Second Grade Mathematics
Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the "how" and the "why" of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase "procedural skill and fluency" is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the "why" behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

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Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:
The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:
Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

Operations and Algebraic Thinking (OA)

Standard 2.OA.A.1 Cluster Heading: A. Represent and solve problems involving additional and subtraction.

Add and subtract within 100 to solve one- and two-step contextual problems, with unknowns in all positions, involving situations of *add to*, *take from*, *put together/take apart*, and *compare*. Use objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Table 1 - Addition and Subtraction Situations)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract within 20 to solve contextual problems, involving any of the problem types, including start unknown situations.	Given a solved one-step addition or subtraction contextual problem, relate the solution to the context provided. Given a solved two-step addition or subtraction contextual problem, relate the solution to the context provided. Match an equation with a symbol for the unknown number to a contextual problem.	Add and subtract within 100 to solve one-step contextual problems, involving all situations, using mathematical drawings, diagrams, and equations with a symbol for the unknown number. Add and subtract within 100 to solve two-step contextual problems, involving all situations, using mathematical drawings, diagrams, and equations with a symbol for the unknown number.	Add and subtract within 100 to solve one-step contextual problems, involving all situations, using mathematical drawings, diagrams, and equations with a symbol for the unknown number. Explain the reasonableness of the chosen solutions. Add and subtract within 100 to solve two-step contextual problems, involving all situations, using mathematical drawings, diagrams, and equations with a symbol for the unknown number. Explain the reasonableness of the chosen solutions.

TN Department of Education

Instructional Focus Statements

Level 3:

In grade 1, students developed an understanding of adding and subtracting within 20 through interacting with a wide variety of problem-solving situations. Students also began adding a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100) in standard 1.NBT.C.5. In grade 2, there are three significant differences in how students interact with contextual problems. The first is extending the range of numbers students use for addition and subtraction from within 20 to within 100 encompassing a much larger range of sums and differences. The second is that students are expected to be exposed to all types of common addition and subtraction situations. See Table 1 - Addition and Subtraction Situations for examples. Finally, the standard explicitly calls out two-step problems for the first time.

As students begin to work with a larger range of numbers and more complex problem-solving situations, they should continue to make use of models, drawings, and multiple representations in order to demonstrate their understanding. Students should also begin representing problems with equations using symbols for unknown number. To solve, students may employ strategies that involve counters, linking cubes, ten frames, base ten blocks, part-part-whole models, number lines, bar models, etc. In working with larger numbers, students should begin to transition to more efficient representations of problem situations, looking for and realizing that some representations are easier to use with larger numbers. For example, students should understand that using a bar model, number line, or the numeral itself may be more efficient when working with larger numbers rather than drawing out the number of objects.

In transitioning all students to working with two-step contextual problems, instruction should initially focus on problems involving smaller, familiar numbers and operations allowing students to focus on the conceptual understanding of multiple operations within the problem as opposed to focusing on computation with less familiar numbers. Additionally, it is easier for students to begin with problems that call for the same operation within the problem and then move on to working with two-step problems that involve using both addition and subtraction. It is important to call out that students should continue to use manipulatives, multiple strategies, and written equations when solving two-step contextual problems. To demonstrate their understanding, they should be able to explain the connections between the visual representation and the equation(s) that represents the problem. Additionally, students should be encouraged to use multiple strategies and make connections between each strategy. For example, students may write individual equations for each step in a two-step problem or write both steps in one equation. This is a good opportunity for students to compare their work to others and explain why both are correct or in some cases incorrect, explain the connection between the two strategies, and explain the connection to equations using symbols to represent unknown numbers.

Teaching key words to associate with addition and subtraction should not be an instructional focus. Instruction should focus on developing an understanding of what operation is needed to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Level 4:

As students deepen their understanding of operations with addition and subtraction with a larger range of numbers and two-step problems, they should not only be able to represent these problems with mathematical drawings, diagrams, and equations with a symbol for the unknown number, but also able to explain their thinking and the reasonableness of their solutions. Students should make connections between the visual representations as well as the problem represented as an equation. As an extension, students should be able to create their own two-step contextual problem and explain the solution using visual presentations, equations, and precise mathematical vocabulary.

Operations and Algebraic Thinking (OA)

Standard 2.OA.B.2 Cluster Heading: B. Add and subtract within 30.

Fluently add and subtract within 30 using mental strategies. By the end of 2nd grade, know all sums of two one-digit numbers and related subtraction facts.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Fluently add and subtract within 20 using mental strategies. Know all sums up to 10.	Use mathematical drawings to add and subtract for sums and differences between 20-30. Know all sums up to 10 and their related subtraction facts.	Fluently add and subtract within 30 using mental strategies. Know all sums of two one-digit numbers and related subtraction facts.	Fluently add and subtract within 30 using mental strategies and explain why the chosen strategies work.

Instructional Focus Statements

Level 3:

Fluency is the ability to apply procedures accurately, efficiently, and flexibly. By the end of grade 1, students fluently added and subtracted within 20 using mental strategies and knew from memory all sums up to 10. By the end of grade 2, students should extend this understanding to fluently add and subtract within 30 using mental strategies.

Building fluency that is based on mental strategies is a process. Students begin by developing a conceptual understanding of the operations of addition and subtraction through direct modeling. The next natural progression is for students to work with student-driven, invented strategies that are deeply rooted in place value and number sense. Students began working with invented strategies with adding and subtracting within 20 in standard 1.OA.C.5. Before they reach fluency with mental strategies, students must be given the opportunity to interact with direct modeling and/or student-invented strategies in order to have the mathematical foundation needed to move along the continuum towards reaching fluency with mental strategies. This process takes time. Students should be exposed to various strategies and choose the one that is most efficient and makes the most sense to them, ultimately utilizing their strategies for mental computation as they progress in their learning. It is important to note that timed tests do not build fluency in students. Exposure to flexible thinking, explaining their thoughts, and appropriate scaffolding over time builds fluency.

As students become more fluent with adding and subtracting numbers within 30, they should start to produce answers without recording their thinking and explaining their mental thought process. Students should explain or defend their answer, such as decomposing and composing the numbers, properties of operations, place value, or describing mental images used to obtain the answer. Additionally, students should have many opportunities to practice, explain their thinking, compare and make connections with multiple strategies. Number Talks, written explanations, and selecting the strategy that makes the most sense to them will allow students to develop a conceptual understanding to become fluent with adding and subtracting within 30 and know from memory all sums of two one-digit numbers and related subtraction facts.

One final note, algorithms for addition and subtraction are not introduced within the standards until grade 3.

Level 4:

As students develop a wider range of mental strategies that they are comfortable with and can explain, they should be able to explain the connections that exist between multiple strategies. It is imperative that as students transition to using mental strategies, they are asked questions that press for the underlying mathematics and that they can provide an explanation using precise mathematical vocabulary.

Operations and Algebraic Thinking (OA)

Standard 2.OA.C.3 Cluster Heading: C. Work with equal groups of objects to gain foundations for multiplication.

Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by 2s. Write an equation to express an even number as a sum of two equal addends.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Skip count by twos. Split a group of even numbered objects into two equal subgroups counting by ones. Choose representations of adding within 20 using doubles.	Determine whether a group of objects (up to 20) has an odd or even number of members by breaking the group of objects into two subgroups with the same number in each subgroup (even) or if it can be broken into two equal subgroups with a leftover object (odd). Give examples of adding within 20 using the strategy of doubles.	Determine whether a group of objects (up to 20) has an odd or even number of members by pairing objects or counting them by twos. Write an equation to express an even number as a sum of two equal addends.	Generalize whether a group of objects or given numeral is an odd or even number based on patterns and justify their thinking. Create a group of objects that is even and explain why in more than one way. Create a group of objects that is odd and explain why in more than one way.

Instructional Focus Statements

Level 3:

Students develop an understanding of odd and even numbers by using concrete materials to explore and discover the unique properties held by numbers classified as even and numbers classified as odd.

Instruction should focus first on students determining if a number can be broken into two equal parts. For example, students may use counters to represent the numbers four, seven, and ten. They find that the counter representations of four and ten can be shared equally in two groups while the group of seven counters cannot be evenly split into two equal groups (i.e., one group always has at least 1 more counter than the other). Students then

begin looking for patterns as they represent additional numbers in order to self-define what it means for a number to be even and what it means for a number to be odd.

Instruction should also focus on students determining if a number is even or odd by pairing objects and looking to see if there are leftover items. For example, a student may be given 18 counters with which they make nine pairs of counters. A student may also be given 19 counters in order to discover that they can make nine pairs of counters with one leftover. This provides another avenue for students to develop an understanding of what it means for a number to be even or odd.

As students move along a continuum of learning from concrete to abstract, they can be challenged to make connections relating the concept of odd and even to the addition facts. Students are able to show that an even number can be expressed by a doubles fact (e.g., 9 + 9 = 18). Students may even make the generalization that when two like addends are combined, the sum is always even because the addends represent two equal groups. Odd numbers can be expressed using a doubles + 1 facts (e.g., 19 = 9 + 9 + 1 or 19 = 9 + 10). Students are also able to show that an even number can be expressed by a sum of twos. For example, 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 18. Likewise, an odd is a sum of repeatedly added twos plus one.

The categorization of numbers as odd or even is an important structure in our number system. All too often children are simply told that the even numbers are those that end in 0, 2, 4, 6, or 8 and odd numbers are those that in 1, 3, 5, 7, or 9. Although this is true, and students may discover this pattern and even have discussions about it, it is only an attribute of even and odd numbers rather than a definition that explains what “even” or “not even” (i.e., odd) really means.

Level 4:

Students extend their understanding of even and odd numbers by generalizing whether a group of objects or given numeral is an odd or even number based on patterns. Students make a claim if a number is even or odd and justify their reasoning with representations and/or verbal reasoning. Some students may generalize by the next number in a counting set, counting by twos, pairing, recognizing the value of the ones digit, addition and/or subtraction strategies, etc. Additionally, students can be challenged to create a group of objects with a given condition that it is either even or odd and provide multiple justifications for why the group they created can be classified as either even or odd. It is important to note that students’ generalizations should be based on student determined rules, not provided rules.

Operations and Algebraic Thinking (OA)

Standard 2.OA.C.4 Cluster Heading: C. Work with equal groups of objects to gain foundations for multiplication.

Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. For example*, a 3 by 4 array can be expressed as 3 + 3 + 3 + 3 = 12 or 4 + 4 + 4 = 12.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Construct an array when given up to 25 objects.	Count to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to represent the sum.	Use repeated addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	Given a rectangular array of objects with up to 5 rows and up to 5 columns, write an equation to express the total as a sum of equal addends; rearrange the objects to make a new rectangular array; write a new equation to express the total as a sum of equal addends; and provide mathematical justification for why the two equations are equivalent.

Instructional Focus Statements

Level 3:

Students should extend their understanding of addition with equal groups to discover the benefit of intentionally using repeated addition in order to determine the total number of objects in a rectangular array. This understanding is a critical building block for multiplication in grade 3.

As students initially engage with this standard, they should simultaneously develop academic vocabulary around the concept of an array while discovering efficient methods to determine the total number of objects in arrays of various sizes. Real-world examples such as a tray of doughnuts or an egg carton can be particularly helpful for students as they are initially visualizing and working with arrays. Students may count by ones or skip count by either rows or columns to determine the total number of objects. Students should be able to construct a viable argument justifying to their classmates (MP 3) how and

why their counting method is efficient. One mathematical understanding that all students should discover is that skip counting by rows and skip counting by columns for the same array will both lead to the same total count of objects in the array. This sets students up with a strong foundational understanding for what will become the commutative property of multiplication in subsequent grades.

Once students are able to skip count to find the total number of objects in arrays, they should be challenged to write an equation to represent their thinking and provide justification for how the equation matches their skip counting patterns. Ultimately, students should be able to represent the total number of objects in any given array with an equation that expresses the total as a sum of equal addends.

Instruction should integrate this standard and standard 2.G.A.2 where students partition a rectangle into rows and columns of same-sized squares and find the total number of squares. The geometry standard provides a concrete learning experience for students as they physically build arrays. Students can then find the total number of squares using the same methods as have been used with standard 2.OA.C.4. With both partitioned rectangles and arrays of objects, it is imperative that students express their thinking as a sum of repeated addends.

Level 4:

Students at this level should be challenged to discover deeper connections between rectangular arrays with the same number of objects but different numbers of rows and columns. For example, students could be challenged to find as many arrays that contain twelve objects as possible (i.e., 1 by 12, 12 by 1, 2 by 6, 6 by 2, 3 by 4, and 4 by 3), write equations with repeated addends that would represent each, and then mathematically justify why all of these yield a count of twelve objects. As students create hypotheses and test their thinking, they will interact with arrays that are larger than five rows by five columns (e.g., in the example provided students interact with a row of twelve). Extending the range allows students to make connections over all combinations. This level of flexible thinking will support students as they develop multiplication strategies in grade 3.

Operations and Algebraic Thinking (OA)

Standard 2.OA.D.5 Cluster Heading: D. Solve problems involving addition and subtraction and identify and explain patterns in arithmetic.
Identify arithmetic patterns in an addition or hundreds chart and explain them using properties of operations. *For example, analyze patterns in the addition chart and observe an alternating pattern of even and odd numbers (because each time we move to the right one box or down one box, we are adding one more to our sum: (2 + 3) + 1 = 2 + (3 + 1) = 2 + 4 which uses the associative property of addition).* (See Table 3 - Properties of Operations)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Fill in missing elements in a simple pre-identified mathematical pattern. Extend a pre-identified mathematical pattern.	Recognize, describe, extend, and create patterns when skip counting by ones, twos, fives, and tens. Analyze the structure of a repeating pattern by identifying the unit (core) of the pattern.	Identify arithmetic patterns in an addition or hundreds chart. Explain arithmetic patterns in an addition or hundreds chart using properties of operations.	Identify arithmetic patterns in an addition or hundreds chart and explain them using properties of operations and repeated addition/subtraction equations.

Instructional Focus Statements

Level 3:

Patterns are a way in which students connect with the world around them. They have looked at and extended patterns as a way to build spatial recognition skills since they were very young. Instruction for this standard should focus on students using that same level of curiosity to discover a wide variety arithmetic patterns that exist in mathematics and then dig deeper to explain mathematically why the pattern occurs. Arithmetic patterns in grade two should include both addition and subtraction patterns in addition and hundreds charts.

Kindergarten patterning standards required students to recognize, describe, extend, and create patterns and explain a simple rule for a pattern using concrete materials. Students also analyzed the structure of a repeating pattern by identifying the unit (core) of the pattern. In grade 1, students discovered, described, and extended patterns when counting by ones, twos, fives, and tens, and then used those patterns to extend the skip counting sequence. Instruction in grade 2 focuses on patterns that exist in addition tables and hundreds charts. Students analyze patterns in the addition chart and observe an alternating pattern of even and odd numbers (because each time we move to the right one box or down one box to the right, we are adding one more

to our sum: (2 + 3) + 1 = 2 + (3 + 1) = 2 + 4 which uses the associative property of addition). In the hundreds chart, students may make the following discoveries: for each row, left to right, numbers increase by one; for each row, right to left, numbers decrease by one; numbers increase and decrease by ten in each column; on the diagonal (starting at the top) from left to right numbers increase by 11; on the diagonal (starting at the bottom) from right to left numbers decrease by 11; on the diagonal (starting at the top) from right to left numbers increase by 9; on the diagonal (starting at the bottom) from left to right numbers decrease by 9. It is not the expectation for students in grade 2 to use the formal terms for these properties.

Students should discover and make their own sense of arithmetic patterns as opposed to being directed to them. For example, they may discover patterns that exist in pairs of numbers that yield the same sum in an addition chart. It is crucial that students do not stop at the discovery of a pattern, but that they investigate the math of the pattern and communicate that understanding verbally and with equations.

As many mathematical concepts build on patterns and students' ability to identify those patterns, the more experience children have with looking for and making sense of patterns the better prepared that they will be to access concepts requiring patterned thinking in subsequent grades. As students notice and wonder about the patterns they see in the tables and in number sequences, continue to ask questions such as, "How do you know?" and "Does that always work?" to push their thinking about patterns.

Level 4:

As students develop a deeper conceptual understanding of patterns and the mathematics behind them, they should be able to identify increasingly more complex arithmetic patterns that exist in mathematics and explain the patterns using properties of operations. Additionally, they should be challenged to not only explain the relationship between the patterns using properties of operations, but also with repeated addition and subtraction equations. This understanding will prepare students for conceptually understanding multiplication patterns.

Number and Operations in Base Ten (NBT)

Standard 2.NBT.A.1 Cluster Heading: A. Understand place value.

Know that the three digits of a three-digit number represent amounts of hundreds, tens, and ones (*e.g., 706 can be represented in multiple ways as 7 hundreds, 0 tens, and 6 ones; 706 ones; or 70 tens and 6 ones*).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a two-digit number as groups of tens and ones in up to three different ways.	Represent a three-digit number as groups of hundreds, tens, and ones in up to three different ways.	Represent a three-digit number as groups of hundreds, tens, and ones in four or more ways.	Represent a three-digit number as groups of hundreds, tens, and ones in four or more ways and justify how the representations are equivalent.

Instructional Focus Statements

Level 3:

In grade 1, students learned to represent a two-digit number in more than one way and began to develop place value understanding involving tens and ones. In grade 2, students extend their understanding of place value and the base-ten number system to include three-digit numbers. Students develop an understanding of the structure of our base-ten system by building tens out of 10 ones, building hundreds out of 10 tens, and building a thousand out of 10 hundreds. Students understand that every time you have 10 of a particular item, you group it to make the next place value unit. In grade 4, students solidify their understanding that the base-ten number system is tens time greater than each place to the right.

It is important for students to initially work with unitizable manipulatives such as Unifix cubes, straws, craft sticks, beans, and cups so that they physically bundle groups of ten before moving to unitized manipulatives such as base ten blocks. Representations with place value manipulatives (e.g., base ten blocks, place value chips, layered three-digit place value cards) as well as math drawings provide students the opportunity to discover connections between written three-digit numbers and place value of numbers composed of hundreds, tens, and ones. Instruction might further progress to non-proportional manipulatives such as place value chips to represent the numbers. When using all of these manipulatives, it is important for students to develop their understanding across the full continuum from concrete to abstract representations of three-digit numbers.

Through interacting with multiple representations, students develop the conceptual understanding that the digit in the hundreds place represents that many groups of 100. They further extend their understanding to realize that when there is a "0" in a place there are zero bundles of that group. Students also develop flexibility in thinking as they discover non-traditional ways to decompose numbers. They realize that a number may be represented with more than 10 tens (e.g., 428 could be represented by a student as 3 hundreds, 12 tens, and 8 ones). They also develop the understanding that when recording the value of the number in standard form, regrouping must take place in order to write the appropriate digit in the appropriate place (e.g., they cannot write 12 in the tens place and must "fair trade" the ten 10s in for one 100 in order to record the number's value).

Instruction for this standard should be integrated with standard 2.NBT.A.3 where students read and write numbers in various forms. Students should connect the words to the written numeral and be guided to make explicit connections between concrete and pictorial representations of place value.

As students begin to make sense of the base ten number system, they should flexibly compose and decompose three-digit numbers in multiple ways. In working with numbers such as 543, students may use representations such as 5 hundreds, 4 tens, and 3 ones; 4 hundreds 14 tens, and 3 ones; 3 hundreds, 24 tens, and 3 ones, etc. To encourage discovery of multiple representations, pose questions similar to the following:

- "What if you only had 4 hundred flats/chips, could you still make this number?"
- "What if you only had ten rods and ones, could you still make this number?"

Once these variety of representations have been created, they should be shared and discussed in a whole group setting to solidify understanding. This ability to flexibly compose and decompose numbers is foundational for students working with addition and subtraction problems involving regrouping.

Level 4:

At this level, students should have a good understanding of the base ten number system and compose and decompose three-digit numbers in a wide variety of ways. Students should explain not only how they are representing a three-digit number, but also how each of those multiple representations are equivalent to each other.

Number and Operations in Base Ten (NBT)

Standard 2.NBT.A.2 Cluster Heading: A. Understand place value.

Recognize, describe, extend, and create patterns when counting by ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 through counting. *For example: 111, 113, 115, …; 82, 84, 86, …; 370, 380, 390, …; 100, 200, 300, …; etc.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize, describe, extend, and create patterns when counting by ones, twos, fives, and tens and use those patterns to predict the next number in the counting sequence up to 120 through counting.	Recognize, describe, extend, and create patterns when counting by ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 using a representation (e.g., number path, number line, etc.).	Recognize patterns when counting by ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 through counting. Describe patterns when counting by ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 through counting. Extend patterns when counting by ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 through counting. Create patterns when counting by	Identify missing number(s) in a given counting sequence within 1000 when counting by twos, fives, tens, and hundreds starting from any number in the counting sequence, when the pattern (rule) is not provided. Students can describe the pattern (rule) and explain their reasoning.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		ones, twos, fives, tens, and hundreds and use those patterns to predict the next number in the counting sequence up to 1000 through counting.	

