



Dublin City Schools K-12 Mathematics Education Grade Course of Study Vision

Vision

In Dublin City Schools, we believe in an education where mathematics is not only a fundamental part of every student's education, but a source of inspiration, curiosity, and real-world problem solving skills that can prepare all students for success in an increasingly complex and interconnected world.

We believe in developing mathematicians by providing students high quality experiences designed to:

- View mathematics as a powerful tool for understanding and improving the world around them.
- Refine critical thinking skills in order to analyze problems, assess information, and make informed decisions.
- Think flexibly and creatively with numbers in order to connect conceptual understanding with concrete situations.
- Develop skills such as perseverance, grit, and tenacity in order to embrace challenges and learn from mistakes.
- Cultivate a mathematical mindset that emphasizes the balance of fluency (efficiency, flexibility, accuracy) alongside conceptual understanding.
- Reflect upon their thinking, and the thinking of others, to examine the reasonableness of strategies and solutions while gaining self-efficacy as a lifelong learner.

Instructional Agreements for Mathematical Learning within the Dublin City Schools

- Content standards will be learned in partnership with the Standards for Mathematical Practice in order to empower students to reason abstractly and quantitatively, make sense of problems and persevere in solving them, use appropriate tools strategically, attend to precision, think critically, communicate clearly, and collaborate effectively.
- Teachers will provide opportunities for students to share ideas, communicate their mathematical thinking, and learn from each other's perspectives, fostering a supportive learning community where students can take risks with confidence.
- A variety of instructional techniques and tools will be utilized to support students to create and connect different mathematical representations.
- Teachers will design experiences for students to recognize the real-world relevance of math by exploring and applying mathematical principles to solve authentic problems.
- Teachers will provide students with actionable feedback to support their growth as mathematicians.
- Differentiated instruction will ensure a challenging yet supportive learning environment that supports all students in achieving their full potential.

Together, we will create mathematically empowered learners prepared to navigate the challenges and opportunities ahead with a love of learning, a sense of curiosity, and the skills necessary for success.



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K–12 Course Mathematical Practices:

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about



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data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.



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6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



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Algebra II & Honors Algebra II Course Goals:

Building on their work with linear, quadratic, and exponential functions from Algebra I, mathematicians in this course extend their repertoire of functions to include polynomial, rational, radical, and logarithmic functions and transformations of each of these. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using logarithms. Students extend their knowledge of trigonometry and its applications beyond right triangles. Students will be introduced to matrices and matrix operations as well as applications such as solving systems to equations.

Course Content Standards:

Domain	Cluster	Standard
THE COMPLEX NUMBER SYSTEM	Perform arithmetic operations with complex numbers.	N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
		N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
	Use complex numbers in polynomial identities and equations.	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
		N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
THE REAL NUMBER SYSTEM	Extend the properties of exponents to rational exponents.	N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
		N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</i>
		N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.



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	Use properties of rational and irrational numbers.	N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS	Perform arithmetic operations on polynomials.	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic.
	Understand the relationship between zeros and factors of polynomials.	A.APR.2 Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.
	Rewrite rational expressions	A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
		A.APR.7(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
CREATING EQUATIONS	Create equations that describe numbers or relationships	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.</i> c. Extend to include more complicated function situations with the option to solve with technology.
		A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.



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		<p>c. Extend to include more complicated function situations with the option to graph with technology.</p> <p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</p> <p>a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.</p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</p> <p>d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations.</p>
REASON WITH EQUATIONS AND INEQUALITIES	Understand solving equations as a process of reasoning and explain the reasoning.	A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	Represent and solve equations and inequalities graphically	A.REI.11 Explain why the x -coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
SEEING STRUCTURE IN EXPRESSIONS	Interpret the structure of expressions.	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity.</p>
		A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the " $x - 5$ " is common to both expressions being added, so it simplifies to $(3x +$



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		<p>$2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p>
	Write expressions in equivalent forms to solve problems.	<p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. <i>For example, $8t$ can be written as 2^{3t}.</i></p>
BUILDING FUNCTIONS	Build a function that models a relationship between two quantities.	<p>F.BF.1 Write a function that describes a relationship between two quantities.</p> <p>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>
	Build new functions from existing functions.	<p>F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>F.BF.4 Find inverse functions.</p> <p>b. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>c. (+) Verify by composition that one function is the inverse of another.</p> <p>d. (+) Find the inverse of a function algebraically, given that the function has an inverse.</p>
INTERPRETING FUNCTIONS	Interpret functions that arise in applications in terms of the context.	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p>



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		<p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>c. Emphasize the selection of a type of function for a model based on behavior of data and context.</p>
		<p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>
	<p>Analyze functions using different representations.</p>	<p>F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.</p> <p>c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior.</p> <p>f. Graph exponential functions, indicating intercepts and end behavior.</p> <p>g. (+) Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior.</p>
		<p>F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change</i></p>



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		<p><i>in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay.</i></p> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
LINEAR, QUADRATIC, AND EXPONENTIAL MODELS	Construct and compare linear, quadratic, and exponential models, and solve problems.	F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY	Define trigonometric ratios, and solve problems involving right triangles.	G.SRT.8 Solve problems involving right triangles. b. (+) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
	Apply trigonometry to general triangles.	G.SRT.9 (+) Derive the formula $A = 1/2ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10 (+) Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
		G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA	Summarize, represent, and interpret data on two categorical and quantitative variables.	S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by discussing residuals.
VECTOR AND MATRIX QUANTITIES	Perform operations on matrices, and use matrices in applications.	N.VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
		N.VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.



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		N.VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
		N.VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication of square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
		N.VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
REASONING WITH EQUATIONS AND INEQUALITIES	Solve systems of equations.	A.REI.6 Solve systems of linear equations algebraically. b. Extend to include solving systems of linear equations in three variables algebraically.
		A.REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
		A.REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).