Dublin City Schools
K-12 Mathematics Education
Grade Course of Study
Vision

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In Dublin City Schools, we believe in an education where mathematics is not only a fundamental part of every student's education, but a source of inspiration, curiosity, and real-world problem solving skills that can prepare all students for success in an increasingly complex and interconnected world.

We believe in developing mathematicians by providing students high quality experiences designed to:

- View mathematics as a powerful tool for understanding and improving the world around them.
- Refine critical thinking skills in order to analyze problems, assess information, and make informed decisions.
- Think flexibly and creatively with numbers in order to connect conceptual understanding with concrete situations.
- Develop skills such as perseverance, grit, and tenacity in order to embrace challenges and learn from mistakes.
- Cultivate a mathematical mindset that emphasizes the balance of fluency (efficiency, flexibility, accuracy) alongside conceptual understanding.
- Reflect upon their thinking, and the thinking of others, to examine the reasonableness of strategies and solutions while gaining self-efficacy as a lifelong learner.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

- Content standards will be learned in partnership with the Standards for Mathematical Practice in order to empower students to reason abstractly and quantitatively, make sense of problems and persevere in solving them, use appropriate tools strategically, attend to precision, think critically, communicate clearly, and collaborate effectively.
- Teachers will provide opportunities for students to share ideas, communicate their mathematical thinking, and learn from each other's perspectives, fostering a supportive learning community where students can take risks with confidence.
- A variety of instructional techniques and tools will be utilized to support students to create and connect different mathematical representations.
- Teachers will design experiences for students to recognize the real-world relevance of math by exploring and applying mathematical principles to solve authentic problems.
- Teachers will provide students with actionable feedback to support their growth as mathematicians.
- Differentiated instruction will ensure a challenging yet supportive learning environment that supports all students in achieving their full potential.

Together, we will create mathematically empowered learners prepared to navigate the challenges and opportunities ahead with a love of learning, a sense of curiosity, and the skills necessary for success.

K-12 Course Mathematical Practices:

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about

data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Precalculus Course Goals:

Precalculus emphasizes the development of a strong foundation in advanced algebra, trigonometry, and analytical skills. Mathematicians in this course will build upon previous coursework to explore polynomial, radical, exponential, logarithmic, and trigonometric functions through the lens of change. Students will analyze and represent functions using multiple representations including a foundational knowledge of limits. Throughout the course, students will also develop rigorous symbolic manipulation skills that will support future learning of mathematics.

Course Content Standards:

Domain	Cluster	Standard
POLYNOMIAL AND RATIONAL FUNCTIONS	Change in Tandem	**1.1.A** Describe how the input and output values of a function vary together by comparing function values.
		1.1.B Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.
	Rates of Change	**1.2.A** Compare the rates of change at two points using average rates of change near the points.
		1.2.B Describe how two quantities vary together at different points and over different intervals of a function.
	Rates of Change in Linear and Quadratic Functions	**1.3.A** Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.
		1.3.B Determine the change in the average rates of change for linear, quadratic, and other function types.
	Polynomial Functions and Rates and Change	**1.4.A** Identify key characteristics of polynomial functions related to rates of change.
	Polynomial Functions and Complex Zeros	**1.5.A** Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology
		1.5.B Determine if a polynomial function is even or odd.