Instructional Focus Statements

Level 3:

Students began counting in kindergarten to 100 by ones, fives, and tens and extended that work in grade 1 counting to 120 by ones, twos, and fives starting at any multiple of that number. Grade 2 students expand the range of numbers by counting in a wide variety of ways within 1,000. Additionally, they not only count by ones, but also by twos, fives, tens, and hundreds starting from any number in the counting sequence. Students should be able to recognize and describe the counting sequence pattern (rule) and then explain their reasoning when extending the sequence.

Instruction should provide opportunities for students to describe and extend patterns of numbers when they count forward and backward using models, hundreds charts, number paths, and open number lines. For example, students learn that the ones digit alternates between five and zero when counting by fives when starting the counting sequence with a multiple of five. They should be counting starting not only at five, but from a different number in the counting sequence such as 150 and 123 observing if their predicted pattern holds true. When students count by hundreds, they learn that the hundreds digit is the only digit that changes and that the digit increases by one hundred with each subsequent number in the counting sequence. Students should be challenged to recognize and explain why the tens place changes when counting by ones when crossing over a decade number (e.g., 349 to 350). It is essential that instruction includes discussions of patterns within counting sequences coupled with student-provided, mathematical explanations of why those patterns occur. Particular attention needs to be shown to developing conceptual understanding around crossing over hundreds place values. It can be challenging for some students to mentally regroup to create a hundred. For example, a student counting by 10s from 295 may struggle mentally regrouping the tens to get the third hundred in 305. Before students are able to use mental strategies to solve this type of problem, they should be given the opportunity to interact with direct modeling in order to have the mathematical foundation needed.

Level 4:

Students at this level extend their understanding of patterns by identifying a missing number(s) in a given counting sequence within 1,000 when counting by twos, fives, tens, and hundreds. Students should be able to identify the counting sequence pattern (rule) and then explain their reasoning.

Number and Operations in Base Ten (NBT)

2.NBT.A.3 Cluster Heading: A. Understand place value.

Read and write numbers to 1000 using standard form, word form, and expanded form. *For example, write 234 as 200 + 30 + 4.*

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Read and write numbers in standard form from 0-120. Write numerals from 0-9 in word form.	Read and write numbers to 120 using standard form, word form, and expanded form. Given a number written in either standard form, expanded form, or word form, choose one other representation.	Read and write numbers to 1000 using standard form, word form, and expanded form. *For example, write 234 as 200 + 30 + 4.* Given a number written in either standard form, expanded form, or word form, choose all other representations.	Explain why standard form and expanded form of a number are equivalent. Explain how the digit zero in a number affects standard form, word form, and expanded form of a number.

Instructional Focus Statements

Level 3:

As students develop an understanding that the third digit of a three-digit number represents the amount of hundreds, they can expand the skill set of reading and writing numerals to 120 from grade 1 to reading and writing numbers to 1000 in standard form, word form, and expanded form in grade 2. This is a conceptual standard and Instruction should focus on seeing the various forms of a given number as a cohesive set, discussing the similarities and differences between the forms, as opposed to discrete, isolated representations.

Concrete models, drawings, and place value cards are particularly helpful as students connect numbers in word form, standard form, and expanded form. Specifically, layered three-digit place value cards foster student-made connections between written three-digit numbers and expanded form. A conceptual understanding of expanded form is essential as students utilize place value strategies to add and subtract large numbers in standard 2.NBT.7. Place value concepts provide a convenient way to compose and decompose numbers to facilitate addition and subtraction computations. A common misconception is

that students do not recognize the value of the digits in 134 and write the expanded form as 1 + 3 + 4 instead of 100 + 30 + 4. Also, when working with expanded form, students need to develop the understanding that a 0 represents no groups of tens or ones as opposed to simply representing a placeholder.

One final note, teachers and students should be cognizant that when reading and writing whole numbers, the word “and” should not be used as this will cause confusion when students read decimal numbers in subsequent grades (e.g., 532 is stated and written as “five hundred thirty-two” as opposed to “five hundred and thirty-two”).

Level 4:

Students at this level build on their understanding of place value to articulate the meaning of zero in a number with respect to word form, standard form, and expanded form. For example, given the number 507, a student can represent the number with concrete objects or drawings, read and write the number using standard form, word form and expanded form, and state that zero in this number indicates there will not be tens represented in any of these three forms of the number. It is important to differentiate this from stating that there are no tens as a student could correctly decompose 507 to 50 tens and seven ones.

The symbol 0 helps us distinguish among numbers such as 507, 57, and 570. Without 0, these numbers would have the same nonzero digits in the same order. Zero is often thought of meaning nothing, a notion that can present difficulties in understanding the value of multi-digit numbers. Zero is one of the most important digits in the base-ten system.

Numbers and Operations in Base Ten (NBT)

Standard 2.NBT.A.4 Cluster Heading: A. Understand place value.

Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols >, =, and < to show the relationship.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Given a comparison of two two-digit numbers, accurately explain the comparison based on the meanings of the digits in each place and the symbols (>, =, or <) used.	Compare two three-digit numbers based on the meanings of the digits in each place verbalizing the comparison with comparative language including greater than, more than, less than, fewer than, equal to, or same as.	Compare two three-digit numbers based on the meanings of the digits in each place and use the symbols >, =, and < to show the relationship.	Accurately order a set of three or more three-digit numbers from least to greatest or greatest to least based on the meanings of the digits in each place and uses the symbols > or < to show the relationships and provide justification for the comparison.

Instructional Focus Statements

Level 3:

Students developed an understanding of comparing two two-digit numbers utilizing the correct inequality symbol in grade 1. Also, in working with standards 2.NBT.1 and 2.NBT.3, students are building, drawing, and writing numbers with hundreds, tens, and ones. Standard 2.NBT.A.4 is conceptual in nature and builds on these understandings as learners compare two three-digit numbers using the appropriate inequality symbol.

Since students have an understanding of comparing two-digit numbers from grade 1, instruction on this standard should challenge them to discover what happens when a third digit is included as a part of the numbers they are comparing. Students should be encouraged to utilize concrete representations, place value representations, and number paths as they compare three-digit numbers. Ultimately, students should come to the realization that the number with the most hundreds is greater. If the number of hundreds is the same, the number with more tens is greater. If the number of hundreds and tens is the same, the number with more ones is greater. When students truly understand this concept, it makes sense to compare three-digit numbers by looking

at the hundreds place first. It is important that students come to this realization through discovery and conversation as opposed to a procedural set of rules. It is equally important that students are able to articulate and justify their thinking. Conversations should focus on discussing how numbers are related and how to determine which number is greater or less.

Students should have ample experiences communicating their comparisons in words before using symbols. Students were introduced to the symbols greater than (>), less than (<) and equal to (=) in first grade and continue to use them in grade 2 to compare numbers within 1,000. It is important for students to associate the symbols < and > with their real meaning. Rather than use tricks such as alligators or Pac Man, it may help students who confuse the symbols to remember that the open end of the symbol is always closest to the greater number and the closed end is always closest to the lesser number. Students could also be challenged to examine what happens to their comparisons when the order of the digits within numbers is changed.

Level 4:

Students at this level can accurately order a set of three or more three-digit numbers from least to greatest or greatest to least based on the meanings of the digits in each place and uses the symbols > or < to show the relationships. The student provides justification for the comparison (oral or written) by explaining the reasoning used. Students may also choose to use drawings and/or manipulatives to justify their explanations.

Number and Operations in Base Ten (NBT)

Standard 2.NBT.B.5 Cluster Heading: B. Use place value understanding and properties of operations to add and subtract.

Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract within 20 using mental strategies.	Fluently add and subtract within 100 using objects or drawings.	Fluently add within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. Fluently subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction.	Fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. Students can explain their answers and provide justifications for why one strategy is more efficient than another.

Instructional Focus Statements

Level 3:

2.NBT.B.5 begins to build procedural fluency from previous conceptual standards. Fluency is about a student being able to flexibly think about the problem posed in order to efficiently answer by employing a strategy from their toolbox that makes sense in that particular situation leading to an accurate answer. There is no one strategy that works every time for every student. Each child develops fluency from the strategies that individually work best for them.

In kindergarten and grade 1, students build an understanding of the base-ten number system. In 1.NBT.C.4, students added a two-digit number to a one-digit number and a two-digit number to a multiple of ten (within 100). In grade 2, students continue developing this understanding so that by the end of grade 2 they can fluently add and subtract within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. It is important to note that solving using the standard algorithm is not introduced to students until grade 3 and mastered until grade 4. This work will be foundational as students extend their learning to adding and subtracting within 1,000 using concrete models in 2.NBT.B.7. The use of place value and physical objects from previous course work should be continued in this standard. It is important that students who still need direct modeling in order to grasp the mathematics be allowed to do so. With these students, it will be important over the course of the year to help them move from direct modeling to more strategy-based approaches. Ultimately strategy-based approaches are what builds fluency for students. The foundation of this standard, and ultimately fluency for addition and subtraction, relies on the understanding of three major concepts; computation of sums and differences of all one-digit numbers, the ability to flexibly compose and decompose numbers, and combining like units (regrouping).

Initially, students will need direct modeling with visual models and diagrams to help them grasp the conceptual understanding of decomposing, composing, and combining like units (regrouping). This may be accomplished with craft sticks, linking cubes, base ten blocks, or other hands-on manipulatives. Once a student no longer needs the manipulatives, they move to more strategy-based thinking. For example, as students find the sum of 36 + 27, they should understand that the decomposition of each number is 3 tens and 6 ones + 2 tens and 7 ones, respectively. From previous work in grade 1, students should understand that the ones can be combined into a new group of ten. In this example, 7 ones and 6 ones are combined to form a new group of ten and 3 ones. It is imperative that students develop a deep understanding of combining units by using visual representations such as base ten blocks, ten frames, and bundling manipulatives first before moving to a purely strategy-based approach. Students should also understand that the composition, including the newly formed unit, results in 3 tens + 2 tens + 1 ten + 3 ones, resulting in a total of 6 tens and 3 ones, or 63. It is also beneficial to provide experiences using open number lines and the hundreds chart using benchmarks of tens to help students develop strategies for adding and subtracting. Additionally, written equations should be intentionally connected to visual representations.

