Effective Mathematics Teaching Practices Teacher and Student Behaviors

Mathematics Teaching Practices	What is the teacher doing?	What are students doing?
1. Establish mathematics goals to focus learning.	• Consider broader goals, as well as the goals of the actual lesson, including the following: • What is to be learned? • Why is the goal important? • Where are students coming from? Where do students need to go? • How can learning be extended?	• Make sense of new concepts and skills; including connections to concepts/big ideas learned in previous grades. • Experience connections among the strands, overall and specific expectations. • Deepen their understanding and expect mathematics to make sense.
2. Implement tasks that promote reasoning and problem solving.	• Chooses task that: • are built on current student understandings. • have various entry points with multiple ways for the problems to be solved. • are interesting to students (e.g., evolve from students' thinking; connect to real world mathematics)	• Work to make sense out of the task and persevere in solving problems. • Use a variety of models and materials to make sense of the mathematics in the task. • Convince themselves and others the answer is reasonable.
3. Use and connect mathematical representations.	• Uses tasks that allow students to use a variety of representations. • Encourages the use of different representations, including concrete models, pictures, words, and numbers, that support students in explaining their thinking and reasoning.	• Use materials to make sense out of problem situations. • Connect representations to mathematical ideas and structures of big ideas, including operational sense with whole numbers, fractions and decimals.
4. Facilitate meaningful mathematical discourse.	• Engages students in explaining their mathematical reasoning in small groups and classroom situations.	• Explain the ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of

	• Facilitates discussion among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	others. • Ask questions of others to make sense of their ideas.
5. Pose purposeful questions.	• Asks questions that build on and extend student thinking. • Facilitates discussion among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Think more deeply about the process of the mathematics rather than simply focusing on the answer. • Listen to and comment on the explanations of others in the class.
6. Build procedural fluency from conceptual understanding.	• Provides opportunities for students to reason about mathematical ideas. • Expects students to explain why their strategies work. • Connects student methods to efficient procedures as appropriate.	• Understand and explain the procedures they are using and why they work. • Use a variety of strategies to solve problems and make sense of the mathematical tasks. • Do not rely on shortcuts or tricks to do mathematics.
7. Support productive efforts of learning in mathematics.	• Supports students in building their understanding without showing and telling a procedure but rather focusing on the important mathematical ideas. • Asks questions that scaffold and advance student thinking. • Builds questions and plans lessons based on important student misconceptions rather than focusing on the correct answer. • Recognize the importance of effort as students work to make sense of new ideas.	• Stick to tasks and recognize that effortful learning is part of making sense. • Ask questions that will help to better understand the task. • Support each other with ideas rather than telling others the answer or how to solve the problem.
8. Elicit and use evidence of student thinking.	• Determines what to look for in gathering evidence of student learning. • Poses questions and	• Accept reasoning and understanding are as important as the answer to the problem.

	answers student questions that provide information about student understanding and reasoning. ● Uses evidence to determine next steps of instruction.	● Use mistakes and misconceptions to rethink understanding. ● Ask questions to clarify confusion or misunderstanding. ● Assess progress toward developing mathematical understanding.

Adapted from John Hattie's (2017, p. 244) summation from Principles to Action (National Council of Teachers of Mathematics, 2014)

The *Kindergarten Guide for Effective Mathematics Instruction* provides additional resources for evidence-based practices that support student learning.

SECTION 4: Essential Instructional Guidance

Understanding the Mathematical Practices

Mathematical Practices are listed with each grade mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The Mathematical Practices describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

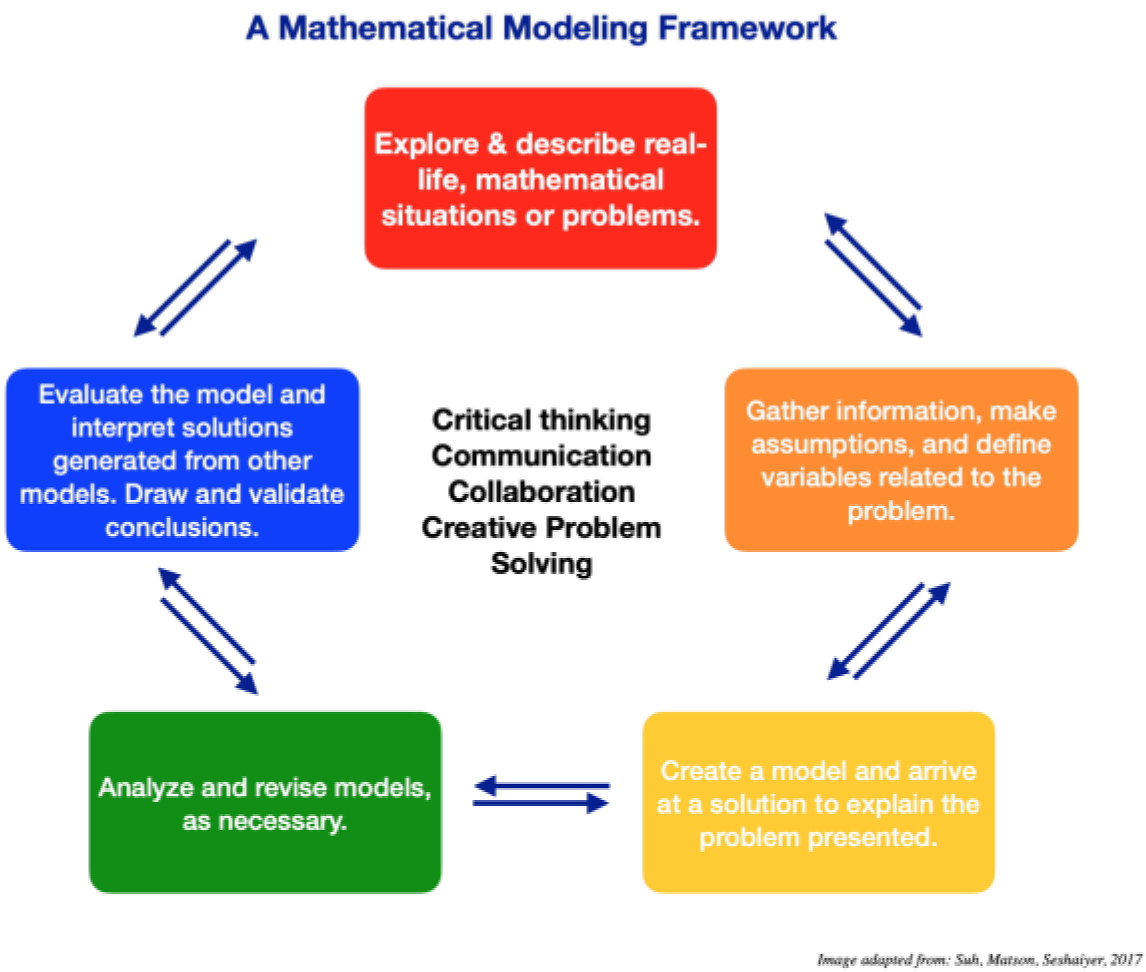
***Mathematical Practices** – These mathematical practices describe how students should engage with the mathematics content for the grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers.*		
K.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.		
	Expectations	**Evidence of Student Learning**
K.MP.1	**Make sense of problems and persevere in solving them.**	*Fundamentals* ● Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). ● Students should be able to use both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. *Strategies and Methods* ● As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, Kindergarten students begin to reason as they become more conscious of what they know and how they solve problems.
K.MP.2	**Reason abstractly and quantitatively.**	*Fundamentals* ● Mathematically proficient students in Kindergarten begin to use numerals to represent a specific amount (quantity). *Example* ● A student may write the numeral “11” to represent an amount of objects counted, select the correct number card “17” to follow “16” on the calendar, or build a pile of counters depending on the number drawn. In addition, Kindergarten students begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation (Mary has 3 bears. Asia gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation (Mary had 5 bears. She gave some to Asia.)
K.MP.3	**Construct viable arguments and critique the**	*Fundamentals* ● In Kindergarten, mathematically proficient students begin to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Through opportunities that encourage exploration,

