Year Long Mathematical Practices (MP):

Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.

MP.1 – Make sense of problems and persevere in solving them.

MP.2 – Reason abstractly and quantitatively

MP.3 – Construct viable arguments and critique reasoning of others.

MP.4 – Model with mathematics.

MP.5 – Use appropriate tools strategically.

MP.6 – Attend to precision.

MP.7 – Look for and make use of structure.

MP.8 – Look for and express regularity in repeated reasoning.

Semester 1 (August – December)

Unit 1: Modeling with Rational and Piecewise-Defined (4-5 weeks)

Functions Students will reason abstractly and quantitatively while modeling contextual mathematical problems using rational and piecewise functions. This unit requires the development of attention to precision, notational fluency, and problem-solving perseverance. Students will extend prior solving and graphical analysis work with rational functions from Advanced Algebra to include division methods, analysis of extraneous solutions, and the use of limit notation to describe vertical and horizontal asymptotes. Students will utilize both interval notation and inequality notation. Work with one-sided limits, two-sided limits, and continuity will occur in graphical, numerical, and analytical situations. Analysis of absolute value functions will occur as piecewise-defined functions both algebraically and graphically. Students will analyze piecewise-defined functions graphically, numerically, and algebraically by building upon prior knowledge of functions. This work provides reinforcement of prior function study in the new piecewise context in order to increase proficiency and promote Calculus readiness.

Overarching Standards for Unit 1

FGR.2: Analyze the behaviors of rational and piecewise functions to model contextual mathematical problems.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 1

FGR.2.1: Graph piecewise-defined functions, including step functions and absolute value functions.
FGR.2.2 Describe characteristics by interpreting the algebraic form and graph of a piecewise-defined function.
FGR.2.3: Represent the limit of a function using both the informal definition and the graphical interpretation in the context of piecewise-defined functions; interpret limits expressed in analytic notation.

FGR.2.4: Divide polynomials using various methods.

FGR.2.5: Graph rational functions and identify key characteristics.
FGR2.6: Represent the behavior of a rational function using limit notation for vertical and horizontal asymptotes and end behavior.
FGR.2.7: Represent the limit of a function using both the informal definition and the graphical interpretation in the context of rational functions; interpret limits expressed in analytic notation.

FGR.2.8: Solve simple rational equations in one variable and give examples showing how extraneous solutions may arise.
FGR.2.9: Perform partial fraction decomposition of rational functions using non-repeated linear factors.

MM.1.1: Explain mathematically applicable problems using a mathematical model.
MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 2: Modeling with Trigonometric Expressions and Functions (5-6 weeks)

In Advanced Algebra, students investigated radian measure conceptually and visually on a circle and developed the one counterclockwise revolution radian measures corresponding to reference angles of $\pi/6$, $\pi/4$, and $\pi/3$ through conversions from degrees. This unit will build the unit circle, connect radian measures to portions of 2π, work with radian measure not containing π, and utilize angles both in the clockwise and counterclockwise direction. Students will define the six trigonometric ratios in terms of x, y, and r using a circle centered at the origin of the coordinate plane and use the parametric interpretation of the coordinates on the unit circle as (cos(t), sin(t)). The six trigonometric functions will be developed and utilized for modeling periodic phenomena. Characteristics of the six trigonometric functions and their graphs will be investigated and used to classify these functions based on commonalities. These trigonometric relationships will be used to derive the fundamental trigonometric identities. This unit requires the development of attention to precision, notational fluency, and problem-solving perseverance as the trigonometric functions are used in a variety of contexts. The need to restrict the domain of each trigonometric function to develop its corresponding inverse function will be explored. The use of the inverse function will be related to prior inverse studies with restriction awareness developed for understanding subsequent contextual situations such as vectors, technology use, etc.

Overarching Standards for Unit 2

FGR.3: Utilize trigonometric expressions to solve problems and model periodic phenomena with trigonometric functions.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 2

FGR.3.1: Use the concept of a radian as the ratio of the arc length to the radius of a circle to establish the existence of 2π radians in one revolution.