As students work with multiple representations and strategies, they should be able to make connections between the representations and strategies, including written equations, and explain which strategy is most efficient for them for a particular problem. It is also important to note that this standard integrates with 2.MD.B.6 where students work with sums and differences on a number path or number line. Number path/line strategies are a very visual way to help build operational fluency for students.

Students should encounter equations that are written both vertically and horizontally and make estimates when adding and subtracting to determine if their solution is reasonable. Using estimation and the understanding of reasonableness will be a foundational skill that students will use throughout future grade levels/courses. As students become fluent with addition and subtraction within 100, they should be able to accurately compute the correct answer, flexibly use multiple representations, and choose the strategy that is most efficient.

Level 4:

Students should solidify their understanding by flexibly, accurately, and efficiently adding multiple addends within 100 using properties of operations, strategies based on place value, and/or the relationship between addition and subtraction. The multiple addends or subtrahends should naturally elicit the use of different strategies. Students must explain or defend the accuracy of their answer by not only showing multiple strategies, but also by explaining why one strategy is more efficient than another strategy. As students express their thinking in written and verbal formats, they should use precise mathematical language, diagrams, and written equations.

Number and Operations in Base Ten (NBT)

Standard 2.NBT.B.6 Cluster Heading: B. Use place value understanding and properties of operations to add and subtract.
Add up to four two-digit numbers using properties of operations and strategies based on place value.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Accurately add three or four numbers with sums less than 20 using properties of operations and strategies based on place value.	Accurately add three or four numbers with sums within 100 which do not require composing or decomposing tens, using properties of operations and strategies based on place value.	Add four two-digit numbers with sums within 100 which require composing or decomposing tens, using properties of operations and strategies based on place value.	Create an equation involving the sum of at least three addends and requires regrouping that add up to a given whole number within 100. Explain the properties of operations or place-value based strategies that could be used to solve the created equation.

Instructional Focus Statements

Level 3:

As students expand their toolbox with strategies to efficiently add two two-digit numbers, they begin to work with a greater number of addends. In the conceptual standard 2.NBT.B.6, students add up to four two-digit numbers using properties of operations and strategies based on place value. This should be scaffolded by first introducing students to adding three two-digit numbers and encouraging the use of previously learned strategies that include the commutative, associative, and identity properties. The commutative and associative properties are particularly helpful as they allow students to reorder a string of numbers in a way that makes combining the numbers more efficient. For example, when adding 24 + 18 + 32, one student might add 18 and 32 first to get 50 and then add 24 to get a total of 74. Another student might decompose the 18 to 16 + 2 giving 24 + 16 + 2 + 32 in order to get 40 + 2 + 32 yielding 74. Both solution paths are correct and demonstrate ways in which students can flexibly combine numbers, one using a property and one not.

As students use different strategies and properties, they should continue to use visual representations such as a number path and/or number line as they make sense of which addend order best matches their thinking. As students begin to add with four two-digit numbers, it is easier for them to start with

problems with no regrouping and over time transition to working with cases where regrouping is needed. As in previous work, students will initially need to use manipulatives and drawings to help them move them towards procedural fluency with problems that require regrouping.

Level 4:

As students solidify their understanding of adding up to four two-digit numbers, they should be flexible in using multiple strategies and representations. Students should be able to construct a viable argument (MP 3) as to why their way is the most efficient way to add the numbers. Further, they should be able to critique the reasoning of others (MP 3) as their explanations are provided. Additionally, students should be able to produce a string of numbers whose sum is a given whole number within100. They should also be able to explain the properties of operations or place-value based strategies that could be used to solve the equation.

Numbers and Operations in Base Ten (NBT)

Standard 2.NBT.B.7 Cluster Heading: B. Use place value understanding and properties of operations to add and subtract.
Add and subtract within 1000 using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to explain the reasoning used. (Explanations may include words, drawing, or objects.)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract two whole numbers with sums/differences within 1000 which do not require composing or decomposing tens or hundreds. Explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract two whole numbers with sums/differences within 1000 which require composing or decomposing tens or hundreds, but not both. Explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract two whole numbers with sums/differences within 1000 which require composing or decomposing tens and hundreds. Explain their answer using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	Add and subtract three or more whole numbers with sums/differences within 1000 which require composing or decomposing both tens and hundreds. Explain the thinking of other's answers using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Instructional Focus Statements

Level 3:

In previous grades, students began to develop a conceptual understanding of adding and subtracting through a three-stage progression of learning: concrete, representational, and abstract. Instruction within grade 1 exclusively focused on understanding addition and subtraction through concrete and representational learning. In standard 2.NBT.A.1, students represent 3-digit numbers in multiple ways and in standard 2.NBT.B.5 students demonstrate fluency with addition and subtraction within 100 using properties of operations, strategies based on place value, and/or the

relationship between addition and subtraction. As students build on these understandings, they continue to use concrete and representational learning as they transition to adding and subtracting a larger range of numbers within 1000 over time.

In the concrete learning stage, instruction should focus on direct modeling. Students should be physically manipulating objects to solve math problems and explaining the connections between the model and the problem. The initial focus for this standard should be on having students use direct modeling techniques and strategies, moving towards representational strategies with place value charts as they add and subtract within 1000. When thinking about choosing numbers to add and subtract within 1000, consider that direct modeling can become very cumbersome. That said, it is important to select friendly numbers. Students should be encouraged to look for patterns in their processes and answers when adding and subtracting as this will also cause them to think about more efficient ways of combining numbers. It is important to note that direct modeling is a necessary developmental phase which allows children who are not ready for more efficient methods a way to explore the same problems as classmates who have progressed beyond this stage.

Base ten blocks are very helpful in helping model hundreds, tens, and ones for students who have developed the conceptual understanding that the ten rod is comprised of 10 ones and a 100 flat is comprised of 10 tens. As students have developed this understanding in working with addition and subtraction in grade 1, and due to the magnitude of the numbers being added and subtracted in grade 2, it is appropriate to utilize base 10 blocks and place value charts with students at this point. As students are working with them, make sure to ask students to explain how they are using the blocks to check their understanding of composing and decomposing hundreds and tens.

As students are solidifying their understanding of adding and subtracting within 1000, they should build upon their own “invented strategies” developed in grade 1. Invented strategies are when students begin to compose and decompose numbers in flexible ways to more quickly figure out a computation. Some examples of invented strategies are compensation, counting on, counting back, and making a ten. It is important to note that for many students, these strategies are easier, more intuitive, and quicker than the standard algorithm that is not introduced until grade 3. For example, when finding the sum of 234 and 126, students should make use of previous composing and decomposing strategies to make tens and hundreds. Decomposing the addition problem by place value as (200 + 30 + 4) + (100 + 20 + 6) is one way that students may efficiently find the sum. In doing so, students should use previous understandings of regrouping and making tens and hundreds resulting in 300 + 50 + 10 = 360. This is just one example of a student-invented strategy. Students should be encouraged to compare and contrast to find the strategy that works the best for them. Over time, students should increase the range of numbers with which they are adding and subtracting.

Students can also subtract within 1000 by viewing subtraction problems as unknown addend problems, e.g., 276 + ? = 425. Counting-on and adding-on methods for addition can also be used for subtraction. Additionally, many students struggle with subtracting numbers that have zero tens and/or zero ones, sometimes referred to as “subtraction across zeros.” It is imperative that students conceptually understand how to compose and decompose numbers when regrouping. For example, when subtracting 400 – 274, students should be able to “break apart” a hundred as ten tens and a ten as ten ones. This should be done with direct modeling with manipulatives and drawings for students to grasp the conceptual understanding that will be needed in future grades.

As students work with concrete and representational strategies, including student-invented strategies, they should be able to explain their reasoning, make connections to different solution paths, and explain the similarities and differences and relationship between the two using precise mathematical vocabulary.

Level 4:

As students extend their understanding, they should be able to add and subtract three or more whole numbers with sums and differences within 1000 which require composing or decomposing both tens and hundreds. Students should also be able to explain their answers and the answers of others using concrete models, drawings, strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Additionally, students should be able to explain, using precise mathematical vocabulary, the connections that exists between multiple strategies across both addition and subtraction. Students should be able to construct a viable argument (MP 3) to justify when strategies are more efficient.

Number and Operations in Base Ten (NBT)

Standard 2.NBT.B.8 Cluster Heading: B. Use place value understanding and properties of operations to add and subtract.

Mentally add or subtract 10 or 100 to/from any given number within 1000.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Use concrete manipulatives and counting by ones to find the number that is ten more than a given two-digit number. Use concrete manipulatives and counting by ones to find the number that is ten less than a given two-digit number.	Add 10 or 100 to a given number within 1000 using an appropriate tool (i.e., number path, hundreds chart, number line, etc.). Subtract 10 or 100 to a given number within 1000 using an appropriate tool (i.e., number path, hundreds chart, number line, etc.).	Mentally add 10 or 100 to a given number within 1000. Mentally subtract 10 or 100 from a given number within 1000.	Mentally add multiples of 10 or multiples of 100 and explain the reasoning used when given a number within 1000. Mentally subtract multiples of 10 or multiples of 100 and explain the reasoning used when given a number within 1000.

Instructional Focus Statements

Level 3:

The instructional focus for this standard should build on the conceptual understanding students developed with standard 1.NBT.C.5, mentally finding 10 more or 10 less than a given two-digit number. As students deepen their understanding of place value, they extend the range of numbers to 1000, mentally finding 10 more or 10 less and 100 more or 100 less than any number in that range.

As students extend their understanding of mentally finding 10 more and 10 less to a larger range of numbers, particular attention needs to be shown to developing conceptual understanding around crossing over hundreds place values. It can be challenging for some students to mentally regroup to create a hundred. For example, a student mentally adding 295 + 10 may struggle regrouping the tens to get the third hundred in 305. Before students are able to use mental strategies to solve this type of problem, they must be given the opportunity to interact with direct modeling in order to have the mathematical foundation needed to move along the learning continuum.

With this standard, finding 100 more or 100 less will most likely be the easier of the two concepts for students to grasp as the pattern is very similar to the pattern students would have discovered when mentally finding 10 more or 10 less to a two-digit number in grade 1. That said, students need to experience a wide variety of both concrete math materials and representational mathematical strategies when adding or subtracting 100. Students should be exposed to various strategies and then choose the one that is most efficient and makes the most sense to them. This process will allow students to grow so that they are eventually able to not only add and subtract 100 using mental strategies, but also conceptually understand the computation. This process takes time. Additionally, students need opportunities to explain their reasoning using place value understanding and patterns on an open number line or extended hundreds chart as the basis for their explanation. Instruction should not focus on tricks or procedures that have little mathematical connections. Instead, instruction should focus on helping students conceptually understand our place value system, discovering patterns that exist, and mathematically discussing the implications of those patterns as they relate to addition and subtraction.

