

Happy summer! Mrs. Homan would hate for you to forget calculus over the summer, so here are some problems to help refresh your memory. Please have these completed by the first day of school. We will have a pre-test upon return from summer. We will go through an in depth review of calculus AB topics, but here are a few problems to get you started. Have a good summer...and make good choices!

Solutions will be posted towards the end of summer on Mrs. Homan's website:

<https://sites.google.com/site/mrshomanpapiosouth/home>

1. Notation is very important. Improper use of notation confuses the meaning of a statement and indicates a lack of conceptual understanding. Each of the following examples contains at least one notational error. Identify the error(s) made, then rewrite the examples using proper notation.

a. $\int x^2 = \frac{x^3}{3} + C$

b. $\int x^2 - 2dx = \frac{x^3}{3} - 2x + C$

c. $\int \frac{1}{x} dx = \ln x + C$

d. $\int_0^5 \frac{2x}{x^2+1} dx = \int_0^5 \frac{du}{u} = [\ln|u|]_0^5 = [\ln(x^2+1)]_0^5 = \ln 26$

2. True or False?

a. $\lim_{x \rightarrow 2} \frac{1}{x-2} = \infty$

b. $\lim_{x \rightarrow 2} \frac{1}{x-2}$ does not exist

c. $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = \infty$

d. $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$

e. $\lim_{x \rightarrow 2^-} \frac{1}{x-2}$ does not exist

f. $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = \infty$

g. $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = -\infty$

h. $\lim_{x \rightarrow 2^+} \frac{1}{x-2}$ does not exist

3. Find the following limits:

a. $\lim_{x \rightarrow 2} \frac{3x^2 - 7}{3x^2 + 2x - 1}$

b. $\lim_{x \rightarrow 0} \frac{2x}{\sin 7x}$

c. $\lim_{x \rightarrow \infty} \frac{\sqrt{x+1} + 2}{5x - 6}$

d. $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x)$

e. $\lim_{x \rightarrow \frac{\pi}{2}^-} (\tan x)$

f. $\lim_{x \rightarrow \frac{\pi}{2}^+} (\tan x)$

g. $\lim_{x \rightarrow \infty} \frac{2 + x - 4x^2}{5x^2 - 1}$

h. $\lim_{x \rightarrow \infty} (\arctan x)$

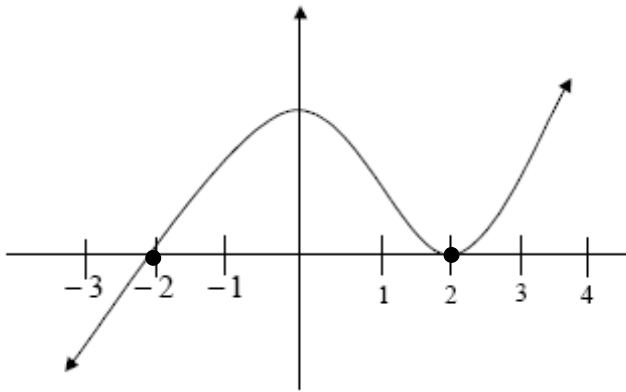
i. $\lim_{x \rightarrow 0} \frac{2x}{3 - \sqrt{x+9}}$

j. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

k. $\lim_{x \rightarrow -1} f(x)$ where $f(x) = \begin{cases} 2x+1, & x \leq -1 \\ 3, & -1 < x < 1 \\ 2x+1, & x \geq 1 \end{cases}$

4. Use the definition of the derivative to find $f'(x)$ where $f(x) = \frac{1}{2x+1}$.
5. Find $\frac{dy}{dx}$ for the following functions. (I'll even give you a break...don't simplify ☺)
- a. $y = 2x^5 - \sqrt{3x} - \cos 5x$
- b. $y = (\ln(x^2 + 1))(\tan(4 - 3x))$
- c. $y = \sin^4(3x + e^{2x})$
- d. $y = \frac{x^2 - 3x + 2}{5x^4 - 2x}$
- e. $y = \int_x^2 \frac{dt}{1-t^5}$
- f. $y = x^{\cos 2x}$
6. Let $y(x)$ be defined implicitly by $x^2 y^3 - 7e^{3x} + 4y = 19$. Find $\frac{dy}{dx}$.