FGR.3.2: Utilize right triangles on the unit circle to determine the values of the six trigonometric ratios for $\pi/6$, $\pi/4$, and $\pi/3$. Use reflections of the triangles as reference angles to establish known values in all four quadrants of the coordinate plane.

FGR.3.3: Define the six trigonometric ratios in terms of x, y, and r using the unit circle centered at the origin of the coordinate plane. Interpret radian measures of angles as a rotation both counterclockwise and clockwise around the unit circle.

FGR.3.4: Derive the fundamental trigonometric identities.

FGR.3.5: Determine the value(s) of trigonometric functions for a set of given conditions.

FGR.3.6: Graph and write equations of trigonometric functions using period, phase shift, and amplitude in modeling contexts.

FGR.3.7: Classify the six trigonometric functions as even or odd and describe the symmetry.

FGR.3.8: Restrict the domain of a trigonometric function to create an invertible function and graph the inverse function. Evaluate inverse trigonometric expressions.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 3: Applying Trigonometric Identities and Equations (4-5 weeks)

This unit requires students to build upon their trigonometric knowledge and use the fundamental trigonometric identities to simplify expressions for easier application in contextual situations as well as to develop and verify new identities. Students will make use of structure, use abstract reasoning, and attend to precision in the development of the sum, difference, double-angle, and half-angle formulas for sine, cosine, and tangent. Identities will be used in problem solving situations and modeling contexts. The unit strengthens the development of notational fluency and problem-solving perseverance. Right triangular trigonometry is extended to oblique triangle situations using the Law of Sines, Law of Cosines, and Area Formula. These tools will be used strategically and appropriately in multiple and varied contextual situations.

Overarching Standards for Unit 3

AGR.4: Manipulate, prove, and apply trigonometric identities and equations to solve contextual mathematical problems.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 3

AGR.4.1: Apply the fundamental trigonometric identities to simplify expressions and verify other identities.
AGR.4.2: Use sum, difference, double-angle, and half-angle formulas for sine, cosine, and tangent to establish other identities and apply them to solve problems.

AGR.4.3: Solve trigonometric equations arising in modeling contexts.

AGR.4.4: Prove and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.
AGR.4.5: Determine the area of an oblique triangle.

MM.1.1: Explain mathematically applicable problems using a mathematical model.
MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Semester 2 (January – May)

Unit 4: Modeling with Conic Sections and Polar Equations (4-5 weeks)

Students will contrast their prior study of functions with the algebraic study of implicit forms of conic sections. This unit requires the students to make use of structure, use abstract reasoning, and attend to precision when working with conic sections graphically and algebraically. Students will use the tool of completing the square to arrive at the standard form of a conic section. Work with implicit curves reinforces and develops concepts needed for future STEM field studies. The polar coordinate system will be defined and related to the rectangular coordinate system through appropriate conversions. Investigation of special polar equations, their application to contextual situations, and an appreciation for the simpler polar form will occur. The connection between the graph of a trigonometric function like $y = 3\cos(x)$ on a rectangular plane and its manifestation on the polar plane as $r = 3\cos(\theta)$ with a rotational input and radial distance output will be analyzed. This unit will require students to make sense of problems, develop perseverance, and strategically use appropriate tools.

Overarching Standards for Unit 4

GSR.5: Analyze the behaviors of conic sections and polar equations to model contextual mathematical problems.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 4

GSR.5.1: Identify and graph different conic sections given the equations in standard form.
GSR.5.2: Identify different conic sections in general form and complete the square to convert the equation of a conic section into standard form.