	reasoning of others.	discovery, and discussion, Kindergarten students begin to learn how to express opinions, become skillful at listening to others, describe their reasoning and respond to others' thinking and reasoning. They begin to develop the ability to reason and analyze situations as they consider questions such as, "Are you sure…?", "Do you think that would happen all the time?", and "I wonder why…?"
K.MP.4	**Model with mathematics.**	*Fundamentals* ● Mathematically proficient students in Kindergarten begin to experiment with representing real-life problem situations in multiple ways such as with numbers, words (mathematical language), drawings, objects, acting out, charts, lists, and number sentences. *Example* ● When making toothpick designs to represent the various combinations of the number "5", the student writes the numerals for the various parts (such as "4" and "1") or selects a number sentence that represents that particular situation (such as 5 = 4 + 1)*.
K.MP.5	**Use appropriate tools strategically.**	*Strategies and Methods* ● In Kindergarten, mathematically proficient students begin to explore various tools and use them to investigate mathematical concepts. Through multiple opportunities to examine materials, they experiment and use both concrete materials (e.g. 3- dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, calculators, interactive websites) to explore mathematical concepts. Based on these experiences, they become able to decide which tools may be helpful to use depending on the problem or task. *Example* ● When solving the problem, "There are 4 dogs in the park. 3 more dogs show up in the park. How many dogs are in the park?", students may decide to act it out using counters and a story mat; draw a picture; or use a handful of cubes.
K.MP.6	**Attend to precision.**	*Fundamentals* ● Mathematically proficient students in Kindergarten begin to express their ideas and reasoning using words. As their mathematical vocabulary increases due to exposure, modeling, and practice, kindergarteners become more precise in their communication, calculations, and measurements. In all types of mathematical tasks, students begin to describe their actions and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise explanations and reasoning regarding their process of finding solutions. *Example* ● A student may use color words (such as blue, green, light blue) and descriptive words (such as small, big, rough, smooth) to accurately describe how a collection of buttons is sorted.
K.MP.7	**Look for and make use of structure.**	*Fundamentals* ● Mathematically proficient students in Kindergarten begin to look for patterns and structures in the number system and other areas of mathematics. For example, when searching for triangles around the room, kindergarteners begin to notice that some triangles are larger than others or come in different colors- yet they are all triangles. While exploring the part-whole relationships of a number using a number balance, students begin to realize that 5 can be broken down into sub-parts, such as 4 and 1 or 3 and 2, and still remain a total of 5.
K.MP.8	**Look for and express regularity in repeated reasoning.**	*Fundamentals* ● In Kindergarten, mathematically proficient students begin to notice repetitive actions in geometry, counting, comparing, etc. For example, a kindergartener may notice that as the number of sides increase on a shape, a new shape is created (triangle has 3 sides, a rectangle has 4 sides, a pentagon has 5 sides, a hexagon has 6 sides). When counting out loud to 100, kindergartners may recognize the pattern 1-9 being repeated for each decade (e.g., Seventy-ONE, Seventy-TWO, Seventy- THREE… Eighty-ONE, Eighty-TWO, Eighty-THREE…). When joining one more cube to a pile, the child may realize that the new amount is the next number in the count sequence.

SECTION 5: Mathematical Modeling

Mathematical Modeling Framework

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create. The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.)



The Mathematical Modeling Framework offers some insight into what modeling with mathematics looks like when implemented in K-12 classrooms.

- Real-life, mathematical situations or problems are investigated.
- Students gather information, make assumptions, and define unknowns (variables).
- Mathematical models are created and used to arrive at a solution to explain the real-life, mathematical situation or problem.
- Models are analyzed and revised as needed.
- Models are evaluated by students. Solutions using different models are interpreted and conclusions are drawn and validated.

When choosing mathematical modeling tasks and activities, keep in mind that these tasks should (1) be interesting and/or important for students to experience, (2) exemplify specific components of the modeling cycle, (3) be doable by real students in real classrooms in real time.

When planning to engage students in learning through mathematical modeling, please note the following:

MODELING (LIKE REAL LIFE) IS OPEN-ENDED AND MESSY.
It may seem like a good idea to help students by distilling a problem so they can immediately see which are the important factors to be considered. However, doing so prevents them from doing this on their own and robs them of the feelings of investment and accomplishment in their work. Also, mathematical models are not perfect and multiple models can provide very different results. Mathematician George Box summed this up beautifully when he said, “All models are wrong, but some are useful.” (Box and Draper 1987, p. 424).

WHEN STUDENTS ARE MODELING, THEY MUST BE MAKING GENUINE CHOICES.
The best problems involve making decisions about things that matter to the students and help them see how using mathematics can help them make good, informed decisions.

START BIG, START SMALL, JUST START.
You may feel ready to jump in and make big changes, and if so, that is great! However, even small changes to things you already do in your classroom can encourage students to engage in mathematical modeling. To start small, choose a mathematical modeling task that you feel comfortable with – maybe one that you and your colleagues tackle while engaging in the mathematical modeling framework.

