Course: *Algebra II Functions* **Unit #2:** *Polynomial Functions*	**Year of Implementation:** 2024-2025

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Stage One - Desired Results

Link(s) to New Jersey Student Learning Standards for this course:
https://www.state.nj.us/education/cccs/2020/

Unit Standards: *(keep each of the following headings in place)*

Content Standards

The Complex Number System N -CN

A. Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Seeing Structure in Expressions A-SSE

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P

B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions.

Creating Equations A -CED

A. Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Arithmetic with Polynomials and Rational Expressions A -APR

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

Interpreting Functions F-IF

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key

features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

21st Century Life & Career Standards

All curriculum writers/revisionists need to include standards that apply to "Career Readiness, Life Literacies, and Key Skills". This should include a brief description of the standard and the standard number. Document only those standards and practices that apply to each unit. Use the following link to assist you [see pages of 31-36; 41-42; 53-56 for specific standard #'s and strands]

https://www.state.nj.us/education/cccs/2020/2020%20NJSLS-CLKS.pdf

- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

Interdisciplinary Content Standards

- SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
- SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- L.11-12.6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

NJ Statutes: NJ State law mandates the inclusion of the following topics in lesson design and instruction as aligned to elementary and secondary curriculum.

Amistad Law: N.J.S.A. 18A 52:16A-88 Every board of education shall incorporate the information regarding the contributions of African-Americans to our country in an appropriate place in the curriculum of elementary and secondary school students.

Holocaust Law: N.J.S.A. 18A:35-28 Every board of education shall include instruction on the Holocaust and genocides in an appropriate place in the curriculum of all elementary and secondary school pupils. The instruction shall further emphasize the personal responsibility that each citizen bears to fight racism and hatred whenever and wherever it happens.

LGBT and Disabilities Law: N.J.S.A. 18A:35-4.35 A board of education shall include instruction on the political, economic, and social contributions of persons with disabilities and lesbian, gay, bisexual, and transgender people, in an appropriate place in the curriculum of middle school and high school students as part of the district's implementation of the New Jersey Student Learning Standards (N.J.S.A.18A:35-4.36) A board of education shall have policies and procedures in place pertaining to the selection of instructional materials to implement the requirements of N.J.S.A. 18A:35-4.35.

Diversity and Inclusion (N.J.S.A. 18A:35-4.36a) A board of education shall incorporate instruction on

diversity and inclusion in an appropriate place in the curriculum of students in grades kindergarten through 12 as part of the district's implementation of the New Jersey Student Learning Standards.

Asian American and Pacific Islanders (AAPI) P.L.2021, c.410 Ensures that the contributions, history, and heritage of Asian Americans and Pacific Islanders (AAPI) are included in the New Jersey Student Learning Standards (NJSLS) for Social Studies in kindergarten through Grade 12 (P.L.2021, c.416)

For additional information, see

NJ Amistad Curriculum: ***https://www.nj.gov/education/amistad/about/***

Diversity and Inclusion: ***https://www.nj.gov/education/standards/dei/index.shtml***

(Sample Activities/ Lessons): ***https://www.nj.gov/education/standards/dei/samples/index.shtml***

Asian American and Pacific Islanders:

Asian American and Pacific Islander Heritage and History in the U.S.

A Teacher's Guide from EDSITEment offering a collection of lessons and resources for K-12 social studies, literature and arts classrooms that center around the experiences, achievements and perspectives of Asian Americans and Pacific Islanders across U.S. history.

Transfer Goal: Students will be able to independently use their learning to predict behaviors based on patterns.

As aligned with LRHSD Long Term Learning Goal(s):https://www.lrhsd.org/Page/6163

Patterns: analyze data and recognize patterns in a variety of situations

Enduring Understandings	*Essential Questions*
Students will understand that. . . *EU 1* the characteristics of polynomials and their representations can be examined depending on what information is sought.	How can the correlation between the graphical and algebraic representations of a polynomial function be communicated?

EU 2 factors, solutions, and zeros of a polynomial function are related.	How can you use the relationships between models to predict real world situations?
<u>Knowledge</u> *Students will know . . .* *EU 1* ● the leading coefficient dictates the end behavior of a polynomial function. (A-SSE A1, F-IF B4, C7c) ● the maximum number of real zeros of a function coincide with its degree and are the x-intercepts of its graph. (A -APR B, F-IF B4, C7c) ● imaginary zeros are not indicated on the graph of a polynomial function. (N -CN C, F-IF B4, C7c) ● the solutions to a polynomial function represent the x-intercept. (A -APR B, F-IF B4, C7c) ● verbal models can be translated into an algebraic model. (A -CED 2, F-IF B4,5, C7c) *EU 2* ● polynomial equations can be translated from standard form to factored form and factored form to standard form in a polynomial equation. (N -CN C, A-SSE B, A -APR B) ● solutions to various polynomial equations can be found by factoring. (N -CN C, A-SSE B, A -APR B, C4, F-IF C8a) ● the relationship between a zero and a factor. (A -APR B, F-IF B4, C7c)	*<u>Skills</u>* *Students will be able to. . .* *EU 1* ● add, subtract, multiply, evaluate, simplify, compose, and (long and synthetic) divide polynomial functions. (A -APR A1, B2, D6) ● sketch a function's end behavior using the leading coefficient test. (A-SSE A1, F-IF B4, C7c) ● give a precise graph using zeros and end behavior. (F-IF B4, C) ● identify the number of real and imaginary zeros based on the degree and the graph of a polynomial function. (A -APR B, N -CN C, F-IF B4, C7c) ● recognize the relationship between the zeros of a function and the x-intercepts of a graph. (A -APR B, F-IF B4, C7c) ● model real life situations with algebraic functions (differentiate between significant and extraneous information in a given situation and distinguish between linear, quadratic, and polynomial models for a given situation). (A -CED 2, F-IF B4,5, C7c) *EU 2* ● factor a polynomial function (include grouping and sum/diff cubes). (N -CN C, A-SSE B, A -APR B, C4, F-IF C8a) ● solve a polynomial function to find the zeros (include: factor theorem, remainder theorem, synthetic division). (N -CN C, A-SSE B, A -APR B, D6)

