# Dublin City Schools
# K-12 Mathematics Education
# Grade Course of Study
# Vision

## Vision

In Dublin City Schools, we believe in an education where mathematics is not only a fundamental part of every student's education, but a source of inspiration, curiosity, and real-world problem solving skills that can prepare all students for success in an increasingly complex and interconnected world.

We believe in developing mathematicians by providing students high quality experiences designed to:

- View mathematics as a powerful tool for understanding and improving the world around them.
- Refine critical thinking skills in order to analyze problems, assess information, and make informed decisions.
- Think flexibly and creatively with numbers in order to connect conceptual understanding with concrete situations.
- Develop skills such as perseverance, grit, and tenacity in order to embrace challenges and learn from mistakes.
- Cultivate a mathematical mindset that emphasizes the balance of fluency (efficiency, flexibility, accuracy) alongside conceptual understanding.
- Reflect upon their thinking, and the thinking of others, to examine the reasonableness of strategies and solutions while gaining self-efficacy as a lifelong learner.

## Instructional Agreements for Mathematical Learning within the Dublin City Schools

- Content standards will be learned in partnership with the Standards for Mathematical Practice in order to empower students to reason abstractly and quantitatively, make sense of problems and persevere in solving them, use appropriate tools strategically, attend to precision, think critically, communicate clearly, and collaborate effectively.
- Teachers will provide opportunities for students to share ideas, communicate their mathematical thinking, and learn from each other's perspectives, fostering a supportive learning community where students can take risks with confidence.
- A variety of instructional techniques and tools will be utilized to support students to create and connect different mathematical representations.
- Teachers will design experiences for students to recognize the real-world relevance of math by exploring and applying mathematical principles to solve authentic problems.
- Teachers will provide students with actionable feedback to support their growth as mathematicians.
- Differentiated instruction will ensure a challenging yet supportive learning environment that supports all students in achieving their full potential.

Together, we will create mathematically empowered learners prepared to navigate the challenges and opportunities ahead with a love of learning, a sense of curiosity, and the skills necessary for success.

## K-12 Course Mathematical Practices:

***1. Make sense of problems and persevere in solving them.***
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

***2. Reason abstractly and quantitatively.***
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

***3. Construct viable arguments and critique the reasoning of others.***
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about

data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

***4. Model with mathematics.***
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

***5. Use appropriate tools strategically.***
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

***6. Attend to precision.***

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

***7. Look for and make use of structure.***

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

***8. Look for and express regularity in repeated reasoning.***

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Data Science Foundations Course Goals:

Acquiring foundational knowledge in data science and basic programming skills are the primary objectives and outcomes of the Data Science Foundations course. It includes the use of mathematics, statistics, and computer science methods in the analysis and interpretation of data in all forms. In the context of real-world situations students will make predictions and decisions using data. Students combine problem solving and reasoning skills with statistics and modeling to analyze big data to find patterns and communicate meaning in data. Ohio's Learning Standards related to Statistics and Probability relevant to data science are taught along with the data demands of good citizenship in the 21st century. These habits and skills cut across disciplines, promote perseverance, and provide a gateway into successful postsecondary education and a variety of careers.

## Course Content Standards:

| Domain | Cluster | Standard |
|---|---|---|
| **QUANTITIES** | Reason quantitatively and use units to solve problems | **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |
| | | **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. |
| | | **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| **VECTOR AND MATRIX QUANTITIES** | Perform operations on matrices, and use matrices in applications | **N.VM.6(+)** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. |
| **CREATING EQUATIONS** | Create equations that describe numbers or relationships | **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* |

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| | | **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| | | **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. |
| | | **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |
| **REASONING WITH EQUATIONS AND INEQUALITIES** | Represent and solve equations and inequalities graphically | **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |
| **INTERPRETING FUNCTIONS** | Understand the concept of a function, and use function notation | **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. |
| | | **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| | Interpret the functions that arise in applications in terms of the context | **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| | | **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* |

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| | | **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. |
| | | **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| **BUILDING FUNCTIONS** | Build a function that models a relationship between two quantities | **F.BF.1** Write a function that describes a relationship between two quantities.<br>**a.** Determine an explicit expression, a recursive process, or steps for calculation from context.<br>i. Focus on linear and exponential functions.<br>ii. Focus on situations that exhibit quadratic or exponential relationships.<br>**b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*<br>**c.** **(+)** Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time . |
| **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** | Construct and compare linear, quadratic, and exponential models, and solve problems | **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| | | **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| | Interpret expressions for functions in terms of the situation they model | **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. |

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| **MODELING WITH GEOMETRY** | Apply geometric concepts in modeling situations | **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. |
| **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** | Summarize, represent, and interpret data on a single count or measurement variable | **S.ID.1** Represent data with plots on the real number line (dot plotsG, histograms, and box plots) in the context of real-world applications using the GAISE model. |
| | | **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. |
| | | **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |
| | | **S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. |
| | Summarize, represent, and interpret data on two categorical and quantitative variables | **S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. |
| | | **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>**a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.<br>**b.** Informally assess the fit of a function by discussing residuals. |

