Course: *Algebra II Functions* **Unit #3:** *Exponential & Logarithmic Functions*	**Year of Implementation:** 2024-2025

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Stage One - Desired Results

Link(s) to New Jersey Student Learning Standards for this course:
{provide all applicable links to standards here}
https://www.state.nj.us/education/cccs/2020/

- **Unit Standards:**

Content Standards

Structure in Expressions A-SSE
B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to transform expressions for exponential functions.

Interpreting Functions F-IF
C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
b. Use the properties of exponents to interpret expressions for exponential functions.

Building Functions F-BF
A. Build a function that models a relationship between two quantities
1. Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations.
c. Compose functions

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing by f(x), f(h)+k, kf(x), f(kx) and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.

5. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models F-LE

A. Construct and compare linear and exponential models and solve problems

4.Understand the inverse relationship between exponents and logarithms.

B. Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

a. 21st Century Life & Career Standards

- All curriculum writers/revisionists need to include standards that apply to "Career Readiness, Life Literacies, and Key Skills". This should include a brief description of the standard and the standard number. Document only those standards and practices that apply to each unit. Use the following link to assist you [see pages of 31-36; 41-42; 53-56 for specific standard #'s and strands] https://www.state.nj.us/education/cccs/2020/2020%20NJSLS-CLKS.pdf

    - 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3)
    - 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
    - 9.4.12.IML.4: Assess and critique the appropriateness and impact of existing data visualizations for an intended audience (e.g., S-ID.B.6b, HS-LS2-4).
    - 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

***b.* Interdisciplinary Content Standards**

- SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

- SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- L.11-12.6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate

c. ***NJ Statutes:*** NJ State law mandates the inclusion of the following topics in lesson design and instruction as aligned to elementary and secondary curriculum.

Amistad Law: N.J.S.A. 18A 52:16A-88 Every board of education shall incorporate the information regarding the contributions of African-Americans to our country in an appropriate place in the curriculum of elementary and secondary school students.

Holocaust Law: N.J.S.A. 18A:35-28 Every board of education shall include instruction on the Holocaust and genocides in an appropriate place in the curriculum of all elementary and secondary school pupils. The instruction shall further emphasize the personal responsibility that each citizen bears to fight racism and hatred whenever and wherever it happens.

LGBT and Disabilities Law: N.J.S.A. 18A:35-4.35 A board of education shall include instruction on the political, economic, and social contributions of persons with disabilities and lesbian, gay, bisexual, and transgender people, in an appropriate place in the curriculum of middle school and high school students as part of the district's implementation of the New Jersey Student Learning Standards (N.J.S.A.18A:35-4.36) A board of education shall have policies and procedures in place pertaining to the selection of instructional materials to implement the requirements of N.J.S.A. 18A:35-4.35.

Diversity and Inclusion (N.J.S.A. 18A:35-4.36a) A board of education shall incorporate instruction on diversity and inclusion in an appropriate place in the curriculum of students in grades kindergarten through 12 as part of the district's implementation of the New Jersey Student Learning Standards.

Asian American and Pacific Islanders (AAPI) P.L.2021, c.410 Ensures that the contributions, history, and heritage of Asian Americans and Pacific Islanders (AAPI) are included in the New Jersey Student Learning Standards (NJSLS) for Social Studies in kindergarten through Grade 12 (P.L.2021, c.416)

For additional information, see

NJ Amistad Curriculum: ***https://www.nj.gov/education/amistad/about/***

Diversity and Inclusion: ***https://www.nj.gov/education/standards/dei/index.shtml***

- *(Sample Activities/ Lessons):* ***https://www.nj.gov/education/standards/dei/samples/index.shtml***

Asian American and Pacific Islanders:

- ***Asian American and Pacific Islander Heritage and History in the U.S.***
  A Teacher's Guide from EDSITEment offering a collection of lessons and resources for K-12 social studies, literature and arts classrooms that center around the experiences, achievements and perspectives of Asian Americans and Pacific Islanders across U.S. history.

Transfer Goal: Students will be able to independently use their learning to manipulate forms to communicate meaningful information.

As aligned with LRHSD Long Term Learning Goal(s):https://www.lrhsd.org/Page/6163

Structure: Students will be able to use multiple representations, critical thinking skills and prior knowledge to solve problems in new situations.

Enduring Understandings	*Essential Questions*
Students will understand that. . . *EU 1* ● expressions can be rewritten to produce other statements of equivalent forms. *EU 2*	● What can be done at an "unsolvable" stage of a problem?

● the characteristics of exponential and logarithmic functions and their representations are useful when solving real world problems.	
<u>Knowledge</u> *Students will know . . .* *EU 1* ● key features of the graphs of exponential and logarithmic functions (domain, range, intercepts, increasing/decreasing intervals, asymptotes). (A-SSE B3, A-CED A, F-IF, C7e, 8b) ● transformations for exponential and logarithmic functions. (A-SSE B3, A-CED A, F-IF, C8b) ● logarithmic functions are the inverse of the exponential functions. (F-BF 4C) ● exponential rules are used to simplify exponential and logarithmic expressions. (F-LE 4,5) *EU 2* ● exponential and logarithmic functions and their equations can be used to solve real world problems.(F-IF B) ● problems can be modeled using exponential or logarithmic functions. (F-IF B, F-IF C)	*<u>Skills</u>* *Students will be able to. . .* *EU 1* ● graph exponential and logarithmic functions, with and without transformations. (F-IF C7) ● identify domain, range, intercepts, asymptotes, and increasing/decreasing of exponential and logarithmic functions. (F-IF C) ● find the inverse of logarithmic and exponential functions. (F-BF B4) ● Expand and condense logarithmic expressions and equations. (A-SSE B3) *EU 2* ● use properties of logarithms to evaluate logarithmic expressions and equations. (F-IF C) ● use properties of exponents (including rational exponents) to evaluate exponential expressions and equations. (F-LE 4,5) ● translate and solve exponential functions to model real-world phenomena. (F-IF B, F-IF C)
Stage Two - Assessment	

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Stage Three - Instruction

Learning Plan: **Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections: Each learning activity listed must be accompanied by a learning goal of A= Acquiring basic knowledge and skills, M= Making meaning and/or a T= Transfer.** *{place A, M and/or T along with the applicable EU number in parentheses after each statement} All knowledge and skills must be addressed in this section with a corresponding lesson/activity which teaches each concept. The following color codes are used to notate activities that correspond with interdisciplinary connections and 21st Century Life & Career Connections (which involves Technology Literacy): Red = Interdisciplinary Connection; Purple = 21st Century Life & Career Connection*

- Activity #1 - Go around the class and have each student double the previous number, starting with the number 1. Write each number on the board. Note how quickly the numbers increase. Then start at 1 and have students give the fraction that is half the previous number. Write these alongside the doubled numbers. See if students recognize that these are reciprocals of the first set and note how quickly the numbers decrease. Tell them they will study growth and decay patterns in this lesson. (A, M, EU1)
- Activity #2 – The NCAA holds a championship basketball tournament each spring. The nation's top 64 teams in Division 1 are invited to play. When a team loses, it is out of the tournament. Determine the number of teams left in the tournament after Round 5. (M, EU1)
- Activity #3 – Nspire Activity – Graphing Logarithmic Functions - (M, EU1, EU2) http://education.ti.com/calculators/timathnspired/US/Activities/Detail?sa=1010&t=1168&id=16057
- Activity #4 – Graphing Calculator – Determining whether data best fits an exponential model – (M, T, EU2)

Exponential Functions in the Real World

The United States has continued to grow since the late 1700's. The following chart shows how the number of people per square mile changed from 1790-1980.

Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile
1790	4.5	1840	9.8	1890	17.8	1940	37.2
1800	6.1	1850	7.9	1900	21.5	1950	42.6
1810	4.3	1860	10.6	1910	26.0	1960	50.6
1820	5.5	1870	10.9	1920	29.9	1970	57.5
1830	7.4	1880	14.2	1930	34.7	1980	64

1) Graph the data on your calculator. (Remember, enter the numbers in the list, then go back to the Menu and use Stat to graph). What is happening in the graph?
2) What is the line of best fit? Draw it as well to see if it works.
3) a) Using the equation from Question 2, what would be the number of people per square mile in 2000? Go online and find out the actual amount of people per square mile in the United States. How close is the prediction?

Prediction: _____ Actual: _____ Difference: _____
b) Was the prediction equation a good estimator? Why or why not?
4) As stated in the title, this is an exponential function. What other type of function would create a similar outcome? Why?

- Activity #5 – Research – Have students describe two specific situations, one for exponential growth and one for exponential decay. For each, they should produce a function and a graph, and use them to generate a meaningful value. (T, EU1, EU2)
- Activity #6 – Concentration Game – On index cards write 10 Exponential expressions and on 10 other write the Logarithmic equivalent – (A, EU1)
- Activity #7 - Natural Log Applications (A, M, T EU1, EU2) https://teacher.desmos.com/activitybuilder/custom/58752d0d39f53222057422c3
- Activity #8 – Simon Says – Have students demonstrate graphs of various functions using their hands – (T, EU1, EU2)

Suggested Sequence of Learning Activities:

- Critical Vocabulary: see below (A, EU1, EU2)
- Activity #1 outlined above (A, M, EU1)
- Graph Exponential Growth and Decay functions (A, EU1)
- NCAA Activity Activity #2 outlined above (M, EU 1)
- Graph logarithmic equations (A, EU1)
- Activity #3, link above (M, EU1, EU2)
- Condense and expand Logarithmic functions (A, EU1))
- Simplify/evaluate exponential and logarithmic expressions using inverse properties and properties of logarithms (A, EU1)
- Activity #4 outlined above (M, T, EU2)
- Solve exponential and logarithmic equations using common logarithms and properties of logarithms (A, EU1, EU2)
- Research Activity #5 outlined above (T, EU1, EU2)
- Concentration Game Activity #6 outlined above (A, EU1)
- Evaluate expressions and solve equations using base e and natural logarithm (A, EU1, EU2)
- Activity #7 outlined above (A, M, T, EU1, EU2)
- Simon Says Activity #8 outlined above (T, EU1, EU2)

Critical Vocabulary:

Asymptote	Change of Base Formula	Common Logarithm	Continuously Compounded Interest
Exponential Decay	Exponential Growth	Exponential Equation	Exponential Function
Logarithm	Logarithmic Equation	Logarithmic Function	Natural Logarithm

Pacing Guide

Unit #	*Title of Unit*	*Approximate # of teaching days*
1	Unit 1 Quadratic Functions	30
2	Unit 2 Polynomial Functions	18
3	Unit 3 Exponential & Logarithmic Functions	18
4	Unit 4 Radical Functions	18
5	Unit 5 Rational Functions	18
6	Unit 6 Functions	14
7	Unit 7 Statistics & Probability	18

Instructional Materials

TInSpire Calculator
DESMOS online graphing calculator and activities
Khan Academy
Kuta Infinite Software

Accommodations

Special Education: The curriculum will be modified as per the Individualized Education Plan (IEP). Students will be accommodated based on specific accommodations listed in the IEP.

Students with 504 Plans: Students will be accommodated based on specific accommodations listed in the 504 Plan.

English Language Learners: Students will be accommodated based on individual need and in consultation with the ELL teacher.

Students at Risk of School Failure: Students will be accommodated based on individual need and provided various structural supports through their school.

Gifted and Talented Students: Students will be challenged to enhance their knowledge and skills through acceleration and additional independent research on the subject matter.