Honors Pre-Calculus Summer Work 2024-2025 **Name:** ______________________________

This packet is to help you review various topics that are considered to be prerequisite knowledge upon entering Honors Pre-Calculus. This packet will be due within the first week of the semester and will be graded as multiple daily grades. An assessment will also be within the first week from the review material on this packet.

- **All work must be completed NEATLY and organized!**
- **Show all work for credit! Use additional sheets of paper if needed.**
- **Box your answers!**
- **NO CALCULATOR UNLESS OTHERWISE STATED!**

I. Equations of Lines

1. Slope-Intercept: $y = mx + b$ where $m = \frac{\Delta y}{\Delta x}$

(Note: Δ means "change in")

2. Point-slope: $y - y_1 = m(x - x_1)$

3. Standard Form: $Ax + By + C = 0$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Directions: State all linear equations in Slope-Intercept Form unless otherwise stated.

1. Write the equation of the line parallel to the line $4x - 6y = -1$ containing the x-intercept of $3x - 2y = 12$.

2. Write an equation of the line with x-intercept of 3 and y-intercept of 5.

3. Write the equation of the line through $(2, -4)$ and perpendicular to $x - 2y = 7$.

4. Find the value of "a" if a line containing the point $(a, -3a)$ has a y-intercept of 7 and a slope of $-\frac{2}{3}$.

II. Quadratics/Polynomials

A. Factoring – Strategies to try when factoring:

- Look for a common factor - Difference of two squares: $a^2 - b^2 = (a-b)(a+b)$ - Perfect Square Trinomials: $a^2 \pm 2ab + b^2 = (a \pm b)^2$	- Guess and Check - Grouping - Sum/Difference of Cubes $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

1. Factor completely each of the following:

a. $4x^2 + 27x + 35$ b. $-28y^2 + 7t^2$ c. $x^3 - 2x^2 - 9x + 18$

d. $4x^3 + 8x^2 - 5x - 10$ e. $8x^3 - 27$ f. $x^4 - 16$ g. $x^2 + 9$

B. Solving by Factoring – since the following are equations, we can go a step further and solve for the variable by factoring or using the quadratic formula (Reminder: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

1. Solve each of the following:

a. $-3x^2 - 5x + 12 = 0$ b. $4y - 2 = y^2$ c. $225 - b^2 = 0$ d. $x^2 = x$

e. $4x^2 = 25$ f. $(x-3)^2 = 10$ g. $3x^2 + 2x - 1 = 0$ h. $3x^3 + 3x^2 - 27x - 27 = 0$

2. Graphing – to graph a quadratic equation in standard form, $y = ax^2 + bx + c$, find the important points of the graph and plot.

- **Y-intercept:** If a point is the y-intercept of the curve, then that is the point at which the graph crosses the y-axis. Since this point is on the y-axis, then the x-coordinate must be 0. Substitute zero in for x and solve for y.

- **Vertex:** x-coordinate of the vertex (axis of symmetry): $x = -\frac{b}{2a}$

y-coordinate of the vertex: substitute the value found for the x-coordinate into the original equation and solve for y

- **X-Intercepts:** If a point is an x-intercept of the curve, then it is a point at which the graph crosses the x-axis. Since these points are on the x-axis, then the y-coordinates must be 0. Substitute zero in for y and solve for x by factoring or using the quadratic formula.

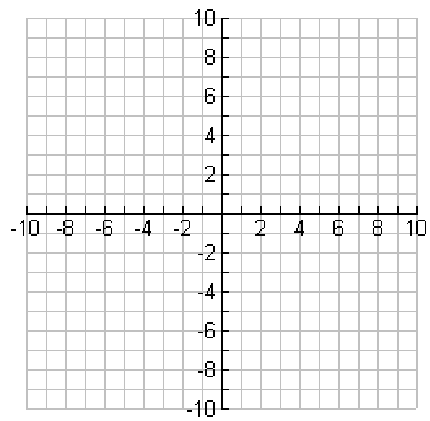
No Calculator, but you should also be able to graph with the use of your graphing calculator

1. Given $y = -3x^2 + x + 2$, find and graph.

a. y-intercept: ______________

b. Vertex: ______________

c. x-intercepts: ______________



III. Systems

Substitution or Elimination can be used to solve systems of equations.

- If there is a solution to the system, then the equations are representing intersecting lines.

- If both variables cancel out and an equation is formed that is never true, then there is no solution and the lines never intersect. Lines that never intersect are parallel lines.

- If both variables cancel out and an equation is formed that is always true, then there are infinitely many solution and the equations must represent the same line.

Directions – Solve each of the following.

- ***Explain what the solution tells us about the lines represented by the equations.***
- ***No calculator, but you need to be able to solve with the use of a calculator as well.***

1. $\begin{cases} 3x - 4y = 2 \\ -x + 3y = 1 \end{cases}$

2. $\begin{cases} -x + y = 3 \\ 2x - 2y = -6 \end{cases}$

Solution: ___________________

Explanation:

Solution: ___________________

Explanation:

IV. Exponents

Directions – Simplify using only positive exponents and no calculator!

