

Year Long Mathematical Practices (MP):

Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.

MP.1 – Make sense of problems and persevere in solving them.

MP.2 – Reason abstractly and quantitatively

MP.3 – Construct viable arguments and critique reasoning of others.

MP.4 – Model with mathematics.

MP.5 – Use appropriate tools strategically.

MP.6 – Attend to precision.

MP.7 – Look for and make use of structure.

MP.8 – Look for and express regularity in repeated reasoning.

Semester 1 (August – December)

Unit 1: Polynomial Expression (2-3 weeks)

In Unit 1 students will be able to use geometric shapes to justify operations with polynomial expressions. Contextual situations, which utilize expressions, will be used to add, subtract and multiply polynomials. Students will be able to make connections between integers and polynomials by using a concrete-representational-abstract (C-R-A) approach to problem solving.

Overarching Standards for Unit 1

PAR.2: Interpret the structure of and perform operations with polynomials within a geometric framework.

MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 1

PAR.2.1: Interpret polynomial expressions of varying degrees that represent a quantity in terms of its given geometric framework. (See the Mathematical Modeling Framework and Statistical Reasoning Framework for contextual connections.)

PAR.2.2: Perform operations with polynomials and prove that polynomials form a system analogous to the integers in that they are closed under these operations.

PAR.2.3: Using algebraic reasoning, add, subtract, and multiply single variable polynomials.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 2: Geometric Foundations, Construction and Proof (3-4 weeks)

In this unit, students will be introduced to the basic building blocks of geometry. Students will further develop their understanding of basic elements by learning constructions using a straightedge and a compass. Students will begin the fundamental geometric practice of writing proofs.

Overarching Standards for Unit 2

- GSR.4:** Establish facts between angle relations and generate valid arguments to defend established facts. Prove theorems and solve geometric problems involving lines and angles to model and explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 2

- GSR.4.1:** Use the undefined notions of point, line, line segment, plane, distance along a line segment, and distance around a circular arc to develop and use precise definitions and symbolic notations to prove theorems and solve geometric problems.
 - GSR.4.3:** Make formal geometric constructions with a variety of tools and methods.
 - GSR.4.2** Classify quadrilaterals in the coordinate plane by proving simple geometric theorems algebraically.
 - GSR.4.4:** Prove and apply theorems about lines and angles to solve problems.
 - GSR.4.5:** Use geometric reasoning to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
 - MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
 - MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
 - MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 3: Congruence (4-5 weeks)

In this unit, students will be introduced to transformations in the coordinate plane, describe a sequence of transformations that will map one figure onto another, and describe transformations that will map a figure onto itself. Students will use transformations to develop an understanding of congruence and use this to prove theorems involving triangles.

Overarching Standards for Unit 3

- GSR.3:** Experiment with transformations in the plane to develop precise definitions for translations, rotations, and reflections and use these to describe symmetries and congruence to model and explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 3

- GSR.3.1:** Use geometric reasoning and symmetries of regular polygons to develop definitions of rotations, reflections, and translations
 - GSR.3.2:** Verify experimentally the congruence properties of rotations, reflections, and translations: lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure; parallel lines are taken to parallel lines.
 - GSR.3.3:** Use geometric descriptions of rigid motions to draw the transformed figures and to predict the effect on a given figure. Describe a sequence of transformations from one figure to another and use transformation properties to determine congruence.
 - GSR.3.4:** Explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions. Use congruency criteria for triangles to solve problems and to prove relationships in geometric figures.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
 - MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 4: Similarity (4-5 weeks)

In unit 4, students will explore nonrigid transformations and proportional reasoning to develop an understanding of similarity. Students will use the definition of dilation to describe similarity and the criterion for triangles to be similar. They will use this to prove similarity involving triangles.

