Math III Foundations

5 credits - Level: II
Grade: 11
Prerequisite: Qualifying students will be assigned by the District Supervisor of Math. Placement decision will be based upon deficiency in mathematics as demonstrated on the standardized test administered in tenth grade

Scheduled for the 1st semester of the junior year, this course will provide at-risk HSPA math students with an intensive program of preparation for the spring HSPA. It is designed to promote confidence in taking standardized tests in mathematics, as well as to strengthen the students' ability in the five content clusters covered in the 11th grade HSPA. ***This course does not fulfill one of the three units of math required for graduation***.

PROFICIENCIES

1. NUMBER AND NUMERICAL OPERATIONS:
A. Number Sense
1. Extend understanding of the number system to all real numbers.
2. Compare and order rational and irrational numbers.
3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

B. Numerical Operations
1. Extend understanding and use of operations to real numbers and algebraic procedures.
2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.
3. Perform operations on matrices.
   - Addition and subtraction
   - Scalar multiplication
4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

C. Estimation
1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

2. GEOMETRY AND MEASUREMENT:
A. Geometric Properties
1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).
2. Draw perspective views of 3D objects on isometric dot paper, given 2D representations (e.g., nets or projective views).
3. Apply the properties of geometric shapes.
   o Parallel lines - transversal, alternate interior angles, corresponding angles
   o Triangles
      ▪ Conditions for congruence
      ▪ Segment joining midpoints of two sides is parallel to and half the length of the third side
      ▪ Triangle Inequality
   o Minimal conditions for a shape to be a special quadrilateral
   o Circles - arcs, central and inscribed angles, chords, tangents
   o Self-similarity
4. Use reasoning and some form of proof to verify or refute conjectures and theorems.
   o Verification or refutation of proposed proofs
   o Simple proofs involving congruent triangles
   o Counterexamples to incorrect conjectures

B. Transforming Shapes
1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic object, and, conversely, determine whether and how one object can be transformed to another by a transformation or a sequence of transformations.
2. Recognize three-dimensional figures obtained through transformations of two-dimensional figures (e.g., cone as rotating an isosceles triangle about an altitude), using software as an aid to visualization.
3. Determine whether two or more given shapes can be used to generate a tessellation.
4. Generate and analyze iterative geometric patterns.
   o Fractals (e.g., Sierpinski's Triangle)
   o Patterns in areas and perimeters of self-similar figures
   o Outcome of extending iterative process indefinitely
C. Coordinate Geometry
1. Use coordinate geometry to represent and verify properties of lines.
   o Distance between two points
   o Midpoint and slope of a line segment
   o Finding the intersection of two lines
   o Lines with the same slope are parallel
   o Lines that are perpendicular have slopes whose product is -1
2. Show position and represent motion in the coordinate plane using vectors.
   o Addition and subtraction of vectors

D. Units of Measurement
1. Understand and use the concept of significant digits.
2. Choose appropriate tools and techniques to achieve the specified degree of precision and error needed in a situation.
   - Degree of accuracy of a given measurement tool
   - Finding the interval in which a computed measure (e.g., area or volume) lies, given the degree of precision of linear measurements

E. Measuring Geometric Objects
1. Use techniques of indirect measurement to represent and solve problems.
   - Similar triangles
   - Pythagorean theorem
   - Right triangle trigonometry (sine, cosine, tangent)
2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.
   - Approximation of area using grids of different sizes
   - Finding which shape has minimal (or maximal) area, perimeter, volume, or surface area under given conditions using graphing calculators, dynamic geometric software, and/or spreadsheets
   - Estimation of area, perimeter, volume, and surface area

3. PATTERNS AND ALGEBRA:

A. Patterns
1. Use models and algebraic formulas to represent and analyze sequences and series.
   - Explicit formulas for nth terms
   - Sums of finite arithmetic series
   - Sums of finite and infinite geometric series
2. Develop an informal notion of limit.
3. Use inductive reasoning to form generalizations.

