UNIT 9: SOLVING QUADRATIC EQUATIONS

Part A: Video Tutorial Section

Video 1 and 2:

https://www.youtube.com/watch?v=saXEJW1Y1Oo (**Solving Quadratic Equations by Graphing**)

https://www.youtube.com/watch?v=JHUju_Qkqbg (**Using a Graphic Calculator to Solve Quadratic Functions)**)

Video 3:

https://www.youtube.com/watch?v=55G8037gsKY (**Solving Quadratic Functions Using Square Roots**)

 Videos 4 and 5:

https://www.youtube.com/watch?v=9UgvReIRsp8 (**Completing the Square**)

https://www.youtube.com/watch?v=xGOQYTo9AKY (**More examples of Completing the Square**)

Videos 6 and 7:

 https://www.youtube.com/watch?v=3ayhvAI3IeY (**Using the Quadratic Formula**)

https://www.youtube.com/watch?v=i7idZfS8t8w (**More Examples of Using the Quadratic Formula**)

Video 8:

https://www.youtube.com/watch?v=HNVY6IqwM74 (**The Basics of Solving Systems of Linear & Quadratic Equations**)

Videos 9 and 10:

https://www.youtube.com/watch?v=7dOLwi_Hpq0 **(Using Substitution to Solve Systems of Linear and Quadratic Equations)**

https://www.youtube.com/watch?v=V7H1oUHXPkg **(More Examples of Using Substitution to Solve Systems of Linear and Quadratic Equations)**

Video 11:

https://www.youtube.com/watch?v=1dZCK8QW07Y **(Solving Quadratic Equations Using Graphing, Substitution and Elimination Method)**

https://www.youtube.com/watch?v=PMwsNDbKCh4 **(More Examples of Solving Quadratic Equations Using Graphing, Substitution and Elimination Method)**

Part B : Vocabulary, Hints and Explanations

Solving quadratic equations by graphing

As students have already learned, a **quadratic equation** is a nonlinear equation (graphs as a parabola), that can be written in standard form as:

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

At this stage, the student should become familiar and capable with the graphing calculator!!

In unit 8 the student learned:

- A quadratic equation has two roots if its graph has two x-intercepts
- A quadratic equation has one root it its graph has one x-intercept
- A quadratic equation has no real solutions if its graph has no x-intercepts.

In solving real life scenarios, the student may determine both a positive and a negative solution. Most real world scenarios are only solved with a positive solution.

Solving quadratic equations using square roots

Any time would see a number squared (x^2), you can determine the answer by taking the square root of both sides of the equation. This is the same as any balanced equation, in which you perform the same operation to both sides of the equation/equal sign.

When solving quadratic equations, an answer may be two real solutions, one real solution, or no real solutions.

Solve $x^2 = d$ (d being the answer) by taking the square root of each side.

Note: the words "square root" have been used because the format does not allow for use of the sign. When teaching, use the square root sign.

Two Real Solutions: When d is > 0, $x^2 = d$ had two real solutions x = +/- square root of d

Example: $3x^2 - 75 = 0$

$3x^2 = 75$

$x^2 = 25$

x = square root of 25

$x = +/- 5$

One real solution: When $d = 0$, $x^2 = d$ has one real solution, $x = 0$

Example: $x^2 - 27 = -27$

$x^2 = 0$

$x = 0$

No real solution: When $d < 0$, $x^2 = d$ has no real solution

Think – can any number square equal a negative number? NO!

Example: $-3x^2 + 10 = 13$

$-3x^2 = 3$

$x^2 = -1$

Hint: Be sure the student understands that you can square a negative number to achieve a positive answer ($-3^2 = 9$). BUT you cannot square a number and achieve a negative answer!

Solving Quadratic Equations by Completing the Square

Students may be confused by this process.

To complete the square, you are solving an equation that does not have the last term of the equation as in a quadratic equation. You have the first two terms:

$$x^2 + bx$$

But lack the final term: c

To create a final term:

1. Find half of b
2. Square the result from step 1
3. Add that to the equation

Example: $x^2 + 10x$

1. Find half of 10 = 5
2. Square the result $5^2 = 25$
3. Add to the equation $x^2 + 10x + 25$
4. Solve - $(x + 5)^2$

The solution will be $(x + \frac{1}{2} b)^2$

You can encourage students to memorize the quadratic formula. However, be sure they know that it is in the reference sheet on the Regents. (please see Regents reference table for the formula)

The quadratic equation is represented by:

$$ax^2 + bx + c$$

When using real numbers, it may help a student to write the letters over the terms.

$$a \quad + b \quad + c$$

$$2x^2 + -5x \quad + 3$$

$$a = 2 \quad b = -5 \quad c = 3$$

Choosing a Solution Method

There are five methods for solving a quadratic equation. There are advantages and disadvantages to each. The table below offers suggestions:

METHOD	ADVANTAGES	DISADVANTAGES
Factoring	• Straightforward when equation can be factored easily	• Some equations are not factorable
Graphing	• Can easily see the number of solutions • Use when approximate solutions are sufficient • Can use a graphing calculator	• May not give exact solutions
Using Square Roots	• Use to solve equations in the form of $x^2 = d$	• Can only be used with certain equations
Completing the Square	• Best used when a = 1 and b is even	• May involve difficult calculations
Quadratic Formula	• Can be used for any quadratic equation • Gives exact solutions	• Takes time to do the calculations

Solving a System of Linear and Quadratic Equations

The student learned to solve a system of linear equations in Unit 4 by:

- Graphing
- Substitution
- Elimination

The same steps are applied to solving a linear and a quadratic equation.