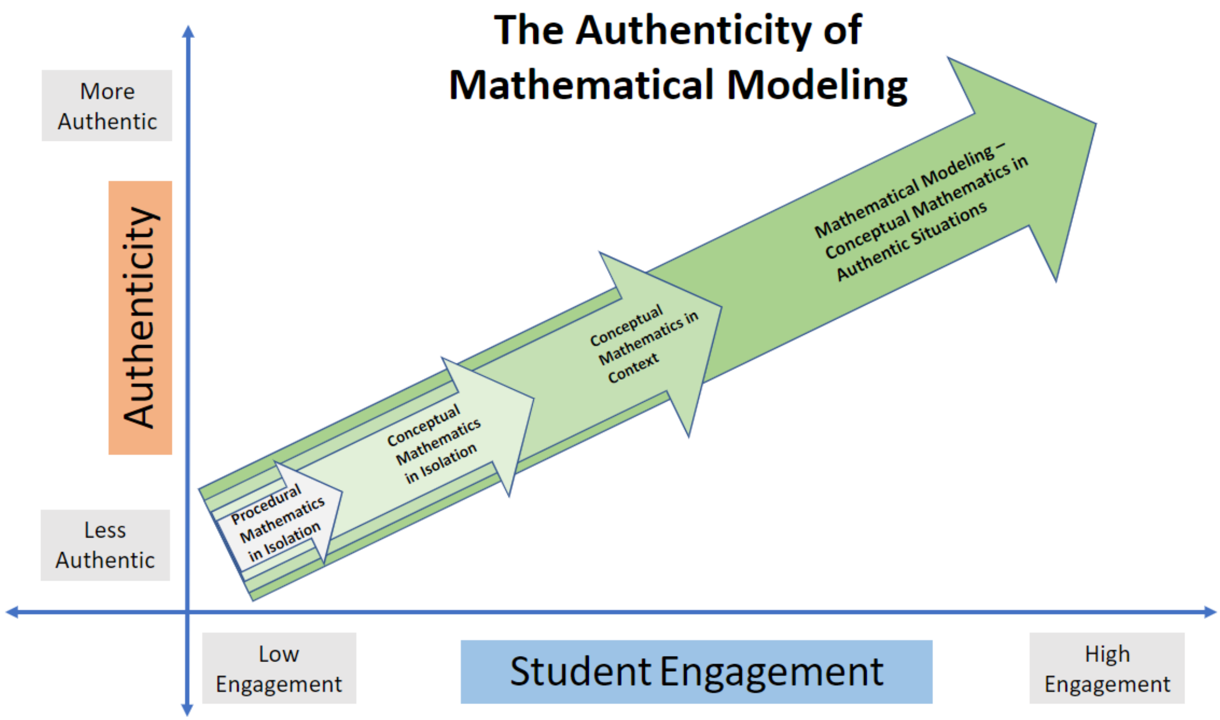
ASSESSMENT SHOULD FOCUS ON THE PROCESS, NOT THE PRODUCT.
Mathematical models (and the results they produce) are intimately tied to the assumptions made in creating the models. Assessment should be in service of helping students improve their ability to model, which will, in time, translate to a better product.

MODELING DOES NOT HAPPEN IN ISOLATION.
Whether students are working in teams, sharing ideas with the whole class, or going online to do research or collect data, modeling is not about working in a vacuum. The problems are challenging, and it helps to know you have support as you seek answers.

Adapted from GAIMME, 2016

Mathematical Modeling Continuum

The Mathematical Modeling Continuum provides a visual and applicable flow map that illustrates how we can address 21st century (cognitive, intrapersonal, interpersonal) competencies domains through a progression of the four stages of mathematical problem development: Mathematics Problems, Word Problems, Application Problems and Mathematical Modeling Problems. The eventual objective of the continuum is to develop a process in which students learn to transfer knowledge in one situation and apply it to new situations.



Framework for Statistical Reasoning

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The [Framework for Statistical Reasoning]() should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

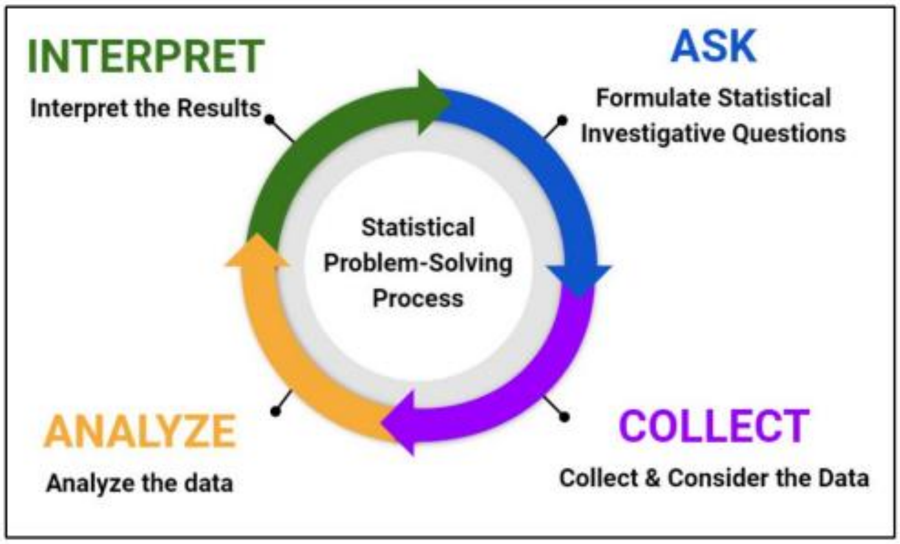


FIGURE 1: GEORGIA FRAMEWORK FOR STATISTICAL REASONING

- **I.** **Formulate a statistical investigative question:** Oftentimes questions will naturally come about in various content areas. In the beginning, questions that can be answered with each class member contributing a piece of data is easiest. As students become familiar with formulating questions, they may begin venturing outside of the classroom such as questions comparing boys versus girls, or adults versus children (Van de Walle, Karp, Lovin, Bay-Williams, 2014).
- **II.** **Collect and Consider Data:** Depending on the question asked, you will collect numerical or categorical data. Examples of categorical data are favorite colors, ice cream flavors, transportation home, etc. Examples of numerical data may include measurements of plants, distance traveled to school, temperatures throughout the week, etc.
- **III.** **Analyze the Data:** Make sense of the data and communicate what the data means using graphs and words.
- **IV.** **Interpret the Data:** Answer the original question based on the data that was collected. You may also find that once you answer one question, others will develop. This is a great way to continue the statistical reasoning.

Specific expectations for data in Kindergarten include:

- Expectations in this grade level should be taught throughout the year and applied contextually to the current expectation and everyday events.
- Relevant problems can include word problems that are meaningful to a student's real environment. It is important for the problems presented to be relevant and interesting for the learners to pique their natural, intellectual curiosity.
- Limit category counts to be less than or equal to ten.
- At this grade level, more support is needed with formulating statistical questions. Students should be given guidance when developing statistical investigative questions. Students should be provided with support strategies for collecting and organizing their data.
- Students will display their data using objects and pictures. In later grades, students will represent data in pictographs and bar graphs.
- In Kindergarten, students should be able to use friendly language to explain their data and answer the overall question.
- The terminology below is used to clarify expectations for the teaching professional. Students are not required to use this terminology when engaging with the learning objective.
- A statistical investigative question is one that requires data that will vary. Examples: "How did you get to school today?"; "What is your favorite ____?"

For more information about this Georgia Framework for Statistical Reasoning, please see the Essential Instructional Guidance at the end of the K-8 Mathematics Standards document.

SECTION 6: Progressions

Vertical Learning Progressions

The K-12 Mathematics Learning Progressions provides a visual progression of mathematics expectations within Georgia's K-12 Mathematics Standards across all levels for students, parents, and educators to make connections among key concepts as students move from one grade level to the next. There are many reasons to devote a bit of your time to the progression of standards.

The progressions will:

- deepen your understanding of how the development of mathematical thinking has proven to be a critical element of a student's mathematics success as they matriculate through elementary school.
- inform you of your grade-level expectations compared to other grade levels in the progression
- help you plan more effectively with same-grade and other grade-level colleagues
- deepen your understanding of the mathematics you teach
- help you understand how mathematical thinking develops in order to support deep student learning
- inform you of where students should be in their mathematical journey based on what they should understand from the prior years
- show you how conceptual understanding develops, making it easier to help students who have gaps in learning
- help your students see the connections between ideas in mathematics in your grade level and beyond by helping them connect what they already know to what is to come
- help you assess understanding more completely and develop better assessments

The progressions document can be found here: K-12 Mathematics Learning Progressions.