Level 4:

Instruction at this level should focus on students exploring adding or subtracting multiples of 10 or 100 from a given number withing 1000. Students should connect adding and subtracting multiples of 10 or 100 to skip counting. For example, to mentally add 300 to 456, students may skip count 456, 556, 656, 756 and determine that the sum is 756. Open number lines may be helpful when developing a mental image of skip counting.

Measurement and Data (MD)

Standard 2.MD.A.1 Cluster Heading: A. Measure and estimate lengths in standard units.

Measure the length of an object in whole number units by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure the length of an object using whole number non-standard units.	When given an appropriate tool, (e.g., rulers, yardsticks, meter sticks, and measuring tapes) measure the length of an object in whole number units.	Select and use an appropriate tool (e.g., rulers, yardsticks, meter sticks, and measuring tapes) to measure the length of an object in whole number units.	Select and use an appropriate tool (e.g., rulers, yardsticks, meter sticks, and measuring tapes) to measure the length of an object in whole number units and provide an explanation as to why the measuring tool chosen is the best for a given situation. Correct the error and explain why the error may have occurred when given an incorrect measure for an object.

Instructional Focus Statements

Level 3:

Grade 2 students build upon their non-standard measurement experiences in grade 1 (standard 1.MD.A.2) to measure in standard units for the first time. On the surface, measurement seems like a relatively simple concept for students to understand. However, there are conceptual and procedural concepts that must be an intentional focus of instruction. Students are used to counting discrete objects where numbers represent those objects (standard K.CC.B.5). For both this standard and the number line standard 2.MD.B.6, it is important for students to develop the understanding that the "1" on the

ruler represents the distance between 0 and 1 as opposed to it representing the tick mark (an object) on the ruler. This is conceptually abstract for students. One manipulative that can help develop this understanding is the use of physical objects, typically squares that are one inch or one centimeter in length. These are often called "length-units" and can help bridge a student from the non-standard units used in grade 1 to using a ruler in grade 2. One effective application of this manipulative is for students to use these length-unit manipulatives to build their own rulers. They provide a concrete representation to help students see the abstract concept of distance.

Other important mathematical understandings for students to develop include developing an understanding that there is no space between successive length-units, the importance of correctly aligning the zero-point on the ruler at the beginning of the total length, that the number on the ruler represents all of the accumulated distance from zero to that number, and the importance of the units being the exact same size. Often with counting discrete objects, those objects may have slight variance in size. For example, when counting cars on a table, they may not all be exactly the same and there still be eight cars. In measurement, it is crucially important that the units be the same. A teacher might challenge students by lining up 4 larger squares next to 4 smaller squares to say that an object is eight units long to provide students the opportunity for discussion and discourse around if unity in size is important (MP 3).

It is important to note that grade 2 students should engage in multiple activities where they are measuring with rulers, yardsticks, and meter sticks. The tools should contain only *whole numbers* as the instructional focus for this grade is on solidifying an understanding of how to measure using standard units. These rulers can be found with an internet search and printed or simply cover any decimals or fractions with tape. As students have not been introduced to either formal fraction or decimal notation, they are not prepared to integrate the concepts of halves, thirds, quarters, or eighths with measurement.

As students are choosing appropriate tools and units to measure objects, it is appropriate that they be allowed to measure in both inches and feet or both centimeters and meters. (MP 5) This provides two things for students. First, they develop an understanding of which is more efficient and second they begin to understand the connection between the size of a unit and the number of units needed providing a natural lead in to standard 2.MD.A.2. Students also need to develop an understanding of when a measuring tape is more useful than a ruler or yardstick. One specific example would be in measuring the circumference of a basketball. Students should be given the opportunity to measure numerous items including items that are curved or distances that are not straight lines. Conceptual understanding of measurement can also be supported through grade appropriate science activities (2.ETS2.1) and other STEM related work.

Level 4:

As students engage in measurement activities, they develop an understanding of the relative size of units within the same system and should be able to justify their choice of unit when measuring. Students may begin relating the units within a system such as noticing that there are 12 inches in a foot and incorporate that as a part of their justification for choosing a specific unit. Additionally, students should be able to look at an incorrect measure, correct the mistake, and provide an explanation for the mistake that was made.

Measurement and Data (MD)

Standard 2.MD.A.2 Cluster Heading: A. Measure and estimate lengths in standard units.

Measure the length of an object using two different whole number units of measure and describe how the two measurements relate to the size of the unit chosen.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Measure the length of an object using non-standard whole number units.	Measure the length of an object using two different whole number units of measure.	Measure the length of an object using two different whole number units of measure and describe how the two measurements relate to the size of the unit chosen.	Match the correct units of measure to two measurements for the same object when given two lengths of an object without the unit of measure labeled and justify their reasoning by describing how the measures are related.

Instructional Focus Statements

Level 3:

Grade 2 students develop an understanding of the inverse relationship that exists between the relative size of a unit and the number of units needed to measure an object. Students need opportunities to explore measurements to discover that the larger the unit, the fewer number of units in a given measurement and the smaller the unit, the more units in a given measurement. Students should measure the same object using different units and reflect on how the overall measurements relate to each other and then on how the measurements relate to the size of the unit chosen. It is important to note that activities can involve both standard and non-standard units of measure to help students develop this understanding. For example, when measuring the height of a door, if one unit selected to measure is small such as inches, students should notice that it takes many of these small units to measure the height. Conversely, when measuring the same door with a larger unit such as yards, it will take many less units to measure the height. Instruction should provide ongoing measurement experiences and activities to help students solidify their understanding that the unit used to measure is as important as the attribute being measured. This will be important when students estimate with standard units of measure in standard 2.MD.A.3. Additionally, this standard provides a good opportunity for students to attend to precision (MP 6) by labeling units when they measure.

It is important to note that grade 2 students should engage in multiple activities where they are measuring with rulers, yardsticks, and meter sticks. The tools should contain only *whole numbers* as the instructional focus for this grade is on solidifying an understanding of how to measure using standard units. These rulers can be found with an internet search and printed. As students have not been introduced to either formal fraction or decimal notation, they are not prepared to integrate the concepts of halves, thirds, quarters, or eighths with measurement.

Level 4:

Students at this level should be able to extend their understanding by matching the correct units of measure to two measurements for the same object and justify their reasoning. For example, students may be told that a cabinet is both 79 units and 2 units long. When asked which unit is meters and which unit is inches, students should match 79 to inches and 2 to meters and be able to justify their reasoning. Additionally, students can extend their understanding of units to make inferences about the relative size of objects. For example, if students are told that a red string is 10 regular paper clips long and a blue string is 10 jumbo paper clips long, they should be able to reason that since the number of units used is the same (10) and since the units used have different sizes (jumbo and regular), that the two strings are actually different in length.

Measurement and Data (MD)

Standard 2.MD.A.3 Cluster Heading: A. Measure and estimate lengths in standard units.

Estimate lengths using whole number units of inches, feet, yards, centimeters, and meters.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify an object with a length of 1 unit without measuring (i.e., identify something in the room that is 1 inch long) for at least one unit of measure in inches, feet, yards, centimeters, or meters. Choose a reasonable whole number estimate for a measurement provided in inches, feet, yards, centimeters, or meters for 1 unit.	Identify an object with a length of 1 unit without measuring (i.e., identify something in the room that is 1 inch long) for inches, feet, yards, centimeters, and meters. Choose a reasonable whole number estimate for a measurement provided in inches, feet, yards, centimeters, and meters.	Estimate lengths using whole number units of inches, feet, yards, centimeters, and meters.	Estimate the length of a given object with more than one whole number unit of measure and justify their reasonableness for each estimate.

Instructional Focus Statements

Level 3:

Instruction for this standard should not only focus on helping students be able to accurately estimate, but also on helping them understand what estimating is and the importance of estimation particularly in everyday life. It is important to note that this is the first time estimation is explicitly called out in the standards. Students will have to reason not only with the relative sizes of measurement, but also with the concept of estimation and the idea that it is okay to not have an exact answer. Both measurement and estimation vocabulary should be modeled for students using terms such as a little more than, a little less than, about, and close to.

As students are developing their conceptual understanding of estimating, they should have ample opportunities to estimate a length and then actually measure to check their accuracy. For that reason, this standard integrates well with both standards 2.MD.A.1 and 2.MD.A.2 once students have solidified

their understanding of the mechanics of measuring objects. As students use standard units to measure objects, they can also begin estimating lengths before they actually measure the length. Students should estimate the lengths of objects using inches, feet, centimeters, and meters prior to measuring. Once a student has made an estimate and measured the object, they can then reflect on the accuracy of the estimate made and consider this information for the next estimate. This iterative process not only helps them become better estimators, but also supports the development of measurement skills. Another by-product of estimating is that as students estimate, they must consider the size of the unit helping them to become more familiar with unit size in general.

Another helpful tool for students is for them to develop their own benchmarks of measure for each unit. Students may say a paperclip is about an inch or the desk is about a meter off the floor. Visual benchmarks help students when they estimate as length is very abstract to grade 2 students. It is important that students develop their own benchmarks as one visual benchmark may work better for a student than another one.

It is important to note that grade 2 students should engage in multiple activities where they are estimating and measuring with rulers, yardsticks, and meter sticks. The tools should contain only *whole numbers* as the instructional focus for this grade is on solidifying an understanding of how to measure using standard units. These rulers can be found with an internet search and printed or by simply covering any decimal fractions with tape. As students have not been introduced to either formal fraction or decimal notation, they are not prepared to integrate the concepts of halves, thirds, quarters, or eighths with measurement.

Level 4:

As students extend this understanding of estimation, they should be challenged to estimate the length of one object using two different units of measure and justify the reasonableness of each estimate. This allows students to think flexibly about length and continue making connections between different units of measure within the same system of measure. Students may estimate that a desk is two feet long and also 24 inches long. When asked to explain their estimates, students may begin using repeated addition as a part of their justification incorporating their knowledge that there are 12 inches in a foot. Additionally, students should be challenged to critique estimates and explanations provided by their peers.

Measurement and Data (MD)

Standard 2.MD.A.4 Cluster Heading: A. Measure and estimate lengths in standard units.

