AP Calculus (BC) Summer Assignment

I can't tell you how excited I am that you have chosen to take this course next year. I hope you are excited as well and up for the challenge. I will be looking for your full cooperation as we prepare for this course next spring. I have assignments I would like you to do over the next few months to keep you fresh and ready for our course starting January.

It goes without saying that lack of completion of these assignments will jeopardize your enrollment in the course. I do not expect this to be a problem since I know most of you already.

I will be posting problems and exercises on the first day of each month leading up to our course. That means you must access my web site each month to get your new month's assignment. **Problems over the summer are due to Mr. Godack by the first full week of school**. Stop by my room B228 to drop it off or to ask questions. We will discuss handing in problems over the fall.

If you have any questions, please email me over the summer. I highly encourage you to work with other people in the class. If you do not know or remember how to do a problem, use the internet and other students as help. Blank answers will not be accepted. I expect that all parts of the assignment will be completed or at least attempted. If you happen to find a particular problem on the internet, don't copy it. All you are hurting is yourself. Get a hint then continue to try it on your own.

The first thing you must do is to go to Mr. Godack's summer assignment google site for instructions and links for the summer assignment. You must go there to access files, download documents and get links to other pages.

YOU MUST FIRST GO TO THE PAGE BELOW FOR THE SUMMER ASSIGNMENT!!

https://sites.google.com/gasd-pa.org/godack-bc-calculus

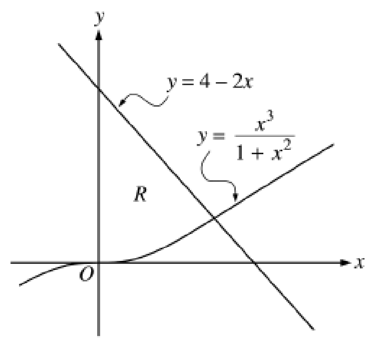
You must complete the online survey BEFORE THE LAST DAY OF SCHOOL and get your monthly assignments here.

AP Calculus BC Summer Assignment (June)

Solve each problem on a separate sheet of paper as if they are open ended AP problems. This means you must include all justifications necessary as on the AP AB exam. PLEASE BE NEAT!! You have the summer to work on these and I am looking for a neat, clean, clear copy of your solution! Problems (1-3) are with calculator and (4-6) are <u>without</u> calculator.

Question 1

Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

Question 3

A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

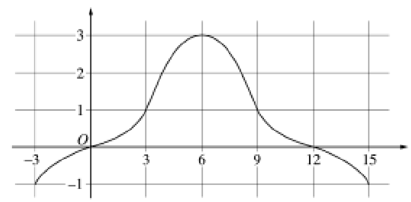
(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

(d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

Question 5

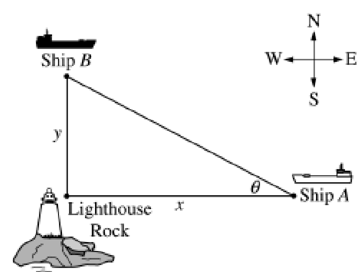
Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

Question 6

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.