SECTION 7: Big Ideas

Big Ideas, K - 5

The image below illustrates the big ideas for Georgia's K-12 Mathematics Standards. For each big idea in Kindergarten through High School Advanced Algebra: Concepts & Connections, the grade levels where it is addressed is indicated. Most of the big ideas span multiple grade levels, supporting the progression of mathematics and the coherence across grade levels. Each big idea is unpacked at the standard level.

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS Algebra: Concepts & Connections</th><th>HS Geometry: Concepts & Connections</th><th>HS Advanced Algebra: Concepts & Connections</th></tr>
<tr><td colspan="12">Mathematical Modeling (MM)</td></tr>
<tr><td colspan="12">Mathematical Practices (MP)</td></tr>
<tr><td colspan="12">Data & Statistical Reasoning (DSR)</td></tr>
<tr><td colspan="12">Numerical Reasoning (NR)</td></tr>
<tr><td colspan="12">Patterning & Algebraic Reasoning (PAR)</td></tr>
<tr><td colspan="12">Geometric & Spatial Reasoning (GSR)</td></tr>
<tr><td colspan="6">Measurement & Data Reasoning (MDR)</td><td colspan="6"></td></tr>
<tr><td colspan="8"></td><td colspan="4">Functional & Graphical Reasoning (FGR)</td></tr>
<tr><td colspan="7"></td><td>Probability Reasoning (PR)</td><td colspan="2"></td><td colspan="2">Probabilistic Reasoning (PR)</td></tr>
</table>

SECTION 8: Standards Analysis

Content Standards

Georgia's K-12 Mathematics Standards are for ALL learners. All students, including students with disabilities, English Learners (EL) and students identified as gifted, must be challenged to excel within the general curriculum and be prepared for success in their post-school lives, including college, military enlistment, and/or careers.

Each student is expected to meet the high academic standards and demonstrate the level of mathematical reasoning needed to fully develop their conceptual understanding and procedural fluency; therefore, their instruction must incorporate supports and accommodations.

Promoting a culture of high expectations for all students is a fundamental goal of Georgia's K-12 Mathematics Standards.

KINDERGARTEN STANDARDS	
K.MP: **Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.**	
K.NR.1: **Demonstrate and explain the relationship between numbers and quantities up to 20; connect counting to cardinality (the last number counted represents the total quantity in a set).**	
K.NR.2: **Use count sequences within 100 to count forward and backward in sequence.**	
K.NR.3: **Use place value understanding to compose and decompose numbers from 11–19.**	
K.NR.4: **Identify, write, represent, and compare numbers up to 20.**	
K.NR.5: **Explain the concepts of addition, subtraction, and equality and use these concepts to solve real-life problems within 10.**	
K.PAR.6: **Explain, extend, and create repeating patterns with a repetition, not exceeding 4 and describe patterns involving the passage of time.**	
K.MDR.7: **Observe, describe, and compare the physical and measurable attributes of objects and analyze graphical displays of data to answer relevant questions.**	Measurement Data
K.GSR.8: **Identify, describe, and compare basic shapes encountered in the environment, and form two-dimensional shapes and three-dimensional figures.**	

Understanding the Content Standards

Clicking on each of the standards below will provide a brief description of the standard along with a breakdown of the standard through its learning objectives. For more detailed information about how to help students build toward mastery of these standards and background information, review the *Explanation of the Mathematics Content Standards*.

Kindergarten Mathematics	
Mathematical Practice K.MP	
Numerical Reasoning K.NR.1 K.NR.2 K.NR.3 K.NR.4 K.NR.5	**Patterning & Algebraic Reasoning** K.PAR.6
Measurement & Data Reasoning K.MDR.7	**Geometric & Spatial Reasoning** K.GSR.8

Understanding the Content Standards

MATHEMATICAL PRACTICES STANDARD/KEY COMPETENCY
MATHEMATICAL PRACTICES – *reasoning and explaining, modeling and using tools, seeing structure and generalizing*
MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the “habits of mind” that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories. These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Mathematical Practices present.
Breakdown of Standard/Key Competency (Expectation/Learning Objective)
K.MP.1 Make sense of problems and persevere in solving them.
K.MP.2 Reason abstractly and quantitatively.
K.MP.3 Construct viable arguments and critique the reasoning of others.
K.MP.4 Model with mathematics.
K.MP.5 Use appropriate tools strategically.
K.MP.6 Attend to precision.
K.MP.7 Look for and make use of structure.
K.MP.8 Look for and express regularity in repeated reasoning.

STANDARD/KEY COMPETENCY 1
NUMERICAL REASONING – *counting, money, place value, numbers to 20, addition, subtraction and fluency*
K.NR.1: Demonstrate and explain the relationship between numbers and quantities up to 20; connect counting to cardinality (the last number counted represents the total quantity in a set). **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will explore how numbers up to 20 are used to explain the quantity of objects. Students will be able to count and answer "how many?" when given up to 20 objects arranged in a variety of ways. When counting, students should know that the last number counted represents the total quantity in a set. Students will also be able to say the name of the number that is one more, and the number that is one less than a given number. Students will also be able to name and tell the value of a penny, nickel, or dime.
Breakdown of Standard/Key Competency 1 **(Expectation/Learning Objective)**
K.NR.1.1 Count up to 20 objects in a variety of structured arrangements and up to 10 objects in a scattered arrangement.
K.NR.1.2 When counting objects, explain that the last number counted represents the total quantity in a set (cardinality), regardless of the arrangement and order.
K.NR.1.3 Given a number from 1-20, identify the number that is one more or one less.
K.NR.1.4 Identify pennies, nickels, and dimes and know their name and value.

STANDARD/KEY COMPETENCY 2
***NUMERICAL REASONING –** counting, money, place value, numbers to 20, addition, subtraction and fluency*
K.NR.2: Use count sequences within 100 to count forward and backward in sequence. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will rote count forward to 100. Starting at 10, students will rote count forward by tens to 100. Starting at 10, students will rote count backward to 1. Once students have mastered rote counting backward from 10, students will start at 20 and rote count backward to 1. Students will also count forward within 100 and backward within 20 from a given number.
Breakdown of Standard/Key Competency 2 (Expectation/Learning Objective)
K.NR.2.1 Count forward to 100 by tens and ones and backward from 20 by ones.
K.NR.2.2 Count forward beginning from any number within 100 and count backward from any number within 20.

STANDARD/KEY COMPETENCY 3
***NUMERICAL REASONING –** counting, money, place value, numbers to 20, addition, subtraction and fluency*
K.NR.3: Use place value understanding to compose and decompose numbers from 11–19. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through one learning objective. This learning objective is not meant to be taught in isolation, but rather within a cluster of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will use tools such as objects, 5 frames, 10 frames, or rekenreks, to compose (to put together) and decompose (break apart) numbers into a group of ten ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
Breakdown of Standard/Key Competency 3 (Expectation/Learning Objective)
K.NR.3.1 Describe numbers from 11 to 19 by composing (putting together) and decomposing (breaking apart) the numbers into ten ones and some more ones.