Measure, using whole number lengths, to determine how much longer one object is than another and express the difference in terms of a standard unit of length.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Visually identify which of two objects is longer/shorter.	Determine how much longer one object is than another in terms of a standard unit of length when given the measure of two objects in whole number units.	Measure, using whole number lengths, to determine how much longer one object is than another and express the difference in terms of a standard whole number unit of length.	Measure more than two objects to determine the length of each in terms of a standard whole number unit of length. Students should make multiple comparative statements about the length of the objects in the set including not only which objects are longer/shorter than others, but also around specifically how much longer or shorter. Students should encounter sets that have some objects with the same length. Explain why it is important for objects to be measured using the same standard whole number unit of length when determining how much longer one object is than another.

Instructional Focus Statements

Level 3:

In grade 1, students learned to compare the length of objects directly and indirectly *without* measuring them (1.MD.A.1). Standard 2.MD.A.4 will extend a student's understanding when they measure to find the length of two objects and then calculate the difference in lengths. They are moving from knowing that one is longer than the other in grade 1 to finding exactly how much longer in grade 2. Students learn that to find the difference in length, it is essential that they use the same unit to measure both objects. This standard is a natural extension of standard 2.MD.A.1.

It is important to note that grade 2 students should engage in multiple activities where they are measuring with rulers, yardsticks, and meter sticks to measure in inches, feet, yards, centimeters, or meters. The tools should contain only *whole numbers* as the instructional focus for this grade is on solidifying an understanding of how to measure using standard units. These rulers can be found with an internet search and printed. As students have not been introduced to either formal fraction or decimal notation, they are not prepared to integrate the concepts of halves, thirds, quarters, or eighths with measurement.

Instructional opportunities should focus on students choosing two objects to measure, identifying an appropriate tool and unit, measuring both objects, and then determining the difference in length. Students should make a comparative statement about the differences in the length of two objects. For example, "This object is shorter by 3 centimeters" or "This object is longer by 5 inches". This also provides an opportunity for students to practice estimation (standard 2.MD.A.3) as they can provide an estimation for how much longer one object is than another before actually calculating the difference.

Instruction in measurement intersects with other strands in mathematics. Measurement situations can provide a real-world connection to the computation skills students are learning in standards 2.OA.A.1 and 2.NBT.B.7. Additionally, measurement applications offer opportunities for interdisciplinary learning in subjects such as social studies and science. For example, students could measure the daily growth of a bean sprout and determine the difference from one day to another.

Level 4:

Students should be able to measure more than two objects to determine the length of each in terms of a standard unit of length and make comparative statements about the length of the objects in the collection including not only which objects are longer/shorter than others, but also around specifically how much longer or shorter. Students should be able to justify their thinking using appropriate mathematical vocabulary (MP 3). Additionally, students should be able to explain why it is important that objects be measured with the same standard unit of length in order to provide accurate comparisons.

Students may also be challenged with a set of three paths including a zig-zag path, curved path, and straight path. Students can be challenged to find the length of each path and make comparative statements about the three paths. This will challenge them to use appropriate tools strategically (MP 5) as they determine the length of each.

Measurement and Data (MD)

Standard 2.MD.B.5 Cluster Heading: B. Relate addition and subtraction to length.

Add and subtract within 100 to solve contextual problems, with the unknown in any position, involving lengths that are given in the same units by using drawings and equations with a symbol for the unknown to represent the problem. (See Table 1 – Addition and Subtraction Situations)

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract within 20 to solve contextual problems, involving same unit lengths.	Given a solved addition or subtraction contextual problem, involving same unit lengths, relate the solution to the context provided. Match an equation with a symbol for the unknown number to a contextual problem.	Add and subtract within 100 to solve contextual problems, involving same unit lengths, with unknowns in any position, using drawings and equations with a symbol for the unknown number.	Add and subtract within 100 to solve contextual problems, involving same unit lengths, using drawings and equations with a symbol for the unknown number. Explain the reasonableness of the chosen solutions.

Instructional Focus Statements

Level 3:

In standard 2.OA.A.1, students develop an understanding of adding and subtracting within 100. The focus of this standard is to specifically integrate lengths that are given in the same units into contextual situations with the same numeric boundaries. Students should continue to use drawings and equations with a symbol for the unknown in any position to represent the problem. Students should use their understanding of addition and subtraction to combine and compare lengths of objects in both one and two step problems. Students should be able to add two lengths to obtain the length of the whole. Students should also be able to subtract one length from another to find out the difference in lengths. In doing so, students should be provided with a variety of opportunities and multiple representations to apply addition and subtraction to solve problems that involve length. For example, in the contextual problem, *"Jill built a toy train that is 25 inches long. Sammy built a toy train that is 16 inches long. How much longer is Jill's train than Sammy's train, in inches?"* Students should use concrete models and drawings to represent the length of the trains and determine the difference between the two. As with other grade-level standards, this contextual problem can be solved with either addition or subtraction. Students should write an equation that represents

the contextual problem such as 25 – 16 = n or 16 + n = 25. As students use addition and subtraction equations to represent problems, they should be able to explain the relationship between the two representations.

It is also important that students work with the same unit of measurement in contextual problems with the unknown in any position. As students attend to precision, they should be presented with realistic contextual problems and manipulatives to make a real-world connection. Students may use student-created number lines and drawings that are not true to size. Although this is appropriate, non-precise representations may be confusing to students. That said, this standard is a nice integration of 2.MD.B.6 where students represent whole numbers from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Using graph paper and folded paper can be useful tools when drawing equally spaced points on a number line.

As students solve contextual problems, they should be able to justify their choice of operation. As mentioned above, both addition and subtraction equations can be used to solve problems and it is imperative that students are able to explain their reasoning. Teaching key words to associate with addition and subtraction should not be an instructional focus. Instruction should focus on developing an understanding of what operation can be used to solve the problem rather than focusing on key words that sometimes, but not always, associate with the operation.

Level 4:

As students extend their understanding of adding and subtracting within 100 to solve contextual problems involving lengths, they should be flexible with representing problems using multiple representations such as a number line, equation, and written explanation. Instruction should focus on extending student understanding as they are challenged to work with three and four objects embedded in multi-step problems. These situations should include a mix of addition and subtraction in order to solve the problem. Students should also be able to explain their chosen solution path and the reasonableness of their answer using precise mathematical vocabulary.

Measurement and Data (MD)

Standard 2.MD.B.6 Cluster Heading: B. Relate addition and subtraction to length.

Represent whole numbers as lengths from 0 on a number line and know that the points corresponding to the numbers on the number line are equally spaced. Use a number line to represent whole number sums and differences of lengths within 100.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Locate a whole number and explain that the number represents the length from zero to that number, given a number line model marked equally with whole number increments.	Represent addition or subtraction of whole number lengths within 100 when given a partially completed number line model.	Use an equally spaced number line to represent sums of whole number lengths within 100. Use an equally spaced number line to represent differences of whole number lengths within 100.	Use an equally spaced double number line to represent equivalent sums and differences of whole number lengths within 100. Use a provided number line to create a contextual situation that explains the related model.

Instructional Focus Statements

Level 3:

In previous grades, students may have encountered and worked with number lines as a strategy with counting and cardinality, addition, and subtraction. Standard 2.MD.B.6 will be a student's first formal introduction to number lines. The instructional focus for this standard is for students to understand and represent whole numbers as lengths from 0, know that the points corresponding to the numbers on the number line are equally spaced, and use the number line for addition and subtraction. Because this is students' first experience with number lines, they may have used number paths in previous grades. Instruction should include a brief comparison of number lines and number paths, with the teacher intentionally connecting the two representations.

Students should understand that the points corresponding to the numbers on the number line are equally spaced. That said, it is important for students to be given ample opportunity to work equally with pre-generated and student-generated number lines. Instruction should start with pre-generated number lines. It is important for students to understand that the points on the number line are equally spaced. As students begin to transition to student-

generated number lines, they should be given opportunities and methods to create number lines with equally spaced points. Drawing equally spaced points on a number line is difficult for students. Using graph paper and folded paper can be useful tools when attending to precision to create number lines with equally spaced points.

As students use number lines to represent whole number sums and differences, they should model counting on and counting back by drawing "jumps" on the number line to represent the problem. Students should also use previous addition and subtraction strategies of adding to, taking from, putting together, taking apart, and comparing using illustrated "jumps" on the number line.

As students utilize number lines to represent addition and subtraction, it is appropriate to work with a variety of increments, including ones, fives, and tens. It is easier for students to visualize the illustration of addition and subtraction by one-unit "jumps" when using a number line in increments of 1s. However, when working with a larger range of numbers it may be easier for students to work with an open number line. The focus for this part of the standard is on addition and subtraction and it is appropriate to use non-precise, student-generated number lines. For example, when finding the difference of 83 – 29, students may use an open number line and place a point on 83 and "jump" back 4 increments of 10 units and "jump" back 4 increments of 1 resulting in the difference of 54. This may result in an accurate solution without paying attention to the equally spaced increments of tens and ones on the same number line, which could be confusing to students. It is more important that students label their 'jumps" and understand the difference between the two numbers rather than the spacing accuracy on the number line. In this case, an open number line may be more efficient as students are working with increments of 10s and 1s. Students should explore multiple solution paths and be able to explain why they have chosen their method. Additionally, students should be able to make connections between the context, their solution path, and their equation and explain their reasoning with precise mathematical vocabulary.

Level 4:

As students extend their understanding, they should solve problems relating addition and subtraction within 100 increasing in rigor over time. Students should be able to model the problem by drawing a double number line model and equivalent equations to represent the situation. Students should also be able to defend their strategies and solutions with a verbal or written explanation. Additionally, students should use a given number line model depicting addition or subtraction within 100, write the equation modeled, and then generate a contextual situation connected to the provided model.

Measurement and Data (MD)

Standard 2.MD.C.7 Cluster Heading: C. Work with time and money.

Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Tell and write time in hours using analog and digital clocks.	Tell and write time in hours and half-hours using analog and digital clocks.	Tell and write time in quarter hours and to the nearest five minutes (in a.m. and p.m.) using analog and digital clocks.	Show the time using analog and digital clocks, given a time in quarter hours or a time to the nearest five minutes.

Instructional Focus Statements

Level 3:

Students are formally introduced to telling time in grade 1 where they read clocks to the nearest hour and then to the half hour using digital and analog clocks (standard 1.MD.B.3). In grade 2, students learn to read to the quarter hour and 5-minute intervals. Students should have the opportunity to relate telling time to everyday experiences at school such as the time school starts and ends, lunch time, recess time, etc. Students should also link time to their home experiences such as the time they get up, eat dinner, go to sleep, and so on.

Students should have opportunities for telling time embedded throughout the day. They should learn that there are twenty-four hours in each day and should explore the two cycles of twelve hours in a day - a.m. and p.m. For example, students should be able to state that they ate breakfast at 6:30 a.m. but eat dinner at 6:30 p.m. Vocabulary such as noon or midnight are also helpful for students when thinking about a.m. and p.m. As in grade 1, teachers should model for students how to write time using colon notation.

When students are telling time to quarter hours, it is helpful to connect this to partitioning a circle into halves and fourths or quarters (standard 2.G.A.3) as students do not intuitively connect 15 minutes with a quarter hour. To this point, quarters to them are related to the number 25 due to their interaction with monetary amounts in grade 1. Placing a quarter hour at 25 is a very common mistake for grade 2 students and instruction should focus on helping them understand why it is at 15. When students are working on telling time to the nearest five minutes, they should be encouraged to skip-count by fives

to help them tell time by five-minute intervals. It is important to help students understand first why they are skip counting by fives and second why they stop skip-counting at 55 and what happens at 60.

Level 4:

Students should be able to demonstrate conceptual understanding of time with digital and analog clocks by representing the same time written or spoken in words in both the analog and digital format. Students need to make sure they are using the correct placement of the hour hand when showing time to the hour and half hour. Students can duplicate the time on a clock model or draw the hands on a printed copy of a clock.

Measurement and Data (MD)

Standard 2.MD.C.8 Cluster Heading: C. Work with time and money.

Solve contextual problems involving amounts less than one dollar including quarters, dimes, nickels, and pennies using the ¢ symbol appropriately. Solve contextual problems involving whole number dollar amounts up to $100 using the $ symbol appropriately.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Count the value of a set of like coins less than one dollar using the ¢ symbol only. Count the value of a set of like bills using the dollar sign.	Count the value of a set of mixed coins less than one dollar using the ¢ symbol. Count the value of a collection of mixed bills using the dollar sign appropriately up to $100.	Solve one-step contextual problems involving dollar amounts up to $100 using the $ symbol appropriately. Solve one-step contextual problems involving amounts less than one dollar including quarters, dimes, nickels, and pennies using the ¢ appropriately.	Solve simple two-step contextual problems involving dollar amounts up to $100 using the $ symbol appropriately. Solve simple two-step contextual problems involving amounts less than one dollar including quarters, dimes, nickels, and pennies using the ¢ appropriately.

Instructional Focus Statements

Level 3:

In grade 1, students counted the value of like sets of coins comprised of pennies, nickels, dimes, and quarters that was restricted to being less than a dollar. In grade 2, students solve contextual problems many of which will require them to work with the value of a mixed set of coins, compare the value of two sets of coins, make and recognize equivalent collections of coins (same amount but different arrangements), select coins for a given amount, and make change. Students should have a wide range of experiences with money in grade 2. Students in grade also solve one-step contextual problems involving amounts up to $100. Since students have not been introduced to decimals, problems should not be notated in decimal notation. Thus, students should notate using the $ or ¢ symbol appropriately. This allows students to interact with more authentic problems without interacting with decimal notation. With previous instruction on coins, the placement of the cents symbol is typically a much easier concept for students to grasp. Due to this

concept and the placement of the dollar sign in metric measure, students might have misconception about placing the dollar sign before the amount. Instruction should focus heavily on the placement of the dollar symbol with students understanding the differences of the two symbols.

Working with groups of mixed coins can be a challenge for young children as money is a non-proportional model (the value of the coins is not physically related to the value of the other coins). Providing opportunities for students to solve a problem involving the same quantity of like coins such as, *"Peter has 5 dimes. John has 5 nickels and Steve has 5 pennies. Who has the most money?"*, can help students make sense of the total value of each set is disproportionate from the quantity. The use of a hundreds chart or number line to count both like and mixed coin values can help students make sense of the non-proportional coin values. As with finding the value of sets of like coins, skip counting remains a good strategy for students to employ as they work with mixed coin sets. Working with money offers a real-world opportunity for students to use the strategies they have developed for adding and subtracting within 100 (standards 2.OA.A.1 and 2.NBT.B.7).

In contextual situations, students learn that a number (38) can be represented different ways (3 tens and 8 ones; 2 tens and 18 ones) and still remain the same amount (38), students can apply this understanding to money. For example, 25 cents can look like a quarter, two dimes and a nickel, and it can look like 25 pennies, and still all remain 25 cents. This concept of equivalent worth takes time and requires numerous opportunities to create different sets of coins, count sets of coins, and recognize the “purchase power” of coins (a nickel can buy the same things a 5 pennies). This same exploration should be applied to mixed bill combinations. For example, "How many different ways can you make $12 using $1, $5 or $10 bills?"

As teachers provide students with sufficient opportunities to explore coin values (25 cents) and actual coins (2 dimes, 1 nickel), teachers will help guide students over time to learn how to: mentally give each coin in a set a value, place the random set of coins in order, and use mental math by adding on to find differences, and skip counting to determine the final amount.

After success with counting coins from a contextual situation, students should be given the opportunity to solve contextual problems involving addition or subtraction using all problem types, which involve counting sets of mixed coins or mixed dollar amounts. Teachers should ensure students have opportunities to talk and make sense of the problems they are solving. Students should communicate their thinking and justify their answers for the contextual problems.

Level 4:

As students become efficient with solving contextual problems with money, they should be challenged to solve a wide range of simple, two-step contextual problems involving addition and subtraction. These contextual problems should involve both addition and subtraction and should encompass a wide variety of situations. Challenging problem-solving situations offer students the opportunity to further solidify their understanding of working with monetary amounts. Students should be encouraged to attend to precision (MP 6), look for and express regularity in repeated reasoning (MP 8), and make sense of problems and persevere in solving them (MP 1). Students should also be able to explain their reasoning to others.

Measurement and Data (MD)

Standard 2.MD.D.9 Cluster Heading: D. Represent and interpret data.

Given a set of data, create a line plot, where the horizonal scale is marked off in whole-number units.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add pre-sorted data to a pre-drawn line plot with the horizontal scale labeled in whole number units.	Add unsorted data to a pre-drawn line plot with the horizontal scale labeled in whole number units.	Create a line plot from given data where the horizontal scale is marked off in whole-number units.	Generate data and create a line plot where the horizontal scale is marked off in whole-number units.

Instructional Focus Statements

Level 3:

Students in grade 2 have had no previous experience creating line plots. A line plot is a graph that shows the frequency of data occurring along a portion of a number line diagram and, at this grade, marked in whole number units. This method of organizing data allows for easier comparisons of the frequency of each number on the horizontal axis. To create a line plot, a partial number line is drawn and a mark (typically an X or a dot) is made above the corresponding value on the line for every data point. Students may struggle with determining the beginning and ending numbers on the partial number line and then creating equal intervals between them because this is their first experience considering the range of a data set. Instruction could include a gradual release method of initially supplying students with a blank template then moving them towards creating their own line plots. Students should discuss the importance of making uniform “Xs” so that comparisons can be made easily.

Level 4:

As students are collecting data, they have a choice of making a record of their data first and then creating the line plot or creating the line plot as they collect data which will require them to reason quantitatively (MP 2) as they set up their partial number line diagram. This is also a good opportunity for students to try both ways and then construct a viable argument for which method is better (MP 3). Another interesting discussion can focus on outliers within data. Students can be challenged to think through if the outlier should be there or if the data point was generated by inaccurate measurement. Ultimately it is important to note that the focus of this standard is more on creating and interpreting line plots than generating data.

Measurement and Data (MD)

Standard 2.MD.D.10 Cluster Heading: D. Represent and interpret data.

Draw a pictograph (with a key of values of 1, 2, 5, or 10) and a bar graph (with intervals of one) to represent a data set with up to four categories. Solve addition and subtraction problems related to the data in a graph.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Organize, represent, and interpret data with up to three categories. Ask and answer questions about the total number of data points, how many in each category, and "how many more or less" are in one category than in another. Complete a pictograph which represents a limited data set with up to three categories. Each picture symbol represents a single element in the data set. Complete a bar graph with intervals of one which represents a limited data set with up to three categories.	Draw a pictograph (with a key of values of 1 or 10) to represent a sorted data set with up to three categories. Each picture symbol represents a single element in the data set. Draw a bar graph with intervals of one to represent a sorted data set with up to three categories. Solve one-step addition and subtraction problems related to the data presented in a provided pictograph or bar graph.	Draw a pictograph (with a key of values of 1, 2, 5, or 10) to represent a data set with up to four categories. Solve one and simple two-step addition and subtraction problems related to the data in a graph. Draw a bar graph with intervals of one to represent a data set with up to four categories. Solve one and simple two-step addition and subtraction problems related to the data in a graph.	Draw a pictograph to represent a data set with up to four categories. Create and solve one and two-step addition and subtraction contextual problems related to the data in the graph. Draw a bar graph with intervals of one to represent a data set with up to four categories. Create and solve one and two-step addition and subtraction contextual problems related to the data in the graph. Decide if a pictograph or bar graph better represents a data set and provide justification.

TN Department of Education

Instructional Focus Statements

Level 3:

In grade 1, students organized data with up to three categories and asked and answered questions about the data. In grade 2, students expand on this to collect and organize data in four categories and then represent that data specifically in pictographs with a key of values of 1, 2, 5, or 10 and/or bar graphs with intervals of one. Additionally, students should be solving one and two-step addition and subtraction problems related to the data in the graph. This supports standard 2.OA.A.1. Instruction should focus on how to draw a pictograph or bar graph and then on answering questions related to the graphs, incorporating a variety of problem types.

Pictographs are one of the most intuitive types of graphs for grade 2 students. Due to experiences with skip counting and one-to-one correspondence coupled with an understanding of non-proportional representations from working with monetary value of coins, students tend to have an easier time developing a conceptual understanding of and answering questions about data represented in pictographs. That said, as this is a student's first experience reading and drawing pictographs, students will need instruction on how to read a pictograph, how to physically represent data on the graph, what a scale is, how to use a scale, why the scale is important, and when to use a scale of one and when to use a different scale such as two, five, or ten. Students need opportunities to interact both with vertical and horizontal graphs. Students should experience a balance of opportunities both collecting their own data prior to creating pictographs and creating pictographs from provided data.