Overarching Standards for Unit 4

- GSR.5:** Describe dilations in terms of center and scale factor and use these terms to describe properties of dilations; use the precise definition of a dilation to describe similarity and establish the criterion for triangles to be similar; use these terms, definitions, and criterion to prove similarity, model, and explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 4

- GSR.5.1:** Verify experimentally the properties of dilations.
- GSR.5.2:** Given two figures, use and apply the definition of similarity in terms of similarity transformations.
- GSR.5.3** Use the properties of similarity transformations to establish criterion for two triangles to be similar. Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- GSR.5.4:** Construct formal proofs to justify and apply theorems about triangles.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 5: Right Triangle Trigonometry (2-3 weeks)

In this unit, students will use similarity in right triangles to understand right triangle trigonometry. They will use the relationship between the sine and cosine of complementary angles to solve problems involving right triangles.

Overarching Standards for Unit 5

- GSR.6:** Examine side ratios of similar triangles; use the relationship between right triangles to develop an understanding of sine, cosine, and tangent to solve geometric problems and to model and explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 5

- GSR.6.1:** Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- GSR.6.3:** Use trigonometric ratios and the Pythagorean Theorem to solve for sides and angles of right triangles in applied problems.
- GSR.6.2:** Explain and use the relationship between the sine and cosine of complementary angles.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Semester 2 (January – May)

Unit 6: Circles (5-6 weeks)

In this unit, students will examine and apply theorems involving angle relationships, find arc lengths, and find the area of sectors of circles. Students will graph and write equations of circles. Students will extend their understanding of arc length in circles and begin exploring angles within the unit circle as a fraction of the circumference all the way around the unit circle. They will fluently convert between degrees and radians. Students will use special right triangles concepts to define the value of the sine, cosine, and tangent and understand the x (cosine), y (sine), and r (1) values of each angle measure found at all terminal angles that are multiples of $\pi/6$, $\pi/3$ and $\pi/4$ around the unit circle, and will be able to find the sine, cosine, and tangent at all of these radian measures, as well.

Overarching Standards for Unit 6

- GSR.8:** Examine and apply theorems involving circles; describe and derive arc length and area of a sector; and model and explain real-life situations involving circles.
- GSR.7:** Explore the concept of a radian measure and special right triangles.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 6

- GSR.8.1:** Identify and apply angle relationships formed by chords, tangents, secants and radii with circles.
- GSR.8.3:** Write and graph the equation of circles in standard form.
- GSR.8.2:** Using similarity, derive the fact that the length of the arc (arc length) intercepted by an angle is proportional to the radius; derive the formula for the area of a sector. Solve mathematically applicable problems involving applications of arc length and area of sector.
- GSR.7.1:** Explore and interpret a radian as the ratio of the arc length to the radius of a circle.

- GSR.7.2:** Explore and explain the relationship between radian measures and degree measures and convert fluently between degree and radian measures.
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- GSR.7.3:** Use special right triangles on the unit circle to determine the values of sine, cosine, and tangent for 30° ($\pi/6$), 45° ($\pi/4$), and 60° ($\pi/3$) angle measures. Use reflections of triangles to determine reference angles and identify coordinate values in all four quadrants of the coordinate plane.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 7: Equations and Measurement (3-4 weeks)

In this unit, students will develop informal arguments for geometric formulas and solve contextual problems involving volume.

Overarching Standards for Unit 7

- GSR.9:** Develop informal arguments for geometric formulas using dissection arguments, limit arguments, and Cavalieri's principle; solve realistic problems involving volume; explore and visualize relationships between two-dimensional and three-dimensional objects to model and explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 7

- GSR.9.1:** Use volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems including right and oblique solids.
- GSR.9.2:** Use geometric shapes, their measures, and their properties to describe objects and approximate volumes.
- GSR.9.3:** Apply concepts of density based on area and volume in modeling situations.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 8: Probability and Statistics (6-7 weeks)

In this unit, students will organize real-life data in two-way frequency tables. They will use the two-way frequency tables to find probabilities. Students calculate, model, and interpret probabilities of compound events. Students will calculate permutations and combinations within real-world contexts and develop probability distributions based on the entire sample space. Students will calculate expected value of a probability distribution and understand it to be the mean of that probability distribution. Using expected value, students will make decisions about risk vs. reward in real-world situations such as games of chance and insurance.