STANDARD/KEY COMPETENCY 4
NUMERICAL REASONING – *counting, money, place value, numbers to 20, addition, subtraction and fluency*
K.NR.4: Identify, write, represent, and compare numbers up to 20. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will engage in multiple opportunities to count various objects. Students will identify written numerals to represent a given set of objects up to 20. Students will be able to write the numeral that represents a set of 0-20 objects. When given two groups of objects, students will verbally identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.
Breakdown of Standard/Key Competency 4 (Expectation/Learning Objective)
K.NR.4.1 Identify written numerals 0- 20 and represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
K.NR.4.2 Compare two sets of up to 10 objects and identify whether the number of objects in one group is more or less than the other group, using the words “greater than,” “less than,” or “the same as”.

STANDARD/KEY COMPETENCY 5
NUMERICAL REASONING – *counting, money, place value, numbers to 20, addition, subtraction and fluency*
K.NR.5: Explain the concepts of addition, subtraction, and equality and use these concepts to solve real-life problems within 10. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will explore the operations of addition and subtraction and use addition and subtraction to solve problems within 10 from real-life where the result or total is unknown. They will represent situations in various ways using objects, fingers, drawings, expressions, or equations. Students will use a variety of strategies to solve addition and subtraction problems within 10. Students will fluently add and subtract using a variety of strategies.
Breakdown of Standard/Key Competency 5 **(Expectation/Learning Objective)**
K.NR.5.1 Compose (put together) and decompose (break apart) numbers up to 10 using objects and drawings
K.NR.5.2 Represent addition and subtraction within 10 from a given authentic situation using a variety of representations and strategies.
K.NR.5.3 Use a variety of strategies to solve addition and subtraction problems within 10.
K.NR.5.4 Fluently add and subtract within 5 using a variety of strategies to solve practical, mathematical problems

STANDARD/KEY COMPETENCY 6
PATTERNING & ALGEBRAIC REASONING – *repeating patterns and time*
K.PAR.6: Explain, extend, and create repeating patterns with a repetition, not exceeding 4 and describe patterns involving the passage of time. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will identify a pattern with no more than 4 repetitions (iterations) created using shapes and numbers and extend the pattern. Students will describe patterns related to time from real-life.
Breakdown of Standard/Key Competency 6 **(Expectation/Learning Objective)**
K.NR.6.1 Create, extend, and describe repeating patterns with numbers and shapes, and explain the rationale for the pattern.
K.NR.6.2 Describe patterns involving the passage of time using words and phrases related to actual events.

STANDARD/KEY COMPETENCY 7
MEASUREMENT & DATA REASONING – *attributes of objects, classifying objects*
K.MDR.7: Observe, describe, and compare the physical and measurable attributes of objects and analyze graphical displays of data. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will observe, describe, and compare the measurable attributes of objects and sort objects into categories by an attribute. Students will use language such as heavier, lighter, longer, taller, shorter, wider, larger, smaller to compare, describe, and order common objects. Students may use a variety of techniques and tools to compare, describe, and order objects. Students may use another object as the standard of comparison to describe object(s).
Breakdown of Standard/Key Competency 7 **(Expectation/Learning Objective)**
K.NR.7.1 Directly compare, describe, and order common objects, using measurable attributes (length, height, width, or weight) and describe the difference.
K.NR.7.2 Classify and sort up to ten objects into categories by an attribute; count the number of objects in each category and sort the categories by count.
K.NR.7.3 Ask questions and answer them based on gathered information, observations, and appropriate graphical displays to solve problems relevant to everyday life.

STANDARD/KEY COMPETENCY 8
GEOMETRIC & SPATIAL REASONING – *2D and 3D shapes, relative locations, attributes*
K.GSR.8: Identify, describe, and compare basic shapes encountered in the environment, and form two-dimensional shapes and three-dimensional figures. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Kindergarten curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will observe shapes in their environment and describe the shapes based on the number of sides, vertices, and other attributes. They will identify basic two-dimensional shapes (squares, circles, triangles, rectangles, hexagons, and octagons) and three-dimensional shapes (cubes, cones, cylinders, and spheres) and describe how these shapes are similar and different. Students will begin to understand how three-dimensional figures are composed of two-dimensional shapes. They will form larger shapes by putting two or more basic shapes together and will also explain the location of an object in relation to another object using positional language.
Breakdown of Standard/Key Competency 8 **(Expectation/Learning Objective)**
K.GSR.8.1 Identify, sort, classify, analyze, and compare two-dimensional shapes and three-dimensional figures, in different sizes and orientations, using informal language to describe their similarities, differences, number of sides and vertices, and other attributes.
K.GSR.8.2 Describe the relative location of an object using positional words.
K.GSR.8.3 Use basic shapes to represent specific shapes found in the environment by creating models and drawings.
K.GSR.8.4 Use two or more basic shapes to form larger shapes.

SECTION 9: Instructional Supports

Curriculum Units

SEMESTER 1

Unit 1: Wondering About My World and Investigating to Find Answers *(Introduction to Project-Based Learning)*	4 - 5 Weeks
In this unit, students will explore how numbers up to 10 are used to explain the quantity of objects in their world. Through multiple opportunities to count various objects, they will identify written numerals to represent a given set of objects up to 10. Students will begin learning to rote count to 100 forward and backward from 20. Based on their curiosity and interests, students will generate questions to investigate situations. They will collect data to answer the questions they generated and represent and explain their data. (See Framework for Statistical Reasoning.)	
K.NR.1, K.NR.2, K.NR.4, K.MDR.7, K.MP.1-8	
Unit 2: 2-D Shapes in My World	3 – 4 weeks
Students will observe shapes in their environment and describe the shapes based on the number of sides, vertices, and other attributes. They will identify basic two-dimensional shapes (squares, circles, triangles, rectangles, hexagons, and octagons) and form larger shapes by putting two or more basic shapes together. They will explain the location of shapes by saying where a shape is in relation to another shape. Students will identify a pattern created by shapes and extend the pattern. They will observe, describe, and compare the measurable attributes of objects and sort objects into categories by an attribute.	
K.GSR.8, K.MDR.7, K.PAR.6, K.MP.1-8	
Unit 3: How Many? (Numbers Up to 20)	5 – 6 weeks
In this unit, students will extend the work with numbers and quantities as they explore and count sets of objects up to 20. They will begin to explore sets up to 20 as they see the numbers as 10 and some more. They will use numerals 0 - 20 to represent the number of objects and be able to count out a given number of objects. Students will compare two sets of objects using the phrases “greater than,” “less than”, or “the same as.” When given a number 1-20, they will be able to say the number that is one more than or one less than the number. They will count forward to 100 by ones, and backward from 20. In order to see the sequence in counting by tens, students will count to 50 by tens. Students will identify pennies, nickels, and dimes and know their value. They will ask questions and answer them as they explore coins.	
K.NR.1, K.NR.2, K.NR.3, K.NR.4, K.MDR.7, K.MP.1-8	
Unit 4: Understanding and Using Addition and Subtraction in My Life	4 – 5 weeks
Students will explore the operations of addition and subtraction and use addition and subtraction to solve problems within 10 from real-life where the result or total is unknown. They will represent the situations in various ways using objects, fingers, drawings, expressions, or equations. Students will solve problems they create by generating questions and gathering information. Students will use a variety of strategies to solve addition and subtraction problems within 10. Students will identify and describe patterns with addition of numbers. Students will identify and extend patterns with numbers and shapes. As they have conversations about their days, they will describe patterns related to time from real-life (yesterday, today, tomorrow).	
K.NR.5, K.PAR.6, K.MDR.7, K.MP.1-8	