GSR.5.3: Define polar coordinates and relate polar coordinates to Cartesian coordinates.
GSR.5.4: Classify special polar equations and apply to contextual situations.
GSR.5.5: Graph equations in the polar coordinate plane with and without the use of technology.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 5: Modeling with Vector Quantities (4-5 weeks)

Application of trigonometry to modeling contextual mathematical problems with the use of vectors and parametric equations will require students to reason abstractly and quantitatively, make sense of problems, and attend to precision. Notational fluency with different vector representations and situations should be developed. Work with vectors in component form as well as magnitude and direction form and the articulation of each meaning as related to their geometric and graphical interpretation will occur. Vector operations and use of varied vector addition methods will be incorporated. This unit provides the opportunity to work with an abundance of contextual problems involving force, velocity, etc. Parametric equations will be articulated as models for the pathway of a moving object with appropriate indications of direction. Parametric equation sets of the unit 4 conic sections and their corresponding relationship to the Pythagorean identities will be explored. Application of parametric equations to model contextual situations and extensive work with vectors develops relevant knowledge for implementation in STEM college and career fields.

Overarching Standards for Unit 5

AGR.6: Represent and model vector quantities to solve problems in contextual situations.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 5

AGR.6.1: Represent vector quantities as directed line segments; represent magnitude and direction of vectors in component form using appropriate mathematical notation.
AGR.6.2: Add and subtract vectors and multiply vectors by a scalar to find the resultant vector.
AGR.6.3: Add and subtract vectors on a coordinate plane using different methods.
AGR.6.4: Solve contextual vector problems, such as those involving velocity, force, and other quantities.

AGR.6.5: Sketch the graph of a curve represented parametrically, indicating the direction of motion.
AGR.6.6: Apply parametric equations to contextual problems.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 6: Modeling with Sequences and Series (4-5 weeks)

Articulation in the difference between a sequence and a series and working with them from a number sense perspective is critical for success in this unit. Sequences will be investigated graphically, numerically, and symbolically. Fluent work with sequences in contextual situations requires students to make sense of problems and attend to precision. The meaning of convergence and divergence of a sequence using technology and number sense will be explored. Divergence will be linked to non-convergence and to the meaning of the non-existence of a limit. Understanding a series as the sum of a sequence and the limit of its partial sums is crucial for students to fully explore. Series will be explored graphically, numerically, and symbolically. This unit requires the development of notational fluency and precision in thinking. Derivation of the sum formulas for both the finite geometric series and infinite geometric series will occur. Applying the finite and infinite geometric sum formulas to contextual modeling situations will require students to reason abstractly and quantitatively. Extensive work with sequences and series increases the development of number sense and provides foundational work of relevant knowledge needed in STEM college and career fields.

Overarching Standards for Unit 6

PAR.7: Demonstrate how sequences and series apply to mathematical models in real-life situations.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 6

PAR.7.1: Demonstrate that sequences are functions whose domain is the set of natural numbers.
PAR.7.2: Represent sequences graphically, numerically, and symbolically.
PAR.7.3: Determine the limit of a sequence if it exists.

PAR.7.4: Demonstrate that a series is the sum of the sequence and represent series graphically, numerically, and symbolically.
PAR.7.5: Describe the behavior of a series in terms of the limit of its partial sums.
PAR.7.6: Derive and use the sum formula of a finite geometric series to solve contextual problems to model real-life situations
PAR.7.7: Derive and use the sum formula of an infinite geometric series to solve contextual problems to model real-life situations.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 7: Culminating Capstone Unit (1-2 weeks)

The capstone unit applies content that has already been learned in previous interdisciplinary PBLs and units throughout the school year. The capstone unit is an interdisciplinary unit that allows students to create a presentation, report, or demonstration that could include their models used to answer an overarching driving question. (e.g., Students can present their solution(s), findings, project, or answer to the driving question to a larger audience during the culminating capstone unit.)

FGR.2: Analyze the behaviors of rational and piecewise functions to model contextual mathematical problems.
FGR.3: Utilize trigonometric expressions to solve problems and model periodic phenomena with trigonometric functions.
AGR.4: Manipulate, prove, and apply trigonometric identities and equations to solve contextual mathematical problems.
GSR.5: Analyze the behaviors of conic sections and polar equations to model contextual mathematical problems.
AGR.6: Represent and model vector quantities to solve problems in contextual situations.
PAR.7: Demonstrate how sequences and series apply to mathematical models in real-life situations.
MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.