

Year Long Mathematical Practices (MP):

Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.

MP.1 – Make sense of problems and persevere in solving them.

MP.2 – Reason abstractly and quantitatively

MP.3 – Construct viable arguments and critique reasoning of others.

MP.4 – Model with mathematics.

MP.5 – Use appropriate tools strategically.

MP.6 – Attend to precision.

MP.7 – Look for and make use of structure.

MP.8 – Look for and express regularity in repeated reasoning.

Semester 1 (August – December)

Unit 1: Descriptive and Inferential Statistics (5-6 weeks)

This unit delves into interpretation of statistics, rather than pure computation of statistics. Students will learn best practices to plan, interpret, and critique studies using samples within a population to make inferences about the population, at-large.

Overarching Standards for Unit 1

DSR.2: Communicate descriptive and inferential statistics by collecting, critiquing, analyzing, and interpreting real-world data.

MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 1

DSR.2.1: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Distinguish between primary and secondary data and how it affects the types of conclusions that can be drawn.

DSR.2.2: When collecting and considering data, critically evaluate ethics, privacy, potential bias, and confounding variables along with their implications for interpretation in answering a statistical investigative question. Implement strategies for organizing and preparing big data sets.

DSR.2.8: Summarize and evaluate reports based on data for appropriateness of study design, analysis methods, and statistical measures used.

DSR.2.3: Distinguish between population distributions, sample data distributions, and sampling distributions. Use sample statistics to make inferences about population parameters based on a random sample from that population and to communicate conclusions using appropriate statistical language.

DSR.2.4: Calculate and interpret z-scores as a measure of relative standing and as a method of standardizing units.

DSR.2.5: Given a normally distributed population, estimate percentages using the Empirical Rule, z-scores, and technology.

DSR.2.6: Model sample-to-sample variability in sampling distributions of a statistic using simulations taken from a given population.

DSR.2.7: Given a margin of error, develop and compare confidence intervals of different models to make conclusions about reliability.

MM.1.1: Explain mathematically applicable problems using a mathematical model.

MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

MM.1.3: Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.

MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 2: Modeling Polynomial Functions (4-5 weeks)

This unit begins with a deeper exploration into quadratic functions to include those with non-real solutions. Students will solve systems of quadratic equations and perform quadratic regressions. They will perform computations with complex numbers (addition, subtraction, and multiplication) using properties of operations. Moving into exploration of polynomial functions, students will identify the number of zeros and end behavior for any polynomial, or to write a viable equation for the polynomial, given its zeros. Students will graph and identify the key features such as zeros of polynomials of degree greater than 2 either by inspection of a pre-graphed or pre-factored equation, or by using technology.

Overarching Standards for Unit 2

- FGR.5:** Extend exploration of quadratic solutions to include real and non-real numbers and explore how these numbers behave under familiar operations and within real-world situations; create polynomial expressions, solve polynomial equations, graph polynomial functions, and model real-world phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 2

- FGR.5.1:** Graph and analyze quadratic functions in contextual situations and include analysis of data sets with regressions.
 - FGR.5.2:** Define complex numbers i such that $i^2 = -1$ and show that every complex number has the form $a + bi$ where a and b are real numbers and that the complex conjugate is $a - bi$.
 - FGR.5.3:** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
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- FGR.5.4:** Use the structure of an expression to factor quadratics.
 - FGR.5.5:** Write and solve quadratic equations and inequalities with real coefficients and use the solution to explain a contextual situation.
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- FGR.5.6:** Solve systems of quadratic and linear functions to determine points of intersection.
 - FGR.5.7:** Create and analyze quadratic equations to represent relationships between quantities as a model for contextual situations.