Mathematical Practices (K.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

SEMESTER 2

Unit 4: Understanding and Using Addition and Subtraction in My Life *(Continued from Semester 1)*	3 – 4 weeks
In this unit, students will explore the operations of addition and subtraction and use addition and subtraction to solve problems within 10 from real-life where the result or total is unknown. They will represent the situations in various ways using objects, fingers, drawings, expressions or equations. Students will solve problems they create by generating questions and gathering information. Students will use a variety of strategies to solve addition and subtraction problems within 10. Students will identify and describe patterns with addition of numbers. Students will identify and extend patterns with numbers and shapes. As they have conversations about their days, they will describe patterns related to time from real-life (yesterday, today, tomorrow).	
K.NR.5, K.PAR.6, K.MDR.7, K.MP.1-8	
Unit 5: Using Numbers within 20	5 – 6 weeks
In this unit, students will continue to explore numbers and develop understanding of numbers (number sense). They will use place value as they compose (put together) and decompose (break apart) numbers into ten and some more. Students will represent the numbers as ten and some more using objects and drawings. They will count to 100 by tens and ones and count backward from 20 by ones.	
K.NR.3, K.NR.2, K.MP.1-8	
Unit 6: 3-D Shapes in My World	3 – 4 weeks
In this unit, students will revisit shapes in their environment and identify three-dimensional shapes (cubes, cones, cylinders, and spheres) in their environment. Students will explore and compare two-dimensional shapes and three-dimensional shapes in various sizes and orientations. They will describe how shapes are similar and different. They will order common objects based on measurable attributes and sort objects by an attribute. Students will generate statistical questions about shapes in their world and collect, represent, analyze, and explain their findings. (See Framework for Statistical Reasoning.)	
K.GSR.8, K.MDR.7, K.MP.1-8	
Unit 7: Using Numbers and Data to Make Sense of My World	4 – 5 weeks
In this unit, students will further investigate place value and solve addition and subtraction problems in the real-world. They will explain patterns they see and have additional experiences in creating, extending, and describing patterns with numbers and shapes. Students will describe patterns related to the passage of time in their lives (yesterday, today, and tomorrow). Based on their interests and curiosity, they will create investigative statistical questions, collect data, analyze the data, and explain the data to answer their questions. (See Framework for Statistical Reasoning.)	
K.NR.3, K.NR.5, K.PAR.6, K.MDR.7, K.MP.1-8	
Unit 8: Culminating Capstone Unit	2 – 3 weeks
(applying concepts in real-life contexts through a culminating interdisciplinary unit) The capstone unit applies content that has already been learned in previous interdisciplinary PBLs and units throughout the school year. The capstone unit is an interdisciplinary unit that allows students to create a presentation, report, or demonstration that could include their models used to answer an overarching driving question. (e.g., Students can present their solution(s), findings, project, or answer to the driving question to a larger audience during the culminating capstone unit.)	
ALL STANDARDS AND LEARNING OBJECTIVES	

Mathematical Practices (K.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

Instructional Design

The learning plans within the units follow a modified 5-E model. The components of this model include diagnostic assessments, Engage, Explore, Apply, and Reflect sections. Also included are sections focusing on Evidence of Student Success, Student Learning Supports, and Engaging Families. Below is a list of each component included in the instructional design along with a brief description.

- **Diagnostic Assessment –** A brief diagnostic assessment for the standard is included in each learning plan. One diagnostic may be used for multiple tasks that address a specific standard.
- **Engage -** Within this section, the learning experiences include evidence-based instructional strategies that can be used as an introduction that mentally engages students to capture their interest, provides an opportunity to communicate what they know, and allows them to connect what they know to new ideas.
- **Explore -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to engage in hands-on activities to explore the new concept/big idea at a deep level.
- **Apply -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to apply what they have learned in a new situation to develop a deeper understanding of the big idea.
- **Reflect -** Within this section, the learning experiences include evidence-based instructional strategies that allow students the opportunity to review and reflect on their own learning and new understandings.
- **Evidence of Student Success –** Within this section, formative assessment questions have been provided as a tool for teachers to determine the level of student understanding of the standard.
- **Student Learning Supports –** Within this section, suggested strategies are outlined to support learners as they progress towards instructional goals. Teachers should use frequent formative assessment information to determine which students need additional support. For more information on supporting the learning, extending the learning and language supports, please review the information under Instructional Support Strategies within the Comprehensive Grade Level Overview.
- **Engaging Families –** Within this section, activities are included for families to use to build on mathematical ideas and continue to extend mathematics learning outside of school.

Mathematical Instructional Tools

Use of manipulatives

Students at all grade levels should use concrete manipulatives and virtual simulations to develop the mathematical ideas that are presented in the standards in order to create a basis for the abstract reasoning necessary.

Concrete materials help students make sense of mathematical representations. Mathematical representations include the use of mathematical symbols to represent the concrete mathematical idea, thought process, or situation. This is a very important, yet often neglected, step along the way. Mathematical representations can be concrete, representational, and abstract. Each type of representation is important to student understanding. Mathematizing a situation or problem is also necessary to build understanding. Mathematizing means to take any situation and view it through the lens of mathematics.

The following are simple rules of thumb for using manipulatives:

- Introduce new manipulatives/tools by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available manipulatives/tools to use in solving problems.
- Encourage the use of a manipulative or tool at the beginning of student development of a concept and at any point when you believe it would be helpful to a student to conceptualize the mathematics presented.

Concrete-Representational-Abstract (CRA) Instructional Approach

Concrete-Representational-Abstract (CRA) is a systematic and sequential Research-Based Educational approach to teaching and learning mathematics. With CRA, students are introduced to mathematics concepts using manipulatives or hands-on materials (concrete), pictorial representations or “seeing” stage involves using images, charts, and graphs to represent objects (representational), and, lastly, abstract figures, or “symbolic’ stage such as numbers and mathematical symbols that can be used to represent and solve contextual problems mathematically. Teachers help students bridge the connection between the concrete, representational, and abstract representations of the mathematics used to solve the problems.

Why is CRA so important? All students learn differently. Some students may need very little time with concrete objects or representations before they can jump to the abstract. Other students need the visuals for a longer period to make sense of mathematical ideas. We have a variety of learning styles in our classrooms, so varying the way that we approach teaching mathematics with the students is helpful in that respect. In addition, students at each stage of this progression can deepen their understanding of the concepts being learned by sharing their learning experiences during mathematics lessons.