	• write a polynomial function of least degree given the zeros. (A-SSE , A -CED A2, F-IF 8a) • use the quadratic formula to find all solutions for the polynomial equation. (N -CN 7) • transform polynomial equations to expedite graphing and/or solving the polynomial equation. (A-SSE B3, A - APR B, C, D, F-IF 8a)

Stage Two - Assessment

Stage Three - Instruction

Learning Plan: **Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections: Each learning activity listed must be accompanied by a learning goal of A= Acquiring basic knowledge and skills, M= Making meaning and/or a T= Transfer.** *{place A, M and/or T along with the applicable EU number in parentheses after each statement} All knowledge and skills must be addressed in this section with a corresponding lesson/activity which teaches each concept. The following color codes are used to notate activities that correspond with interdisciplinary connections and 21st Century Life & Career Connections (which involves Technology Literacy): Red = Interdisciplinary Connection; Purple = 21st Century Life & Career Connection*

Synthetic Division Ship Battling (A, EU 1)
https://docs.google.com/document/d/1lURpf2vco905VnV3_AHWWl4kZ3FpbAHtCZTT3HqUDvQ/edit?usp=sharing

Desmos, Polynomial End Behavior (A, EU 1&2)
https://teacher.desmos.com/activitybuilder/custom/63c9440073eb64692aa16385

Desmos, Polynomial Equation Challenges (finding roots) (A, EU 1&2)
https://teacher.desmos.com/activitybuilder/custom/561582ecbd554ea00760f933?collections=5e8daca7ba47980c870d2e02

Desmos Graphing Calculator Lab- Analyzing Polynomial Functions (A, M, EU 1 & 2)

https://docs.google.com/document/d/1r7qGUJ2XZjPaLe_KWOo8WtgNC46c3dkfFibPioaX3SU/edit?usp=sharing

Desmos, SAVVAS envisions "What are the Rules?" 3 Act Task (M, T, EU 1&2)
https://teacher.desmos.com/activitybuilder/custom/6205428ab039da0a88467ad5?collections=5cd58a825a7de70cbb55328e

Desmos, Match My Polynomial (M, T, EU 1&2)
https://teacher.desmos.com/activitybuilder/custom/63b6c6640b21eef767d46259

Suggested Sequence of Learning Activities:

- perform operations on polynomials (+, -, x, evaluate) (A, EU1)
- perform composition on functions (A, EU1)
- perform long division (A, EU1)
- perform synthetic division (A, EU1)
- *Synthetic Division Ship Battling, link above (A, EU 1)*
- sketch graphs of polynomial functions using end behavior and degree (only real solutions) (A, M, EU 1 & 2)
- Desmos, Polynomial End Behavior, link above (A, M, EU 1 & 2)
- sketch graphs of polynomial functions using end behavior, degree, multiplicities (real and imaginary solutions) (M)
  Desmos Graphing Calculator Lab- Analyzing Polynomial Functions, see above (A, M, EU 1 & 2)
- factor and solve quadratic and cubic (sum/diff cubes, grouping) polynomials (A, EU2)
- apply the factor and remainder theorems to polynomials to find the zeros and/or factors (A,M, EU2)
- Desmos, Polynomial Equation Challenges (finding roots), link above (A, EU 1 & 2)
- use the rational root theorem to find all rational zeros of a polynomial (M, EU2)
- *Desmos, SAVVAS envisions "What are the Rules?" 3 Act Task, link above (M, T, EU 1 & 2)*
- find ALL zeros (real and imaginary) of a polynomial function (A)
- write a polynomial function of least degree given all zeros of the polynomial (M)
- sketch the graph of a polynomial function using end behavior and zeros (T)
- Desmos, Match My Polynomial, link above (M, T, EU 1 & 2).

Pacing Guide

{This chart will be identical in all of the units for this course.}

Unit #	*Title of Unit*	*Approximate # of teaching days*
1	Unit 1 Quadratic Functions	30
2	Unit 2 Polynomial Functions	18
3	Unit 3 Exponential & Logarithmic Functions	18
4	Unit 4 Radical Functions	18
5	Unit 5 Rational Functions	18
6	Unit 6 Functions	14
7	Unit 7 Statistics & Probability	18

Instructional Materials

TInSpire Calculator
DESMOS online graphing calculator and activities
Khan Academy
Kuta Infinite Software

Accommodations

Special Education: The curriculum will be modified as per the Individualized Education Plan (IEP). Students will be accommodated based on specific accommodations listed in the IEP.

Students with 504 Plans: Students will be accommodated based on specific accommodations listed in the 504 Plan.

English Language Learners: Students will be accommodated based on individual need and in consultation with the ELL teacher.

Students at Risk of School Failure: Students will be accommodated based on individual need and provided various structural supports through their school.

Gifted and Talented Students: Students will be challenged to enhance their knowledge and skills through acceleration and additional independent research on the subject matter.