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| | | **c.** Fit a linear function for a scatterplot that suggests a linear association. |
| | Interpret linear models | **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. |
| | | **S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit. |
| | | **S.ID.9** Distinguish between correlation and causation. |
| **MAKING INFERENCES AND JUSTIFYING CONCLUSIONS** | Understand and evaluate random processes underlying statistical experiments | **S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. |
| | | **S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* |
| | | **S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. |
| | | **S.IC.6** Evaluate reports based on data. |
| **CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY** | Understand independence and conditional probability, and use them to interpret data | **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). |
| | | **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* |
| | | **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For* |

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| | | *example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* |
| **USING PROBABILITY TO MAKE DECISIONS** | Use probability to evaluate outcomes of decisions | **S.MD.7** Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game. |
| **DATA AND ANALYSIS** | Data collection and storage | **DA.DCS.9-12.F.a** Analyze patterns in a real-world data store through hypothesis, testing and use of data tools to gain insight and knowledge. |
| | | **DA.DCS.9-12.F.b** Investigate data storage systems to compare and contrast how data is stored and accessed. |
| | | **DA.DCS.9-12.A.a** Create multidimensional data collections that can be utilized through various methods to solve complex data problems. |
| | | **DA.DCS.9-12.A.b** Investigate data storage and collection tools to analyze tradeoffs and limitations. |
| | Visualization and communication | **DA.VC.9-12.F.a** Analyze the benefits and limitations of data visualization or multisensory artifacts and tools to communicate which is most appropriate to solve a real-world problem. |
| | | **DA.VC.9-12.A.a** Create visualization or multisensory artifacts to communicate insights and knowledge gained from complex data analysis that answers real-world questions. |
| | Inference and modeling | **DA.IM.9-12.F.a** Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data. |
| | | **DA.IM.9-12.A.a** Create a model that simulates a complex system and uses extracted data to hypothesize, test and refine the model to discover connections or trends. |
| **ALGORITHMIC THINKING AND PROGRAMMING** | Algorithms | **ATP.A.9-12.F.a** Define and use appropriate problem solving strategies and visual artifacts to create and refine a solution to a real world problem. |
| | | **ATP.VDR.9-12.F.a** Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways. |

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| | | **ATP.A.9-12.A.a** Define and explain recursive algorithms to understand how and when to apply them. |
| | | **ATP.A.9-12.A.b** Use recursion to effectively solve problems. |
| | | **ATP.A.9-12.A.d** Use sorting and searching to analyze and organize data. |
| | Variables and data representation | **ATP.VDR.9-12.F.a** Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways. |
| | | **ATP.VDR.9-12.A.a** Utilize different data storage structures to store larger and more complex data than variables can contain. |
| | | **ATP.VDR.9-12.A.b** Identify the appropriate data structures or variables to use to design a solution to a complex problem |
| | Control structures | **ATP.CS.9-12.F.a** Define control structures and Boolean logic and use them to solve real-world scenarios. |
| | | **ATP.CS.9-12.F.b** Use appropriate syntax to create and use a method. |
| | | **ATP.CS.9-12.F.c** Analyze the collection and generation of data through automated processes to explain the legal concerns that are not always evident to users. |
| | | **ATP.CS.9-12.A.a** Write programs that use library methods and control structures and methods to solve a problem. |
| | Modularity | **ATP.M.9-12.F.a** Break down a solution into procedures using systematic analysis and design. |
| | | **ATP.M.9-12.F.b** Create computational artifacts by systematically organizing, manipulating and/or processing data. |
| | | **ATP.M.9-12.A.a** Construct solutions to problems using student created components (e.g., procedures, modules, objects). |
| | | **ATP.M.9-12.A.b** Design or redesign a solution to a large-scale computational problem by identifying generalizable patterns. |

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| | | **ATP.M.9-12.A.c** Create programming solutions by reusing existing code (e.g., libraries, Application Programming Interface (APIs), code repositories). |
| | | **ATP.VDR.9-12.A.a** Utilize different data storage structures to store larger and more complex data than variables can contain. |
| | | **ATP.VDR.9-12.A.b** Identify the appropriate data structures or variables to use to design a solution to a complex problem |
| **IMPACTS OF COMPUTING** | Culture | **IC.Cu.9-12.F.b** Explore other professions to understand how computing has and will impact them positively and negatively. |
| | | **IC.Cu.9-12.A.b** Analyze the equity, access and influence of the distribution of computing resources to see their global impact. |
| | Social interactions | **IC.SI.9-12.F.b** Analyze the collection and generation of data through automated processes to explain the privacy concerns that are not always evident to users. |
| | Safety, law and ethics | **IC.SLE.9-12.F.c**. Analyze the collection and generation of data through automated processes to explain the legal concerns that are not always evident to users. |