Bar graphs are less intuitive for students. As with pictographs, students will need instruction on how to read a bar graph, how to physically represent data on the graph, what a scale is, how the scale on bar graphs differs from the scale on pictographs, and why the scale is important. Students should discuss ways in which bar orientation (horizontal or vertical), order, thickness, spacing, shading, colors, and so forth make the bar graphs easier or more difficult to read and interpret. With bar graphs, one of the axes will have numerical meaning and will represent segments of a number line diagram. As this scale represents counts in whole numbers (sometimes called a count scale), when students answer "how many more" or "how many less" questions, they are supporting their understanding of finding differences on a number line diagram (standard 2.MD.B.6). Students should experience a balance of opportunities both collecting their own data prior to creating bar graphs and creating pictographs from provided data.

This standard integrates well with standard 2.G.A.1 as students can make graphs from sorted shapes. Additionally, it provides the opportunity to review and reinforce standard K.MD.B.3 by sorting coins into categories followed by answering addition and subtraction questions about the data (value of various sets of coins) which supports standard 2.MD.C.8.

Level 4:

Students at this level should be challenged to graphically represent a data set with up to four categories. With pictographs, students should thoughtfully choose the scale for their graph and be able to justify their choice. Additionally, students should create their own one and two-step addition and subtraction contextual problems related to the data in their graph and provide solutions for their problems. Students should also be challenged to choose which type of graph best represents their data and provide justification for their choice (MP 3).

Geometry (G)

Standard 2.G.A.1 Cluster Heading: A. Reason about shapes and their attributes.

Identify triangles, quadrilaterals, pentagons, and hexagons. Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles/vertices or a given number of sides of equal length.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Correctly name two-dimensional shapes regardless of their orientation or size. Describe similarities and differences between two-dimensional shapes.	Identify at least one attribute of a triangle, quadrilateral, pentagon, or hexagon. Draw two-dimensional shapes given a defining attribute such as number of sides or number of vertices.	Identify regular and irregular triangles, quadrilaterals, pentagons, and hexagons. Draw two-dimensional shapes having specified attributes (as determined directly or visually, not by measuring), such as a given number of angles/vertices or a given number of sides of equal length.	Draw two-dimensional shapes having at least two specified attributes (as determined directly or visually, not by measuring), such as a given number of angles/vertices and a given number of sides of equal length. Draw a non-example of a triangle, quadrilateral, pentagon, and hexagon.

Instructional Focus Statements

Level 3:

In previous grades, students interacted with naming two-dimensional and three-dimensional shapes, drawing two-dimensional shapes, and identifying what constitutes defining attributes of shapes. Students will have previously worked with triangles, a wide variety of different types of quadrilaterals, and hexagons. The terms quadrilateral, rhombus, trapezoid, and pentagon may be unfamiliar to students and instruction may need to focus on helping students develop a conceptual understanding of the uniqueness of each of these shape classifications. As students are developing an understanding of new types of shapes, instruction could present opportunities for students to sort and classify shapes. For example, students could sort two-dimensional shapes by the number of vertices. Additionally, students should be challenged to identify and articulate their own sorting criteria for a set of shapes as this

will help reinforce the concept of which attributes are defining and which ones are not. It is important to note that students do not need to know the formal names of triangles but should be able to identify that any three-sided closed figure with three vertices is a triangle and recognize that all triangles do not look alike. Students should be given opportunities to interact with a variety of quadrilaterals, pentagons, and hexagons that do not look alike.

As students have had experiences drawing two-dimensional shapes in previous grades, instruction in grade 2 should focus on students drawing shapes with very specific attributes such as a given number of angles or a given number of sides of equal length. This standard builds on work from standard 1.G.A.1 where students drew two-dimensional shapes with defining attributes, such as the number of sides or number of vertices. Students should explore and draw shapes in a variety of orientations and configurations as it is important that they have a diversified lens for looking at shapes.

The precision of language is key to students developing correct understandings of the properties of shapes and solids. While it is not an expectation for students at this developmental stage to understand the classification of squares as rectangles, it is important that they are not exposed to the misconception that squares are not rectangles. This misunderstanding creates confusion in grade 5 (5.G.B.3) when students classify two-dimensional figures by a hierarchy based on attributes. Instruction should instead focus on the attributes of a square that make it unique.

Level 4:

Students at this level should be able to articulate attributes that define a shape and extend this understanding to state attributes that are not present in a shape. To take this concept a step further, students could be asked to draw a non-example of a given shape (e.g., draw a shape that is not a quadrilateral and explain why it is not a quadrilateral). Students can also be challenged to draw two-dimensional shapes having at least two specified attributes (e.g., draw a shape with four angles and with two sides of equal length).

Geometry (G)

Standard 2.G.A.2 Cluster Heading: A. Reason about shapes and their attributes.

Partition a rectangle into rows and columns of same-sized squares and find the total number of squares.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Construct a wide variety of different rectangles when given squares and count to determine how many squares compose each rectangle.	Count to identify how many squares are in a given rectangle that has been pre-partitioned into rows and columns of same-sized squares. Partition a rectangle into two or four equal shares.	Partition a rectangle into rows and columns of same-sized squares and find the total number of squares.	Partition a simple, rectilinear shape into rows and columns of same sized squares and find the total number of squares.

Instructional Focus Statements

Level 3:

The instructional focus for this standard should be centered on students partitioning a rectangle into squares (or square-like regions) and then determining the total number of squares. It is more important that the regions be the same basic size than that they be perfect squares. This standard is foundational to the concept of area introduced in grade 3 and that area is measured in square units (3.MD.C.5). This standard also supports standard 2.OA.C.4 where students are finding the number of objects arranged in rectangular arrays using repeated addition. Both standards play an important part in building a necessary foundation prior to formally defining multiplication in grade 3 (3.OA.A.1).

This standard explicitly connects equal sharing and partitioning with rectangles of various sizes supporting students' necessary foundational understanding of topics precursory to multiplication. Teachers should plan activities with square tiles providing students the opportunity to discovery how many tiles it will take to cover an entire rectangle followed by a discussion of why all rows have the same number of tiles, why all columns have the same number of tiles, and the most efficient way to calculate the number of tiles needed. Initially it is helpful to intentionally have students work with rectangles where either the rows or columns contain a number of tiles that fit common, familiar skip-counting patterns. Also supporting standard 2.OA.C.4, students

can write an equation to express the total number of tiles as a sum of equal addends. Once students are comfortable building rectangles when given square tiles, instruction should shift to where students are using drawings and partitioning rectangles into rows and columns. Graph paper can be a particularly helpful tool for students struggling with the “same-sized squares” portion of this standard.

It is important to note that initially some students may not be able to distinguish between a row and a column. Precise vocabulary should be modeled during instruction and encouraged in class discussions, so all students are able to make a distinction between rows and columns.

Level 4:

Instruction at this level extends students’ understanding of partitioning rectangles in rows and columns to partitioning simple, rectilinear shapes into rows and columns of same-sized squares and then finding the total number of squares. This is precursory to student formally finding the area of rectilinear shapes in grade 3. Students should be able to explain how finding the total number of squares it takes to cover a rectilinear shape differs from a simple rectangle and provide justification for the methods they use that exceed simple counting by ones.

Geometry (G)

Standard 2.G.A.3 Cluster Heading: A. Reason about shapes and their attributes.

Partition circles and rectangles into two, three, and four equal shares. Describe the shares using the words halves, thirds, fourths, half of, a third of, and a fourth of, and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain that circles and rectangles can be partitioned into smaller pieces. Partition circles and rectangles in 2 equal pieces and identify them as halves. Compose a circle from half or quarter circles. Compose a rectangle from 2 smaller, congruent rectangles.	Partition circles and rectangles into two equal shares, describe the shares using the words halves, and use the phrases half of to describe the relationship between a share and the whole. Partition circles and rectangles into four equal shares, describe the shares using the words fourths and quarters, and use the phrases fourth of or quarter of to describe the relationship between a share and the whole. Describe the whole as two of, or four of the shares. Explain that partitioning circles or rectangles into more equal shares creates smaller shares.	Partition circles and rectangles into two, three, and four equal shares, describe the shares using the words halves, thirds, fourths, half of, a third of, and a fourth of. Describe the whole as two halves, three thirds, and four fourths. Recognize different representations and orientations of halves of the same rectangle as being equal. Recognize different representations and orientations of thirds of the same rectangle as being equal. Recognize different representations and orientations of fourths of the same rectangle as being equal.	Partition the same rectangle into halves in more than one way and explain why all the halves are equal despite them not having the same shape. Partition the same rectangle into thirds in more than one way and explain why all the thirds are equal despite them not having the same shape. Partition the same rectangle into fourths in more than one way and explain why all the fourths are equal despite them not having the same shape. Recognize and articulate that two halves, three thirds, and four

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
			fourths each referencing the same whole are equivalent.

Instructional Focus Statements

Level 3:

In grade 1 (1.G.A.3), students partitioned circles and rectangles into halves and fourths. In grade 2, students extend their understanding of partitioning to include thirds. Students also increase their academic vocabulary with respect to fractional amounts using the words, halves, thirds, fourths, and quarters, and the phrases half of, third of, fourth of, and quarter of to describe their thinking and solutions. Additionally, students recognize that a whole is made up of two halves, three thirds, and four fourths.

Creating experiences where students explore shapes that are cut into two, three, or four pieces that are not equal and facilitating discourse around why they are not halves, thirds, or fourths can help reinforce student understanding from grade 1 that fractional pieces must be equal shares of the whole. This is foundational to the concept of unit fractions in grade 3 (3.NF.A.1).

Students continue to explore and divide shapes to reinforce their understanding developed in grade 1 (1.G.A.3) that as they create more shares within a whole, the shares get smaller (e.g., fourths are smaller than halves). Students add to this understanding as they explore decomposing fractional regions in different ways. Students should experience multiple representations of halves, thirds, and fourths of the same shape in various orientations to help them solidify their understanding that equal shares do not have to have the same shape. For example, a square may be split first into four equal pieces that are smaller rectangles and then second into equal pieces along the diagonals that are small triangles. The rectangular fourth and the triangular fourth each represent an equal share of the whole square even though the shares are different shapes.

Level 4:

As students solidify their understanding that a rectangle can be split into the same fractional amount in multiple ways and that equal shares of identical wholes need not have the same shape, they should be able to explain why this occurs. Students should also be challenged to think about how equal shares of identical circles can have different shapes. Students should not be expected to decompose circles in this way, but they should recognize and be able to explain that it is possible.