Domain	Cluster	Standard
	Polynomial Functions and End Behavior	**1.6.A** Describe end behaviors of polynomial functions.
	Rational Functions and End Behavior	**1.7.A** Describe end behaviors of rational functions.
	Rational Functions and Zeros	**1.8.A** Determine the zeros of rational functions.
	Rational Functions and Vertical Asymptotes	**1.9.A** Determine vertical asymptotes of graphs of rational functions
	Rational Functions and Holes	**1.10.A** Determine holes in graphs of rational functions.
	Equivalent Representations of Polynomial and Rational Expressions	**1.11.A** Rewrite polynomial and rational expressions in equivalent forms.
		1.11.B Determine the quotient of two polynomial functions using long division.
		1.11.C Rewrite the repeated product of binomials using the binomial theorem.
	Transformations of Functions	**1.12.A** Construct a function that is an additive and/or multiplicative transformation of another function.
	Function Model Selection and Assumption Articulation	**1.13.A** Identify an appropriate function type to construct a function model for a given scenario.
		1.13.B Describe assumptions and restrictions related to building a function model.
	Function Model Construction and Application	**1.14.A** Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model.
		1.14.B Construct a rational function model based on a context.
		1.14.C Apply a function model to answer questions about a data set or contextual scenario
EXPONENTIAL AND LOGARITHMIC FUNCTIONS	Change in Arithmetic and Geometric Sequences	**2.1.A** Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers.
		2.1.B Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.

Domain	Cluster	Standard
	Change in Linear and Exponential Functions	**2.2.A** Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.
		2.2.B Describe similarities and differences between linear and exponential functions.
	Exponential Functions	**2.3.A** Identify key characteristics of exponential functions.
	Exponential Function Manipulation	**2.4.A** Rewrite exponential expressions in equivalent forms.
	Exponential Function Context and Data Modeling	**2.5.A** Construct a model for situations involving proportional output values over equal-length input-value intervals.
		2.5.B Apply exponential models to answer questions about a data set or contextual scenario.
	Compositions of Functions	**2.7.A** Evaluate the composition of two or more functions for given values.
		2.7.B Construct a representation of the composition of two or more functions.
		2.7.C Rewrite a given function as a composition of two or more functions.
	Inverse Functions	**2.8.A** Determine the input-output pairs of the inverse of a function.
		2.8.B Determine the inverse of a function on an invertible domain.
	Logarithmic Expressions	**2.9.A** Evaluate logarithmic expressions.
	Inverses of Exponential Functions	**2.10.A** Construct representations of the inverse of an exponential function with an initial value of 1.
	Logarithmic Functions	**2.11.A** Identify key characteristics of logarithmic functions.
	Logarithmic Function Manipulation	**2.12.A** Rewrite logarithmic expressions in equivalent forms.
	Exponential and Logarithmic Equations	**2.13.A** Solve exponential and logarithmic equations

Domain	Cluster	Standard
		2.13.B Construct the inverse function for exponential and logarithmic functions.
	Logarithmic Function Context and Data Modeling	**2.14.A** Construct a logarithmic function model.
TRIGONOMETRIC FUNCTIONS	Periodic Phenomena	**3.1.A** Construct graphs of periodic relationships based on verbal representations.
		3.1.B Describe key characteristics of a periodic function based on a verbal representation.
	Sine, Cosine, and Tangent	**3.2.A** Determine the sine, cosine, and tangent of an angle using the unit circle.
	Sine and Cosine Function Values	**3.3.A** Determine coordinates of points on a circle centered at the origin.
	Sine and Cosine Function Graphs	**3.4.A** Construct representations of the sine and cosine functions using the unit circle.
	Sinusoidal Functions	**3.5.A** Identify key characteristics of the sine and cosine functions.
	Sinusoidal Function Transformations	**3.6.A** Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.
	Sinusoidal Function Context and Data Modeling	**3.7.A** Construct sinusoidal function models of periodic phenomena.
	The Tangent Function	**3.8.A** Construct representations of the tangent function using the unit circle.
		3.8.B Describe key characteristics of the tangent function.
		3.8.C Describe additive and multiplicative transformations involving the tangent function.
	Inverse Trigonometric Functions	**3.9.A** Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain
	Trigonometric Equations	**3.10.A** Solve equations involving trigonometric functions

Domain	Cluster	Standard
	The Secant, Cosecant, and Cotangent Functions	**3.11.A** Identify key characteristics of functions that involve quotients of the sine and cosine functions.
	Equivalent Representations of Trigonometric Functions	**3.12.A** Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.
		3.12.B Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.
		3.12.C Solve equations using equivalent analytic representations of trigonometric functions.