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- FGR.5.8:** Identify the number of zeros that exist for any polynomial based upon the greatest degree of the polynomial and the end behavior of the polynomial by observing the sign of the leading coefficient.
- FGR.5.9:** Identify zeros of polynomial functions using technology or pre-factored polynomials and use the zeros to construct a graph of the function defined by the polynomial function. Analyze identify key features of these polynomial functions.
- FGR.5.10:** Use the structure of an expression to factor polynomials, including the sum of cubes, the difference of cubes, and higher-order polynomials that may be expressed as a quadratic within a quadratic.
- FGR.5.11:** Using all the zeros of a polynomial function, list all the factors and multiply to write a multiple of the polynomial function in standard form.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 3: Exponential and Logarithmic Functions (5-6 weeks)

Students will find inverses of functions by hand, in models, charts, and graphs and verify by composition or graphing that one function is an inverse of another. Introduction of composition of functions to verify inverses is included as a strategy/method for this unit (in addition to numerical and graphical verification). Based on what students already know, students will explore logarithmic functions as inverses of exponential functions. They will move into graph logarithmic and exponential functions and identify key features. Students will use logarithmic properties and inverses to solve real-life exponential and logarithmic problems in one variable.

Overarching Standards for Unit 3

- FGR.3:** Explore and analyze structures and patterns for exponential and logarithmic functions and use exponential and logarithmic expressions, equations, and functions to model real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 3

- FGR.3.1:** Find the inverse of exponential and logarithmic functions using equations, tables, and graphs, limiting the domain of inverses where necessary to maintain functionality, and prove by composition or verify by inspection that one function is the inverse of another.
- FGR.3.2:** Analyze, graph, and compare exponential and logarithmic functions.
- FGR.3.3:** Use the definition of a logarithm, logarithmic properties, and the inverse relationship between exponential and logarithmic functions to solve problems in context.
- FGR.3.4:** Create exponential equations and use logarithms to solve contextual problems for which only one variable is unknown.
- FGR.3.5:** Create and interpret logarithmic equations in one variable and use them to solve problems.
- FGR.3.6:** Create, interpret, and solve exponential equations to represent relationships between quantities and analyze the relationships numerically with tables, algebraically, and graphically.
- FGR.3.7:** Create, interpret, and solve logarithmic equations in two or more variables to represent relationships between quantities.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.

Advanced Algebra

Mathematics



- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Semester 2 (January – May)

Unit 4: Investigating Radical Functions (3-4 weeks)

In this unit, students will write radical functions as functions with rational exponents and use these to solve real-world problems. Students will analyze key features of radical graphs and will select tools (including technology) to model radical functions. Given real-world situations, students should solve radical/rational exponent equations in one variable (recognizing extraneous solutions) or to graph and analyze radical/rational exponent functions in two variables to arrive at conclusions to real-world problems.

Overarching Standards for Unit 4

- FGR.4:** Explore and analyze structures and patterns for radical functions and use radical expressions, equations, and functions to model real-life phenomena.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 4

- FGR.4.1:** Rewrite radical expressions as expressions with rational exponents. Extend the properties of integer exponents to rational exponents.
- FGR.4.2:** Solve radical equations in one variable and give examples showing how extraneous solutions may arise.
- FGR.4.3:** Analyze and graph radical functions.
- FGR.4.4:** Create, interpret and solve radical equations with one unknown value and use them to solve problems that model real-world situations.
- FGR.4.5:** Create, interpret, and solve radical equations in two or more variables to represent relationships between quantities.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 5: Investigating Linear Algebra and Matrices (2-3 weeks)

Students will represent real-world data into matrices and perform calculations within a real-world context. Students will have the opportunity to use technology for matrix calculations involving matrices greater than 2×2 in dimension. Students will use linear programming to solve real-world optimization problems.