CRA is also connected to mathematical modeling. Mathematical Modeling means to “mathematize” a situation or problem, to take a situation which might, at first glance, not seem mathematical, and view it through the lens of mathematics. Modeling may include the use of mathematical symbols, in addition to concrete materials, to represent the situation or problem. Mathematical modeling can be concrete, representational, and abstract and each type of model is important to student understanding.

Almost all topics in mathematics can be taught using CRA. However, students do not have to progress through the concrete to get to the representational and abstract stages. Students often work at the concrete and abstract or representational and abstract stages simultaneously. The reason for engaging students in this progression is to help them make sense of mathematics.

Evidence of Student Success

The value of formative assessments and feedback cannot be overstated. Continuous progress monitoring with both feedback and commentary should occur throughout instructional lessons.

Summative Assessments

A summative assessment is an evaluation tool generally used at the end of an assignment, unit, project, or at the end of the course. Evaluative criteria should be incorporated to assess student learning. In an educational setting, summative assessments tend to be more formal kinds of assessments (e.g., unit tests, final exams, projects, reports, and state assessments) and are typically used to assign students a course grade or to certify student mastery of intended learning outcomes for the Georgia Mathematics Standards.

Guiding Questions:

- What are the evaluative criteria (or rubric) and how do they measure student proficiency for your objectives?
- Are the assessments aligned with approved standards and learning targets?

Formative Assessments

A formative assessment is an evaluation tool used to guide and monitor the progress of student learning during instruction. Formative assessments should align to the rigor of Georgia's K-12 Mathematics Standards and the corresponding summative assessment. Its purpose is to provide continuous feedback to both the student and the teacher concerning learning successes and failures. Formative assessments diagnose skill and knowledge gaps, measure progress, and evaluate instruction. Teachers use formative assessments to determine what concepts require more teaching and what teaching techniques require modification. Educators use results of these assessments to improve student performance. Formative assessments would not necessarily be used for grading purposes. Examples include (but are not limited to): pre/posttests, portfolios, benchmark assessments, quizzes, teacher observations, teacher/student conferencing, teacher commentary and feedback.

Guiding Questions:

- How will students demonstrate their understanding?
- Why should there be more than one form of assessment for students?
- In what ways will student learning be monitored during the lesson and how will this guide your instruction?
- How will feedback support students meeting the goals of the lesson?

Learning Targets and Success Criteria

Create Learning Targets: Learning targets frame a lesson from the student point of view. A learning target helps students grasp the lesson's purpose-- why it is crucial to learn this chunk of information, on this day, and in this way. Learning targets written in a student friendly way are often posted beginning with the words "I CAN..." Learning targets should clearly state what you expect students to know, understand and/or be able to do at the end of the lesson. This is called the "Learning Intention".

Learning targets are written using observable, measurable actions and should align with the content standards identified. This component is called the **"Success Criteria"**.

Types of Learning Targets:

1. Content Knowledge
2. Strategy Development
3. Thinking/Reasoning Development
4. Procedural
5. Investigative or Inquiry
6. Reflective
7. Skills
8. Product

Guiding Questions:

- As a result of the lesson, what should students know and be able to do?
- Why is it important that students achieve this new learning – what will they be able to do as a result of this new learning?
- How is the learning target meaningful and relevant beyond the specific task/activity? Does it relate to the content standards?
- Is the task or activity aligned with the learning target?

For a detailed description of each element of the Evidence of Student Learning section, please refer to the Division of School and District Effectiveness for the Georgia Department of Education System for Effective School Instruction.

Standards-Based Grading

- Grading practices should emphasize mastery of standards through the frequent use of aligned assessments, both formative and summative.
- Continual progress monitoring should be used to assess and diagnose each student's strengths and weaknesses, based on the standards of the associated core academic mathematics course, and to provide appropriate interventions.
- Opportunities should be provided for students to review content with a focus on standards not previously mastered.

Suggested Assessment Ideas Choice Board

Formative Assessments		Task-Based Performance Assessments	Formative Assessment Lessons
• Ticket out the door • My favorite no • 3-2-1 • Think-Pair-Share • Jigsaw	• Journals • Gallery Walk • Frayer Model • K-W-L Chart	• 3-Act Tasks • Open Middle • Pattern Talks • Number Talks	• K-5 FALs

Student Learning Supports

Fluency and Numeracy Development

Fluency involves three ideas: accuracy (attending to precision), efficiency (using well-understood strategies with ease), and flexibility (using part-whole strategies rather than less efficient counting strategies to work with numbers).

According to NCTM, fluency is also the ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop fluency, students need experience in integrating concepts and strategies and building on familiar strategies as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through strategic practice. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014).

Research indicates that teachers can best support students' development of automaticity with sums and differences through varied experiences making 10, breaking numbers apart and working on mental strategies, rather than timed tests. Evidence from research has indicated that timed tests cause unhealthy math anxiety with learners as they are developing a solid foundation in numeracy: Link Between Timed Tests and Math Anxiety.

Conceptual Understanding

Conceptual understanding involves three components: comprehension of mathematical concepts, operations, and relations. Students with a conceptual understanding will connect what they know about a mathematical idea to make sense of new mathematical ideas.

From Van de Walle (2016), "Conceptual understanding is a flexible web of connections and relationships within and between ideas, interpretations, and images of mathematical concepts - a relational understanding" (pg. 24).

Georgia Numeracy Project Resources

Georgia Early Numeracy Project

The Georgia Early Numeracy Project is a numeracy development resource provided by the Georgia Department of Education, which introduces teachers and teacher leaders to the trajectory by which learners acquire a solid foundation in numeracy.

The Georgia Early Numeracy Project consists of a two-part universal screener:

- Part 1 of the universal screener is the GloSS assessment which stands for the Global Strategy Stage assessment. This assessment assesses students' strategy proficiency across 3 domains, Addition and subtraction, multiplication and division, and proportions and ratios.
- The second component of the universal screener is the IKAN or Individual Knowledge Assessment of Number. The IKAN is broken into 2 parts. Depending upon the overall strategy stage from the first assessment, which is the GloSS, the student will either take the IKAN Part 1, the Counting Interview or the IKAN Part 2, the Written Assessment.

In alignment with Georgia's Tiered Supports for Students, the Georgia Early Numeracy Project provides tools to target specific skills and provide tiered supports and interventions. To address the identified skills, the Georgia Early Numeracy provides numeracy development intervention tasks and activities. Within each unit in the course, intervention tables are provided to identify specific tasks and activities aligned to the standards and learning objectives discussed within the unit.

The Georgia Early Numeracy Project is the perfect complement to Georgia's K-12 Mathematics Standards, aligning with the multiple big ideas and aiding in the development of mathematical reasoning. The Georgia Early Numeracy Project supports the various mathematical concepts in Kindergarten through Grade 7.

Algebra Readiness from an Elementary Perspective

The kind of mathematical thinking that can provide a foundation for learning algebra must be developed over an extended period of time, starting in kindergarten. (Carpenter, Franke, Levi, 2003) Helping students make connections between arithmetic and algebra through the study of patterns and strategies that incorporate the use of properties empowers students and makes the study of algebra more accessible.