7. Suppose the derivative, $f'(x)$, of a function is given by the following graph:



- a. Find the open intervals where f is increasing and those where f is decreasing.
- b. Find the open intervals where f is concave up and those where f is concave down.
- c. If $f(0) = 3$, draw a possible graph of f .
8. Find where the function $f(x) = x^5 - 2x^3 + 1$ is both increasing and concave down.
9. Determine the equations of the tangent line to the curve of $f(x) = \frac{1}{3}e^{2x}$ at the point where the curve crosses the line at $y = 5$.
10. A man 6 feet tall walks at a rate of 4 feet per second toward a light that is 20 feet above the ground. When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?
11. An airplane is flying in still air with airspeed of 270 miles per hour. If it is climbing at an angle of 28° , find the rate at which it is gaining altitude. (Calculator permitted)
12. Find the absolute maximum and absolute minimum of $f(x) = \frac{2}{3}x^3 - 8x^2 + 14x + 15$ on $[0, 4]$. Give both x and y coordinates.

13. Evaluate the following indefinite integrals.

a. $\int \frac{3}{x \ln x} dx$

b. $\int \csc^2(3x) e^{\cot 3x} dx$

c. $\int (e^{2x} + 2x - \sqrt[6]{x}) dx$

d. $\int \frac{2x}{\sqrt{1-3x}} dx$

e. $\int (x^3 + 1)^2 dx$

f. $\int \frac{1}{1+x^2} dx$

14. Evaluate the following definite integrals

a. $\int_0^4 (x^2 e^{2x^3}) dx$

b. $\int_0^{(\pi/4)^2} \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$

15. Use a right and left endpoint Riemann sum approximation to evaluate $\int_1^3 \frac{1}{x} dx$ using four subintervals.

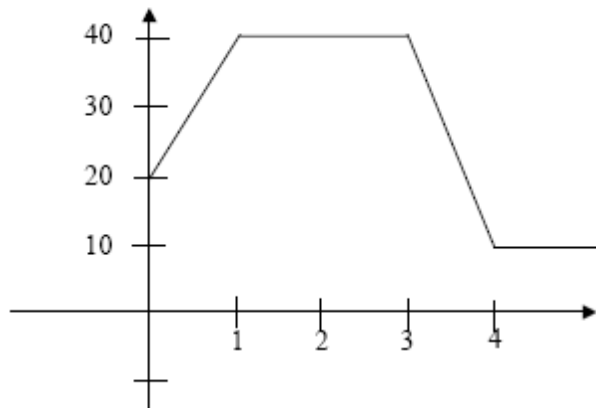
16. Approximate the distance traveled using a midpoint Riemann sum and the trapezoid method using the following data, assuming velocity is always positive: (Calculator Permitted)

t (seconds)	0	1	2	3	4	5	6
$v(t)$ (ft/s)	40	42	40	44	48	50	46

17. Find the area of the region between the graphs of $y = 2x^4$ and $y = x + 1$. (Calculator Permitted)

18. The temperature of a metal rod, 6 meters long, is $5x$ (in C°) at a distance x meters from one end of the rod. What is the average temperature of the rod?

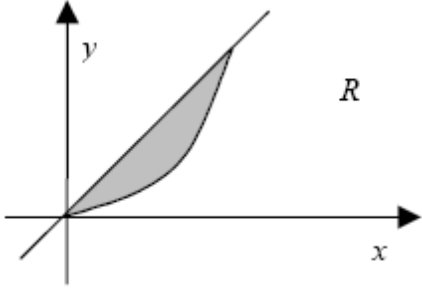
19. Let the velocity of a particle be given by the following graph:



a. Sketch the acceleration

b. What is the total distance traveled between $t = 1$ and $t = 4$?

20. Let R be the region bounded by the curves $y = x^2$ and $y = \frac{2}{3}x$. (Calculator Permitted)



- Find the volume of the solid obtained by revolving this region around the line $y = -1$.
- Find the volume of the solid obtained by revolving this region around the line $x = -1$.
- Find the volume of the solid obtained when the solid is formed when square cross sections are perpendicular to the x -axis.