Properties:	$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$
$a^0 = 1, a \neq 0$	$a^{-n} = \frac{1}{a^n}$	$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$\frac{a^m}{a^n} = a^{m-n}$
	$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$	$(\frac{a}{b})^{-m} = (\frac{b}{a})^m = \frac{b^m}{a^m}$	$\frac{a^n b^{-n}}{c^{-m}} = \frac{a^n c^m}{b^n}$

1. $(\frac{81}{64})^{-\frac{1}{2}}$

2. $(27^{-2})^{-\frac{1}{3}}$

3. $\frac{(3x^2)^{-\frac{1}{2}}}{6x^{-3}}$

4. a. -2^4 b. $(-2)^4$

5. $\frac{3^{-5} \cdot 3^{-10}}{3^2}$

6. $(4^{-1} + 2^{-1})^2$

7. a. $(13y)^{-1}$ b. $13y^{-1}$

8. $8^{-1} \times 8^0$

V. Rational Expressions

1. Simplify the following expressions. (Hint: Factor and cancel)

a. $\frac{x^3-x^2-6x}{x^2-9}$

b. $\frac{y^3-2y^2+y-2}{2-a}$

c. $\frac{z^3}{z^2-2z+4}$

d. $\frac{4ab^2}{ab^2+a}$

2. Perform the following operations and write the result in reduced form.

a. $\frac{a-1}{a} * \frac{a^2}{a^2-1}$

b. $\frac{x^2-9}{x+2} * \frac{x^2+4x+4}{x^2-x-6}$

c. $\frac{x+4}{x^2} \div \frac{x^2-16}{x}$

3. Perform the following operations and write the result in reduced form.

a. $\frac{2}{2x+1} - \frac{5}{(2x+1)^2}$

b. $3 - \frac{4}{x+2}$

c. $\frac{1}{x^2-3x+2} + \frac{2}{x^2-4}$

d. $\frac{3}{x-2} + \frac{5}{2-x}$

e. $\frac{y}{y^2+4y+4} + \frac{3}{y^2+y-2}$

4. Perform the following operations and write the results in reduced form.

a. $\dfrac{1-\frac{y}{3}}{3-y}$ b. $\dfrac{\frac{1}{y+1}-\frac{1}{y}}{\frac{1}{y+1}}$ c. $\dfrac{\frac{x}{x-1}+1}{\frac{x+2}{x}}$

VI. Simplifying Radicals

1. Simplify the following without a calculator.

a. $\sqrt{9a^8b^3}$ b. $\sqrt{\frac{75}{a^6}}$ c. $\sqrt[3]{128}$ d. $\sqrt[3]{24a^4b^8}$ e. $\frac{5}{\sqrt{7}}$ f. $\frac{3y}{4-\sqrt{7}}$

2. Solve the following. Check for extraneous solutions.

a. $\sqrt{7-5x}=8$ b. $\sqrt{2y+15}=\sqrt{4y+1}$ c. $\sqrt[3]{3x+4}+2=0$ d. $(x-3)^{\frac{2}{3}}=4$

VII. Exponentials and Logarithms

$\log_b a = x$ if and only if $b^x = a$, where $b > 0$, but $b \neq 1$ and $a > 0$.

Properties of logarithms:

$\ln(a * b) = \ln a + \ln b$ $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$ $\ln a^b = b * \ln a$

1. Solve the following equations for x.

a. $3\log_2 x = 12$ b. $\log_5 125 = x$ c. $3 + 4\log_x 4 = 5$

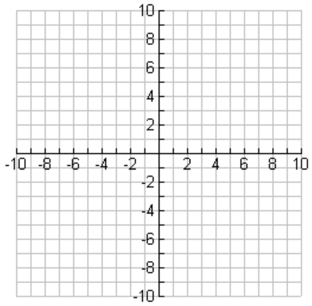
d. $\log_3(x+3) + \log_3 x = \log_3 2$ e. $2\log x - \log(2x-1) = 0$

f. $3(4)^x = 96$ g. $\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$ h. $e^{x^2+5x} = e^{-6}$ i. $3(5)^{-\frac{x}{4}} = 15$

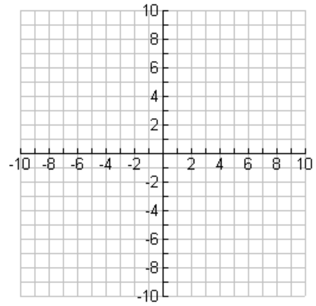
VIII. Parent Functions and Quick Graphs

1. Sketch the following parent functions.

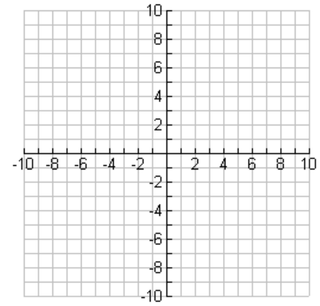
a. $y = x^2$



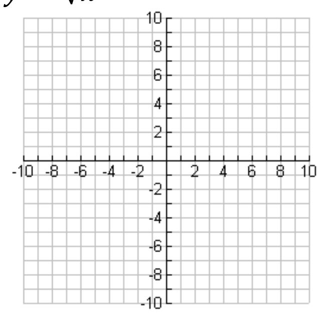
b. $y = x^3$



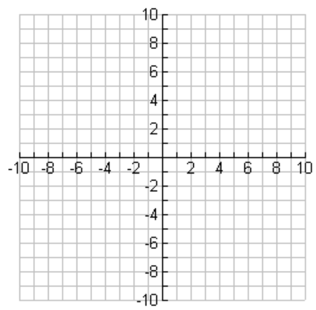
c. $y = \sqrt{x}$



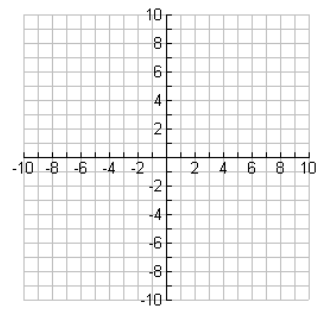
d. $y = \sqrt[3]{x}$



e. $y = 2^x$

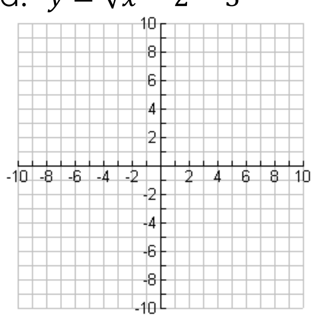


f. $y = \log_2 x$

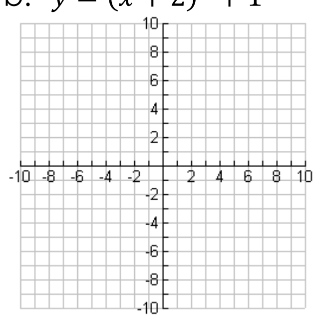


2. Sketch the following using transformations.

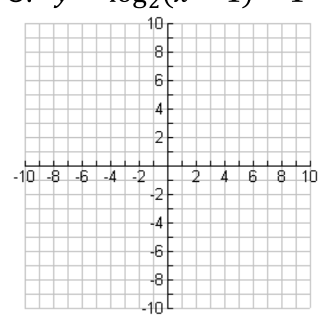
a. $y = \sqrt{x-2} - 3$



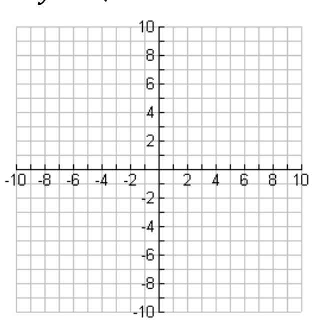
b. $y = (x+2)^2 + 1$



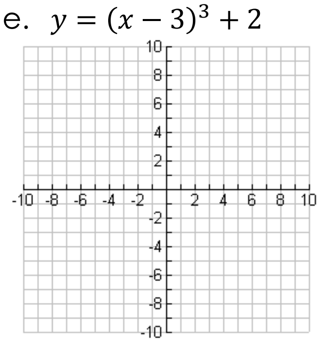
c. $y = \log_2(x-1) - 1$



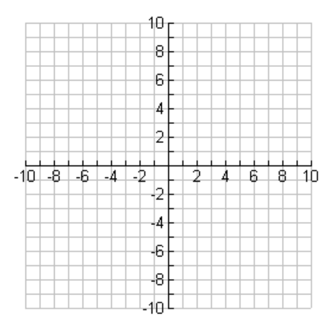
d. $y = \sqrt[3]{x+1} - 2$



e. $y = (x-3)^3 + 2$



f. $y = 2^{x+2} - 2$



IX. Real Domain

Find what does not work. Everything else is the domain.

a) Variable on the bottom	b) Even roots on top or bottom	c) Logs on top or bottom
Ex. $f(x) = \frac{2}{x-3}$	Ex. $f(x) = \sqrt{x}$	Ex. $f(x) = \log(x+2)$
Domain: $(-\infty, 3) \cup (3, \infty)$	Domain: $[0, \infty)$	Domain: $(-2, \infty)$

1. Find the domain of the following functions.

a. $f(x) = x^2 + 4$ b. $g(x) = \frac{x}{x^2-5x}$ c. $f(x) = \sqrt{4-x}$ d. $f(x) = \sqrt{x^2+1}$ e. $\log_2(2x-3)$