To be prepared for foundational algebraic reasoning, students need a foundation in

- Algebra development from kindergarten to high school
- K-5 numeracy (GaDOE strategic goals)
- Mathematical Concepts that need to be developed at each grade level
- Support resources to develop algebraic thinking:
  - Georgia Numeracy Project

Algebra is not confined to a course or set of courses in the school curriculum; rather, it is a strand that unfolds across a pre-K–12 curriculum. Students experience the algebra strand as a way of thinking and valuing structure that develops across grades. At the elementary level, students develop fluency with numbers, explore structure in operations and their properties, and verbalize quantitative relationships (cf. Kieran 2007a; Schifter, Russell, and Bastable 2009). Middle grades students move from verbal descriptions of relationships to proficiency in generalizing numerical relationships and expressing them with symbolic representations and in the language of functions (cf. Kieran 2007b). High school students extend their ability to use and see structure in symbolic expressions as they create and reason with equations, inequalities, and systems (cf. Kieran 2007c).

Each topic within the algebra strand should be experienced as an integration of concepts, procedures, and applications. Concepts such as variable and equivalence and procedures such as solving equations and inequalities are equally important. Multiple strategies, including variations on common procedures and procedures using different representations, are needed to solve problems within mathematics and within other contexts. At all levels, students generalize, model, and analyze situations that are purely mathematical or ones that arise in real-world phenomena. They develop strategic use of a range of representations, tools, and technologies—including calculators, graphing utilities, spreadsheets, and computer algebra systems.

Before students transition into algebra content as a prominent part of their coursework, they need to develop a solid foundation in pre-kindergarten through middle school mathematics. For example, prior to extensive study of linear equations and slope, students should be able to write and interpret equivalent numerical expressions, recognize situations in which quantities are proportionally related, and write ratios to express relationships between those quantities. Characterizing algebra as a strand of the school curriculum highlights the power and usefulness of algebraic thinking and skills—proficiencies that open academic doors and are evident in many professions and careers. Such an algebra strand in the school curriculum is critical and is accessible for all students. (NCTM, 2014)

Interdisciplinary Teaching and Learning

Mathematical modeling can be explored through interdisciplinary teaching and learning. One instructional model for interdisciplinary teaching and learning is project-based learning. There are six powerful project-based instructional practices:

1. Plan Authentic, Intellectually Demanding Project-Based Learning Units Where Students Master Significant Content and Skills
2. Utilize Sustained, In-Depth Inquiry
3. Engage Students in a Collaborative Problem-Solving/Design Process
4. Foster a Classroom Environment That Supports Student Ownership of Learning
5. Engage in Ongoing and Purposeful Feedback, Revision, and Reflection
6. Include Community Partners in Project Planning, Implementation, and Reflection

Process thinking is also an important element of interdisciplinary teaching and learning. Process thinking has six components:

1. **Intellectual Challenge and Accomplishment** – this component focuses on developing students' capacity to learn deeply, think critically, and strive for excellence.
2. **Authenticity** – this component focuses on students working on projects that are meaningful and relevant to their community, their lives, and their future.
3. **Public Product** – students' work is publicly displayed, discussed, and critiqued.
4. **Collaboration** – this component focuses on building students' capacity to collaborate with other students in person or online and/or receive guidance from adult mentors and experts.
5. **Project Management** – students use a project management process that enables them to proceed effectively from project initiation to completion.
6. **Reflection** – students reflect on their work and their learning throughout the project.

The six components of process-based thinking connect nicely to the 8 Mathematical Practices that are embedded in all instructional units. These are a part of the Essential Instructional Guidance included for all grade levels and high school courses.

Instructional Support Strategies

Within each learning plan in the instructional units, specific instructional support strategies are provided in the Student Learning Supports section. These strategies are largely organized into three categories: Supporting the Learning, Extending the Learning, and Language Supports. A description and rationale for these categories is written below; the actual strategies for support can be found within each specific learning plan.

Supporting the Learning

Teachers greatly influence how students perceive and approach struggle in the mathematics classroom. Even young students can learn to value struggle as an expected and natural part of learning, (Principles to Actions, 2014). To close gaps in mathematical understanding, a focus needs to be placed on the structure and teaching strategies implemented in classrooms.

Within the learning plans in each unit, supports designated as Supporting the Learning within the Student Learning Support sections will include, but are not limited to:

- intervention activities specific to the learning experiences within the learning plan
- teacher actions from the K-5 Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan

Extending the Learning

According to Van De Walle, there are four basic strategies for adapting mathematics concepts for students who consistently demonstrate a solid understanding of the concepts of study.

- *Acceleration* is characterized by self-paced learning and frequent exploration of similar topics but include higher-level thinking, more complex or abstract ideas and deeper levels of understanding or content.
- *Enrichment* activities go beyond the topic of study to content that is not specifically a part of the grade level or course curriculum but is aligned with the lesson goals.
- *Sophistication* provides a natural world view of mathematics when the level of complexity is increased or more depth is pursued.
- *Novelty* introduces completely different materials from the regular curriculum.

Within the learning plans in each unit, supports designated as Extending the Learning within the Student Learning Support sections will include, but are not limited to:

- extension activities specific to the learning experiences within the task

- instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GaDOE Talent Development Team

Language Supports

Teachers support student's language development in the context of mathematical sense-making through meaningful "reciprocal" interactions and discourse with others. As ELs explore and connect new math concepts, they will need many well-supported opportunities to use language in listening, speaking, reading, and writing (Baker et al., 2014).

Within the learning plans in each unit, supports designated as Language Supports within the Student Learning Support sections will include, but are not limited to:

- teacher actions from the English Language Proficiency for English (as a 2nd language) Learners section of the K-5 Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan
- Evidence-Based Instructional Strategies: collection of vetted GA Mathematics resources and evidence-based instructional strategies that highlight the benefits of hands-on, relevant experiences in Mathematics that support multilingual learners
- strategies and resources included in the document Mathematics Resources to Support English Learners found in the GaDOE mathematics resources provide specific evidence-based practices that indicate the benefits of hands-on, relevant learning experiences in the mathematics classroom

SECTION 10: Additional Resources

Literature Connections

- For a comprehensive list of Kindergarten Mathematics Literature aligned to Georgia's K-12 Mathematics Standards, visit the Georgia Council of Teachers of Mathematics Webpage www.gctm.org.

Technology

- ✓ Desmos Activities & Explorations
- ✓ Geogebra Activities & Explorations
- ✓ Math Learning Center
- ✓ Digital Learning Objects
- ✓ GaDOE Community

Professional Learning Videos

- ✓ K.NR.1
- ✓ K.NR.2
- ✓ K.NR.3
- ✓ K.NR.4
- ✓ K.NR.5
- ✓ K.PAR.6
- ✓ K.MDR.7 Measurement, K.MDR.7 Data
- ✓ K.GSR.8

General Resources

- ✓ Georgia Home Classroom Digital Learning Plans
- ✓ Georgia Mathematics Strategies Toolkit to Address Learner Variability
- ✓ Kindergarten Guide for Effective Mathematics Instruction