AP Calculus AB Summer Work   **Lucy Beckham HS**   **Name:**_____

You should complete all of your work neatly on a separate sheet of paper. Make sure to label each problem and box in your answers.

I.   Calculator Basics

1.   Find the largest and smallest values of each of the following functions on their given x intervals.

a) $f(x) = 2^x + x^2$ on $[-4, 1]$

b) $y = (\cos x)^x$ on $[-1.5, 4.75]$

c) $f(x) = \frac{3x^2 + x - 5}{x^2 + 1}$ on $[2, 4]$

d) $g(x) = \frac{1}{\sqrt{4 - x^2}}$ on $(-2, 2)$

2.   Graph $y = \frac{x^2 - 9}{x - 3}$ on your calculator.

a)   Explain why this graph appears to be a linear function, rather than a curve with a vertical asymptote at $x = 3$.

b)   Explain what is happening to the function at $x = 3$.

c)   IF you had to fill in a y-value when $x = 3$, what would it be?

3.   An open box is to be made from cutting squares of side length s from each corner of a piece of cardboard 25" by 30".

a)   Write an expression for the volume, V, of the box in terms of s.

b)   Graph $V(s)$ using your calculator. Identify a reasonable domain and range for this graph in the context of this problem.

c)   Find the value of s that will give the maximum volume of the box.

d)   What value(s) of s will give a volume of 1225 cubic inches?

II.   Cartesian Plane Basics

4.   Determine the slope, length and midpoint of the line segment with endpoints $(1, -2)$ and $(3, 2)$.

5.   For what value(s) of k is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what value(s) of k are the two lines perpendicular?

6.   Plot the line $2x - 5y = 10$ indicating your x and y-intercepts. Be sure to label your axes to indicate your scale.

7.   Find the distance from point P $(1,2)$ to the line $x + 2y = 3$.

8. Suppose you want to rent a jet for one day. Swissaire rents jets with a pilot for $2000 per day plus $1.75 per mile and Air France rents jets with a pilot for $1500 per day plus $2.00 per mile.

   a) For each company, write an equation for the cost as a function of distance traveled.

   b) If cost were the only consideration, which company would you choose.

9. Find the line that passes through the point $(-1, 3)$ and the point of interecetion of the lines $x + 3y = 1$ and $2x - y = -5$. Write your answer in point slope from $(y - y_1 = m(x - x_1))$.

III. Basic Functions and Transformations

10. You should be very familiar with the following functions and be able to envision them in your mind without having a graph.

   a. Constant Function: $f(x) = C$, where C is a contestant

   b. Linear Functions: $f(x) = mx + b$

   c. Quadratic Function: $f(x) = ax^2 + bx + c$

   d. Polynomial Function: $f(x) = ax^n + bx^{n-1} + \cdots + mx + n$

   e. Rational Function: $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials

   f. Radical Functions: $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{x}$

   g. Exponential Function: $f(x) = e^x$, Know the domain and range.

   h. Natural Log Function: $f(x) = \ln x$, Know the domain and range.

   i. Absolute Value Function: $f(x) = |x|$

11. Using transformations, describe how the graph of each function can be obtained from the graph of

   $$f(x) = \sqrt{x} \qquad g(x) = \frac{1}{x} \qquad h(x) = |x| \qquad k(x) = x^3 \qquad l(x) = \ln x \qquad \text{OR } m(x) = e^x$$

   a) $y = .5(x - 4)^3 + 2$

   b) $y = |x + 2| - 1$

   c) $y = -\sqrt{3 - x} + 4$

   d) $y = 2\ln(x + 3) - 3$

   e) $y = e^{x-1} - 3$

12. Remember that for even functions, $f(-x) = f(x)$ and that for odd functions, $f(-x) = -f(x)$, show whether the following functions are even, odd or neither.

   a) $y = x^4$

   b) $y = x - x^4$

   c) $y = \frac{1}{x^2 - 4}$

   d) $y = -2x^3 + 4x$

IV. Trig Review

You should know the Unit Circle, basic shapes of the sine, cosine, and tangent functions, their domains and ranges and the following trig identities:

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad\qquad\qquad \sin 2x = 2 \sin x \cos x$$

13. **You MUST be able to evaluate he following WITHOUT a calculator**. We always use radians, not degrees.

    a) $\sin \pi$

    b) $\cos \dfrac{3\pi}{2}$

    c) $\tan \dfrac{5\pi}{4}$

    d) $\sin \dfrac{4\pi}{3}$

    e) $\cos \dfrac{7\pi}{4}$

    f) $\sin \dfrac{2\pi}{3}$

    g) $\tan \dfrac{5\pi}{6}$

    h) $\sin \dfrac{11\pi}{6}$

14. Convert the following to radians:

    a) $510°$

    b) $120°$

    c) $135°$

    d) $-210°$

15. Identify which of the six trig functions are a) even, b) odd, c) neither.

16. Which of the following have the same graph?

    a) $y = \sin x$

    b) $y = -\sin x$

    c) $y = \cos x$

    d) $y = -\sin x$

    e) $y = -\cos x$

17. Consider the function $y = \sqrt{\dfrac{1 + \cos 2x}{2}}$

    a) Can x take on any real value?

    b) How large can $\cos 2x$ become? How small?

    c) How large can $\dfrac{1 + \cos 2x}{2}$ become? How small?

    d) What are the domain and range of the original function?

18. Solve ach of the following on the interval $[0, 2\pi)$.

    a) $2 \sin x - \sqrt{3} = 0$

    b) $\cos^2 x = 2 \sin x - 2$

19. Solve each of the following on the interval $[0, . 2\pi)$.

    a) $\sin^2 x < 2 \cos x + 1$

    b) $2 \sin x \cos x + \sqrt{2} \cos x < 0$

V. Exponents, Radicals and Factoring

20. Simplify each of the following expressions.

a) $\left(\frac{2}{3}x^{-3}\right)(15x^7)$

b) $x^3(2yz^2)^3$

c) $\frac{(3x^3)(4x^5)}{(x^2)^3}$

d) $\frac{(2y^4)(3y^2)^2}{(y^3)^4}$

e) $(3a^{-2}b^3)^{-3}$

21. Simplify the following by removing all possible factors from the radical.

a) $\sqrt{9z^8b}$

b) $\sqrt{24a^4b^8}$

c) $\sqrt{\frac{75}{a^6}}$

22. Factor each of the following completely.

a) $6x^3y^2 + 15x^2y^5 - 30x^7y^4z$

b) $16y^2 - 9$

c) $4x^{16} - 9y^6$

d) $6x^2 + 7x - 20$

e) $3x^2 - 5x + 2$

f) $x^3 - x^2 + 3x - 3$

g) $3a^3 + 3a^2 - 27a - 27$

h) $x^2 + 4x + 4 - 9y^2$