Overarching Standards for Unit 5

- PAR.6:** Represent data with matrices, perform mathematical operations, and solve systems of linear equations leading to real-world linear programming applications.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 5

- PAR.6.1:** Use matrices to represent data, and perform mathematical operations with matrices and scalars, demonstrating that some properties of real numbers hold for matrices, but that others do not.
 - PAR.6.2:** Rewrite a system of linear equations using a matrix representation.
 - PAR.6.3:** Use the inverse of an invertible matrix to solve systems of linear equations.
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- PAR.6.4:** Utilize linear programming to represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as solutions or non-solutions under the established constraints in real-world problems.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
 - MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
 - MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
 - MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 6: Trigonometry and the Unit Circle (3-4 weeks)

Students will begin exploring angles within the unit circle as a fraction of the circumference all the way around the unit circle. They will fluently convert between degree measures and radian measures. They will explore the concepts of terminal angles on the unit circle. Students will define and analyze the x (cosine), y (sine), and r (1) values of each angle measure of 30° ($\pi/6$), 45° ($\pi/4$) and 60° ($\pi/3$), and their associated reflected angles within one counterclockwise revolution of the unit circle. Students will also be able to find the sine, cosine, and tangent at all of these radian measures, as well. Lastly, students will solve simple trigonometric equations.

Overarching Standards for Unit 6

- GSR.7:** Develop an introductory understanding of the unit circle; solve trigonometric equations using the unit circle.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 6

- GSR.7.1:** Define the three basic trigonometric ratios in terms of x, y, and r using the unit circle centered at the origin of the coordinate plane.
 - GSR.7.2:** Apply understanding of the angle measures and coordinates of the unit circle to solve practical, real-life problems involving trigonometric equations.
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- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
 - MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
 - MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
 - MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 7: Exploring Rational Functions (2-3 weeks)

This unit is an introduction to rational functions. Rational functions will be explored in greater depth in Precalculus. In this unit, students will rewrite simple rational expressions, and perform addition, subtraction, multiplication, and division. Students will explore rational functions as models for real-life phenomena.

Overarching Standards for Unit 7

- FGR.8:** Analyze the behaviors of rational functions to model applicable, mathematical problems.
- MM.1:** Apply mathematics to real-life situations; model real-life phenomena using mathematics.

Standards for Student Mastery for Unit 7

- FGR.8.1:** Rewrite simple rational expressions in equivalent forms.
- FGR.8.2:** Add, subtract, multiply and divide rational expressions, including problems in context and express rational expressions in irreducible form.
- FGR.8.3:** Graph rational functions, identifying key characteristics.
- FGR.8.4:** Solve simple rational equations in one variable and give examples showing how extraneous solutions may arise.
- MM.1.1:** Explain mathematically applicable problems using a mathematical model.
- MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.
- MM.1.3:** Using abstract and quantitative reasoning, make decisions about information and data from a mathematically applicable situation.
- MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

Unit 8: Culminating Capstone Unit (1-2 weeks)

The capstone unit applies content that has already been learned in previous interdisciplinary PBLs and units throughout the school year. The capstone unit is an interdisciplinary unit that allows students to create a presentation, report, or demonstration that could include their models used to answer an overarching driving question. (e.g., Students can present their solution(s), findings, project, or answer to the driving question to a larger audience during the culminating capstone unit.)

DSR.2: Communicate descriptive and inferential statistics by collecting, critiquing, analyzing, and interpreting real-world data.

FGR.3: Explore and analyze structures and patterns for exponential and logarithmic functions and use exponential and logarithmic expressions, equations, and functions to model real-life phenomena.

FGR.4: Explore and analyze structures and patterns for radical functions and use radical expressions, equations, and functions to model real-life phenomena.

FGR.5: Extend exploration of quadratic solutions to include real and non-real numbers and explore how these numbers behave under familiar operations and within real-world situations; create polynomial expressions, solve polynomial equations, graph polynomial functions, and model real-world phenomena.

PAR 6: Represent data with matrices, perform mathematical operations, and solve systems of linear equations leading to real-world linear programming applications.

GSR.7: Develop an introductory understanding of the unit circle; solve trigonometric equations using the unit circle.

FGR.8: Analyze the behaviors of rational functions to model applicable, mathematical problems.

MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.