B. Functions and Relationships
1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs.
2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies.
   - Slope of a line or curve
   - Domain and range
   - Intercepts
   - Continuity
   - Maximum/minimum
   - Estimating roots of equations
   - Intersecting points as solutions of systems of equations
   - Rates of change
3. Understand and perform transformations on commonly-used functions.
   - Translations, reflections, dilations
   - Effects on linear and quadratic graphs of parameter changes in equations
   - Using graphing calculators or computers for more complex functions
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.
   - Linear vs. non-linear
   - Symmetry
   - Increasing/decreasing on an interval

C. Modeling
1. Use functions to model real-world phenomena and solve problems that involve varying quantities.
    o Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
    o Direct and inverse variation
    o Absolute value
    o Expressions, equations and inequalities
    o Same function can model variety of phenomena
    o Growth/decay and change in the natural world
    o Applications in mathematics, biology, and economics (including compound interest)
2. Analyze and describe how a change in an independent variable leads to change in a dependent one.
3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).

D. Procedures
1. Evaluate and simplify expressions.
    o Add and subtract polynomials
    o Multiply a polynomial by a monomial or binomial
    o Divide a polynomial by a monomial
2. Select and use appropriate methods to solve equations and inequalities.
    o Linear equations - algebraically
    o Quadratic equations - factoring (when the coefficient of x^2 is 1) and using the quadratic formula
    o All types of equations using graphing, computer, and graphing calculator techniques
    o Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

4. DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS:
A. Data Analysis
1. Use surveys and sampling techniques to generate data and draw conclusions about large groups.
    o Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling)
2. Evaluate the use of data in real-world contexts.
    o Accuracy and reasonableness of conclusions drawn
    o Bias in conclusions drawn (e.g., influence of how data is displayed)
    o Statistical claims based on sampling
3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.
4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.
5. Analyze data using technology, and use statistical terminology to describe conclusions.
    o Measures of dispersion: variance, standard deviation, outliers
    o Correlation coefficient
    o Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

B. Probability
1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair.
2. Use concepts and formulas of area to calculate geometric probabilities.
3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models.
4. Determine probabilities in complex situations.
   - Conditional events
   - Complementary events
   - Dependent and independent events
5. Estimate probabilities and make predictions based on experimental and theoretical probabilities.
6. Understand and use the "law of large numbers" (that experimental results tend to approach theoretical probabilities after a large number of trials).

C. Discrete Mathematics—Systematic Listing and Counting
1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students).
2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.
3. Justify solutions to counting problems.
4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability.

D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms
1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems.
   - Circuits that include every edge in a graph
   - Circuits that include every vertex in a graph
   - Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
   - Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)
2. Explore strategies for making fair decisions.
   - Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)
   - Determining how many Student Council representatives each class (9th, 10th, 11th, and 12th grade) gets when the classes have unequal sizes (apportionment).

5. MATHEMATICAL PROCESSES:
A. Problem Solving
1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts (cf. workplace readiness standard 8.3).
   - Open-ended problems
   - Non-routine problems
   - Problems with multiple solutions
   - Problems that can be solved in several ways
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.
4. Pose problems of various types and levels of difficulty.
5. Monitor their progress and reflect on the process of their problem solving activity.

B. Communication
1. Use communication to organize and clarify their mathematical thinking.
   - Reading and writing
   - Discussion, listening, and questioning
2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
3. Analyze and evaluate the mathematical thinking and strategies of others.
4. Use the language of mathematics to express mathematical ideas precisely.

C. Connections
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

D. Reasoning
1. Recognize that mathematical facts, procedures, and claims must be justified.
2. Use reasoning to support their mathematical conclusions and problem solutions.
3. Select and use various types of reasoning and methods of proof.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
5. Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures
   - Verifying conjectures using informal reasoning or proofs.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

E. Representations
1. Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

F. Technology
1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness standard 8.4-D).
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
5. Use computer software to make and verify conjectures about geometric objects.
6. Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards).