Main Ideas

In this chapter students will classify angle pairs formed by three intersecting lines, study angle pairs formed by a line that intersects two parallel lines, and use angle relationships to prove lines parallel. They will investigate slopes of lines and study the relationship between slopes of parallel and perpendicular lines. Students will find equations of lines. Finally, they will prove theorems about perpendicular lines and find the distance between parallel lines in the coordinate plane.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from *Skills Readiness*
Any of Exs. 3–5 answered incorrectly	**Skill 25** Use angle pair relationships
Any of Exs. 6–7 answered incorrectly	**Skill 22** Sketch lines
Any of Exs. 8–11 answered incorrectly	**Skill 10** Simplify fractions
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

3 Parallel and Perpendicular Lines

COMMON CORE

Lesson	
3.1	CC.9-12.G.CO.1
3.2	CC.9-12.G.CO.9
3.3	CC.9-12.G.CO.9
3.4	CC.9-12.G.GPE.5
3.5	CC.9-12.G.GPE.5
3.6	CC.9-12.G.CO.9

3.1 Identify Pairs of Lines and Angles

3.2 Use Parallel Lines and Transversals

3.3 Prove Lines are Parallel

3.4 Find and Use Slopes of Lines

3.5 Write and Graph Equations of Lines

3.6 Prove Theorems About Perpendicular Lines

Before

Previously, you learned the following skills, which you'll use in this chapter: describing angle pairs, using properties and postulates, using angle pair relationships, sketching a diagram, and simplifying fractions.
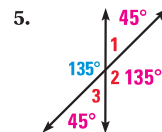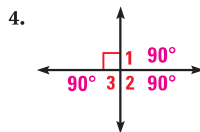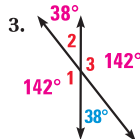
Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. Adjacent angles share a common ? and ? . **vertex, side**

2. Two angles are ? angles if the sum of their measures is 180°. **supplementary**

SKILLS AND ALGEBRA CHECK

Find the measure of each numbered angle.

3. 38° / 2 / 3 142° / 142° 1 / 38°

4. 1 90° / 90° 3 2 90°

5. 45° / 1 / 135° 2 135° / 3 / 45°

Sketch a diagram for the statement.   **6, 7. See margin.**

6. Lines m and n intersect at point P. 7. Line g intersects lines p and q.

Write the fraction in simplest form.

8. $\frac{36}{60}$ **$\frac{3}{5}$**
9. $\frac{54}{24}$ **$\frac{9}{4}$**
10. $\frac{16}{88}$ **$\frac{2}{11}$**
11. $\frac{21}{6}$ **$\frac{7}{2}$**

John Angerson/Alamy

MATHEMATICAL PRACTICES The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. Make sense of problems and persevere in solving them. Pages 162, 180, 192.

2. Reason abstractly and quantitatively. Pages 138, 149, 161.

3. Construct viable arguments and critique the reasoning of others. Pages 145, 149, 157, 158.

4. Model with mathematics. Pages 138, 148, 154, 166, 175.

5. Use appropriate tools strategically. Pages 146, 171, 182, 190.

6. Attend to precision. Pages 156, 157, 176.

7. Look for and make use of structure. Pages 140, 146, 147, 153, 164.

8. Look for and express regularity in repeated reasoning. Pages 172, 173, 174.

COMMON CORE Standards for Mathematical Content–High School

Congruence		Lesson
CC.9-12.G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	3-1
CC.9-12.G.CO.9	Prove theorems about lines and angles.	3-2, 3-3, 3-6
Expressing Geometric Properties with Equations		
CC.9-12.G.GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	3-4, 3-5

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1 **Using properties of parallel and perpendicular lines**
2 **Proving relationships using angle measures**
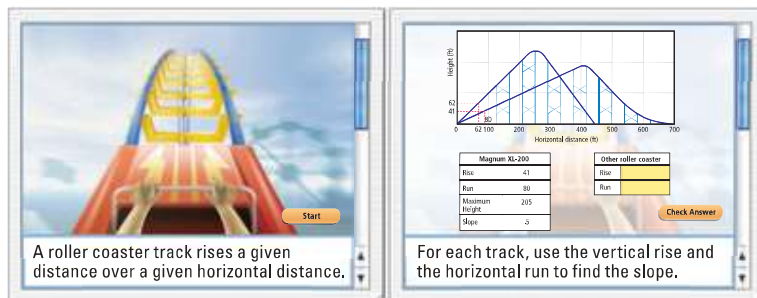**3** **Making connections to lines in algebra**

KEY VOCABULARY

- parallel lines
- skew lines
- parallel planes
- transversal
- corresponding angles

- alternate interior angles
- alternate exterior angles
- consecutive interior angles
- paragraph proof
- slope

- slope-intercept form
- standard form
- distance from a point to a line

Why?

You can use slopes of lines to determine steepness of lines. For example, you can compare the slopes of roller coasters to determine which is steeper.

Animated Geometry

The animation illustrated below helps you answer a question from this chapter: How steep is a roller coaster?

A roller coaster track rises a given distance over a given horizontal distance.

For each track, use the vertical rise and the horizontal run to find the slope.

Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

6.

7.
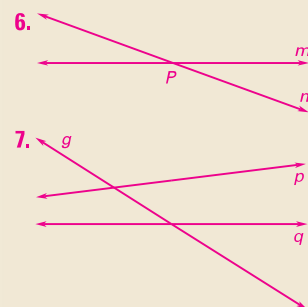
1 PLAN AND PREPARE

Explore the Concept
- Students will draw a representation of a three-dimensional figure.
- This activity leads into the study of the relationships of lines in space in Example 1 in this lesson.

Materials
Each student will need:
- straightedge
- lined paper

Recommended Time
Work activity: 10 min
Discuss results: 5 min

Grouping
Students should work individually.

2 TEACH

Tips for Success
Remind students that in drawings of three-dimensional figures, lines may appear to intersect that in fact do not. Urge students to keep this in mind as they draw their figures.

Alternative Strategy
Use a rectangular box as a model to demonstrate the relationships between the lines in the figure.

Key Discovery
Lines in space may or may not intersect. A pair of lines that intersect lie in the same plane.

3 ASSESS AND RETEACH

If two lines do not intersect, must they lie in different planes? Explain. **No, it is possible for two lines to lie in the same plane and not intersect. For example, see $\overleftrightarrow{JM}$ and $\overleftrightarrow{NR}$ in the diagram.**

COMMON CORE **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).

Investigating Geometry ACTIVITY *Use before Identify Pairs of Lines and Angles*

Draw and Interpret Lines

MATERIALS • pencil • straightedge • lined paper

MATHEMATICAL PRACTICES Model with mathematics.

QUESTION How are lines related in space?

You can use a straightedge to draw a representation of a three-dimensional figure to explore lines in space.

EXPLORE Draw lines in space

STEP 1 *Draw rectangles*
Use a straightedge to draw two identical rectangles.
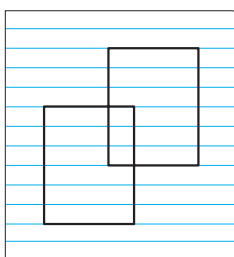
STEP 2 *Connect corners*
Connect the corresponding corners of the rectangles.
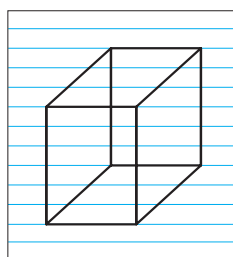
STEP 3 *Erase parts*
Erase parts of "hidden" lines to form dashed lines.
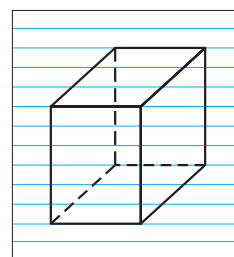
DRAW CONCLUSIONS Use your observations to complete these exercises

Using your sketch from the steps above, label the corners as shown at the right. Then extend $\overrightarrow{JM}$ and $\overline{LQ}$. Add lines to the diagram if necessary.

1. Will $\overleftrightarrow{JM}$ and $\overleftrightarrow{LQ}$ ever intersect in space? (Lines that intersect on the page do not necessarily intersect in space.) **no**

2. Will the pair of lines intersect in space?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{NR}$ **no**   b. $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$ **yes**
   c. $\overleftrightarrow{LM}$ and $\overleftrightarrow{MR}$ **yes**   d. $\overleftrightarrow{KL}$ and $\overleftrightarrow{NQ}$ **no**

3. Does the pair of lines lie in one plane?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{QR}$ **yes**   b. $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$ **yes**
   c. $\overleftrightarrow{JN}$ and $\overleftrightarrow{LR}$ **no**   d. $\overleftrightarrow{JL}$ and $\overleftrightarrow{NQ}$ **yes**

4. Do pairs of lines that intersect in space also lie in the same plane? *Explain* your reasoning. **Yes. Sample answer: Three non-linear points determine a unique plane.**

5. Draw a rectangle that is not the same as the one you used in the Explore. Repeat the three steps of the Explore. Will any of your answers to Exercises 1–3 change? **Check students' work; no.**
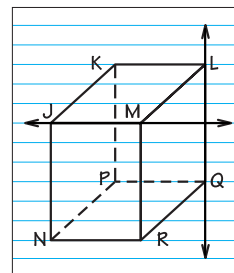
3.1 Identify Pairs of Lines and Angles

| **Before**	You identified angle pairs formed by two intersecting lines.
Now	You will identify angle pairs formed by three intersecting lines.
Why?	So you can classify lines in a real-world situation, as in Exs. 40–42.

Key Vocabulary
- parallel lines
- skew lines
- parallel planes
- transversal
- corresponding angles
- alternate interior angles
- alternate exterior angles
- consecutive interior angles

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** if they do not intersect and are coplanar. Two lines are **skew lines** if they do not intersect and are not coplanar. Also, two planes that do not intersect are **parallel planes**.
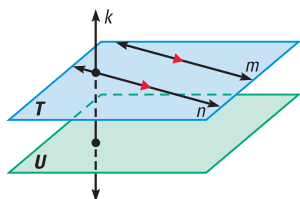
Lines *m* and *n* are parallel lines (*m* ∥ *n*).

Lines *m* and *k* are skew lines.

Planes *T* and *U* are parallel planes (*T* ∥ *U*).

Lines *k* and *n* are intersecting lines, and there is a plane (not shown) containing them.

Small directed triangles, as shown on lines *m* and *n* above, are used to show that lines are parallel. The symbol ∥ means "is parallel to," as in *m* ∥ *n*.

Segments and rays are parallel if they lie in parallel lines. A line is parallel to a plane if the line is in a plane parallel to the given plane. In the diagram above, line *n* is parallel to plane *U*.

EXAMPLE 1 Identify relationships in space

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?

a. Line(s) parallel to $\overleftrightarrow{CD}$ and containing point *A*

b. Line(s) skew to $\overleftrightarrow{CD}$ and containing point *A*

c. Line(s) perpendicular to $\overleftrightarrow{CD}$ and containing point *A*

d. Plane(s) parallel to plane *EFG* and containing point *A*

Solution

a. $\overleftrightarrow{AB}$, $\overleftrightarrow{HG}$, and $\overleftrightarrow{EF}$ all appear parallel to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AB}$ contains point *A*.

b. Both $\overleftrightarrow{AG}$ and $\overleftrightarrow{AH}$ appear skew to $\overleftrightarrow{CD}$ and contain point *A*.

c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{FC}$ all appear perpendicular to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AD}$ contains point *A*.

d. Plane *ABC* appears parallel to plane *EFG* and contains point *A*.

---

1 PLAN AND PREPARE

Warm-Up Exercises
Also available online
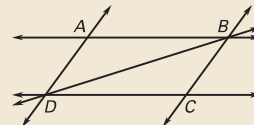
1. Name a line that does not intersect $\overleftrightarrow{AD}$. **$\overleftrightarrow{BC}$**

2. What is the intersection of $\overleftrightarrow{AD}$ and $\overleftrightarrow{DB}$? **D**

Notetaking Guide
Available online
Promotes interactive learning and notetaking skills.

Pacing
Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with next lesson
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question
Big Idea 1
What angle pairs are formed by transversals? **Tell students they will learn how to answer this question by studying three intersecting lines and the relative positions of the angles they determine.**


Motivating the Lesson

Tell students that artists and architects need to understand lines in two and three dimensions that do and do not intersect. Tell them that in this lesson, they will study these lines and the positions of angles formed by intersecting lines.
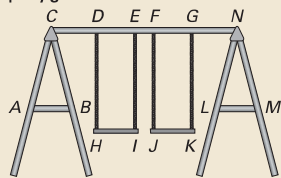
❸ TEACH

Extra Example 1

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?
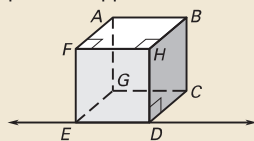
a. Line(s) parallel to $\overleftrightarrow{ED}$ and containing point C $\overleftrightarrow{GC}$

b. Line(s) skew to $\overleftrightarrow{ED}$ $\overleftrightarrow{BC}$, $\overleftrightarrow{AG}$, $\overleftrightarrow{AF}$, $\overleftrightarrow{BH}$

c. Line(s) perpendicular to $\overleftrightarrow{ED}$ $\overleftrightarrow{CD}$, $\overleftrightarrow{GE}$, $\overleftrightarrow{FE}$, $\overleftrightarrow{HD}$

d. Plane(s) parallel to plane ABH **plane CDE**

Extra Example 2

The figure shows a swing set on a playground.

a. Name a pair of perpendicular lines. **Sample answer: $\overleftrightarrow{DH} \perp \overleftrightarrow{HI}$**

b. Name a pair of parallel lines. **Sample answer: $\overleftrightarrow{AB} \parallel \overleftrightarrow{LM}$**

c. Is $\overleftrightarrow{DH}$ perpendicular to $\overleftrightarrow{LM}$? Explain. **No, the lines are skew.**

PARALLEL AND PERPENDICULAR LINES Two lines in the same plane are either parallel or intersect in a point.

Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line.

Animated Geometry at my.hrw.com
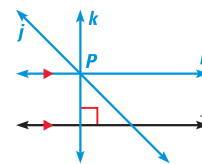
POSTULATES   *For Your Notebook*

POSTULATE 13 Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

There is exactly one line through P parallel to ℓ.
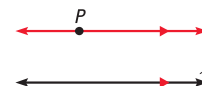
POSTULATE 14 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through P perpendicular to ℓ.
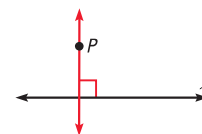
EXAMPLE 2 Identify parallel and perpendicular lines

PHOTOGRAPHY The given line markings show how the roads are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.

Solution

a. $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$ **b.** $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$

c. $\overleftrightarrow{FE}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{MD}$ is parallel to $\overleftrightarrow{FE}$ and by the Parallel Postulate there is exactly one line parallel to $\overleftrightarrow{FE}$ through M.

Niagara Falls, New York

2. Yes; since A is not on $\overleftrightarrow{MD}$ and $\overleftrightarrow{MD}$ is $\perp$ to $\overleftrightarrow{BF}$, the Perpendicular Postulate guarantees that there is exactly one line through a point perpendicular to a line, so $\overleftrightarrow{AC}$ can not be perpendicular to $\overleftrightarrow{BF}$ also.

✓ **GUIDED PRACTICE** for Examples 1 and 2

1. Look at the diagram in Example 1. Name the lines through point H that appear skew to $\overleftrightarrow{CD}$. $\overleftrightarrow{AH}$, $\overleftrightarrow{EH}$

2. In Example 2, can you use the Perpendicular Postulate to show that $\overleftrightarrow{AC}$ is *not* perpendicular to $\overleftrightarrow{BF}$? *Explain* why or why not.

Differentiated Instruction

English Learners Students learning English may confuse *parallel* and *perpendicular*. Tell them to think of the two *l*'s in *parallel* as two lines that do not intersect. Point out that the two *l*'s resemble the symbol for parallel lines, $\parallel$. There is only one *l* in perpendicular. Have them relate this *l* to the symbol for perpendicular, $\perp$, which tells them that two lines meet at a 90° angle.
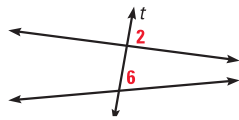
See also the *Differentiated Instruction Resources* for more strategies.

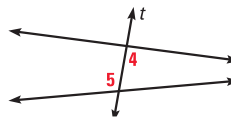**ANGLES AND TRANSVERSALS** A **transversal** is a line that intersects two or more coplanar lines at different points.

KEY CONCEPT　　　　　　　　　　　*For Your Notebook*

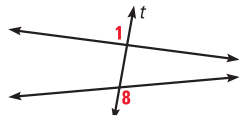### Angles Formed by Transversals


Two angles are **corresponding angles** if they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.

Two angles are **alternate interior angles** if they lie between the two lines and on opposite sides of the transversal.

Two angles are **alternate exterior angles** if they lie outside the two lines and on opposite sides of the transversal.
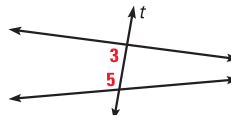
Two angles are **consecutive interior angles** if they lie between the two lines and on the same side of the transversal.

EXAMPLE 3 **Identify angle relationships**

Identify all pairs of angles of the given type.

a. Corresponding　　　　b. Alternate interior
c. Alternate exterior　　　d. Consecutive interior
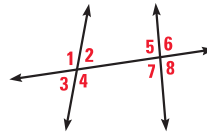
Solution

a. ∠1 and ∠5
   ∠2 and ∠6
   ∠3 and ∠7
   ∠4 and ∠8

b. ∠2 and ∠7
   ∠4 and ∠5

c. ∠1 and ∠8
   ∠3 and ∠6

d. ∠2 and ∠5
   ∠4 and ∠7

✓ **GUIDED PRACTICE** for Example 3

Classify the pair of numbered angles.

3.
corresponding angles

4.
alternate exterior angles

5.
alternate interior angles
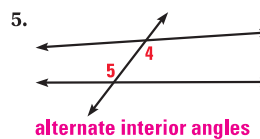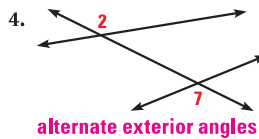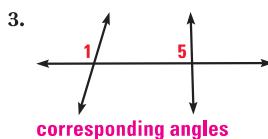
3.1 Identify Pairs of Lines and Angles **141**

Differentiated Instruction

Below level Draw two lines and a transversal. Show students how they can draw an F to identify corresponding angles. Also show them that by drawing a Z they can identify alternate interior angles. Have them use these techniques while doing the exercises.
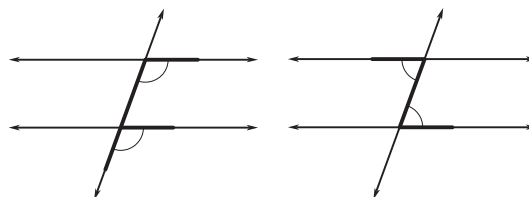
See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3
Identify all pairs of angles of the given type.
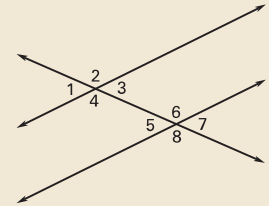
a. Corresponding ∠1 and ∠5; ∠3 and ∠7; ∠2 and ∠6; ∠4 and ∠8
b. Alternate interior ∠3 and ∠5; ∠4 and ∠6
c. Alternate exterior ∠1 and ∠7; ∠2 and ∠8
d. Consecutive interior ∠3 and ∠6; ∠4 and ∠5

MATHEMATICAL PRACTICES *Key Question Example 3*

• Explain the difference between alternate interior angles and consecutive interior angles.
Alternate interior angles are on different sides of the transversal. Consecutive interior angles are on the same side of the transversal.

Closing the Lesson
Have students summarize the major points of the lesson and answer the Essential Question: What angle pairs are formed by transversals?

• **Parallel lines do not intersect and are coplanar.**
• **Skew lines do not intersect and are not coplanar.**
• **Corresponding angles have corresponding positions.**
• **Alternate interior angles lie between the two lines and on opposite sides of the transversal.**

Angle pairs formed by three intersecting lines include corresponding, alternate interior, consecutive interior, and alternate exterior angles.

HOMEWORK
KEY

◯ = See **WORKED-OUT SOLUTIONS**
Exs. 11, 25, and 35

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 28, 36, 37, and 39

④ PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–15, 17–23 odd, 34–39

Average:
Day 1:
Exs. 1, 2, 4–6, 8–10, 12–15, 16–22 even, 24–28, 34–42

Advanced:
Day 1:
Exs. 1, 2, 5, 6, 9, 10, 13, 14, 16–28 even, 29–44*

Block:
Exs. 1, 2, 4–6, 8–10, 12–15, 16–22 even, 24–28, 34–42
(with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:
Basic: 4, 7, 12, 23, 34
Average: 5, 8, 13, 20, 34
Advanced: 6, 10, 14, 22, 35

Extra Practice

• Student Edition
• Chapter Resource Book:
  Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

A 1. **VOCABULARY** Copy and complete: A line that intersects two other lines is a _?_ . **transversal**

2. ★ **WRITING** A table is set for dinner. Can the legs of the table and the top of the table lie in parallel planes? *Explain* why or why not.
No; the legs intersect the tabletop.

EXAMPLE 1
for Exs. 3–6

IDENTIFYING RELATIONSHIPS Think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point B and appear to fit the description?
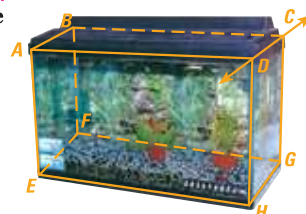
3. Line(s) parallel to $\overleftrightarrow{CD}$ $\overleftrightarrow{AB}$

4. Line(s) perpendicular to $\overleftrightarrow{CD}$ $\overrightarrow{BC}$

5. Line(s) skew to $\overleftrightarrow{CD}$ $\overrightarrow{BF}$

6. Plane(s) parallel to plane CDH   **plane ABF**

EXAMPLE 2
for Exs. 7–10

PARALLEL AND PERPENDICULAR LINES Use the markings in the diagram.

7. Name a pair of parallel lines. $\overleftrightarrow{MK}, \overleftrightarrow{LS}$

8. Name a pair of perpendicular lines. $\overleftrightarrow{PQ}, \overleftrightarrow{PN}$

9. No. *Sample answer:* The lines intersect.

9. Is $\overleftrightarrow{PN} \parallel \overleftrightarrow{KM}$? Explain.
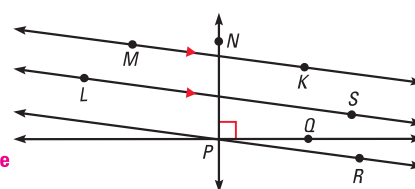
10. Is $\overleftrightarrow{PR} \perp \overleftrightarrow{NP}$? Explain.
No. *Sample answer:* There is no right angle symbol indicating they are perpendicular.

EXAMPLE 3
for Exs. 11–15

11. ∠1 and ∠5, ∠3 and ∠7, ∠2 and ∠6, ∠4 and ∠8

ANGLE RELATIONSHIPS Identify all pairs of angles of the given type.

11. Corresponding

12. Alternate interior
∠3 and ∠6, ∠4 and ∠5

13. Alternate exterior
∠1 and ∠8, ∠2 and ∠7

14. Consecutive interior
∠3 and ∠5, ∠4 and ∠6

15. **ERROR ANALYSIS** *Describe* and correct the error in saying that ∠1 and ∠8 are corresponding angles in the diagram for Exercises 11–14.   ∠1 and ∠8 are not in corresponding positions. ∠1 and ∠8 are alternate exterior angles.
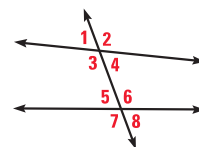
B **APPLYING POSTULATES** How many lines can be drawn that fit each description? Copy the diagram and sketch all the lines.

16. Lines through B and parallel to $\overleftrightarrow{AC}$ **1 line; see margin for art.**

17. Lines through A and perpendicular to $\overleftrightarrow{BC}$
1 line; see margin for art.
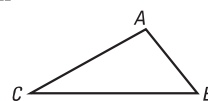
USING A DIAGRAM Classify the angle pair as *corresponding, alternate interior, alternate exterior,* or *consecutive interior* angles.

18. ∠5 and ∠1   **corresponding**

19. ∠11 and ∠13   **consecutive interior**

20. ∠6 and ∠13   **consecutive interior**

21. ∠10 and ∠15   **alternate exterior**

22. ∠2 and ∠11   **alternate interior**

23. ∠8 and ∠4   **corresponding**

Michael Newman/PhotoEdit, Inc.


16.

17.

24.

25.
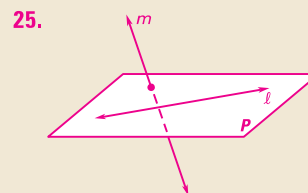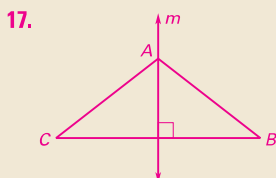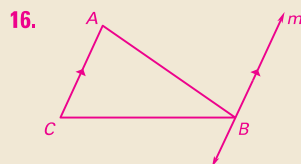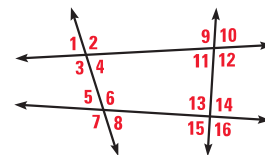
ANALYZING STATEMENTS Copy and complete the statement with *sometimes*, *always*, or *never*. Sketch examples to *justify* your answer. **24–27. See margin for art.**

24. If two lines are parallel, then they are __?__ coplanar. **always**

25. If two lines are not coplanar, then they __?__ intersect. **never**

26. If three lines intersect at one point, then they are __?__ coplanar. **sometimes**

27. If two lines are skew to a third line, then they are __?__ skew to each other. **sometimes**

28. ★ **MULTIPLE CHOICE** ∠*RPQ* and ∠*PRS* are what type of angle pair?

    B

    Ⓐ Corresponding   Ⓑ Alternate interior
    Ⓒ Alternate exterior   Ⓓ Consecutive interior
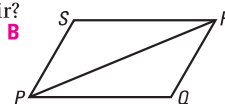
Ⓒ **ANGLE RELATIONSHIPS** Copy and complete the statement. List all possible correct answers.

29. ∠*BCG* and __?__ are corresponding angles. **∠CFJ, ∠HJG**

30. ∠*BCG* and __?__ are consecutive interior angles. **∠CJH**

31. ∠*FCJ* and __?__ are alternate interior angles. **∠DFC, ∠CJH**

32. ∠*FCA* and __?__ are alternate exterior angles. **∠GJH**
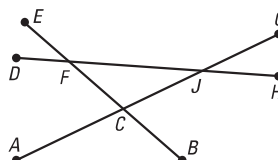
33. **CHALLENGE** Copy the diagram at the right and extend the lines.
    a. Measure ∠1 and ∠2. **80°, 80°**
    b. Measure ∠3 and ∠4. **70°, 70°**
    c. Make a conjecture about alternate exterior angles formed when parallel lines are cut by transversals.
       If parallel lines are cut by a transversal then alternate exterior angles are congruent.
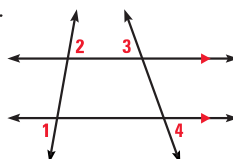
PROBLEM SOLVING

EXAMPLE 2 Ⓐ
for Exs. 34–35

CONSTRUCTION Use the picture of the cherry-picker for Exercises 34 and 35.

34. Is the platform *perpendicular*, *parallel*, or *skew* to the ground? **parallel**

35. Is the arm *perpendicular*, *parallel*, or *skew* to a telephone pole? **skew**
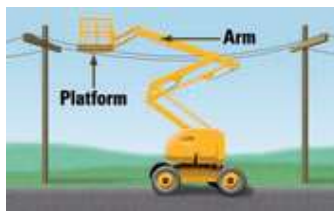
36. ★ **OPEN-ENDED MATH** *Describe* two lines in your classroom that are parallel, and two lines that are skew. **Check students' work.**

37. ★ **MULTIPLE CHOICE** What is the best description of the horizontal bars in the photo? **A**

    Ⓐ Parallel   Ⓑ Perpendicular
    Ⓒ Skew   Ⓓ Intersecting

3.1   Identify Pairs of Lines and Angles   **143**

26.

or

27.

or

Daily Homework Quiz
Also available online

1. Name the pairs of corresponding angles. ∠1 and ∠3; ∠5 and ∠7; ∠2 and ∠4; ∠6 and ∠8

2. Name the pairs of alternate interior angles. ∠2 and ∠7; ∠3 and ∠6

3. Name the pairs of consecutive interior angles. ∠2 and ∠3; ∠6 and ∠7

4. The figure shows a painting easel.
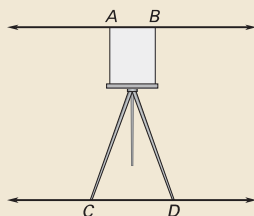
Does $\overleftrightarrow{AB}$ appear to be parallel, perpendicular, or skew to $\overleftrightarrow{CD}$? **parallel**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation
• Practice A, B, C in Chapter Resource Book
• Study Guide in Chapter Resource Book
• Practice Workbook
• @HomeTutor

Challenge
Additional challenge is available in the Chapter Resource Book.

B **38. EASEL** Use the diagram of the easel at the right. An easel is used to display or support an artist's work. The horizontal piece at the bottom that joins the front legs is a transversal.

a. Name four pairs of corresponding angles.
∠5 and ∠9, ∠6 and ∠10, ∠7 and ∠11, ∠8 and ∠12
b. Name two pairs of alternate interior angles.
∠6 and ∠11, ∠8 and ∠9
c. Name two pairs of alternate exterior angles.
∠5 and ∠12, ∠7 and ∠10
d. Name two pairs of consecutive interior angles.
∠6 and ∠9, ∠8 and ∠11
e. In the diagram, the rear leg of the easel appears perpendicular to the transversal. Is it perpendicular? *Explain.*
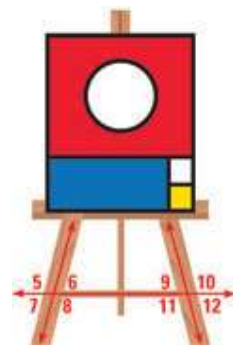
38e. No; the rear leg is skew to the transversal; it does not intersect and is not parallel to the transversal.

39. ★ **SHORT RESPONSE** Two lines are cut by a transversal. Suppose the measure of a pair of alternate interior angles is 90°. Explain why the measure of all four interior angles must be 90°.

39. The adjacent interior angles are supplementary thus the measure of the other two angles must be 90°.

TREE HOUSE In Exercises 40–42, use the photo to decide whether the statement is true or false.

40. The plane containing the floor of the tree house is parallel to the ground. **true**

41. All of the lines containing the railings of the staircase, such as $\overleftrightarrow{AB}$, are skew to the ground. **false**

42. All of the lines containing the *balusters*, such as $\overleftrightarrow{CD}$, are perpendicular to the plane containing the floor of the tree house. **true**
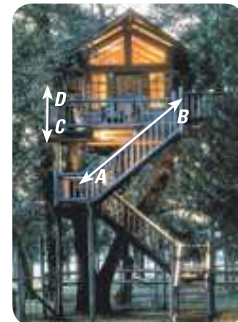
C **CHALLENGE** Draw the figure described. **43, 44. See margin.**

43. Lines ℓ and m are skew, lines ℓ and n are skew, and lines m and n are parallel.

44. Line ℓ is parallel to plane A, plane A is parallel to plane B, and line ℓ is not parallel to plane B.

Paul Rocheleau Photography

43.
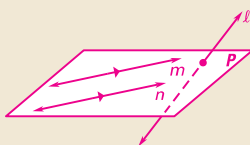
44.
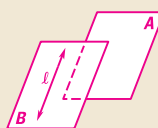
Parallel Lines and Angles

MATERIALS · graphing calculator or computer

MATHEMATICAL PRACTICES
Construct viable arguments and critique the reasoning of others.

QUESTION **What are the relationships among the angles formed by two parallel lines and a transversal?**

You can use geometry drawing software to explore parallel lines.

EXPLORE **Draw parallel lines and a transversal**

STEP 1 **Draw line** Draw and label two points *A* and *B*. Draw $\overleftrightarrow{AB}$.

STEP 2 **Draw parallel line** Draw a point not on $\overleftrightarrow{AB}$. Label it *C*. Choose Parallel from the F3 menu and select $\overleftrightarrow{AB}$. Then select *C* to draw a line through *C* parallel to $\overleftrightarrow{AB}$. Draw a point on the parallel line you constructed. Label it *D*.

STEP 3 **Draw transversal** Draw two points *E* and *F* outside the parallel lines. Draw transversal $\overleftrightarrow{EF}$. Find the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ by choosing Point from the F2 menu. Then choose Intersection. Label the intersection *G*. Find and label the intersection *H* of $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$.

STEP 4 **Measure angle** Measure all eight angles formed by the three lines by choosing Measure from the F5 menu, then choosing Angle.
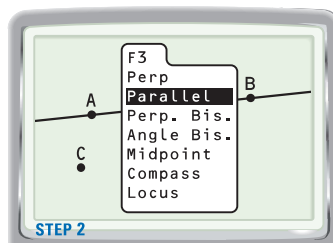
```
F3
Perp
Parallel
Perp. Bis.
Angle Bis.
Midpoint
Compass
Locus
```
STEP 2
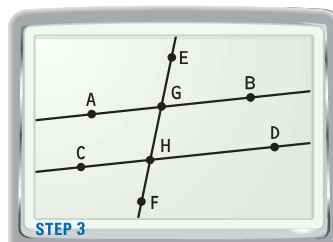
STEP 3

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Record the angle measures from Step 4 in a table like the one shown. Which angles are congruent? **Check students' work; ∠AGE ≅ ∠BGH ≅ ∠CHG ≅ ∠DHF, ∠EGB ≅ ∠AGH ≅ ∠GHD ≅ ∠CHF.**

Angle	∠AGE	∠EGB	∠AGH	∠BGH	∠CHG	∠GHD	∠CHF	∠DHF
Measure 1	?	?	?	?	?	?	?	?

2. Drag point *E* or *F* to change the angle the transversal makes with the parallel lines. Be sure *E* and *F* stay outside the parallel lines. Record the new angle measures as row "Measure 2" in your table. **Check students' work.**

3. Make a conjecture about the measures of the given angles when two parallel lines are cut by a transversal.

   a. Corresponding angles   **b.** Alternate interior angles   **Alternate interior angles are congruent.**
   Corresponding angles are congruent.
4. **REASONING** Make and test a conjecture about the sum of the measures of two consecutive interior angles when two parallel lines are cut by a transversal. **Consecutive interior angles are supplementary.**

3.2   Use Parallel Lines and Transversals   **145**

MATHEMATICAL PRACTICES
Construct viable arguments and critique the reasoning of others.

① PLAN AND PREPARE

Explore the Concept
- Students will use geometric software to find relationships among the angles formed by two parallel lines and a transversal.
- This activity leads into the study of finding angle measures in Example 1 in this lesson.

Materials
Each student will need a graphing calculator or computer with geometry software.

Recommended Time
Work activity: 10 min
Discuss results: 5 min

Grouping
Students should work individually.

② TEACH

Tips for Success
Remind students that when they measure an angle, they need to click on three points of the angle. The second point should always be the vertex of the angle.

Alternative Strategy
Have students construct parallel lines and a transversal with compass and straightedge and measure all the angles with a protractor. Then answer the same questions.

Key Discovery
For two parallel lines and a transversal, corresponding angles and alternate interior angles are congruent.

③ ASSESS AND RETEACH

Make a conjecture about alternate exterior angles when two parallel lines are cut by a transversal. **They are congruent.**

COMMON CORE **CC.9-12.G.CO.9** Prove theorems about lines and angles.

3.2 Use Parallel Lines and Transversals

| Before	You identified angle pairs formed by a transversal.
Now	You will use angles formed by parallel lines and transversals.
Why?	So you can understand angles formed by light, as in Example 4.

Key Vocabulary
• corresponding angles
• alternate interior angles
• alternate exterior angles
• consecutive interior angles

COMMON CORE

CC.9-12.G.CO.9 Prove theorems about lines and angles.

Step 4. ∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8, ∠1 and ∠8, ∠2 and ∠7, ∠3 and ∠6, ∠4 and ∠5, ∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, ∠7 and ∠6; corresponding angles are congruent, alternate exterior angles are congruent, and alternate interior angles are congruent.

ACTIVITY EXPLORE PARALLEL LINES

Materials: lined paper, tracing paper, straightedge

STEP 1 Draw a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 Trace your drawing onto tracing paper.

STEP 3 Move the tracing paper to position ∠1 of the traced figure over ∠5 of the original figure. Compare the angles. Are they congruent? **Yes**

STEP 4 Compare the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal? **See margin.**
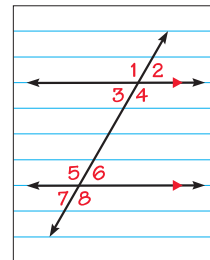
POSTULATE   *For Your Notebook*

POSTULATE 15 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

∠2 ≅ ∠6
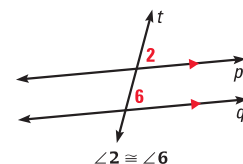
EXAMPLE 1   **Identify congruent angles**

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.

Solution

By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because ∠4 and ∠8 are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.
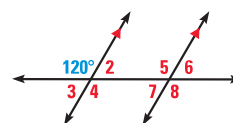
146   Chapter 3   Parallel and Perpendicular Lines

© Goodshoot/JupiterImages Corporation

THEOREMS

THEOREM 3.1 **Alternate Interior Angles Theorem**

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

∠4 ≅ ∠5
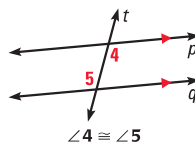
THEOREM 3.2 **Alternate Exterior Angles Theorem**

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

∠1 ≅ ∠8
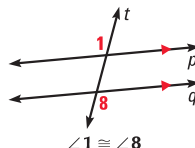
THEOREM 3.3 **Consecutive Interior Angles Theorem**

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

∠3 and ∠5 are supplementary.
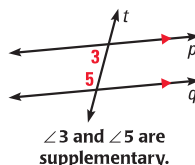
EXAMPLE 2 **Use properties of parallel lines**

xy ALGEBRA Find the value of x.
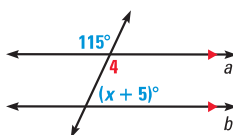
Solution

By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	**Consecutive Interior Angles Theorem**
$115° + (x + 5)° = 180°$	**Substitute 115° for $m\angle 4$.**
$x + 120 = 180$	**Combine like terms.**
$x = 60$	**Subtract 120 from each side.**

Animated **Geometry** at my.hrw.com

✔ **GUIDED PRACTICE** for Examples 1 and 2

Use the diagram at the right.

1. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which postulate or theorem you use in each case.
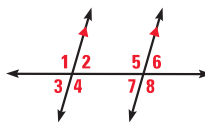
2. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps. **54; $m\angle 7 + m\angle 8 = 180°$, $m\angle 3 = m\angle 7$, 68 + 2x + 4 = 180, 2x + 72 = 180, 2x = 108, x = 54**

1. **$m\angle 4 = 105°$, Vertical Angles Congruence Theorem; $m\angle 5 = 105°$, Corresponding Angles Postulate; $m\angle 8 = 105°$, Alternate Exterior Angles Theorem**

3.2 Use Parallel Lines and Transversals **147**

Differentiated Instruction

Kinesthetic Learners Have students draw a pair of parallel lines cut by a transversal. Then have them identify two alternate interior angles, two alternate exterior angles, and two consecutive interior angles. Have them measure each angle to support their understanding of the theorems.

See also the *Differentiated Instruction Resources* for more strategies.

Motivating the Lesson

Have students visualize a slanting line that crosses all the parallel lines on a sheet of notebook paper. Tell them that in this lesson, they will learn how to find the measures of all the angles formed by measuring just one of the angles.

3 TEACH

Activity Note

The purpose of this activity is to show students that corresponding angles formed by parallel lines and a transversal are congruent.

Extra Example 1

The measure of three of the numbered angles is 55°. Identify the angles. Explain your reasoning.
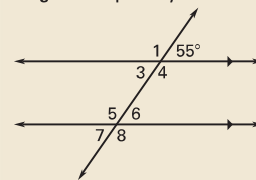
∠6, ∠3, ∠7; $m\angle 6$ = 55° by the Corresponding Angles Postulate. $m\angle 3$ = 55° and $m\angle 7$ = 55° by the Vertical Angles Congruence Theorem.

MATHEMATICAL PRACTICES **Key Question Example 1**

• What is the measure of the other four angles in the figure? **60°**

Extra Example 2

Find the value of x. **165**

**EXAMPLE 3**   **Prove the Alternate Interior Angles Theorem**

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Solution

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.

GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$
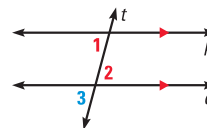
STATEMENTS	REASONS
1. $p \parallel q$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	3. Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	4. Transitive Property of Congruence

EXAMPLE 4   **Solve a real-world problem**

SCIENCE When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?
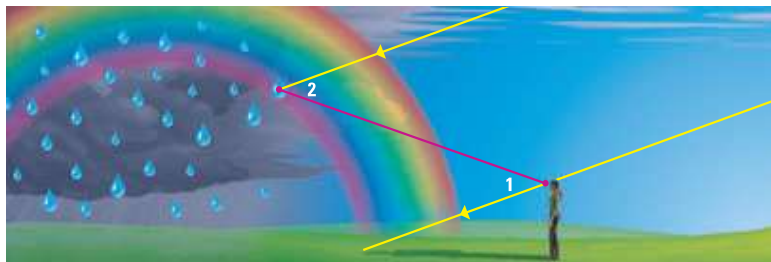
Solution

Because the sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

✓   **GUIDED PRACTICE**   for Examples 3 and 4

3. In the proof in Example 3, if you use the third statement before the second statement, could you still prove the theorem? *Explain.* **Yes. *Sample answer:* $\angle 3$ and $\angle 2$ congruence is not dependent on the congruence of $\angle 1$ and $\angle 3$.**
4. **WHAT IF?** Suppose the diagram in Example 4 shows yellow light leaving a drop of rain. Yellow light leaves the drop at an angle of 41°. What is $m\angle 1$ in this case? How do you know?

4. 41°; $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem $\angle 1 \cong \angle 2$, and by the definition of congruent angles $m\angle 1 = m\angle 2 = 41°$.

3.2 EXERCISES

SKILL PRACTICE

A

1. **VOCABULARY** Draw a pair of parallel lines and a transversal. Label a pair of *corresponding angles*. **See margin.**

2. ★ **WRITING** Two parallel lines are cut by a transversal. Which pairs of angles are congruent? Which pairs of angles are supplementary? **See margin.**

**EXAMPLES
1 and 2**
for Exs. 3–16

3. ★ **MULTIPLE CHOICE** In the figure at the right, which angle has the same measure as ∠1? **C**

  (A) ∠2
  (B) ∠3
  (C) ∠4
  (D) ∠5
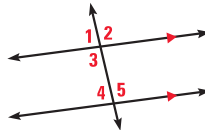
USING PARALLEL LINES Find the angle measure. Tell which postulate or theorem you use.

4. If $m\angle 4 = 65°$, then $m\angle 1 =$ __?__.
   65°; Vertical Angles Congruence Theorem
5. If $m\angle 7 = 110°$, then $m\angle 2 =$ __?__.
   110°; Alternate Exterior Angles Theorem
6. If $m\angle 5 = 71°$, then $m\angle 4 =$ __?__.
   71°; Alternate Interior Angles Theorem
7. If $m\angle 3 = 117°$, then $m\angle 5 =$ __?__.
   63°; Consecutive Interior Angles Theorem
8. If $m\angle 8 = 54°$, then $m\angle 1 =$ __?__.
   54°; Alternate Exterior Angles Theorem
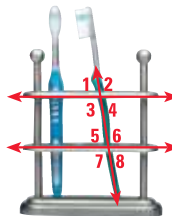
USING POSTULATES AND THEOREMS What postulate or theorem justifies the statement about the diagram?

9. ∠1 ≅ ∠5
10. ∠4 ≅ ∠5 **Alternate Interior Angles Theorem**
11. ∠2 ≅ ∠7
12. ∠2 and ∠5 are supplementary.
    Consecutive Interior Angles Theorem
13. ∠3 ≅ ∠6
14. ∠3 ≅ ∠7
    Corresponding Angles Postulate
15. ∠1 ≅ ∠8 **Alternate Exterior Angles Theorem**
16. ∠4 and ∠7 are supplementary.
    Consecutive Interior Angles Theorem

B
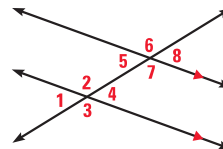
USING PARALLEL LINES Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning. **18, 19. See margin**

17. 150°
18. 140°
19. 122°
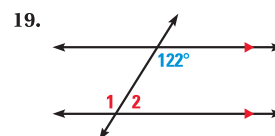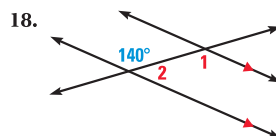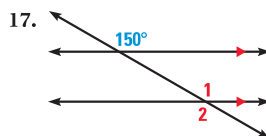
20. **ERROR ANALYSIS** A student concludes that ∠9 ≅ ∠10 by the Corresponding Angles Postulate. Describe and correct the error in this reasoning.
    The lines are not known to be parallel, therefore ∠9 and ∠10 are not necessarily congruent.

    ∠9 ≅ ∠10

Margin notes

2. congruent: alternate interior angles, alternate exterior angles, corresponding angles, vertical angles; supplementary: adjacent angles, consecutive interior angles, exterior angles on the same side of the transversal.

9. Corresponding Angles Postulate

11. Alternate Interior Angles Theorem

13. Alternate Exterior Angles Theorem

17. $m\angle 1 = 150°$, $m\angle 2 = 150°$; Corresponding Angles Postulate, Vertical Angles Congruence Theorem

1. *Sample:*

18. $m\angle 1 = 140°$, $m\angle 2 = 40°$; Alternate Interior Angles Theorem, Consecutive Interior Angles Theorem

19. $m\angle 1 = 122°$, $m\angle 2 = 58°$; Alternate Interior Angles Theorem, Consecutive Interior Angles Theorem

---

4 PRACTICE AND APPLY

Assignment Guide
Answers for all exercises available online
Basic:
Day 1:
Exs. 1–8, 9–15 odd, 17–24, 37–40
Average:
Day 1:
Exs. 1–3, 4–20 even, 21–34, 37–42
Advanced:
Day 1:
Exs. 1–3, 7, 8, 13, 15, 18, 19, 21–37*, 39–43*
Block:
Exs. 1–3, 4–20 even, 21–34, 37–42 (with previous lesson)

Differentiated Instruction
See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check
For a quick check of student understanding of key concepts, go over the following exercises:
Basic: 4, 11, 17, 37, 38
Average: 6, 14, 18, 37, 39
Advanced: 8, 13, 19, 37, 40

Extra Practice
• Student Edition
• Chapter Resource Book:
  Practice levels A, B, C

Practice Worksheet
An easily-readable reduced practice page can be found at the beginning of this chapter.

©HMH

149

22. $m\angle 1 = 100°$, Consecutive Interior Angles Theorem; $m\angle 2 = 80°$, Consecutive Interior Angles Theorem; $m\angle 3 = 100°$, Consecutive Interior Angles Theorem

25. *Sample answer:* $\angle BAC$ and $\angle DCA$, $\angle ABD$ and $\angle CDB$

21. ★ **SHORT RESPONSE** Given $p \parallel q$, *describe* two methods you can use to show that $\angle 1 \cong \angle 4$. *Sample answer:* $\angle 1 \cong \angle 4$ by the Alternate Exterior Angles Theorem; $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$ by Vertical Angles Congruence Theorem, Alternate Interior Angles Theorem, and the Transitive Property of Angle Congruence.
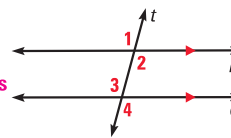
USING PARALLEL LINES Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. *Explain* your reasoning.
23, 24. See margin.

22.   **23.**   **24.**
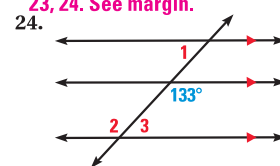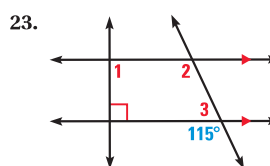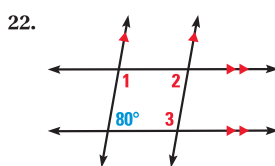
ANGLES Use the diagram at the right.

25. Name two pairs of congruent angles if $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel.

26. Name two pairs of supplementary angles if $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ are parallel. *Sample answer:* $\angle BAD$ and $\angle ABC$, $\angle CDA$ and $\angle DCB$

xy **ALGEBRA** Find the values of x and y.

27. 45, 85

28. 45, 20

29. 65, 60

30. 20, 10

31. 13, 12

32. 10, 25
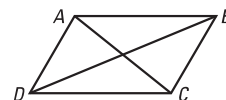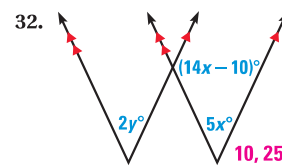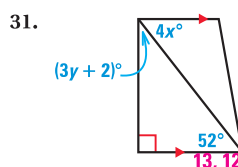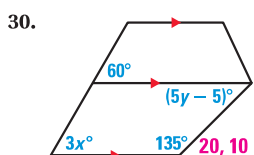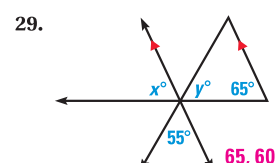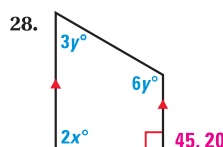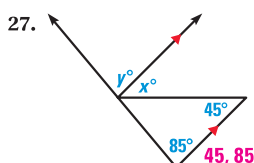
33. ★ **MULTIPLE CHOICE** What is the value of y in the diagram?

Ⓐ 70   Ⓑ 75   Ⓒ 110   Ⓓ 115
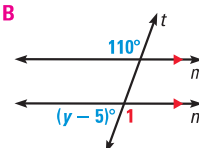
34. **DRAWING** Draw a four-sided figure with sides $\overline{MN}$ and $\overline{PQ}$, such that $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \parallel \overline{NQ}$, and $\angle MNQ$ is an acute angle. Which angle pairs formed are congruent? *Explain* your reasoning. See margin.

Ⓒ **CHALLENGE** Find the values of x and y.

35. 65, 10
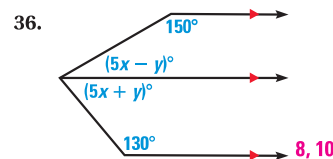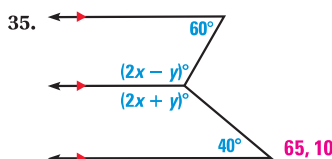
36. 8, 10

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

150

34. $\angle M$ and $\angle Q$, $\angle P$ and $\angle N$. *Sample answer:* $\angle P$ is supplementary to $\angle M$ and $\angle Q$, therefore by the Congruent Supplements Theorem they are congruent; $\angle Q$ is supplementary to $\angle P$ and $\angle N$, therefore by the Congruent Supplements Theorem they are congruent.
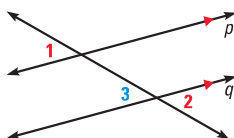
37. PROVING THEOREM 3.2 If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent. Use the steps below to write a proof of the Alternate Exterior Angles Theorem. **See margin.**

GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

a. Show that $\angle 1 \cong \angle 3$.

b. Then show that $\angle 1 \cong \angle 2$.

38. PARKING LOT In the diagram, the lines dividing parking spaces are parallel. The measure of $\angle 1$ is 110°.

a. Identify the angle(s) congruent to $\angle 1$. **$\angle 4, \angle 5, \angle 8$**

b. Find $m\angle 6$. **70°**

B

39. ★ SHORT RESPONSE The *Toddler*™ is a walking robot. Each leg of the robot has two parallel bars and a foot. When the robot walks, the leg bars remain parallel as the foot slides along the surface.

a. As the legs move, are there pairs of angles that are always congruent? always supplementary? If so, which angles?

b. *Explain* how having parallel leg bars allows the robot's foot to stay flat on the floor as it moves.
Sample answer: The transversal stays parallel to the floor.

39a. yes;
$\angle 1$ and $\angle 5$,
$\angle 2$ and $\angle 6$;
yes;
$\angle 1$ and $\angle 2$,
$\angle 1$ and $\angle 6$,
$\angle 2$ and $\angle 5$,
$\angle 5$ and $\angle 6$

40. ★ EXTENDED RESPONSE You are designing a box like the one below.
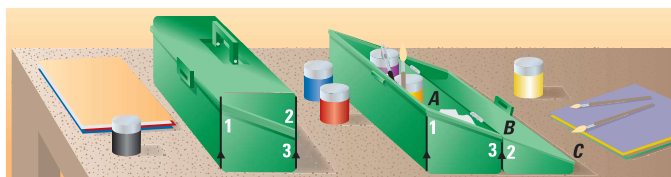
a. The measure of $\angle 1$ is 70°. What is $m\angle 2$? What is $m\angle 3$? **70°, 110°**

b. *Explain* why $\angle ABC$ is a straight angle. **See margin.**

c. **What If?** If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? *Explain.* **Yes; steeper. *Sample answer:* The points are collinear, if $m\angle 1$ is 60°, the angle is smaller and the line becomes steeper.**

41. PROVING THEOREM 3.3 If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary. Write a proof of the Consecutive Interior Angles Theorem. **See margin.**

GIVEN ▶ $n \parallel p$

PROVE ▶ $\angle 1$ and $\angle 2$ are supplementary.

3.2   Use Parallel Lines and Transversals   **151**
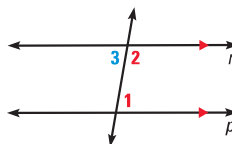
⟳ Internet Reference

Exercise 39 To learn more about walking robots, visit www.news.cornell.edu/releases/Feb05/AAAS.Ruina.bipedal.ws.html

37. Statements (Reasons)

1. $p \parallel q$ (Given)
2. $\angle 1 \cong \angle 3$ (Corresponding Angles Postulate)
3. $\angle 3 \cong \angle 2$ (Vertical Angles Congruence Theorem)
4. $\angle 1 \cong \angle 2$ (Transitive Property of Angle Congruence)

40b. *Sample answer:* $\angle 1$ and $\angle 2$ are corresponding angles and therefore congruent. $\angle 1$ and $\angle 3$ are supplementary so by substitution $\angle 2$ and $\angle 3$ are supplementary, so $\angle ABC$ is a straight angle.
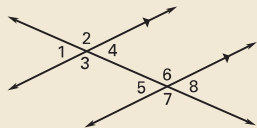
41. Statements (Reasons)

1. $n \parallel p$ (Given)
2. $\angle 1 \cong \angle 3$ (Alternate Interior Angles Theorem)
3. $m\angle 1 = m\angle 3$ (Definition of congruent angles)
4. $\angle 2$ and $\angle 3$ are supplementary (Linear Pair Postulate)
5. $m\angle 2 + m\angle 3 = 180°$ (Definition of supplementary angles)
6. $m\angle 2 + m\angle 1 = 180°$ (Substitution)
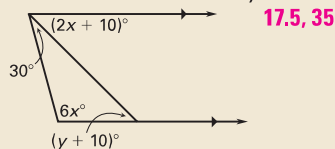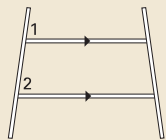7. $\angle 1$ and $\angle 2$ are supplementary. (Definition of supplementary angles)

What theorem justifies each statement?

1. $\angle 3 \cong \angle 6$ **Alt. Interior ⦞ Thm.**

2. $\angle 4$ and $\angle 6$ are supplementary. **Consec. Interior ⦞ Thm.**

3. If $m\angle 2 = 115°$, find $m\angle 7$. **115°**

4. Find the values of x and y. **17.5, 35**

5. The figure shows a plant trellis. If $m\angle 1 = 82°$, find $m\angle 2$. **82°**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation
- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

42. See Additional Answers.

42. **PROOF** The Perpendicular Transversal Theorem states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other. Write a proof of the Perpendicular Transversal Theorem. **See margin.**

    GIVEN ▶ $t \perp r$, $r \parallel s$

    PROVE ▶ $t \perp s$
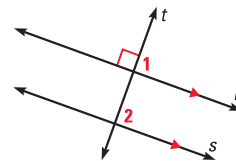
⬚ **43.** **CHALLENGE** In the diagram, $\angle 4 \cong \angle 5$. $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. *Explain* your reasoning. **60°.** *Sample answer:*
$\angle 4 \cong \angle 2$ **by the Alternate Interior Angles Theorem,** $\angle 2 \cong \angle 3$ **by Definition of angle bisector,** $\angle 5 \cong \angle 1$ **by Corresponding Angles Postulate,** $\angle 4 \cong \angle 5$ **is given, so** $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 5$. **Since** $m\angle 1 + m\angle 2 + m\angle 3 = 180°$, **they must each be 60°.**
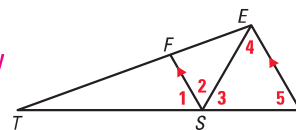
QUIZ

Copy and complete the statement.

1. $\angle 2$ and __?__ are corresponding angles. $\angle 6$

2. $\angle 3$ and __?__ are consecutive interior angles. $\angle 5$

3. $\angle 3$ and __?__ are alternate interior angles. $\angle 6$

4. $\angle 2$ and __?__ are alternate exterior angles. $\angle 7$

Find the value of x.

5.

64

6.

75

7.
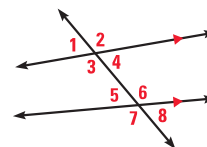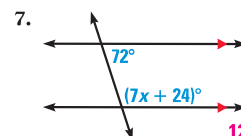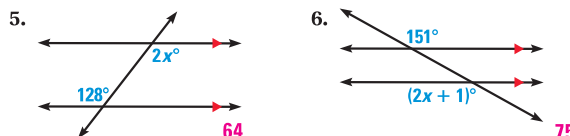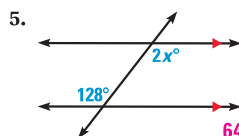
12

3.3 Prove Lines are Parallel

| Before | You used properties of parallel lines to determine angle relationships. |
| Now | You will use angle relationships to prove that lines are parallel. |
| Why? | So you can describe how sports equipment is arranged, as in Ex. 32. |

Key Vocabulary
• paragraph proof
• converse
• two-column proof

Postulate 16 below is the converse of Postulate 15. Similarly, the theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so each converse of a theorem must be proved.

CC.9-12.G.CO.9 Prove theorems about lines and angles.

POSTULATE   *For Your Notebook*

POSTULATE 16 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

$j \parallel k$
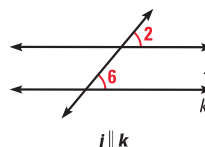
EXAMPLE 1   Apply the Corresponding Angles Converse

ALGEBRA Find the value of x that makes $m \parallel n$.

Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

$(3x + 5)° = 65°$	**Use Postulate 16 to write an equation.**
$3x = 60$	**Subtract 5 from each side.**
$x = 20$	**Divide each side by 3.**

▶ The lines m and n are parallel when $x = 20$.
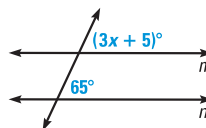
1. Yes. *Sample answer: m ∥ n because the angle corresponding to the angle measuring 75° also measures 75° since it forms a linear pair with the 105° angle. So, corresponding angles are congruent.*

✓ **GUIDED PRACTICE**   for Example 1

1. Is there enough information in the diagram to conclude that $m \parallel n$? *Explain.*

2. *Explain* why Postulate 16 is the converse of Postulate 15.   **Postulate 16 switches the hypothesis and conclusion of Postulate 15.**

3.3 Prove Lines are Parallel   **153**

Standards for Mathematical Content *High School*

CC.9-12.G.CO.9 Prove theorems about lines and angles.

---
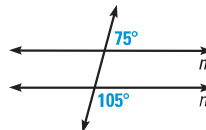
1 PLAN AND PREPARE

Warm-Up Exercises
Also available online
1. Find the value of x. **32**
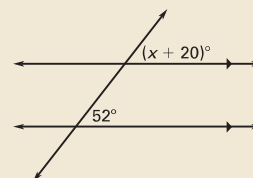
2. Write the converse of the following statement. If it is raining, then Josh needs an umbrella.
If Josh needs an umbrella, then it is raining.

Notetaking Guide
Available online
Promotes interactive learning and notetaking skills.

Pacing
Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
• See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question
Big Idea 2
How do you prove lines parallel?
Tell students they will learn how to answer this question by studying the postulate and theorems in this lesson.

Motivating the Lesson

Tell students that building plans usually include many examples of lines that need to be parallel. Tell them that in this lesson they will learn how to use angle measures to ensure that lines are parallel.

3 TEACH

Extra Example 1

Find the value of *y* that makes *a* ∥ *b*. **23**
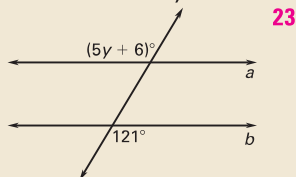
Key Question
Example 1

- What is the difference between what you can prove with the Corresponding Angles Converse and the Corresponding Angles Postulate? **The converse is for proving that lines are parallel. The postulate is for proving that angles are congruent.**

Extra Example 2

Marie was stenciling this design on her kitchen walls. How can she tell if the top and bottom lines of the design are parallel?
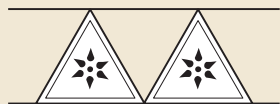
She can measure alternate interior angles and see if they are congruent.

Key Question
Example 2

- What other angles could you measure? **corresponding angles**

THEOREMS *For Your Notebook*

THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

j ∥ *k*
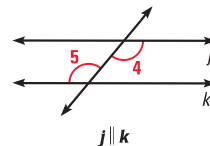
THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

j ∥ *k*
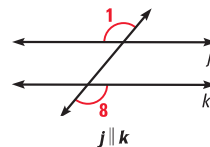
THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

If ∠3 and ∠5 are supplementary, then *j* ∥ *k*.
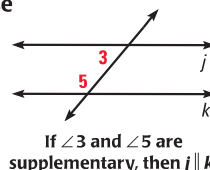
EXAMPLE 2 **Solve a real-world problem**

SNAKE PATTERNS How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?
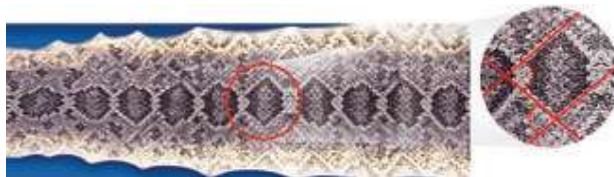
Solution

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.

---

3. yes; Alternate Exterior Angles Converse

4. yes; Corresponding Angles Converse

5. No. *Sample answer:* **Supplementary angles do not have to be congruent.**

✓ **GUIDED PRACTICE** for Example 2

Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.

3.   4.   5. $m∠1 + m∠2 = 180°$

154   Chapter 3   Parallel and Perpendicular Lines
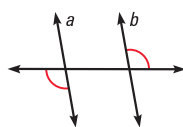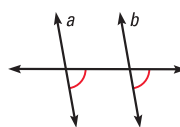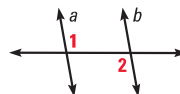
EXAMPLE 3 **Prove the Alternate Interior Angles Converse**

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Solution

GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$
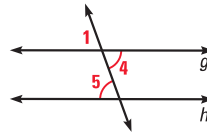
STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	1. Given
2. $\angle 1 \cong \angle 4$	2. Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	3. Transitive Property of Congruence
4. $g \parallel h$	4. Corresponding Angles Converse

Animated Geometry at my.hrw.com

AVOID ERRORS
Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

PARAGRAPH PROOFS A proof can also be written in paragraph form, called a **paragraph proof**. The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.

EXAMPLE 4 **Write a paragraph proof**

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.
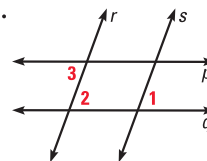
Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

Plan for Proof
a. Look at $\angle 1$ and $\angle 2$.

$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.

If $\angle 2 \cong \angle 3$, then $p \parallel q$.

TRANSITIONAL WORDS
In paragraph proofs, **transitional words** such as *so, then,* and *therefore* help to make the logic clear.
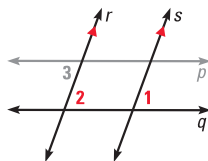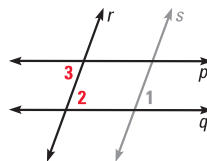
Plan in Action
a. It is given that $r \parallel s$, **so** by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. **Then** $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. **Therefore,** by the Alternate Interior Angles Converse, $p \parallel q$.

3.3 Prove Lines are Parallel **155**

Extra Example 3
Prove that if $\angle 1$ and $\angle 4$ are supplementary, then $a \parallel b$.
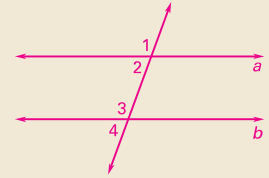
Given: $\angle 1$ and $\angle 4$ are supp.
Prove: $a \parallel b$
Statements (Reasons)
1. $\angle 1$ and $\angle 4$ are supp. (Given)
2. $m\angle 1 + m\angle 4 = 180°$ (Def. of supp. $\angle s$)
3. $m\angle 1 + m\angle 2 = 180°$, $m\angle 3 + m\angle 4 = 180°$ (Linear Pair Post.)
4. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$ (Prop. of =)
5. $m\angle 1 + m\angle 4 + m\angle 2 + m\angle 3 = 360°$ (Prop. of Addn. and =)
6. $180° + m\angle 2 + m\angle 3 = 360°$ (Substitution)
7. $m\angle 2 + m\angle 3 = 180°$ (Subtr. Prop. of =)
8. $a \parallel b$ (Consec. Int. $\angle s$ Conv.)

Extra Example 4
In the figure, $a \parallel b$ and $\angle 1$ is congruent to $\angle 3$. Prove $c \parallel d$. Use a paragraph proof.
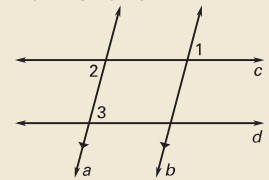
It is given that $a \parallel b$, so $\angle 1 \cong \angle 2$ by Alt. Ext. Ang. Thm. Since $\angle 1 \cong \angle 3$, it follows that $\angle 2 \cong \angle 3$ by Trans. Prop. of $\cong$. Therefore, $c \parallel d$ by the Alt. Int. $\angle s$ Conv.

MATHEMATICAL PRACTICES *Key Question Examples 3, 4*

• Could Example 3 have been done in a paragraph proof and Example 4 in a two-column proof? Explain.
Yes, any proof could be done in either format.

THEOREM *For Your Notebook*

THEOREM 3.7 Transitive Property of Parallel Lines

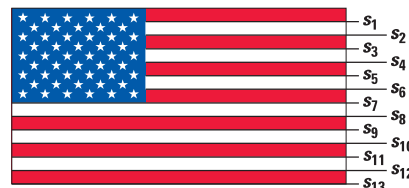If two lines are parallel to the same line, then they are parallel to each other.

If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.
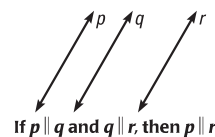
EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.

Solution

The stripes from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

USE SUBSCRIPTS
When you name several similar items, you can use one variable with subscripts to keep track of the items.

✓ **GUIDED PRACTICE** for Examples 3, 4, and 5

6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use? **Given: $\angle 1 \cong \angle 8$, Prove: $j \parallel k$**

7. Vertical Angles Congruence Theorem; $\angle 1 \cong \angle 5$; Corresponding Angles Converse
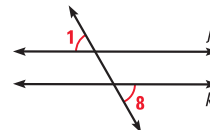
7. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 3.

It is given that $\angle 4 \cong \angle 5$. By the __?__, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, __?__. So, by the __?__, $g \parallel h$.

8. All of the steps are parallel. Since the bottom step is parallel to the ground, the Transitive Property of Parallel Lines applies, and the top step is parallel to the ground.

8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.

Ryan McVay/Getty Images

# 3.3 EXERCISES

SKILL PRACTICE

A 1. **VOCABULARY** Draw a pair of parallel lines with a transversal. Identify all pairs of *alternate exterior angles*. **See margin.**

2. ★ **WRITING** Use the theorems from the lesson *Use Parallel Lines and Transversals* and the converses of those theorems in this lesson. Write three biconditionals about parallel lines and transversals. **See margin.**

EXAMPLE 1
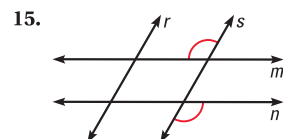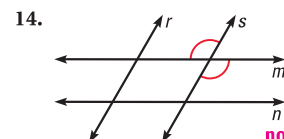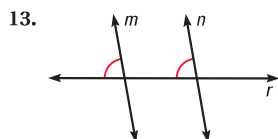for Exs. 3–9

xy ALGEBRA Find the value of x that makes $m \parallel n$.

3.

120° m
3x° n
40

4.

135° m
$(2x + 15)°$ n
60

5.

150° m
$(3x - 15)°$ n
15

6.

$(180 - x)°$
$x°$
90

7.

2$x°$ $x°$
60

8.

$(2x + 20)°$
3$x°$
20
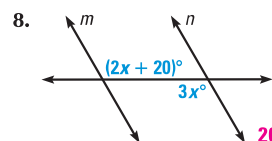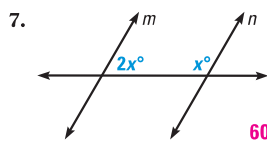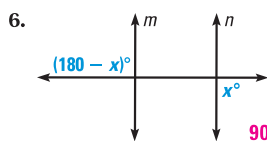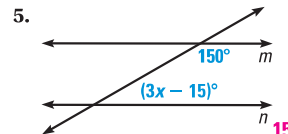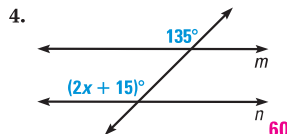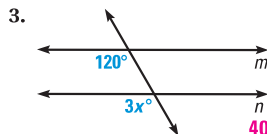
9. **ERROR ANALYSIS** A student concluded that lines a and b are parallel. *Describe* and correct the student's error.
The student believes that $x = y$ but there is no indication that they are equal.

$a \parallel b$

✗
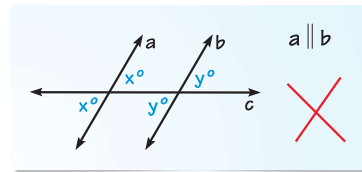
EXAMPLE 2
for Exs. 10–17

IDENTIFYING PARALLEL LINES Is there enough information to prove $m \parallel n$? If so, state the postulate or theorem you would use.

10. yes; Alternate Interior Angles Converse

11. yes; Alternate Exterior Angles Converse

13. yes; Corresponding Angles Converse

15. yes; Alternate Exterior Angles Converse

10.

m n
r

⑪.

m n
r

12.

r
m
n
no

13.

m n
r

14.

r s
m
n
no

15.

r s
m
n
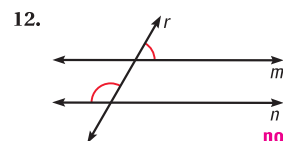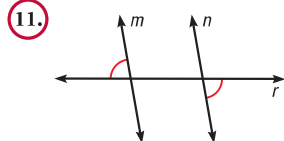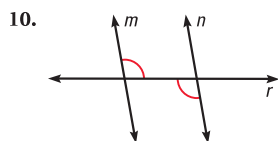
16. ★ **OPEN-ENDED MATH** Use lined paper to draw two parallel lines cut by a transversal. Use a protractor to measure one angle. Find the measures of the other seven angles without using the protractor. Give a theorem or postulate you use to find each angle measure. **See margin.**

1. **Sample:**

n
1 2 ℓ
3 4
5 6 m
7 8

∠1 and ∠8, ∠2 and ∠7

2. **Given two lines cut by a transversal, alternate interior angles are congruent if and only if the lines are parallel; given two lines cut by a transversal, alternate exterior angles are congruent if and only if the lines are parallel; given two lines cut by a transversal, consecutive interior angles are supplementary if and only if the lines are parallel.**

16. See Additional Answers.
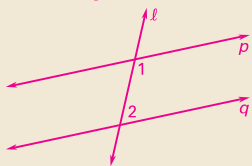
17. **MULTI-STEP PROBLEM** Complete the steps below to determine whether $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ are parallel.

   a. Find $m\angle DCG$ and $m\angle CGH$.   **m∠ DCG = 115°, m∠ CGH = 65°**

   b. *Describe* the relationship between $\angle DCG$ and $\angle CGH$.

   **c.** Are $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ parallel? *Explain* your reasoning.
     yes; Consecutive Interior Angles Converse
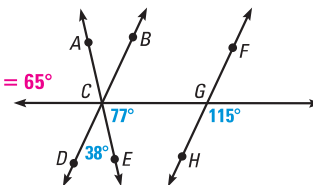
18. **PLANNING A PROOF** Use these steps to plan a proof of the Consecutive Interior Angles Converse: if two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

   a. Draw a diagram you can use in a proof of the theorem.   **See margin.**

   b. Write the GIVEN and PROVE statements.
     Given: ∠1 and ∠2 are supplementary, Prove: p ∥ q

B   **REASONING** Can you prove that lines *a* and *b* are parallel? If so, *explain* how.
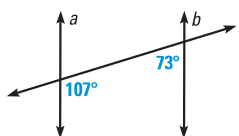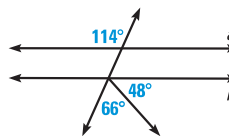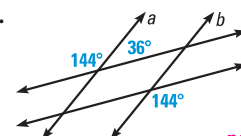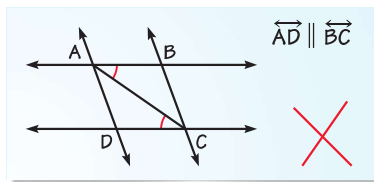
19.   **20.**   **21.**   **no**

22. **ERROR ANALYSIS** A student decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram below. *Describe* and correct the student's error.

The student assumed the congruent angles were alternate interior angles between $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$. By the Alternate Interior Angles Converse, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$.
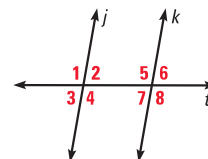
23. ★ **MULTIPLE CHOICE** Use the diagram at the right. You know that $\angle 1 \cong \angle 4$. What can you conclude? **D**

  Ⓐ $p \parallel q$ Ⓑ $r \parallel s$

  Ⓒ $\angle 2 \cong \angle 3$ Ⓓ None of the above
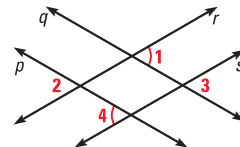
REASONING Use the diagram at the right for Exercises 24 and 25.
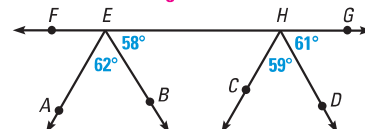
24. ★ **SHORT RESPONSE** In the diagram, assume $j \parallel k$. How many angle measures must be given in order to find the measure of every angle? *Explain* your reasoning.

25. **PLANNING A PROOF** In the diagram, assume $\angle 1$ and $\angle 7$ are supplementary. Write a plan for a proof showing that lines *j* and *k* are parallel.   ***Sample answer:* ∠1 ≅ ∠4 therefore ∠4 and ∠7 are supplementary. Lines *j* and *k* are parallel by the Consecutive Interior Angles Converse.**

26. **REASONING** Use the diagram at the right. Which rays are parallel? Which rays are not parallel? *Justify* your conclusions.
  $\overrightarrow{EA} \parallel \overrightarrow{HC}$; $\overrightarrow{EB}$ **is not parallel to** $\overrightarrow{HD}$, $\angle GHC \cong \angle HEA$, $\angle GHD$ **is not congruent to** $\angle HEB$.

○ = See **WORKED-OUT SOLUTIONS**
   in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

C 27. **VISUAL REASONING** A point R is not in plane ABC.

   a. How many lines through R are perpendicular to plane ABC? **1 line**

   b. How many lines through R are parallel to plane ABC? **an infinite number of lines**

   c. How many planes through R are parallel to plane ABC? **1 plane**

28. **CHALLENGE** Use the diagram.

   a. Find x so that $p \parallel q$. **54**

   b. Find y so that $r \parallel s$. **47.5**

   c. Can r be parallel to s and p be parallel to q at the same time? *Explain.*
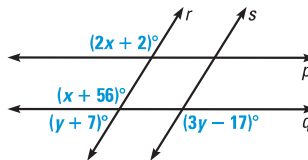
   No. *Sample answer:* **For p to be parallel to q, $x = 54$, then $y = 63$ because of the linear pair formed, but in order for r and s to be parallel, y must equal 47.5.**

PROBLEM SOLVING

EXAMPLE 2 **A**
for Exs. 29–30

29. **PICNIC TABLE** How do you know that the top of the picnic table is parallel to the ground?

   Alternate Interior Angles Converse Theorem
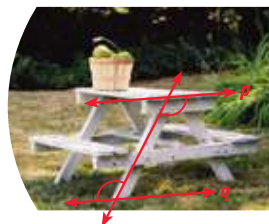
30. **KITEBOARDING** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m? **Corresponding Angles Converse**

31. **DEVELOPING PROOF** Copy and complete the proof.

   GIVEN ▶ $m\angle 1 = 115°$, $m\angle 2 = 65°$
   PROVE ▶ $m \parallel n$
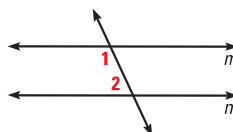
STATEMENTS	REASONS
1. $m\angle 1 = 115°$ and $m\angle 2 = 65°$	1. Given
2. $115° + 65° = 180°$	2. Addition
3. $m\angle 1 + m\angle 2 = 180°$	3. _?_ **Substitution**
4. $\angle 1$ and $\angle 2$ are supplementary.	4. _?_ **Definition of supplementary angles**
5. $m \parallel n$	5. _?_ **Consecutive Interior Angles Converse**

3.3 Prove Lines are Parallel **159**

Study Strategy

Exercises 34–35 If students are having difficulty getting started, suggest that they sketch a copy of the figure and mark the angles that are given congruent.

33. Yes. *Sample answer:*
E 20th is parallel to E 19th by the Corresponding Angles Converse. E 19th is parallel to E 18th by the Alternate Exterior Angles Converse. E 18th is parallel to E 17th by the Alternate Interior Angles Converse. They are all parallel by the Transitive Property of Parallel Lines.

34. Statements (Reasons)

1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Given)
2. $\angle 2 \cong \angle 3$ (Vertical Angles Congruence Theorem)
3. $\angle 1 \cong \angle 4$ (Transitive Property of Angle Congruence)
4. $\overline{AB} \parallel \overline{CD}$ (Alternate Interior Angles Converse)

35. Statements (Reasons)

1. $a \parallel b$, $\angle 2 \cong \angle 3$ (Given)
2. $\angle 2$ and $\angle 4$ are supplementary. (Consecutive Interior Angles Theorem)
3. $\angle 3$ and $\angle 4$ are supplementary. (Substitution)
4. $c \parallel d$ (Consecutive Interior Angles Converse)

36. Statements (Reasons)

1. $\angle 2 \cong \angle 7$ (Given)
2. $\angle 7 \cong \angle 6$ (Vertical Angles Congruence Theorem)
3. $\angle 2 \cong \angle 6$ (Transitive Property of Congruence)
4. $m \parallel n$ (Corresponding Angles Converse)

37. You are given that $\angle 3$ and $\angle 5$ are supplementary. By the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are also supplementary. So $\angle 3 \cong \angle 6$ by the Congruent Supplements Theorem. By the Alternate Interior Angles Converse, $m \parallel n$.

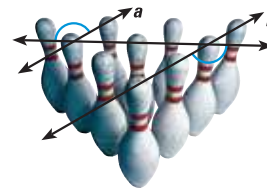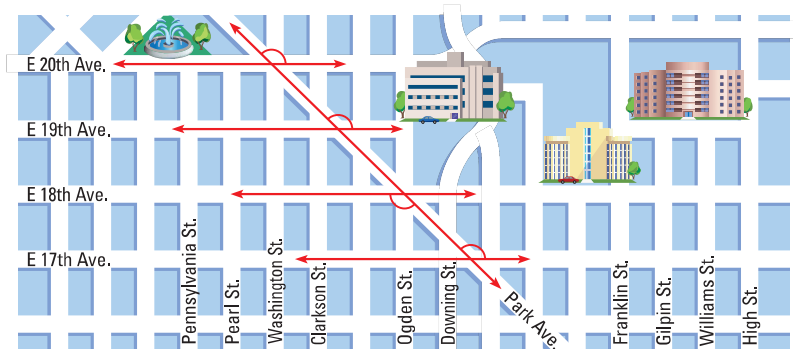**32. BOWLING PINS** How do you know that the bowling pins are set up in parallel lines?
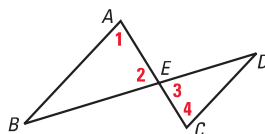**Alternate Exterior Angles Converse Theorem**

EXAMPLE 5
for Ex. 33

33. ★ SHORT RESPONSE The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? *Explain* how you can tell. **See margin.**
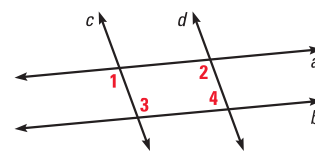
EXAMPLE 3 [B]
for Exs. 34–35

PROOF Use the diagram and the given information to write a two-column or paragraph proof. **34, 35. See margin.**

34. GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
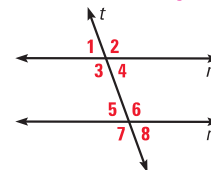**PROVE ▶** $\overline{AB} \parallel \overline{CD}$

35. GIVEN ▶ $a \parallel b$, $\angle 2 \cong \angle 3$
PROVE ▶ $c \parallel d$

EXAMPLE 4
for Exs. 36–37

PROOF In Exercises 36 and 37, use the diagram to write a paragraph proof.
36, 37. See margin.

36. PROVING THEOREM 3.5 In the diagram, assume $\angle 2 \cong \angle 7$. Prove the Alternate Exterior Angles Converse.

(37.) PROVING THEOREM 3.6 In the diagram, assume $\angle 3$ and $\angle 5$ are supplementary. Prove the Consecutive Interior Angles Converse.

38. MULTI-STEP PROBLEM Use these steps to prove Theorem 3.7, the Transitive Property of Parallel Lines.
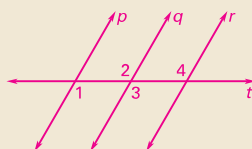
a. Copy the diagram in the Transitive Property of Parallel Lines Theorem box. Draw a transversal through all three lines. **See margin.**

b. Write the GIVEN and PROVE statements. **Given:** $p \parallel q$ and $q \parallel r$, **Prove:** $p \parallel r$

c. Use the properties of angles formed by parallel lines and transversals to prove the theorem. **See margin.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources
★ = **STANDARDIZED TEST PRACTICE**

160

38a.

38c. Statements (Reasons)

1. $p \parallel q$ and $q \parallel r$ (Given)
2. $\angle 1 \cong \angle 2$ (Alternate Interior Angles Theorem)
3. $\angle 2 \cong \angle 3$ (Vertical Angles Congruence Theorem)
4. $\angle 3 \cong \angle 4$ (Alternate Interior Angles Theorem)
5. $\angle 1 \cong \angle 4$ (Transitive Property of Angle Congruence)
6. $p \parallel r$ (Alternate Interior Angles Converse)

C-Squared Studios/Getty Images

39. ★ **EXTENDED RESPONSE** Architects and engineers make drawings using a plastic triangle with angle measures 30°, 60°, and 90°. The triangle slides along a fixed horizontal edge.
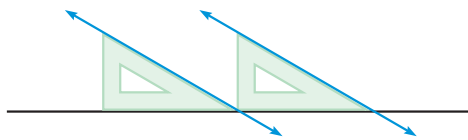
a. *Explain* why the blue lines shown are parallel. *Sample answer:* **Corresponding Angles Converse Theorem**

b. *Explain* how the triangle can be used to draw vertical parallel lines.

39b. Slide the triangle along a fixed horizontal line and use the edge that forms the 90° angle to draw vertical lines.

REASONING Use the diagram below in Exercises 40–44. How would you show that the given lines are parallel? **40–44. See margin.**

40. *a* and *b*

41. *b* and *c*

42. *d* and *f*

43. *e* and *g*

44. *a* and *c*
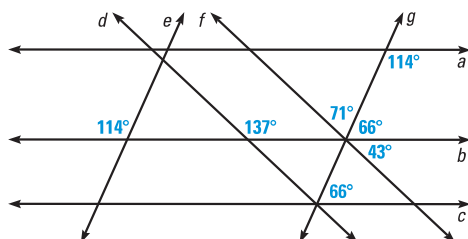
C **45. CHALLENGE** Use these steps to investigate the angle bisectors of angles created by a transversal and parallel lines.

a. Construction Use geometry drawing software to construct line ℓ, point *P* not on ℓ, and line *n* through *P* parallel to ℓ. Construct point *Q* on ℓ and construct $\overrightarrow{PQ}$. Choose a pair of alternate interior angles and construct their angle bisectors. **Check constructions.**

b. Write a Proof Make a conjecture about the angle bisectors. Write a proof of your conjecture. **The angle bisectors are parallel; see margin.**
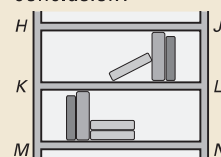
MIXED REVIEW *of Problem Solving*

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** Use the diagram of the tennis court below.

a. Identify two pairs of parallel lines so each pair is on a different plane. **Sample answer; q and p, k and m**
b. Identify a pair of skew lines. **Sample answer; q and m**
c. Identify two pairs of perpendicular lines. **Sample answer; n and m, n and k**

2. **MULTI-STEP PROBLEM** Use the picture of the tile floor below.
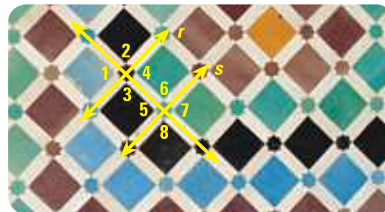
a. Name the kind of angle pair each angle forms with ∠1. **See margin.**

b. Lines *r* and *s* are parallel. Name the angles that are congruent to ∠3. **∠2, ∠6, ∠8**

3. **OPEN-ENDED** The flag of Jamaica is shown. Given that *n* ∥ *p* and *m*∠1 = 53°, determine the measure of ∠2. *Justify* each step in your argument, labeling any angles needed for your justification.
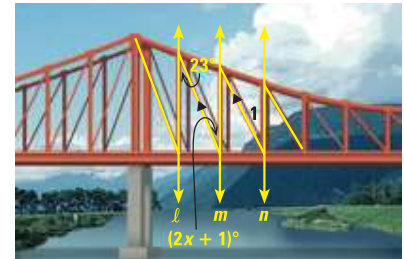**53°; Alternate Exterior Angles Theorem**

**5b. 23°; Transitive Property of Parallel Lines and Alternate Interior Angles Theorem**

4. **SHORT RESPONSE** A neon sign is shown below. Are the top and the bottom of the Z parallel? *Explain* how you know. **See margin.**

5. **EXTENDED RESPONSE** Use the diagram of the bridge below.

a. Find the value of *x* that makes lines ℓ and *m* parallel. **11**

b. Suppose that ℓ ∥ *m* and ℓ ∥ *n*. Find *m*∠1. *Explain* how you found your answer. Copy the diagram and label any angles you need for your explanation. **See below.**

6. **GRIDDED ANSWER** In the photo of the picket fence, *m* ∥ *n*. What is *m*∠1 in degrees? **150°**
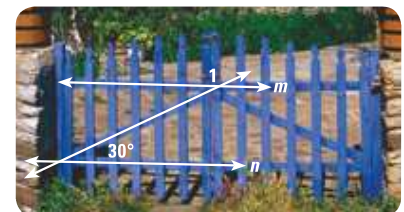
7. **SHORT RESPONSE** Find the values of *x* and *y*. *Explain* your steps. **See margin.**
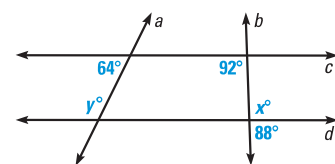
(t) Paul A. Souders/Corbis; (c) Bruno Morandi/Getty Images

3.4 Find and Use Slopes of Lines

Key Vocabulary
- slope
- rise
- run

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line.

If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2) then the slope m is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$
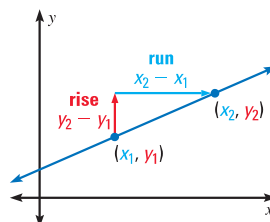
KEY CONCEPT
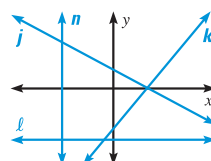*For Your Notebook*

Slope of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

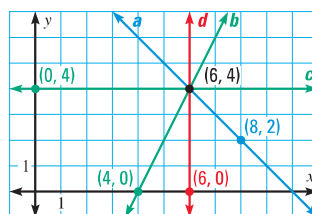**Undefined slope:** vertical, as in line n

---

EXAMPLE 1 Find slopes of lines in a coordinate plane

Find the slopes of line a and line d.

Solution

Slope of line a: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{6 - 8} = \frac{2}{-2} = -1$

Slope of line d: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 6} = \frac{4}{0}$,

which is undefined.

✓ **GUIDED PRACTICE** for Example 1

Use the graph in Example 1. Find the slope of the line.

1. Line b **2**
2. Line c **0**

3.4 Find and Use Slopes of Lines **163**

© Royalty Free/Corbis

COMPARING SLOPES When two lines intersect in a coordinate plane, the steeper line has the slope with greater absolute value. You can also compare slopes to tell whether two lines are parallel or perpendicular.

POSTULATES   *For Your Notebook*

POSTULATE 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

$m_1 = m_2$

POSTULATE 18 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

$m_1 \cdot m_2 = -1$

READ VOCABULARY
If the product of two numbers is -1, then the numbers are called *negative reciprocals*.

EXAMPLE 2   Identify parallel lines

Find the slope of each line. Which lines are parallel?
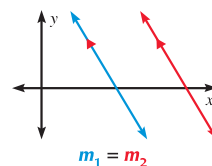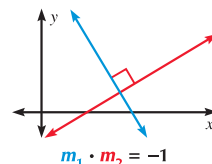
Solution

Find the slope of k_1 through $(-2, 4)$ and $(-3, 0)$.

$$m_1 = \frac{0 - 4}{-3 - (-2)} = \frac{-4}{-1} = 4$$

Find the slope of k_2 through $(4, 5)$ and $(3, 1)$.

$$m_2 = \frac{1 - 5}{3 - 4} = \frac{-4}{-1} = 4$$

Find the slope of k_3 through $(6, 3)$ and $(5, -2)$.

$$m_3 = \frac{-2 - 3}{5 - 6} = \frac{-5}{-1} = 5$$

▶ Compare the slopes. Because k_1 and k_2 have the same slope, they are parallel. The slope of k_3 is different, so k_3 is not parallel to the other lines.
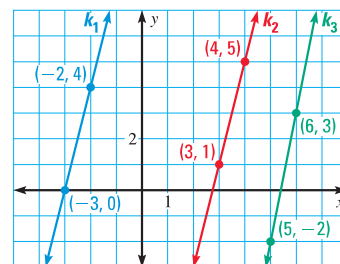
✓ **GUIDED PRACTICE**   for Example 2

3. Line m passes through $(-1, 3)$ and $(4, 1)$. Line t passes through $(-2, -1)$ and $(3, -3)$. Are the two lines parallel? *Explain* how you know.
Yes; they have the same slope.

164   Chapter 3   Parallel and Perpendicular Lines

EXAMPLE 3 Draw a perpendicular line

Line h passes through (3, 0) and (7, 6). Graph the line perpendicular to h that passes through the point (2, 5).

Solution

STEP 1 **Find** the slope m_1 of line h through (3, 0) and (7, 6).

$$m_1 = \frac{6-0}{7-3} = \frac{6}{4} = \frac{3}{2}$$

STEP 2 **Find** the slope m_2 of a line perpendicular to h. Use the fact that the product of the slopes of two perpendicular lines is -1.

$$\frac{3}{2} \cdot m_2 = -1 \quad \text{Slopes of perpendicular lines}$$
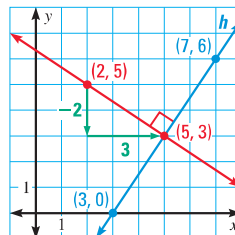
$$m_2 = \frac{-2}{3} \quad \text{Multiply each side by } \frac{2}{3}.$$

**STEP 3** **Use** the rise and run to graph the line.

> **REVIEW GRAPHING**
> Given a point on a line and the line's slope, you can use the rise and run to find a second point and draw the line.

⭐ EXAMPLE 4 Standardized Test Practice

A skydiver made jumps with three parachutes. The graph shows the height of the skydiver from the time the parachute opened to the time of the landing for each jump. Which statement is true?
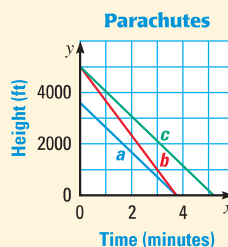
Parachutes

A The parachute opened at the same height in jumps a and b.

B The parachute was open for the same amount of time in jumps b and c.

C The skydiver descended at the same rate in jumps a and b.

D The skydiver descended at the same rate in jumps a and c.

> **ELIMINATE CHOICES**
> The y-intercept represents the height when the parachute opened, so the heights in jumps a and b were not the same. So you can eliminate choice A.

Solution

The rate at which the skydiver descended is represented by the slope of the segments. The segments that have the same slope are a and c.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ ●

✓ GUIDED PRACTICE for Examples 3 and 4

4. Line n passes through (0, 2) and (6, 5). Line m passes through (2, 4) and (4, 0). Is $n \perp m$? *Explain.* **Yes; the product of their slopes is −1.**

5. In Example 4, which parachute is in the air for the longest time? *Explain.*

6. In Example 4, what do the x-intercepts represent in the situation? How can you use this to eliminate one of the choices? **Time of the landing. *Sample answer:* b and c are in the air different amounts of time, so you can eliminate choice B.**

5. Parachute c. *Sample answer:* It was in the air approximately 1.25 minutes longer than either a or b.

3.4   Find and Use Slopes of Lines   **165**

EXAMPLE 5 **Solve a real-world problem**

ROLLER COASTERS During the climb on the Magnum XL-200 roller coaster, you move 41 feet upward for every 80 feet you move horizontally. At the crest of the hill, you have moved 400 feet forward.

a. **Making a Table** Make a table showing the height of the Magnum at every 80 feet it moves horizontally. How high is the roller coaster at the top of its climb?

b. **Calculating** Write a fraction that represents the height the Magnum climbs for each foot it moves horizontally. What does the numerator represent?

c. **Using a Graph** Another roller coaster, the Millennium Force, climbs at a slope of 1. At its crest, the horizontal distance from the starting point is 310 feet. Compare this climb to that of the Magnum. Which climb is steeper?

Solution

a.

Horizontal distance (ft)	80	160	240	320	400
Height (ft)	41	82	123	164	205

The Magnum XL-200 is 205 feet high at the top of its climb.

b. Slope of the Magnum $= \dfrac{\text{rise}}{\text{run}} = \dfrac{41}{80} = \dfrac{41 \div 80}{80 \div 80} = \dfrac{0.5125}{1}$

The numerator, 0.5125, represents the slope in decimal form.

c. Use a graph to compare the climbs. Let x be the horizontal distance and let y be the height. Because the slope of the Millennium Force is 1, the rise is equal to the run. So the highest point must be at (310, 310).

▶ The graph shows that the Millennium Force has a steeper climb, because the slope of its line is greater ($1 > 0.5125$).

Roller Coaster Slopes

Animated **Geometry** at my.hrw.com
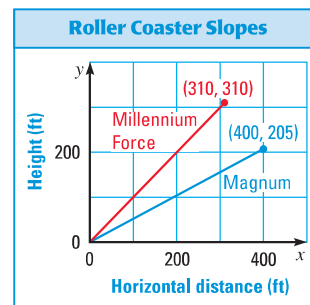
✓ **GUIDED PRACTICE** **for Example 5**

7. Line q passes through the points (0, 0) and (−4, 5). Line t passes through the points (0, 0) and (−10, 7). Which line is steeper, q or t? **line q**

8. **WHAT IF?** Suppose a roller coaster climbed 300 feet upward for every 350 feet it moved horizontally. Is it *more steep* or *less steep* than the Magnum? than the Millennium Force? **more steep; less steep**

13. Perpendicular; the product of their slopes is −1.

14. Neither; the slopes are not equal and their product is not −1.

15. Perpendicular; the product of their slopes is −1.

3.4 EXERCISES

SKILL PRACTICE

A

1. VOCABULARY *Describe* what is meant by the slope of a nonvertical line. **See margin.**

2. ★ WRITING What happens when you apply the slope formula to a horizontal line? What happens when you apply it to a vertical line?
Slope is 0; slope is undefined.

EXAMPLE 1
for Exs. 3–12

1. The slope of a nonvertical line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line.

MATCHING Match the description of the slope of a line with its graph.

3. m is positive. **D**   **4.** m is negative. **A**   **5.** m is zero. **B**   **6.** m is undefined. **C**

A.   **B.**   **C.**   **D.**
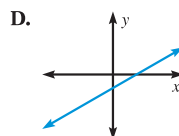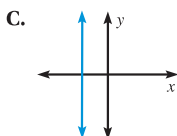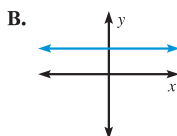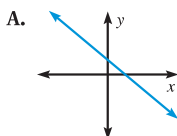
FINDING SLOPE Find the slope of the line that passes through the points.

7. (3, 5), (5, 6) $\frac{1}{2}$ **8.** (−2, 2), (2, −6)   **−2**   **9.** (−5, −1), (3, −1)   **0**   **10.** (2, 1), (0, 6) $-\frac{5}{2}$

11. Slope was computed using $\frac{\text{run}}{\text{rise}}$, it should be $\frac{\text{rise}}{\text{run}}$; $m = \frac{3}{4}$.

ERROR ANALYSIS *Describe* and correct the error in finding the slope of the line.

11.

$m = \frac{4}{3}$

(6, 4)
(2, 1)
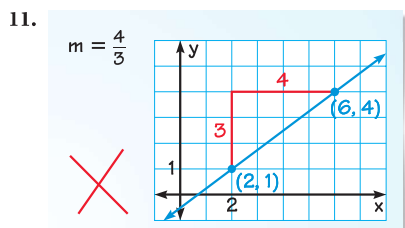
12.

Slope of the line through (2, 7) and (4, 5)

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - 5}{4 - 2} = \dfrac{2}{2} = 1$

x_2 and x_1 were interchanged; $\dfrac{7-5}{2-4} = \dfrac{2}{-2} = -1.$

EXAMPLES 2 and 3
for Exs. 13–18

TYPES OF LINES Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. *Justify* your answer. **13–15. See margin.**

13. Line 1: (1, 0), (7, 4)
    Line 2: (7, 0), (3, 6)

14. Line 1: (−3, 1), (−7, −2)
    Line 2: (2, −1), (8, 4)

15. Line 1: (−9, 3), (−5, 7)
    Line 2: (−11, 6), (−7, 2)

GRAPHING Graph the line through the given point with the given slope.
16–18. See margin.

16. $P(3, −2)$, slope $-\frac{1}{6}$ **17.** $P(−4, 0)$, slope $\frac{5}{2}$ **18.** $P(0, 5)$, slope $\frac{2}{3}$

EXAMPLES 4 and 5
for Exs. 19–22

B

STEEPNESS OF A LINE Tell which line through the given points is steeper.

19. Line 1: (−2, 3), (3, 5)
    Line 2: (3, 1), (6, 5)
    line 2

20. Line 1: (−2, −1), (1, −2)
    Line 2: (−5, −3), (−1, −4)
    line 1

21. Line 1: (−4, 2), (−3, 6)
    Line 2: (1, 6), (3, 8)
    line 1

22. REASONING Use your results from Exercises 19−21. *Describe* a way to determine which of two lines is steeper without graphing them. **Find the slopes and compare them. The one that has a larger absolute value is steeper.**

3.4   Find and Use Slopes of Lines   **167**

16.
(3, −2)
(9, −3)

17.
(−2, 5)
(−4, 0)

18.
(3, 7)
(0, 5)

23. −2;

24. $\frac{1}{3}$;

25. 7;

26.

PERPENDICULAR LINES Find the slope of line n perpendicular to line h and passing through point P. Then copy the graph and graph line n. **23–25. See margin.**

23.

24.

25.
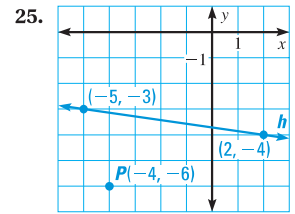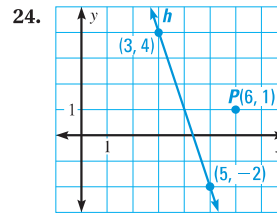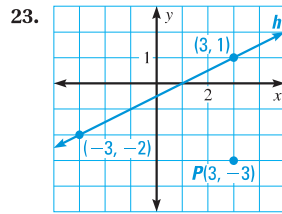
26. No; the slope between points $(-3, 3)$ and $(1, -2)$ is $\frac{-5}{4}$ while the slope between points $(1, -2)$ and $(4, 0)$ is $\frac{2}{3}$; see margin for art.

26. **REASONING** Use the concept of slope to decide whether the points $(-3, 3)$, $(1, -2)$, and $(4, 0)$ lie on the same line. *Explain* your reasoning and include a diagram.

GRAPHING Graph a line with the given description. **27–29. See margin.**

27. Through $(0, 2)$ and parallel to the line through $(-2, 4)$ and $(-5, 1)$

28. Through $(1, 3)$ and perpendicular to the line through $(-1, -1)$ and $(2, 0)$

29. Through $(-2, 1)$ and parallel to the line through $(3, 1)$ and $(4, -\frac{1}{2})$

C **CHALLENGE** Find the unknown coordinate so the line through the points has the given slope.

30. $(-3, 2)$, $(0, y)$; slope -2 **−4** **31.** $(-7, -4)$, $(x, 0)$; slope $\frac{1}{3}$ **5** **32.** $(4, -3)$, $(x, 1)$; slope -4 **3**

PROBLEM SOLVING

A **33.** **WATER SLIDE** The water slide is 6 feet tall, and the end of the slide is 9 feet from the base of the ladder. About what slope does the slide have? **$\frac{2}{3}$**
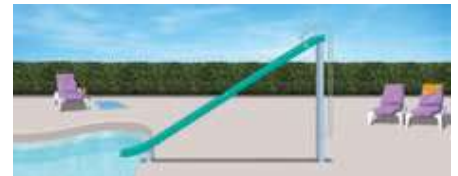
EXAMPLE 5
for Exs. 34–37

34. ★ **MULTIPLE CHOICE** Which car has better gas mileage? **B**

A A   B B

C Same rate   D Cannot be determined
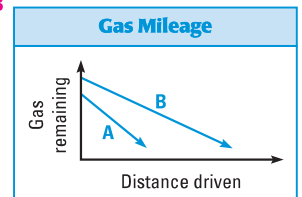
35. ★ **SHORT RESPONSE** *Compare* the graphs of the three lines described below. Which is most steep? Which is the least steep? Include a sketch in your answer. **Line b; line c; see margin for art.**

Line a: through the point $(3, 0)$ with a y-intercept of 4
Line b: through the point $(3, 0)$ with a y-intercept greater than 4
Line c: through the point $(3, 0)$ with a y-intercept between 0 and 4

27.

28.

29.

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**   ◆ = **MULTIPLE REPRESENTATIONS**
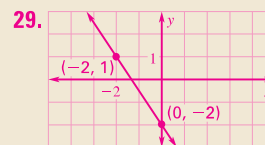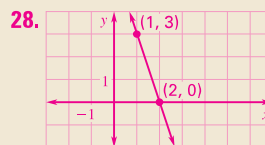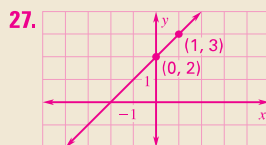
36. MULTI-STEP PROBLEM Ladder safety guidelines include the following recommendation about ladder placement. The horizontal distance h between the base of the ladder and the object the ladder is resting against should be about one quarter of the vertical distance v between the ground and where the ladder rests against the object.

Make sure to place the ladder on a level place. If the ladder is not steady, it is not safe to climb.

Place the base so the distance h to the building is about one quarter of the height v to where the ladder hits the building.

a. Find the recommended slope for a ladder. **4**

b. Suppose the base of a ladder is 6 feet away from a building. The ladder has the recommended slope. Find v. **24 ft**

c. Suppose a ladder is 34 feet from the ground where it touches a building. The ladder has the recommended slope. Find h. **8.5 ft**

[B] **37. ◆ MULTIPLE REPRESENTATIONS** The Duquesne (pronounced "du-KAYN") Incline was built in 1888 in Pittsburgh, Pennsylvania, to move people up and down a mountain there. On the incline, you move about 29 feet vertically for every 50 feet you move horizontally. When you reach the top of the hill, you have moved a horizontal distance of about 700 feet.

a. **Making a Table** Make a table showing the vertical distance that the incline moves for each 50 feet of horizontal distance during its climb. How high is the incline at the top? **See margin for table; 406 ft.**

b. **Drawing a Graph** Write a fraction that represents the slope of the incline's climb path. Draw a graph to show the climb path. $\dfrac{29}{50}$; **see margin for art.**

c. **Comparing Slopes** The Burgenstock Incline in Switzerland moves about 144 vertical feet for every 271 horizontal feet. Write a fraction to represent the slope of this incline's path. Which incline is steeper, the *Burgenstock* or the *Duquesne*? $\dfrac{144}{271}$; **Duquesne**

38. PROVING THEOREM 3.7 Use slopes of lines to write a paragraph proof of the Transitive Property of Parallel Lines: if two lines are parallel to the same line, then they are parallel to each other.

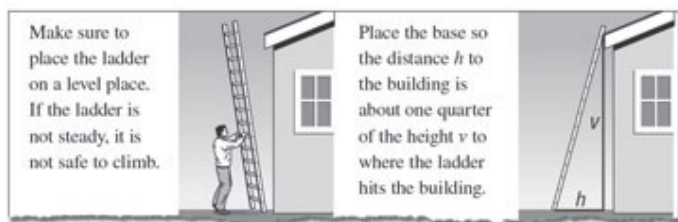**AVERAGE RATE OF CHANGE** In Exercises 39 and 40, slope can be used to describe an *average rate of change*. To write an average rate of change, rewrite the slope fraction so the denominator is one.

39. BUSINESS In 2000, a business made a profit of $8500. In 2006, the business made a profit of $15,400. Find the average rate of change in dollars per year from 2000 to 2006. **$1150 per year**

40. ROCK CLIMBING A rock climber begins climbing at a point 400 feet above sea level. It takes the climber 45 minutes to climb to the destination, which is 706 feet above sea level. Find the average rate of change in feet per minute for the climber from start to finish. **6.8 ft/min**

3.4 Find and Use Slopes of Lines **169**

© Bruce Leighty/Photolibrary.com
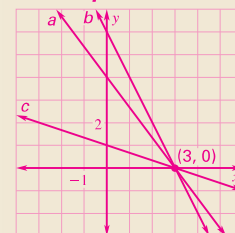
35. Sample:

37a. See below.

37b.

38. Sample answer: Given $p \parallel q$ and $q \parallel r$, prove $p \parallel r$. Since $p \parallel q$, lines p and q have the same slope, by the Slopes of Parallel Lines Postulate. Similarly, since $q \parallel r$, lines q and r have the same slope. So lines p, q, and r all have the same slope. Therefore, lines p and r have the same slope, and $p \parallel r$ by the Slopes of Parallel Lines Postulate.

37a.

Horizontal distance (ft)	50	100	150	200	250	300	350	400	450	500	550	600	650	700
Height (ft)	29	58	87	116	145	174	203	232	261	290	319	348	377	406

41a. 1985 to 1990. *Sample answer:* **about 2 million people per year**

41b. 1995 to 2000. *Sample answer:* **about 3 million people per year**

41c. *Sample answer:* **There was moderate but steady increase in attendance for the NFL over the time period of 1985–2000.** C

41. ★ **EXTENDED RESPONSE** The line graph shows the regular season attendance (in millions) for three professional sports organizations from 1985 to 2000.

   a. During which five-year period did the NBA attendance increase the most? Estimate the rate of change for this five-year period in people per year.

   b. During which five-year period did the NHL attendance increase the most? Estimate the rate of change for this five-year period in people per year.

   c. **Interpret** The line graph for the NFL seems to be almost linear between 1985 and 2000. Write a sentence about what this means in terms of the real-world situation.

PROFESSIONAL SPORTS ATTENDANCE

● National Basketball Association (NBA)
● National Football League (NFL)
● National Hockey League (NHL)
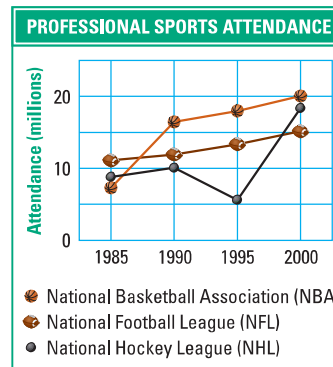
42. **CHALLENGE** Find two values of k such that the points $(-3, 1)$, $(0, k)$, and $(k, 5)$ are collinear. *Explain* your reasoning.

   3, −5. *Sample answer:* Set $\dfrac{k-1}{3} = \dfrac{k-5}{-k}$ and solve for k.

QUIZ

Find the value of x that makes $m \parallel n$.

1.

$2x°$ m
$54°$ n
63

2.

$(3x - 5)°$ m
$145°$ n
50

3.
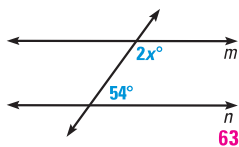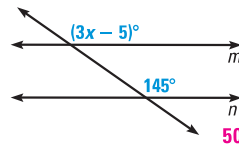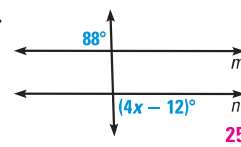
$88°$ m
$(4x - 12)°$ n
25

Find the slope of the line that passes through the given points.

4. $(1, -1), (3, 3)$ **2**

5. $(1, 2), (4, 5)$ **1**

6. $(-3, -2), (-7, -6)$ **1**

my.hrw.com
Keystrokes

Investigate Slopes

MATERIALS • graphing calculator or computer

MATHEMATICAL PRACTICES
Use appropriate tools strategically.

QUESTION How can you verify the Slopes of Parallel Lines Postulate?

You can verify the postulates you learned using geometry drawing software.

EXAMPLE Verify the Slopes of Parallel Lines Postulate

STEP 1 *Show axes* Show the *x*-axis and the *y*-axis by choosing Hide/Show Axes from the F5 menu.

STEP 2 *Draw line* Draw a line by choosing Line from the F2 menu. Do not use one of the axes as your line. Choose a point on the line and label it *A*.

STEP 3 *Graph point* Graph a point not on the line by choosing Point from the F2 menu.

STEP 4 *Draw parallel line* Choose Parallel from the F3 menu and select the line. Then select the point not on the line.

STEP 5 *Measure slopes* Select one line and choose Measure Slope from the F5 menu. Repeat this step for the second line.

STEP 6 *Move line* Drag point *A* to move the line. What do you expect to happen?

STEPS 1–3

```
F5
Hide/Show   ►
Alph-Num
Display
Measure     ►
Coord.&Eq.
Calculate
Clear       ►
```

STEPS 4–5

STEP 6

PRACTICE

1. Use geometry drawing software to verify the Slopes of Perpendicular Lines Postulate.

   a. Construct a line and a point not on that line. Use Steps 1–3 from the Example above. **Check students' work.**

   b. Construct a line that is perpendicular to your original line and passes through the given point. **Check students' work.**

   c. Measure the slopes of the two lines. Multiply the slopes. What do you expect the product of the slopes to be? **−1**

2. **WRITING** Use the arrow keys to move your line from Exercise 1. *Describe* what happens to the product of the slopes when one of the lines is vertical. *Explain* why this happens.
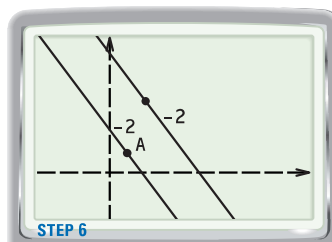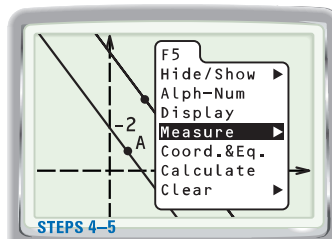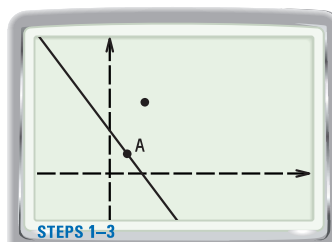   **The result will be undefined. The vertical line has an undefined slope.**

3.4   Find and Use Slopes of Lines   **171**

MATHEMATICAL PRACTICES Use appropriate tools strategically.

①PLAN AND PREPARE

Learn the Method
• Students will verify that parallel lines have the same slope.

Keystroke Help
Keystrokes for several models of calculators are available in blackline format in the *Chapter Resource Book.*

② TEACH

Tips for Success
Be sure to construct the parallel line with the software instead of just drawing it so it looks parallel.

Alternative Strategy
Do the construction as a demonstration with a computer and an overhead projector. Ask students to conjecture about the slopes and then calculate them. Have students move the line and watch the slope values.

Extra Example 1
Construct a different line and choose a point on the line. Label the point *B*. Graph a point not on the line and construct a line through the point, parallel to the first line. Measure the slopes of both lines. How do they compare? **They are equal.**

③ASSESS AND RETEACH

1. If a line has a slope of *m*, what will be the slope of any line parallel to it? *m*

COMMON CORE **CC.9-12.G.GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

3.5 Write and Graph Equations of Lines

Before	You found slopes of lines.
Now	You will find equations of lines.
Why?	So you can find monthly gym costs, as in Example 4.

PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. If $2x + 5y = -20$ and $x = 0$, what is y? **−4**
2. What is the slope of the line containing the points (2, 7) and (3, −10)? **−17**
3. Orphelia bought x apples. If each apple cost $.59, write an equation for y, the total cost of x apples. **$y = 0.59x$**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block

• See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How do you write an equation of a line? **Tell students they will learn how to answer this question by using the slope, the y-intercept and the slope-intercept form for a line.**

Key Vocabulary
• slope-intercept form
• standard form
• x-intercept
• y-intercept

COMMON CORE

CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Linear equations may be written in different forms. The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1   Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.

Solution

STEP 1 **Find** the slope. Choose two points on the graph of the line, (0, 4) and (3, −2).
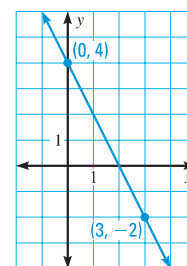
$$m = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

STEP 2 **Find** the y-intercept. The line intersects the y-axis at the point (0, 4), so the y-intercept is **4**.

STEP 3 **Write** the equation.

$y = mx + b$ **Use slope-intercept form.**

$y = -2x + 4$ **Substitute −2 for m and 4 for b.**

EXAMPLE 2   Write an equation of a parallel line

Write an equation of the line passing through the point (−1, 1) that is parallel to the line with the equation $y = 2x - 3$.

Solution

STEP 1 **Find** the slope m. The slope of a line parallel to $y = 2x - 3$ is the same as the given line, so the slope is 2.

LINEAR EQUATIONS
The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

STEP 2 **Find** the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$y = mx + b$ **Use slope-intercept form.**

$1 = 2(-1) + b$ **Substitute for x, y, and m.**

$3 = b$ **Solve for b.**

▶ Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$.

172   Chapter 3   Parallel and Perpendicular Lines

COMMON CORE

Standards for Mathematical Content High School

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

CHECKING BY GRAPHING You can check that equations are correct by graphing. In Example 2, you can use a graph to check that $y = 2x - 3$ is parallel to $y = 2x + 3$.

Animated **Geometry** at my.hrw.com

$y = 2x + 3$
$(-1, 1)$
$y = 2x - 3$
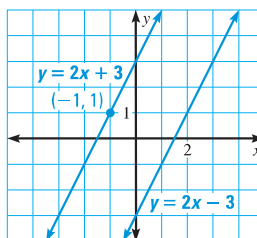
EXAMPLE 3 **Write an equation of a perpendicular line**

Write an equation of the line j passing through the point $(2, 3)$ that is perpendicular to the line k with the equation $y = -2x + 2$.

Solution

STEP 1 Find the slope m of line j. Line k has a slope of -2.

$-2 \cdot m = -1$ **The product of the slopes of ⊥ lines is −1.**
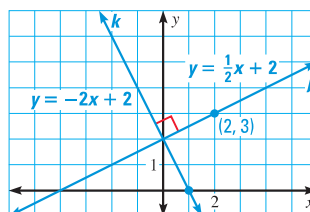
$m = \dfrac{1}{2}$ **Divide each side by −2.**

STEP 2 Find the y-intercept b by using $m = \dfrac{1}{2}$ and $(x, y) = (2, 3)$.

$y = mx + b$ **Use slope-intercept form.**

$3 = \dfrac{1}{2}(2) + b$ **Substitute for x, y, and m.**

$2 = b$ **Solve for b.**

▶ Because $m = \dfrac{1}{2}$ and $b = 2$, an equation of line j is $y = \dfrac{1}{2}x + 2$. You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.

k
$y = \frac{1}{2}x + 2$
$y = -2x + 2$
j
$(2, 3)$

✔ **GUIDED PRACTICE**   for Examples 1, 2, and 3

1. Write an equation of the line in the graph at the right. $y = \dfrac{2}{3}x - 1$

$(3, 1)$
$(0, -1)$

2. Write an equation of the line that passes through $(-2, 5)$ and $(1, 2)$. $y = -x + 3$

3. Write an equation of the line that passes through the point $(1, 5)$ and is parallel to the line with the equation $y = 3x - 5$. Graph the lines to check that they are parallel. $y = 3x + 2$; see margin for art.

4. How do you know the lines $x = 4$ and $y = 2$ are perpendicular? **Sample answer: $x = 4$ is a vertical line while $y = 2$ is a horizontal line.**
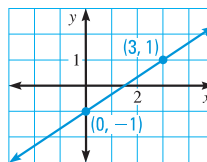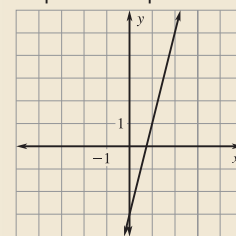
---

3 TEACH

Extra Example 1

Write an equation of the line in slope-intercept form. $y = 4x - 3$

MATHEMATICAL PRACTICES   *Key Question Example 1*

• Could you have used two different points to compute the slope? Explain. **Yes, any two points on the line will give the same slope.**

Extra Example 2

Write an equation of the line passing through the point $(2, -3)$ that is parallel to the line with the equation $y = 6x + 4$. $y = 6x - 15$

Extra Example 3

Write an equation of the line a passing through the point $(3, -4)$ that is perpendicular to the line b with the equation $y = -\dfrac{1}{2}x - 1$. $y = 2x - 10$

3. See Additional Answers.
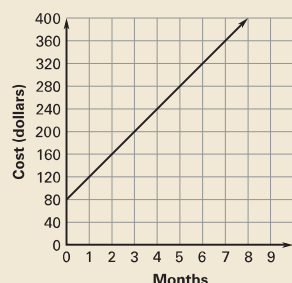

Differentiated Instruction

Advanced Remind students that a line is formed by an infinite number of collinear points. Point out that this set of points can be written using set-builder notation. For instance, the line found in **Example 1** can be written as $\{(x, y) \mid y = -2x + 4\}$, which is read "the set of all ordered pairs (x, y) such that y equals $-2x + 4$." Ask students to suggest the set-builder notations that would represent the lines found in **Examples 2 and 3**.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 4 **Write an equation of a line from a graph**

GYM MEMBERSHIP The graph models the total cost of joining a gym. Write an equation of the line. Explain the meaning of the slope and the y-intercept of the line.

Gym Membership Cost
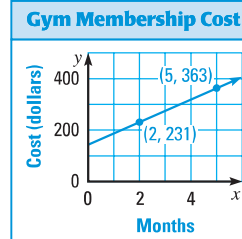
Solution

STEP 1 **Find** the slope.

$$m = \frac{363 - 231}{5 - 2} = \frac{132}{3} = 44$$

STEP 2 **Find** the y-intercept. Use the slope and one of the points on the graph.

$y = mx + b$ Use slope-intercept form.

$231 = 44 \cdot 2 + b$ Substitute for x, y, and m.

$143 = b$ Simplify.

STEP 3 **Write** the equation. Because $m = 44$ and $b = 143$, an equation of the line is $y = 44x + 143$.

▶ The equation $y = 44x + 143$ models the cost. The slope is the monthly fee, \$44, and the y-intercept is the initial cost to join the gym, \$143.

STANDARD FORM Another form of a linear equation is *standard form*. In **standard form**, the equation is written as $Ax + By = C$, where A and B are not both zero.

EXAMPLE 5 **Graph a line with equation in standard form**

Graph $3x + 4y = 12$.

Solution

CHOOSE A METHOD
Another way you could graph the equation is to solve the equation for y. Then the equation will be in slope-intercept form. Use rise and run from the point where the line crosses the y-axis to find a second point. Then graph the line.

The equation is in standard form, so you can use the intercepts.

STEP 1 **Find** the intercepts.

To find the x-intercept, let $y = 0$.   To find the y-intercept, let $x = 0$.

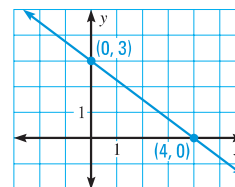$3x + 4y = 12$ $3x + 4y = 12$

$3x + 4(0) = 12$ $3(0) + 4y = 12$

$x = 4$ $y = 3$

STEP 2 **Graph** the line.

The line intersects the axes at $(4, 0)$ and $(0, 3)$. Graph these points, then draw a line through the points.

174 Chapter 3 Parallel and Perpendicular Lines

5. The equation $y = 50x + 125$ models the total cost of joining a climbing gym. What are the meaning of the slope and the y-intercept of the line?
Slope: monthly fee, y-intercept: initial cost to join gym

Graph the equation. 6–8. See margin.

6. $2x - 3y = 6$ 7. $y = 4$ 8. $x = -3$

WRITING EQUATIONS You can write linear equations to model real-world situations, such as comparing costs to find a better buy.

EXAMPLE 6 **Solve a real-world problem**

DVD RENTAL You can rent DVDs at a local store for $4.00 each. An Internet company offers a flat fee of $15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?
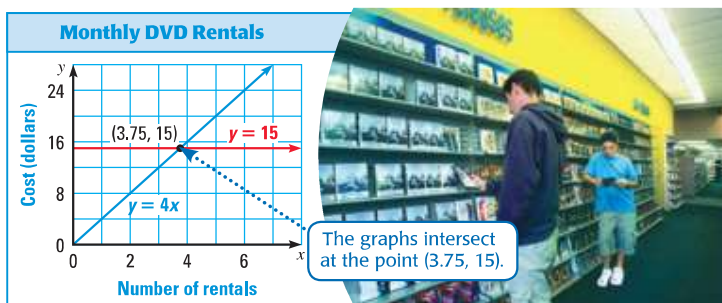
ANOTHER WAY
For alternative methods for solving the problem in Example 6, see the **Problem Solving Workshop**.

Solution

STEP 1 **Model** each rental with an equation.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Graph** each equation.

Monthly DVD Rentals

$(3.75, 15)$ $y = 15$
$y = 4x$
The graphs intersect at the point $(3.75, 15)$.

READ VOCABULARY
The point at which the costs are the same is sometimes called the *break-even point*.

▶ The point of intersection is $(3.75, 15)$. Using the graph, you can see that it is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

9. **WHAT IF?** In Example 6, suppose the online rental is $16.50 per month and the local rental is $4 each. How many DVDs do you need to rent to make the online rental a better buy? **5 DVDs**

10. Online rental would have to increase to 6 DVDs to be a better buy.

10. How would your answer to Exercise 9 change if you had a 2-for-1 coupon that you could use once at the local store?

3.5 Write and Graph Equations of Lines **175**

Bill Aron/PhotoEdit, Inc.

Extra Example 6
One bank charges $1.50 for each use of its debit card. Another bank charges $10 per month for an unlimited number of debit card uses. How many times per month would you need to use your debit card to make the bank that charges a flat rate the better choice?
**7 times or more**

Closing the Lesson
Have students summarize the major points of the lesson and answer the Essential Question: How do you write an equation of a line?

• **The general form of a linear equation in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.**

• **A linear equation in standard form can be graphed if you use its intercepts.**

Use the coordinates of two points on the line to find the slope m. Substitute the coordinates of one of the points and the value of m in the slope-intercept equation $y = mx + b$. Solve the resulting equation for b. Then substitute the values of m and b in $y = mx + b$ to get the slope-intercept form of the equation of the line.

6.

$(3, 0)$
$(0, -2)$

7.

$(-3, 4)$ $(2, 4)$

8.

$(-3, 5)$
$(-3, -1)$
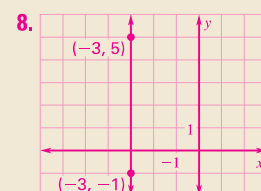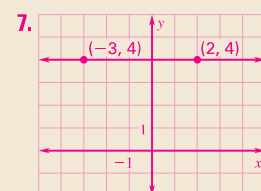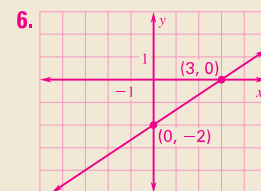
3.5 **EXERCISES**

HOMEWORK
KEY

◯ = See **WORKED-OUT SOLUTIONS**
Exs. 17, 23, and 61

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 9, 29, 64, and 65

SKILL PRACTICE

A 1. **VOCABULARY** What does *intercept* mean in the expression *slope-intercept form*? **The point of intersection on the y-axis when graphing a line.**

2. ★ **WRITING** Explain how you can use the standard form of a linear equation to find the intercepts of a line. **See margin.**

EXAMPLE 1
for Exs. 3–22

WRITING EQUATIONS Write an equation of the line shown.

3.

4.

5.

6.

7.

8.

2. Sample answer: To find the x-intercept let $y = 0$ and solve for x. To find the y-intercept let $x = 0$ and solve for y.
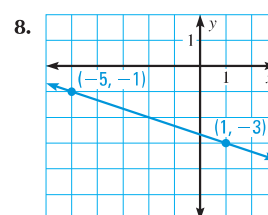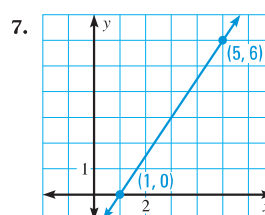
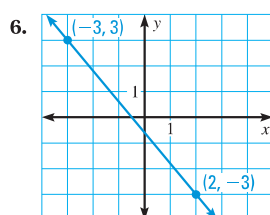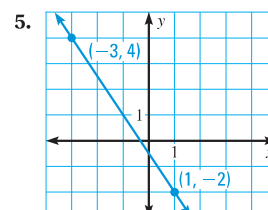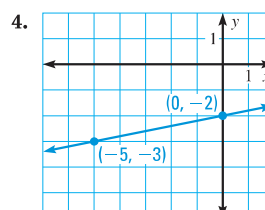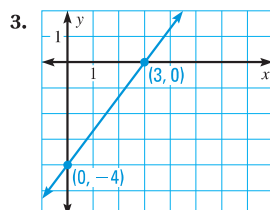3. $y = \frac{4}{3}x - 4$

4. $y = \frac{1}{5}x - 2$

5. $y = -\frac{3}{2}x - \frac{1}{2}$

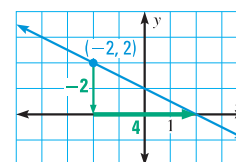6. $y = -\frac{6}{5}x - \frac{3}{5}$

7. $y = \frac{3}{2}x - \frac{3}{2}$

8. $y = -\frac{1}{3}x - \frac{8}{3}$

9. ★ **MULTIPLE CHOICE** Which equation is an equation of the line in the graph? **B**

Ⓐ $y = -\frac{1}{2}x$

Ⓑ $y = -\frac{1}{2}x + 1$

Ⓒ $y = -2x$

Ⓓ $y = -2x + 1$

WRITING EQUATIONS Write an equation of the line with the given slope *m* and *y*-intercept *b*.

10. $m = -5, b = -12$
$y = -5x - 12$

11. $m = 3, b = 2$
$y = 3x + 2$

12. $m = 4, b = -6$
$y = 4x - 6$

13. $m = -\frac{5}{2}, b = 0$
$y = -\frac{5}{2}x$

14. $m = \frac{4}{9}, b = -\frac{2}{9}$
$y = \frac{4}{9}x - \frac{2}{9}$

15. $m = -\frac{11}{5}, b = -12$
$y = -\frac{11}{5}x - 12$

WRITING EQUATIONS Write an equation of the line that passes through the given point *P* and has the given slope *m*.

16. $P(-1, 0), m = -1$
$y = -x - 1$

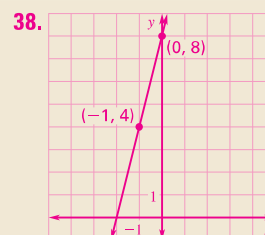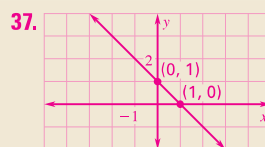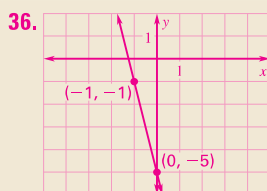17. $P(5, 4), m = 4$
$y = 4x - 16$

18. $P(6, -2), m = 3$
$y = 3x - 20$

19. $P(-8, -2), m = -\frac{2}{3}$
$y = -\frac{2}{3}x - \frac{22}{3}$

20. $P(0, -3), m = -\frac{1}{6}$
$y = -\frac{1}{6}x - 3$

21. $P(-13, 7), m = 0$ $y = 7$

22. **WRITING EQUATIONS** Write an equation of a line with undefined slope that passes through the point $(3, -2)$. $x = 3$

36.

37.

38.

PARALLEL LINES Write an equation of the line that passes through point P and is parallel to the line with the given equation.

23. $P(0, -1), y = -2x + 3$
$y = -2x - 1$
24. $P(-7, -4), y = 16$
$y = -4$
25. $P(3, 8), y - 1 = \frac{1}{5}(x + 4)$

26. $P(-2, 6), x = -5$
$x = -2$
27. $P(-2, 1), 10x + 4y = -8$
28. $P(4, 0), -x + 2y = 12$

29. ★ **MULTIPLE CHOICE** Line a passes through points $(-2, 1)$ and $(2, 9)$. Which equation is an equation of a line parallel to line a? **D**

Ⓐ $y = -2x + 5$ Ⓑ $y = -\frac{1}{2}x + 5$ Ⓒ $y = \frac{1}{2}x - 5$ Ⓓ $y = 2x - 5$

PERPENDICULAR LINES Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.

30. $P(0, 0), y = -9x - 1$
31. $P(-1, 1), y = \frac{7}{3}x + 10$
32. $P(4, -6), y = -3$ $x = 4$

33. $P(2, 3), y - 4 = -2(x + 3)$
34. $P(0, -5), x = 20$ $y = -5$
35. $P(-8, 0), 3x - 5y = 6$

GRAPHING EQUATIONS Graph the equation. **36–44. See margin.**

36. $8x + 2y = -10$
37. $x + y = 1$
38. $4x - y = -8$

39. $-x + 3y = -9$
40. $y - 2 = -1$
41. $y + 2 = x - 1$

42. $x + 3 = -4$
43. $2y - 4 = -x + 1$
44. $3(x - 2) = -y - 4$

45. **ERROR ANALYSIS** *Describe* and correct the error in finding the x- and y-intercepts of the graph of $5x - 3y = -15$.

To find the x-intercept, let x = 0:
$5x - 3y = -15$
$5(0) - 3y = -15$
$y = 5$ ✗

To find the y-intercept, let y = 0:
$5x - 3y = -15$
$5x - 3(0) = -15$
$x = -3$ ✗

PERPENDICULAR BISECTORS Find the midpoint of $\overline{PQ}$. Then write an equation of the line that passes through the midpoint and is perpendicular to $\overline{PQ}$. This line is called the *perpendicular bisector* of $\overline{PQ}$.

46. $P(-4, 3), Q(4, -1)$
$(0, 1); y = 2x + 1$
47. $P(-5, -5), Q(3, 3)$
$(-1, -1); y = -x - 2$
48. $P(0, 2), Q(6, -2)$
$(3, 0); y = 1.5x - 4.5$

USING INTERCEPTS Identify the x- and y-intercepts of the line. Use the intercepts to write an equation of the line.

49.
50.
51.
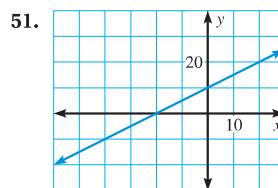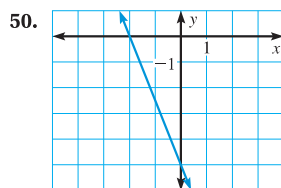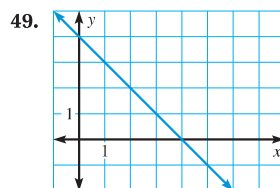
52. **INTERCEPTS** A line passes through the points $(-10, -3)$ and $(6, 1)$. Where does the line intersect the x-axis? Where does the line intersect the y-axis?

39.
40.
41.

53.

(0, 9)

(−1, 5)

(0, −1)

(−1, −5)

no solutions

54.

(1, 4)

(4, 0)

(6, −6)

(5, −8)

one solution

55.

(0, 6)

(1, 1)

infinitely many solutions

59.

$S(−2, 3)$

$R(−7, −3)$

$T(10, −7)$

$\overline{RS}:\ y = \frac{6}{5}x + \frac{27}{5}$

$\overline{ST}:\ y = -\frac{5}{6}x + \frac{4}{3}$

$\overline{RT}:\ y = -\frac{4}{17}x - \frac{79}{17}$

---

56. Check students' work. *Sample answer:* If a false equation occurs the lines are parallel. If the variables drop out, a true equation occurs and the lines are the same line. If a point is found the lines intersect at that point. | C |

SOLUTIONS TO EQUATIONS Graph the linear equations. Then use the graph to estimate how many solutions the equations share. **53–55. See margin.**

53. $y = 4x + 9$
$4x − y = 1$

54. $3y + 4x = 16$
$2x − y = 18$

55. $y = −5x + 6$
$10x + 2y = 12$

56. **xy** **ALGEBRA** Solve Exercises 53–55 algebraically. (For help, see Skills Review Handbook, p. SR12.) Make a conjecture about how the solution(s) can tell you whether the lines intersect, are parallel, or are the same line.

57. **xy** **ALGEBRA** Find a value for k so that the line through (−1, k) and (−7, −2) is parallel to the line with equation $y = x + 1$. **4**

58. **xy** **ALGEBRA** Find a value for k so that the line through (k, 2) and (7, 0) is perpendicular to the line with equation $y = x − \frac{28}{5}$. **5**

59. **CHALLENGE** Graph the points $R(−7, −3)$, $S(−2, 3)$, and $T(10, −7)$. Connect them to make $\triangle RST$. Write an equation of the line containing each side. *Explain* how you can use slopes to show that $\triangle RST$ has one right angle. **See margin.**

PROBLEM SOLVING

EXAMPLE 4 | A |
for Exs. 60–61

59. See margin for art; check the slopes of each line segment. If two of the slopes are negative reciprocals of one another the lines are perpendicular and form a right angle.

60. $y = 23x + 50$; slope: the monthly charge, y-intercept: initial one-time charge; $326

60. **WEB HOSTING** The graph models the total cost of using a web hosting service for several months. Write an equation of the line. Tell what the slop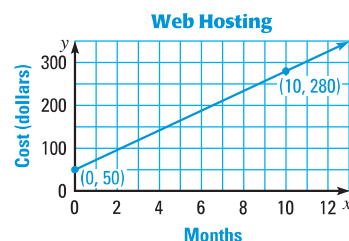e and y-intercept mean in this situation. Then find the total cost of using the web hosting service for one year. **See margin.**

Web Hosting

Cost (dollars)

(10, 280)

(0, 50)

Months

61. **SCIENCE** Scientists believe that a Tyrannosaurus Rex weighed about 2000 kilograms by age 14. It then had a growth spurt for four years, gaining 2.1 kilograms per day. Write an equation to model this situation. What are the slope and y-intercept? Tell what the slope and y-intercept mean in this situation.

$y = 2.1x + 2000$; slope: gain in weight per day, y-intercept: starting weight before the growth spurt

Field Museum, Chicago, Illinois

EXAMPLE 6 | B |
for Exs. 62–65

62. **MULTI-STEP PROBLEM** A national park has two options: a $50 pass for all admissions during the year, or a $4 entrance fee each time you enter.

   a. **Model** Write an equation to model the cost of going to the park for a year using a pass and another equation for paying a fee each time. $y = 50, y = 4x$

   b. **Graph** Graph both equations you wrote in part (a). **See margin.**

   c. **Interpret** How many visits do you need to make for the pass to be cheaper? *Explain.* **13 visits. *Sample answer:* The point of intersection in the graph is (12.5, 50), so if you make 13 or more visits in a year, the pass is cheaper.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

62b.

Cost (dollars)

Number of visits

64.

Distance (mi)

your friend

you

Time (min)

65b.

Price of a package of clasps

$5b + 2c = 27.50$

$2b + c = 13$

Price of a bag of beads

David R. Frazier PhotoLibrary, Inc/Alamy

63. **PIZZA COSTS** You are buying slices of pizza for you and your friends. A small slice costs $2 and a large slice costs $3. You have $24 to spend. Write an equation in standard form $Ax + By = C$ that models this situation. What do the values of A, B, and C mean in this situation?

64. ★ **SHORT RESPONSE** You run at a rate of 4 miles per hour and your friend runs at a rate of 3.5 miles per hour. Your friend starts running 10 minutes before you, and you run for a half hour on the same path. Will you catch up to your friend? Use a graph to support your answer. **No; see margin for art.**

65. ★ **EXTENDED RESPONSE** Audrey and Sara are making jewelry. Audrey buys 2 bags of beads and 1 package of clasps for a total of $13. Sara buys 5 bags of beads and 2 packages of clasps for a total of $27.50.

  a. Let b be the price of one bag of beads and let c be the price of one package of clasps. Write equations to represent the total cost for Audrey and the total cost for Sara. **$2b + c = 13, 5b + 2c = 27.50$**

  b. Graph the equations from part (a). **See margin.**

  c. *Explain* the meaning of the intersection of the two lines in terms of the real-world situation.

66. **CHALLENGE** Michael is deciding which gym membership to buy. Points (2, 112) and (4, 174) give the cost of gym membership at one gym after two and four months. Points (1, 62) and (3, 102) give the cost of gym membership at a second gym after one and three months. Write equations to model the cost of each gym membership. At what point do the graphs intersect, if they intersect? Which gym is cheaper? *Explain.*
First gym: $y = 31x + 50$, second gym: $y = 20x + 42$; $\left(-\dfrac{8}{11}, \dfrac{302}{11}\right)$; second gym: it has a lower initial cost and a lower monthly cost.

Using ALTERNATIVE METHODS

Alternative Strategy

Example 6 in this lesson can be solved by using a table or equation. These methods allow students to visualize the situation by seeing a pattern in the table. They can see when the local renting cost becomes greater than the online renting cost. Some students may understand the process better by writing equations for each rental option and using algebra to solve the system to find when the costs are the same.

Another Way to Solve Example 6

MULTIPLE REPRESENTATIONS In Example 6, you saw how to graph equations to solve a problem about renting DVDs. Another way you can solve the problem is *using a table*. Alternatively, you can use the equations to solve the problem *algebraically*.

PROBLEM

DVD RENTAL You can rent DVDs at a local store for $4.00 each. An Internet company offers a flat fee of $15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

METHOD 1 **Using a Table** You can make a table to answer the question.

STEP 1 **Make** a table representing each rental option.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15

STEP 2 **Add** rows to your table until you see a pattern.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15
3	$12	$15
4	$16	$15
5	$20	$15
6	$24	$15

STEP 3 **Analyze** the table. Notice that the values in the second column (the cost of renting locally) are less than the values in the third column (the cost of renting online) for three or fewer DVDs. However, the values in the second column are greater than those in the third column for four or more DVDs.

▶ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

METHOD 2

Using Algebra You can solve one of the equations for one of its variables. Then substitute that expression for the variable in the other equation.

STEP 1 **Write** an equation for each rental option.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Substitute** the value of y from one equation into the other equation.

$y = 4x$

$15 = 4x$ **Substitute 15 for y.**

$3.75 = x$ **Divide each side by 4.**

STEP 3 **Analyze** the solution of the equation. If you could rent 3.75 DVDs, your cost for local and online rentals would be the same. However, you can only rent a whole number of DVDs. Look at what happens when you rent 3 DVDs and when you rent 4 DVDs, the whole numbers just less than and just greater than 3.75.

▶ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

PRACTICE

1. **IN-LINE SKATES** You can rent in-line skates for $5 per hour, or buy a pair of skates for $130. How many hours do you need to skate for the cost of buying skates to be cheaper than renting them? **27 h**

2. **WHAT IF?** Suppose the in-line skates in Exercise 1 also rent for $12 per day. How many days do you need to skate for the cost of buying skates to be cheaper than renting them? **11 days**

3. **BUTTONS** You buy a button machine for $200 and supplies to make one hundred fifty buttons for $30. Suppose you charge $2 for a button. How many buttons do you need to sell to earn back what you spent? **115 buttons**

4. **MANUFACTURING** A company buys a new widget machine for $1200. It costs $5 to make each widget. The company sells each widget for $15. How many widgets do they need to sell to earn back the money they spent on the machine? **120 widgets**

5. **WRITING** Which method(s) did you use to solve Exercises 1–4? *Explain* your choice(s). **See margin.**

6. **MONEY** You saved $1000. If you put this money in a savings account, it will earn 1.5% annual interest. If you put the $1000 in a certificate of deposit (CD), it will earn 3% annual interest. To earn the most money, does it ever make sense to put your money in the savings account? *Explain.* **no**

3.5 Using Alternative Methods   **181**

3.6 Prove Theorems About Perpendicular Lines

Key Vocabulary
• distance from a point to a line

CC.9-12.G.CO.9 Prove theorems about lines and angles.

2. 90°, 90°, 90°, 90°; all the angles are congruent; all the angles are right angles.

ACTIVITY FOLD PERPENDICULAR LINES

Materials: paper, protractor

STEP 1 | **STEP 2** | **STEP 3**
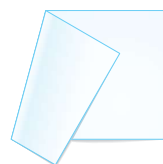
Fold a piece of paper. | **Fold** the paper again, so that the original fold lines up on itself. | **Unfold** the paper.
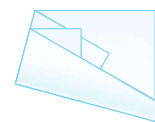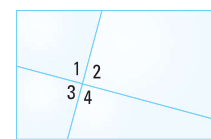
DRAW CONCLUSIONS

1. What type of angles appear to be formed where the fold lines intersect? **right angles**

2. Measure the angles with a protractor. Which angles are congruent? Which angles are right angles? **See margin.**

The activity above suggests several properties of perpendicular lines.

THEOREMS   For Your Notebook

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.
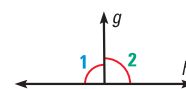
THEOREM 3.9

If two lines are perpendicular, then they intersect to form four right angles.

If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.

Standards for Mathematical Content High School
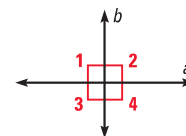
CC.9-12.G.CO.9 Prove theorems about lines and angles.
CC.9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).

First Aid Folly, with waterwheel. Parc de la Villette, Paris, 1986, by Bernard Tschumi. Photo credit: Art on File/Corbis

EXAMPLE 1   Draw conclusions

In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$. What can you conclude about $\angle 1$ and $\angle 2$?
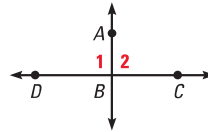
Solution

$\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, so by Theorem 3.9, they form four right angles. You can conclude that $\angle 1$ and $\angle 2$ are right angles, so $\angle 1 \cong \angle 2$.

THEOREM   *For Your Notebook*

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.
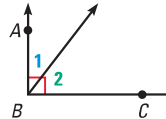
EXAMPLE 2   Prove Theorem 3.10

Prove that if two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

GIVEN ▶ $\overrightarrow{ED} \perp \overrightarrow{EF}$

PROVE ▶ $\angle 7$ and $\angle 8$ are complementary.
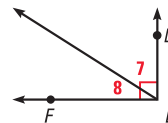
STATEMENTS	REASONS
1. $\overrightarrow{ED} \perp \overrightarrow{EF}$	1. Given
2. $\angle DEF$ is a right angle.	2. $\perp$ lines intersect to form 4 rt. $\angle\!\!\!\angle$. (Theorem 3.9)
3. $m\angle DEF = 90°$	3. Definition of a right angle
4. $m\angle 7 + m\angle 8 = m\angle DEF$	4. Angle Addition Postulate
5. $m\angle 7 + m\angle 8 = 90°$	5. Substitution Property of Equality
6. $\angle 7$ and $\angle 8$ are complementary.	6. Definition of complementary angles

---

✔ GUIDED PRACTICE   for Examples 1 and 2

1. They are complementary. *Sample answer:* $\angle ABD$ is a right angle since 2 lines intersect to form a linear pair of congruent angles (Theorem 3.8), so $\overrightarrow{BA} \perp \overleftrightarrow{CD}$. Then $\angle 3$ and $\angle 4$ are complementary by Theorem 3.10.

1. Given that $\angle ABC \cong \angle ABD$, what can you conclude about $\angle 3$ and $\angle 4$? *Explain* how you know.

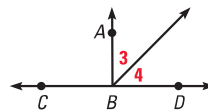2. Write a plan for proof for Theorem 3.9, that if two lines are perpendicular, then they intersect to form four right angles. *Sample answer:* **The definition of perpendicular lines implies that angles formed by the intersecting lines are right angles.**

3.6   Prove Theorems About Perpendicular Lines   **183**

183

THEOREMS For Your Notebook

THEOREM 3.11 Perpendicular Transversal Theorem

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

If *h* ∥ *k* and *j* ⊥ *h*, then *j* ⊥ *k*.
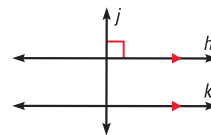
THEOREM 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If *m* ⊥ *p* and *n* ⊥ *p*, then *m* ∥ *n*.
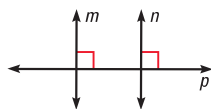
EXAMPLE 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

Solution

Lines *p* and *q* are both perpendicular to *s*, so by Theorem 3.12, *p* ∥ *q*. Also, lines *s* and *t* are both perpendicular to *q*, so by Theorem 3.12, *s* ∥ *t*.
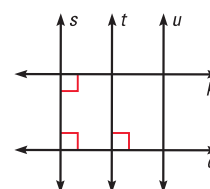
✓ **GUIDED PRACTICE** for Example 3

Use the diagram at the right.

3. Is *b* ∥ *a*? *Explain* your reasoning.

4. Is *b* ⊥ *c*? *Explain* your reasoning.
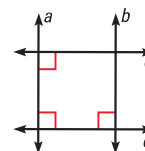
DISTANCE FROM A LINE The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point *A* and line *k* is *AB*.

Distance from a point to a line
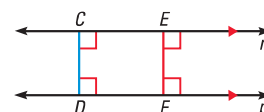
Distance between two parallel lines

The *distance between two parallel lines* is the length of any perpendicular segment joining the two lines. For example, the distance between line *p* and line *m* above is *CD* or *EF*.

Differentiated Instruction

Kinesthetic Learners Give pairs of students a piece of string that is about 3 feet long. Ask them to find a line in the classroom and then choose a point near the line. Direct them to hold one end of the string on the point and stretch the string to the line. Have them compare the distance from the point to several different points on the line with the distance to the line along a perpendicular segment from the point to the line.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 4 **Find the distance between two parallel lines**

SCULPTURE The sculpture below is drawn on a graph where units are measured in inches. What is the approximate length of $\overline{SR}$, the depth of a seat?
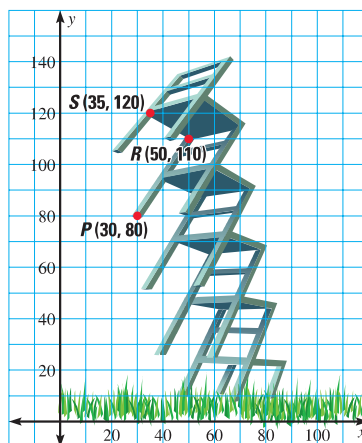
Solution

You need to find the length of a perpendicular segment from a back leg to a front leg on one side of the chair.

Using the points $P(30, 80)$ and $R(50, 110)$, the slope of each leg is

$$\frac{110 - 80}{50 - 30} = \frac{30}{20} = \frac{3}{2}.$$

The segment SR has a slope of

$$\frac{120 - 110}{35 - 50} = -\frac{10}{15} = -\frac{2}{3}.$$

The segment $\overline{SR}$ is perpendicular to the leg so the distance SR is

$$d = \sqrt{(35 - 50)^2 + (120 - 110)^2} \approx 18.0 \text{ inches}.$$

▶ The length of $\overline{SR}$ is about 18.0 inches.

✓ **GUIDED PRACTICE**   **for Example 4**

Use the graph at the right for Exercises 5 and 6.

5. What is the distance from point A to line c?
   about 2.7
6. What is the distance from line c to line d?
   about 1.8
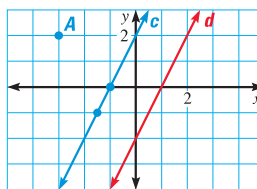
7. Graph the line $y = x + 1$. What point on the line is the shortest distance from the point $(4, 1)$? What is the distance? Round to the nearest tenth. **(2, 3); 2.8**

3.6   Prove Theorems About Perpendicular Lines   **185**

Extra Example 4

What is the distance between the two parallel sides of this table top? **5**
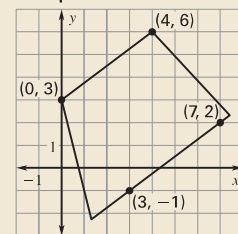
MATHEMATICAL PRACTICES **Key Question Example 4**

• Could you use a point on the other side of the sculpture? Explain.
  No, it must be on a segment perpendicular to the line.

Avoiding Common Errors

For Guided Practice Exercise 6, some students may count spaces between the y-intercepts. Ask students to think of c and d as parallel streets. What is the shortest path from one street to the other?

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the distance between a point and a line?

• **The distance from a point to a line is the length of the perpendicular segment from the point to the line.**
• **Perpendicular lines form four right angles.**
• **Two lines perpendicular to the same line are parallel.**

For the coordinate plane, write an equation for the line through the given point and perpendicular to the given line. Find the point of intersection. Then use the distance formula to calculate the distance from the point of intersection to the given point.

Differentiated Instruction

Advanced Ask students if they can find ways to simplify the algebra and arithmetic when they find the distance between two parallel lines. One possibility is to use a line through the origin that is perpendicular to both lines.

See also the *Differentiated Instruction Resources* for more strategies.

3.6 EXERCISES

HOMEWORK
KEY
◯ = See **WORKED-OUT SOLUTIONS**
Exs. 19, 23, and 29

★ = **STANDARDIZED TEST PRACTICE**
Exs. 11, 12, 21, 22, and 30

2. If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

EXAMPLES 1 and 2
for Exs. 2–7

3. If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

4. If two lines are perpendicular, then they intersect to form four right angles.

EXAMPLE 3
for Exs. 8–12

13. There is no information to indicate that $y \parallel z$ or $y \perp x$.

EXAMPLES 3 and 4
for Exs. 13–14

SKILL PRACTICE

1. VOCABULARY The length of which segment shown is called the distance between the two parallel lines? *Explain.*
$\overline{AB}$; it's $\perp$ to the parallel lines.
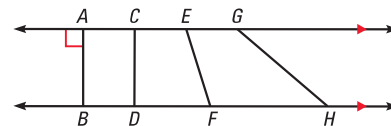
JUSTIFYING STATEMENTS Write the theorem that justifies the statement.

2. $j \perp k$
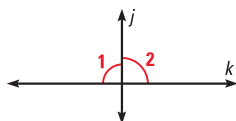
3. $\angle 4$ and $\angle 5$ are complementary.

4. $\angle 1$ and $\angle 2$ are right angles.
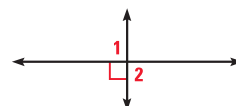
APPLYING THEOREMS Find $m\angle 1$.

5. 65°   25°

6. 90°

7. 38°   52°
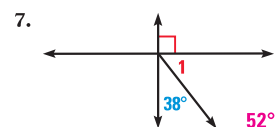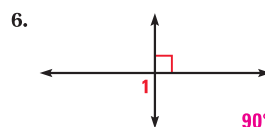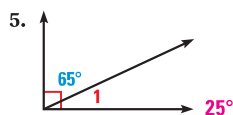
SHOWING LINES PARALLEL *Explain* how you would show that $m \parallel n$.   8–10. See margin.

8.

9. $x°$ $x°$

10. $x°$ $x°$
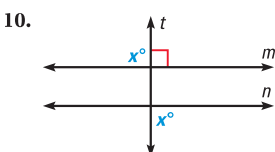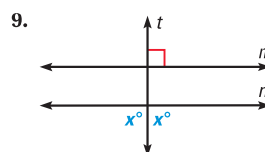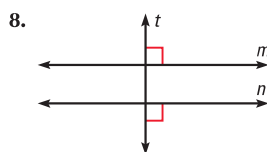
11. ★ **SHORT RESPONSE** *Explain* how to draw two parallel lines using only a straightedge and a protractor. *Sample answer:* **Draw a line. Construct a second line perpendicular to the first line. Construct a third line perpendicular to the second line.**

12. ★ **SHORT RESPONSE** *Describe* how you can fold a sheet of paper to create two parallel lines that are perpendicular to the same line. **See margin.**

ERROR ANALYSIS *Explain* why the statement about the figure is incorrect.

13.
Lines y and z are parallel.
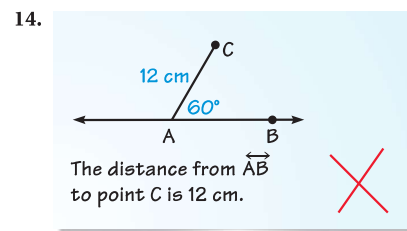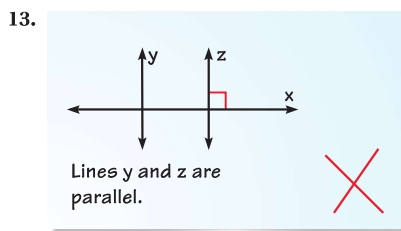
14.
12 cm   60°
The distance from $\overleftrightarrow{AB}$ to point C is 12 cm.
$\overline{AC}$ is not $\perp$ to $\overleftrightarrow{AB}$.

8. Lines Perpendicular to a Transversal Theorem

9. Since the two angles labeled $x°$ form a linear pair of congruent angles, $t \perp n$; since the two lines are perpendicular to the same line, they are parallel to each other.

10. Alternate Exterior Angles Converse

12. Fold the paper into thirds lengthwise and then in half across its width.

18. Lines n and p; they are perpendicular to line k.

19. Lines f and g; they are perpendicular to line d.

20. Lines z and y; they are perpendicular to line w. Lines v, w, and x; lines v and w are perpendicular to line y, and lines w and x are perpendicular to line z.

FINDING ANGLE MEASURES In the diagram, $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$. Find the value of x.

15.
13

F
$63°$
$(x + 14)°$
G — H

16.
95

$20°$
F
$(x - 25)°$
H — G

17.
33

H — $(2x - 9)°$ — G
$x°$
F

DRAWING CONCLUSIONS Determine which lines, if any, must be parallel. *Explain* your reasoning.

18.

n
p
m
k

19.

d
f
g

20.

z
y
v
w
x
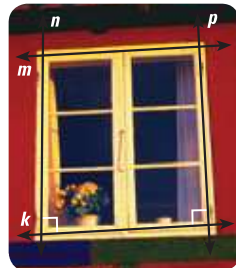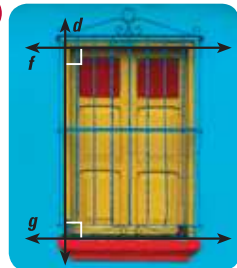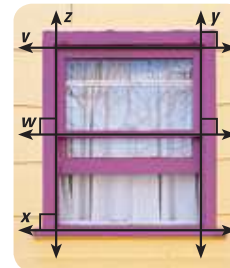
21. ★ **MULTIPLE CHOICE** Which statement must be true if $c \perp d$? **A**

    (A) $m\angle 1 + m\angle 2 = 90°$ (B) $m\angle 1 + m\angle 2 < 90°$

    (C) $m\angle 1 + m\angle 2 > 90°$ (D) Cannot be determined

c
2
d
1
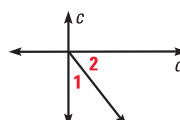
22. ★ **WRITING** *Explain* why the distance between two lines is only defined for parallel lines.
The distance between nonparallel lines is not constant.

EXAMPLE 4
for Exs. 23–24

FINDING DISTANCES Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.

23.

y
1
1
x
4.1

24.

y
1
1
x
2.8

26. Sample answer: $m\angle 1 = 90°$ since it is a right angle; $m\angle 2 + 30° = 90°$, therefore $m\angle 2 = 60°$; $m\angle 5 = 90°$ since it is vertical to $\angle 1$; C $m\angle 3 = 30°$ since it is vertical to the angle measuring 30°; $m\angle 4 = 20°$; since $m\angle 3 + m\angle 4 + 40° = 90°$.
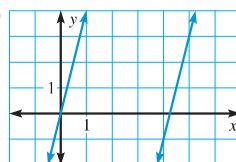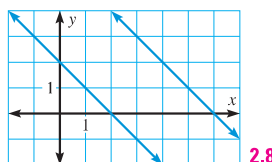
25. FINDING DISTANCES Draw the quadrilateral $ABCD$ with coordinates $A(-4, -1)$, $B(2, 3)$, $C(7, 2)$, and $D(1, -2)$. Find the distance between each pair of parallel sides of $ABCD$. Round to the nearest tenth, if necessary. $\sqrt{13} \approx 3.6$; $\sqrt{26} \approx 5.1$

26. FINDING ANGLES Find all the unknown angle measures in the diagram at the right. *Justify* your reasoning for each angle measure.

1 2 $30°$
3 5
$40°$ 4
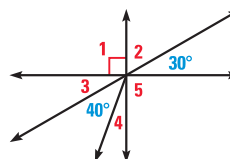
27. FINDING DISTANCES Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$. **2.5**

28. CHALLENGE *Describe* how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? *Explain.*
Construct a perpendicular line to the plane passing through the point. Find the length of the perpendicular segment from the point to the point where the segment intersects the plane; if and only if the line is parallel to the plane.

3.6 Prove Theorems About Perpendicular Lines   **187**

Avoiding Common Errors

Exercise 1 If there are students who say that either $\overline{CD}$ or $\overline{AB}$ would work, review what we are and are not allowed to assume from a diagram. We know from markings that $\overline{AB}$ is perpendicular to the parallel lines. We have no way to be sure that this is so for $\overline{CD}$.

Mathematical Reasoning

Exercises 15–17 Have students tell what theorem from this lesson they are using in these exercises.

Mathematical Reasoning

Exercise 31 Ask students why the angles in the figure are marked with arcs instead of right angle symbols. If they do not have a ready answer, use questioning to help them understand that the figure we use when we start a proof should show only *given* information, not what we are trying to prove.

32. Given: $a \perp b$

Prove: $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.
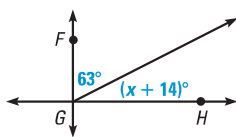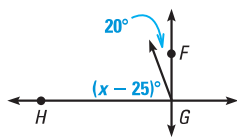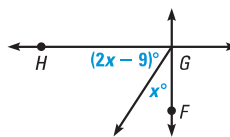
Because $a \perp b$, $\angle 1$ is a right angle by the definition of perpendicular lines, and $m\angle 1 = 90°$. By the Vertical Angles Congruence Theorem, $\angle 1 \cong \angle 4$ so $m\angle 4 = 90°$ and $\angle 4$ is a right angle. Because $\angle 1$ and $\angle 2$ form a linear pair, they are supplementary angles, so $m\angle 1 + m\angle 2 = 180°$. So $m\angle 2 = 90°$, and $\angle 2$ is a right angle. Similarly, $\angle 1$ and $\angle 3$ form a linear pair, so $m\angle 3 = 90°$ and $\angle 3$ is a right angle.

33. See Additional Answers.

---

29. Point *C*; the shortest distance is the length of the perpendicular segment.

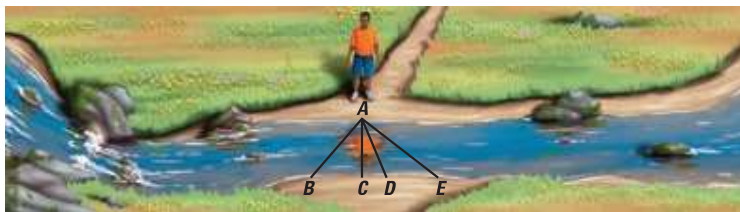**30. Check students sketches;** perpendicular segments; shortest distance between 2 parallel lines is a perpendicular line, so the perpendicular lines would require less paint than the diagonal ones.

EXAMPLE 2
for Exs. 31–34

[A] 29. STREAMS You are trying to cross a stream from point *A*. Which point should you jump to in order to jump the shortest distance? *Explain.*

30. ★ SHORT RESPONSE The segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments would look like if they were perpendicular to the crosswalk. Which method requires less paint? *Explain.*
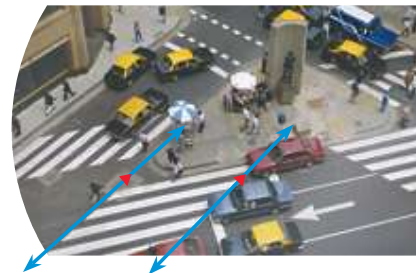
31. PROVING THEOREM 3.8 Copy and complete the proof that if two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

GIVEN ▶ $\angle 1$ and $\angle 2$ are a linear pair.
$\angle 1 \cong \angle 2$

PROVE ▶ $g \perp h$
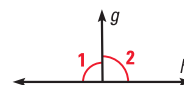
STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are a linear pair.	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary.	2. __?__ **Linear Pair Postulate**
3. __?__ $m\angle 1 + m\angle 2 = 180°$	3. Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	4. Given
5. $m\angle 1 = m\angle 2$	5. __?__ **Definition of congruent angles**
6. $m\angle 1 + m\angle 1 = 180°$	6. Substitution Property of Equality
7. $2(m\angle 1) = 180°$	7. Combine like terms.
8. $m\angle 1 = 90°$ $\angle 1$ is a right angle.	8. __?__ **Division Property of Equality**
9. __?__	9. Definition of a right angle
10. $g \perp h$	10. __?__ **Definition of perpendicular lines**

[B] PROVING THEOREMS Write a proof of the given theorem. **32–34. See margin.**

32. Theorem 3.9

33. Theorem 3.11, Perpendicular Transversal Theorem

34. Theorem 3.12, Lines Perpendicular to a Transversal Theorem

188

○ **= See WORKED-OUT SOLUTIONS** in Student Resources   ★ **= STANDARDIZED TEST PRACTICE**

Stephen Wilkes/Getty Images

---

34. Given: $m \perp p$, $n \perp p$
Prove: $m \parallel n$
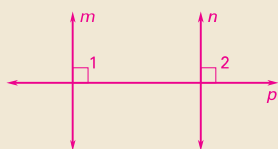
Statements (Reasons)

1. $m \perp p$, $n \perp p$ (Given)
2. $\angle 1$ and $\angle 2$ are right angles. (Perpendicular lines intersect to form four right angles.)
3. $\angle 1 \cong \angle 2$ (Right Angles Congruence Theorem)
4. $m \parallel n$ (Corresponding Angles Converse)

CHALLENGE Suppose the given statement is true. Determine whether $\overrightarrow{AB} \perp \overrightarrow{AC}$.

35. $\angle 1$ and $\angle 2$ are congruent. **no**

36. $\angle 3$ and $\angle 4$ are complementary. **no**

37. $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$ **yes**

38. $m\angle 1 = 40°$ and $m\angle 4 = 50°$ **yes**

QUIZ

7. None. *Sample answer:* It is not known whether *w* is perpendicular to *y*.

8. *b* and *c*; Lines Perpendicular to a Transversal Theorem

9. *m* and *n*; Lines Perpendicular to a Transversal Theorem

Write an equation of the line that passes through point *P* and is parallel to the line with the given equation.

1. $P(0, 0)$, $y = -3x + 1$ **2.** $P(-5, -6)$, $y - 8 = 2x + 10$ **3.** $P(1, -2)$, $x = 15$
$y = -3x$ $y = 2x + 4$ $x = 1$

Write an equation of the line that passes through point *P* and is perpendicular to the line with the given equation.

4. $P(3, 4)$, $y = 2x - 1$ **5.** $P(2, 5)$, $y = -6$ **6.** $P(4, 0)$, $12x + 3y = 9$
$y = -\frac{1}{2}x + \frac{11}{2}$ $x = 2$ $y = \frac{1}{4}x - 1$
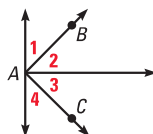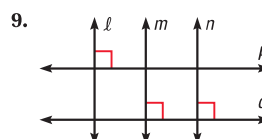
Determine which lines, if any, must be parallel. *Explain.*

7.

8.

9.

⑤ ASSESS AND RETEACH

Daily Homework Quiz
Also available online

1. Find $m\angle 3$. **18°**

72° ... 3
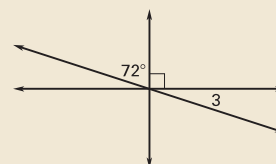
2. How do you know that *a* and *b* are parallel? **Both are perpendicular to *c*.**

c / d / a / b

3. Find the distance between the two parallel lines. Round to the nearest tenth. **6.4**
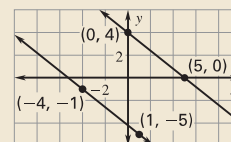
(0, 4), (5, 0), (-4, -1), (1, -5)

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation
• Practice A, B, C in Chapter Resource Book
• Study Guide in Chapter Resource Book
• Practice Workbook
• @HomeTutor

Challenge
Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

1 PLAN AND PREPARE

Explore the Concept
- Students will construct parallel and perpendicular lines using a compass and straightedge.
- This activity supplements the study of parallel lines in previous lessons and also the Parallel Postulate and Perpendicular Postulate in previous lesson.

Materials
Each student will need:
- compass
- straightedge

Recommended Time
Work activity: 20 min
Discuss results: 10 min

Grouping
Students should work individually.

2 TEACH

Tips for Success
Emphasize the need to maintain compass settings in steps 2 and 3 of Explore 1, and also in step 2 of Explore 2. You may wish to tell students that the constructions rely on postulates and theorems about congruent triangles presented in another chapter.

Key Questions
- Ask students how their results would be affected if they changed the compass setting when drawing the arc with center at point *C* in step 3 when constructing parallel lines. **The lines would not be parallel.**
- Ask students how their results would be affected if they changed the compass setting when drawing the arc with center at point *B* in step 2 when constructing perpendicular lines. **The lines would not be perpendicular.**

COMMON CORE **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).

190

Parallel and Perpendicular Lines

MATERIALS • compass • straightedge

MATHEMATICAL PRACTICES Use appropriate tools strategically.

QUESTION How can you construct parallel and perpendicular lines?

You can use a compass and straightedge to construct parallel and perpendicular lines.

EXPLORE 1 Construct parallel lines

Use the following steps to construct a line through a given point *P* that is parallel to a given line *m*.

STEP 1 *Draw point and line* Start by drawing point *P* and line *m*. Choose a point *Q* anywhere on line *m* and draw $\overleftrightarrow{QP}$.
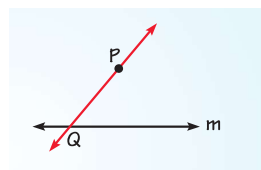
STEP 2 *Draw arcs* Draw an arc with center *Q* that crosses $\overleftrightarrow{QP}$ and line *m*. Label points *A* and *B*.

Using the same compass setting, draw an arc with center *P*. Label point *C*.
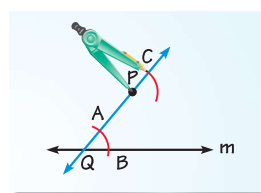
STEP 3 *Copy angle* Draw an arc with radius *AB* and center *A*.

Using the same compass setting, draw an arc with center *C*. Label the intersection *D*.
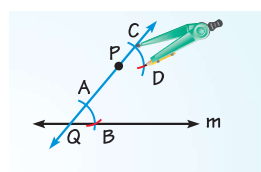
STEP 4 *Draw parallel line* Draw $\overleftrightarrow{PD}$. This line is parallel to line *m*.
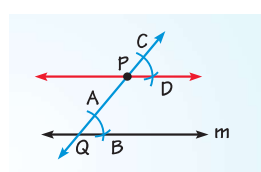
You will justify this construction in the exercises on the next page.

Alternative Strategy
Demonstrate these constructions on the board or on the overhead projector.

EXPLORE 2 | **Construct perpendicular lines**

The diagrams in the first column show how to construct a perpendicular to a line through a point on the line.

The diagrams in the second column show how to construct a perpendicular to a line through a point *not* on the line.

Point *P* is on line *m*. | **Point *P* is *not* on line *m*.**

STEP 1 **Draw arc with center P** Place the compass at point *P* and draw an arc that intersects the line twice. Label the intersections *A* and *B*.

STEP 2 **Draw intersecting arcs** Draw an arc with center *A*. Using the same radius, draw an arc with center *B*. Label the intersection of the arcs *Q*.
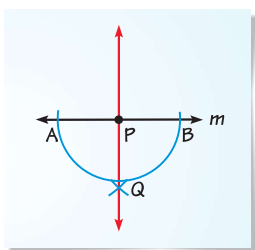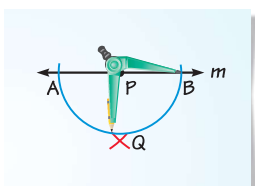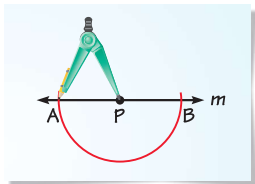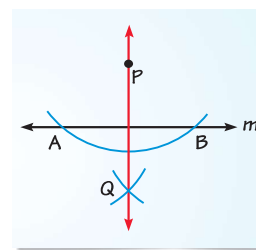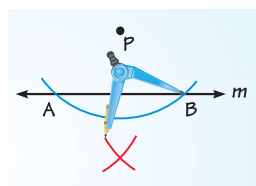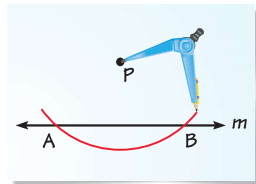
STEP 3 **Draw perpendicular line** Draw $\overleftrightarrow{PQ}$. This line is perpendicular to line *m*.

You will justify these constructions in Exercise 36.

3. Check constructions. The lines are perpendicular. If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line. (Perpendicular Transversal Theorem)

4. Check constructions. The lines are parallel. In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. (Lines Perpendicular to a Transversal Theorem)

DRAW CONCLUSIONS | **Use your observations to complete these exercises**

1. Tell which theorem or postulate justifies the parallel line construction shown in Explore 1. **Postulate 16 (Corresponding Angles Converse)**

2. *Explain* how the construction of the perpendicular line in Explore 2 relates to the construction of a segment bisector. **See margin.**

3. Construct parallel lines *m* and *n*. Then construct a line perpendicular to line *n*. Describe the relationship of this third line to line *m*. Tell which theorem or postulate justifies your answer.

4. Draw a line. Then construct two perpendicular lines to it through different points. Describe the relationship between these two lines. Tell which theorem or postulate justifies your answer.

③ ASSESS AND RETEACH

1. When constructing parallel lines with point *P* above the given line, can point *Q* be located *directly below* point *P*? If not, why not? If so, how is $\overleftrightarrow{PQ}$ related to the two parallel lines when the construction is completed? **Yes; $\overleftrightarrow{PQ}$ is perpendicular to the parallel lines.**

2. When you construct a line perpendicular to a given line, why must the compass setting chosen in step 2 be at least $\frac{1}{2}AB$? **If the compass setting is less than this, the two arcs being drawn will not intersect.**

2. *Sample answer:* In both types of constructions, the intersection of two arcs is used to perform the construction. In the construction of a segment bisector, the intersection points of the arcs determine two points on the segment bisector. In the construction of a perpendicular to a line, the two arcs determine a point that is used with a given point to draw the perpendicular.

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

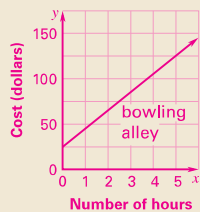## Answer margin (left column)

1a. $y = 35x$, $y = 20x$
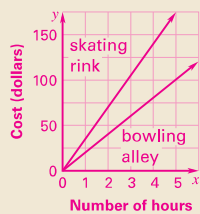
1b. No; they don't have the same slope.

1c. cost per hour of renting each facility

1d. $y = 20x + 25$;

Yes; $y = 20x$ and $y = 20x + 25$ have the same slope.

3. *Sample answer:* $4x + 6y = 8$; *Sample answer:* $2y - 3x = -2$

4. 96.6. *Sample answer:* It's the perpendicular distance to the line.

**5c.** Monongahela Incline. *Sample answer:* $0.7 > 0.68$

Main column

1. MULTI-STEP PROBLEM You are planning a party. You would like to have the party at a roller skating rink or bowling alley. The table shows the total cost to rent the facilities by number of hours.

Hours	Roller skating rink cost ($)	Bowling alley cost ($)
1	35	20
2	70	40
3	105	60
4	140	80
5	175	100

a–d. See margin.

a. Use the data in the table. Write and graph two equations to represent the total cost y to rent the facilities, where x is the number of hours you rent the facility.

b. Are the lines from part (a) parallel? *Explain* why or why not.

c. What is the meaning of the slope in each equation from part (a)?

d. Suppose the bowling alley charges an extra $25 set-up fee. Write and graph an equation to represent this situation. Is this line parallel to either of the lines from part (a)? *Explain* why or why not.

2. GRIDDED ANSWER The graph models the accumulated cost of buying a used guitar and taking lessons over the first several months. Find the slope of the line. **15**
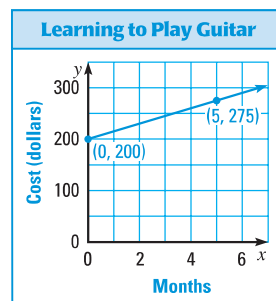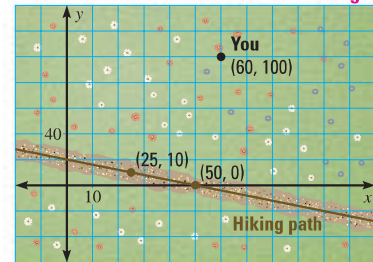

3. OPEN-ENDED Write an equation of a line parallel to $2x + 3y = 6$. Then write an equation of a line perpendicular to your line. **See margin.**

4. SHORT RESPONSE You are walking across a field to get to a hiking path. Use the graph below to find the shortest distance you can walk to reach the path. *Explain* how you know you have the shortest distance. **See margin.**

5. EXTENDED RESPONSE The Johnstown Inclined Plane in Johnstown, Pennsylvania, is a cable car that transports people up and down the side of a hill. During the cable car's climb, you move about 17 feet upward for every 25 feet you move forward. At the top of the incline, the horizontal distance from where you started is about 500 feet.

a. How high is the car at the top of its climb compared to its starting height? **340 ft**

b. Find the slope of the climb. $\frac{17}{25}$

c. Another cable car incline in Pennsylvania, the Monongahela Incline, climbs at a slope of about 0.7 for a horizontal distance of about 517 feet. *Compare* this climb to that of the Johnstown Inclined Plane. Which is steeper? *Justify* your answer. **See margin.**

Dennis MacDonald/Alamy

BIG IDEAS
For Your Notebook

Big Idea 1

Using Properties of Parallel and Perpendicular Lines

When parallel lines are cut by a transversal, angle pairs are formed. Perpendicular lines form congruent right angles.

∠2 and ∠6 are corresponding angles, and they are congruent.

∠3 and ∠6 are alternate interior angles, and they are congruent.

∠1 and ∠8 are alternate exterior angles, and they are congruent.

∠3 and ∠5 are consecutive interior angles, and they are supplementary.

If $a \perp b$, then ∠1, ∠2, ∠3, and ∠4 are all right angles.

Big Idea 2

Proving Relationships Using Angle Measures

You can use the angle pairs formed by lines and a transversal to show that the lines are parallel. Also, if lines intersect to form a right angle, you know that the lines are perpendicular.

Through point A not on line q, there is only one line r parallel to q and one line s perpendicular to q.
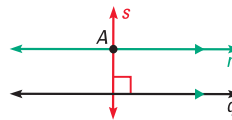
Big Idea 3

Making Connections to Lines in Algebra

In Algebra 1, you studied slope as a rate of change and linear equations as a way of modeling situations.

Slope and equations of lines are also a useful way to represent the lines and segments that you study in Geometry. For example, the slopes of parallel lines are the same ($a \parallel b$), and the product of the slopes of perpendicular lines is -1 ($a \perp c$, and $b \perp c$).
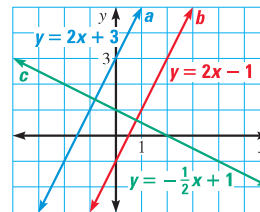
$y = 2x + 3$

$y = 2x - 1$

$y = -\frac{1}{2}x + 1$

Extra Example 1

Think of each segment in the rectangular box as part of a line.
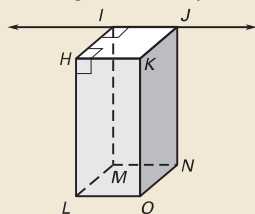
a. $\overleftrightarrow{HI}$, $\overleftrightarrow{KJ}$, $\overleftrightarrow{IM}$, and $\overleftrightarrow{JN}$ are perpendicular to $\overleftrightarrow{IJ}$.

b. $\overleftrightarrow{HK}$, $\overleftrightarrow{MN}$, and $\overleftrightarrow{LO}$ are parallel to $\overleftrightarrow{IJ}$.

c. $\overleftrightarrow{NO}$ and $\overleftrightarrow{ML}$ are skew to $\overleftrightarrow{IJ}$.

d. Plane *HIM* is parallel to plane *KJN*.

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see p. PT2.

- parallel lines
- skew lines
- parallel planes
- transversal
- corresponding angles
- alternate interior angles
- alternate exterior angles
- consecutive interior angles
- paragraph proof
- slope
- slope-intercept form
- standard form
- distance from a point to a line

VOCABULARY EXERCISES

1. Copy and complete: Two lines that do not intersect and are not coplanar are called   ? . **skew lines**

2. Alternate interior angle pairs lie between the two lines and on opposite sides of the transversal while consecutive interior angle pairs lie between the two lines and on the same side of the transversal.

2. **WRITING** *Compare* alternate interior angle pairs and consecutive interior angle pairs.

Copy and complete the statement using the figure at the right.

3. ∠1 and   ?   are corresponding angles. **∠5**

4. ∠3 and   ?   are alternate interior angles. **∠6**

5. ∠4 and   ?   are consecutive interior angles. **∠6**

6. ∠7 and   ?   are alternate exterior angles. **∠2**
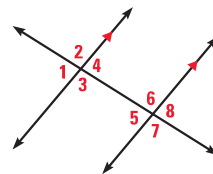
Identify the form of the equation as *slope-intercept form* or *standard form*.

7. $14x - 2y = 26$ **standard form**

8. $y = 7x - 13$ **slope-intercept form**

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

3.1 Identify Pairs of Lines and Angles

EXAMPLE

Think of each segment in the rectangular box at the right as part of a line.

a. $\overleftrightarrow{BD}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BH}$, and $\overleftrightarrow{AG}$ appear perpendicular to $\overleftrightarrow{AB}$.

b. $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$, and $\overleftrightarrow{EF}$ appear parallel to $\overleftrightarrow{AB}$.

c. $\overleftrightarrow{CF}$ and $\overleftrightarrow{EG}$ appear skew to $\overleftrightarrow{AB}$.

d. Plane *EFG* appears parallel to plane *ABC*.
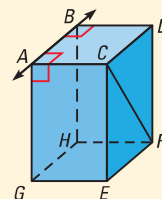
EXERCISES
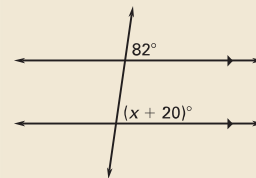
EXAMPLE 1
for Exs. 9–12

Think of each segment in the diagram of a rectangular box as part of a line. Which line(s) or plane(s) contain point *N* and appear to fit the description?

9. Line(s) perpendicular to $\overleftrightarrow{QR}$ $\overrightarrow{NR}$

10. Line(s) parallel to $\overleftrightarrow{QR}$ $\overrightarrow{NP}$

11. Line(s) skew to $\overleftrightarrow{QR}$ $\overrightarrow{JN}$

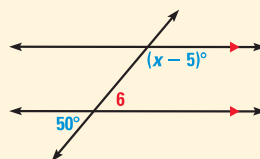12. Plane(s) parallel to plane *LMQ*   **plane *KJN***

3.2   Use Parallel Lines and Transversals
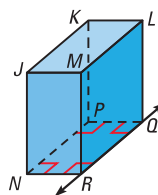
EXAMPLE

Use properties of parallel lines to find the value of *x*.

By the Vertical Angles Congruence Theorem,
$m\angle 6 = 50°$.

$(x - 5)° + m\angle 6 = 180°$	**Consecutive Interior Angles Theorem**
$(x - 5)° + 50° = 180°$	**Substitute 50° for $m\angle 6$.**
$x = 135$	**Solve for *x*.**

EXERCISES

EXAMPLES 1 and 2
for Exs. 13–19

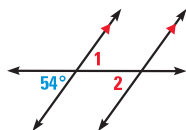**13.** $m\angle 1 = 54°$, vertical angles; $m\angle 2 = 54°$, alternate interior angles

14. $m\angle 1 = 85°$, consecutive interior angles; $m\angle 2 = 95°$, supplementary angles

15. $m\angle 1 = 135°$, corresponding angles; $m\angle 2 = 45°$, supplementary angles
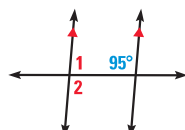
Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.

13.

14.

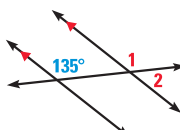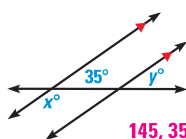
15.
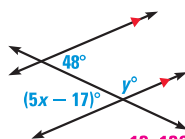

Find the values of *x* and *y*.
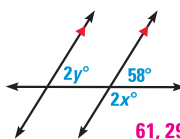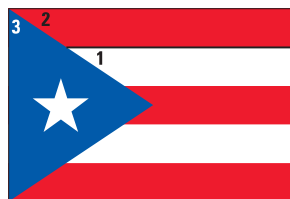
16.

145, 35

17.

13, 132

18.

61, 29

19. **FLAG OF PUERTO RICO** Sketch the rectangular flag of Puerto Rico as shown at the right. Find the measure of $\angle 1$ if $m\angle 3 = 55°$. *Justify* each step in your argument. **35°. *Sample answer:*** $\angle 2$ **and** $\angle 3$ **are complementary, so** $m\angle 2 = 90° -$ **55°** $= 35°$. $m\angle 1 = m\angle 2$ **because** $\angle 1$ **and** $\angle 2$ **are corresponding angles for two parallel lines cut by a transversal.**
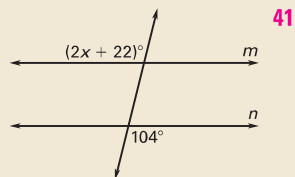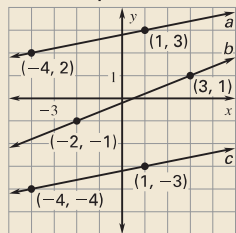

Chapter Review   **195**

Extra Example 3

Find the value of x that makes $m \parallel n$.

41

$(2x + 22)°$ m

$104°$ n

Extra Example 4

Find the slope of each line. Which lines are parallel?

line a: $\frac{1}{5}$; line b: $\frac{2}{5}$; line c: $\frac{1}{5}$; $a \parallel c$

3.3 **Prove Lines are Parallel**

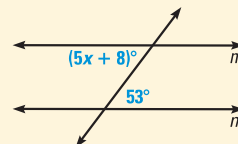### EXAMPLE

Find the value of x that makes $m \parallel n$.

Lines m and n are parallel when the marked angles are congruent.
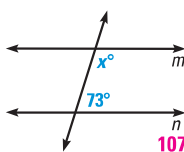
$$(5x + 8)° = 53°$$
$$5x = 45$$
$$x = 9$$

$(5x + 8)°$ m

$53°$ n

▶ The lines m and n are parallel when $x = 9$.

EXERCISES

EXAMPLE 1
for Exs. 20–22

Find the value of x that makes $m \parallel n$.

20.

$x°$ m

$73°$ n

107

21.

$147°$ m

$(x + 14)°$ n

133

22.

$(2x + 20)°$ m

$3x°$ n
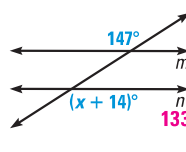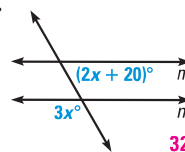
32

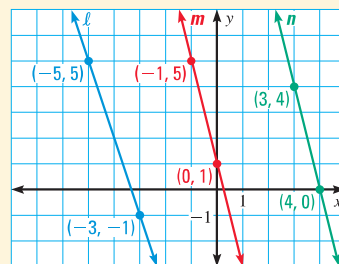**3.4** **Find and Use Slopes of Lines**

EXAMPLE

Find the slope of each line. Which lines are parallel?

Slope of $\ell = \dfrac{-1 - 5}{-3 - (-5)} = \dfrac{-6}{2} = \mathbf{-3}$

Slope of $m = \dfrac{1 - 5}{0 - (-1)} = \dfrac{-4}{1} = \mathbf{-4}$

Slope of $n = \dfrac{0 - 4}{4 - 3} = \dfrac{-4}{1} = \mathbf{-4}$

▶ Because m and n have the same slope, they are parallel. The slope of ℓ is different, so ℓ is not parallel to the other lines.

EXERCISES

EXAMPLES 2 and 3
for Exs. 23–24

Tell whether the lines through the given points are *parallel, perpendicular,* or *neither*.

23. Line 1: (8, 12), (7, −5)
Line 2: (−9, 3), (8, 2)   **perpendicular**

24. Line 1: (3, −4), (−1, 4)
Line 2: (2, 7), (5, 1)   **parallel**

196 Chapter 3 Parallel and Perpendicular Lines

3.5 **Write and Graph Equations of Lines**

EXAMPLE

Write an equation of the line k passing through the point $(-4, 1)$ that is perpendicular to the line n with the equation $y = 2x - 3$.

First, find the slope of line k.   Then, use the given point and the slope in the
Line n has a slope of 2.   slope-intercept form to find the y-intercept.

$2 \cdot m = -1$ $y = mx + b$

$m = -\dfrac{1}{2}$ $1 = -\dfrac{1}{2}(-4) + b$

$-1 = b$

▶ An equation of line k is $y = -\dfrac{1}{2}x - 1$.

EXERCISES

EXAMPLES
2 and 3
for Exs. 25–26

Write equations of the lines that pass through point P and are (a) parallel and (b) perpendicular to the line with the given equation.

25. $P(3, -1)$, $y = 6x - 4$ **a.** $y = 6x - 19$

b. $y = -\dfrac{1}{6}x - \dfrac{1}{2}$

26. $P(-6, 5)$, $7y + 4x = 2$ **a.** $y = -\dfrac{4}{7}x + \dfrac{11}{7}$

b. $y = \dfrac{7}{4}x + \dfrac{31}{2}$

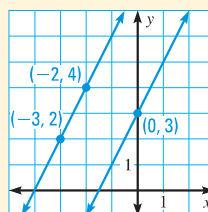**3.6** **Prove Theorems About Perpendicular Lines**

EXAMPLE

Find the distance between $y = 2x + 3$ and $y = 2x + 8$.

Find the length of a perpendicular segment from one line to the other. Both lines have a slope of 2, so the slope of a perpendicular segment to each line is $-\dfrac{1}{2}$.
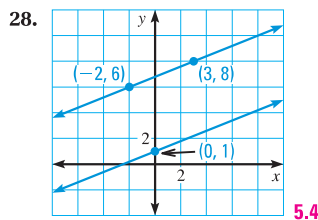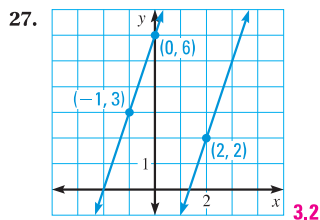
The segment from $(0, 3)$ to $(-2, 4)$ has a slope of

$\dfrac{4 - 3}{-2 - 0} = -\dfrac{1}{2}$. So, the distance between the lines is

$d = \sqrt{(-2 - 0)^2 + (4 - 3)^2} = \sqrt{5} \approx 2.2$ units.

EXERCISES

EXAMPLE 4
for Exs. 27–28

Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.

27.

3.2

28.

5.4

Chapter Review   **197**

Extra Example 5
Write an equation of the line ℓ passing through the point $(6, -4)$ that is perpendicular to the line m with the equation $y = 3x - 5$.

$y = -\dfrac{1}{3}x - 2$

Extra Example 6
Find the distance between $y = 4x - 1$ and $y = 4x + 3$.

$\dfrac{4\sqrt{17}}{17} \approx 0.97$ units