Overarching Standards for Unit 8

- PR.10:** Solve problems involving the probability of compound events to make informed decisions; interpret expected value and measures of variability to analyze probability distributions.
- DSR.11:** Examine real-life situations presented in a two-way frequency table to calculate probabilities, to model categorical data, and to explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 8

- PR.10.1:** Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events. Apply the Addition Rule conceptually, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answers in context.
- PR.10.2** Apply and interpret the general Multiplication Rule conceptually to independent events of a sample space, $P(A \text{ and } B) = [P(A)] \times [P(B|A)] = [P(B)] \times [P(A|B)]$ using contingency tables or tree diagrams.
- PR.10.3:** Use conditional probability to interpret risk in terms of decision-making and investigate questions such as those involving false positives or false negatives from screening tests.
- PR.10.4:** Define permutations and combinations and apply this understanding to compute probabilities of compound events and solve meaningful problems
- PR.10.5:** Interpret the probability distribution for a given random variable and interpret the expected value.
- PR.10.6:** Develop a probability distribution for variables of interest using theoretical and empirical (observed) probabilities and calculate and interpret the expected value.

- PR.10.7:** Calculate the expected value of a random variable and interpret it as the mean of a given probability distribution.
- PR.10.8:** Compare the payoff values associated with the probability distribution for a random variable and make informed decisions based on expected value and measures of variability.

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- DSR.11.1:** Construct and summarize categorical data for two categories in two-way frequency tables.
- DSR.11.2:** Use categorical data in two-way frequency tables to calculate and interpret probabilities within the given framework. (See the Mathematical Modeling Framework and Statistical Reasoning Framework for contextual connections.)

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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 9: Culminating Capstone Unit (1-2 weeks)

The capstone unit applies content that has already been learned in previous interdisciplinary PBLs and units throughout the school year. The capstone unit is an interdisciplinary unit that allows students to create a presentation, report, or demonstration that could include their models used to answer an overarching driving question. (e.g., Students can present their solution(s), findings, project, or answer to the driving question to a larger audience during the culminating capstone unit.)

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- GSR.3:** Experiment with transformations in the plane to develop precise definitions for translations, rotations, and reflections and use these to describe symmetries and congruence to model and explain real-life phenomena.
- GSR.4:** Establish facts between angle relations and generate valid arguments to defend established facts. Prove theorems and solve geometric problems involving lines and angles to model and explain real-life phenomena.
- GSR.5:** Describe dilations in terms of center and scale factor and use these terms to describe properties of dilations; use the precise definition of a dilation to describe similarity and establish the criterion for triangles to be similar; use these terms, definitions, and criterion to prove similarity, model, and explain real-life phenomena.
- GSR.6:** Examine side ratios of similar triangles; use the relationship between right triangles to develop an understanding of sine, cosine, and tangent to solve geometric problems and to model and explain real-life phenomena.
- GSR.7:** Explore the concept of a radian measure and special right triangles.
- GSR.8:** Examine and apply theorems involving circles; describe and derive arc length and area of a sector; and model and explain real-life situations involving circles.
- GSR.9:** Develop informal arguments for geometric formulas using dissection arguments, limit arguments, and Cavalieri's principle; solve realistic problems involving volume; explore and visualize relationships between two-dimensional and three-dimensional objects to model and explain real-life phenomena.
- PR.10:** Solve problems involving the probability of compound events to make informed decisions; interpret expected value and measures of variability to analyze probability distributions.
- DSR.11:** Examine real-life situations presented in a two-way frequency table to calculate probabilities, to model categorical data, and to explain real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.