

PLAN AND PREPARE

Main Ideas

In this chapter, students will use inductive reasoning to make and test conjectures. They will analyze conditional statements and write the converse, inverse, and contrapositive of a conditional statement. Students will use deductive reasoning, the Law of Detachment, and the Law of Syllogism to develop simple logical arguments. Finally, they will use properties of equality and the laws of logic to prove basic theorems about congruence, supplementary angles, complementary angles, and vertical angles.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from <i>Skills Readiness</i>
Any of Exs. 5–8 answered incorrectly	Skill 22 Identify segments, rays, and lines
Any of Exs. 9–11 answered incorrectly	Skill 69 Solve multi-step equations
Any of Exs. 12–13 answered incorrectly	Skill 25 Use angle pair relationships
All exercises answered incorrectly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. Make sense of problems and persevere in solving them. [Pages 78, 95, 112.](#)
2. Reason abstractly and quantitatively. [Pages 64, 71, 79, 86, 114.](#)
3. Construct viable arguments and critique the reasoning of others. [Pages 64, 68, 71, 79, 96.](#)
4. Model with mathematics. [Pages 66, 69, 99, 107.](#)
5. Use appropriate tools strategically. [Pages 78, 87, 113.](#)
6. Attend to precision. [Pages 74, 91, 100.](#)
7. Look for and make use of structure. [Pages 88, 89, 97, 98, 99, 105.](#)
8. Look for and express regularity in repeated reasoning. [Pages 97, 98, 106.](#)

2

Reasoning and Proof

COMMON CORE

Lesson
2.1 SMP 3
2.2 SMP 3
2.3 SMP 3
2.4 CC.9-12.G.CO.9
2.5 CC.9-12.A.REI.1
2.6 CC.9-12.G.CO.9
2.7 CC.9-12.G.CO.9

2.1 Use Inductive Reasoning

2.2 Analyze Conditional Statements

2.3 Apply Deductive Reasoning

2.4 Use Postulates and Diagrams

2.5 Reason Using Properties from Algebra

2.6 Prove Statements about Segments and Angles

2.7 Prove Angle Pair Relationships

Before

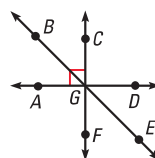
Previously, you learned the following skills, which you'll use in this chapter: naming figures, using notations, solving equations, and drawing diagrams.

Prerequisite Skills

VOCABULARY CHECK

Use the diagram to name an example of the described figure.

1. A right angle $\angle AGC$ **1–4. Sample answers are given.**
2. A pair of vertical angles $\angle BGC, \angle EGF$
3. A pair of supplementary angles $\angle BGC, \angle CGE$
4. A pair of complementary angles $\angle AGB, \angle BGC$



SKILLS AND ALGEBRA CHECK

Describe what the notation means. Draw the figure.

5. \overline{AB} segment AB
6. \overleftrightarrow{CD} line CD
7. EF length of segment EF
8. \overrightarrow{GH} ray GH **5–8. See margin for art.**

Solve the equation.

9. $3x + 5 = 20$ **5**
10. $4(x - 7) = -12$ **4**
11. $5(x + 8) = 4x$ **-40**

Draw the angles.

12. $\angle 1$ and $\angle 2$ are vertical angles. **Check drawings.**
13. $\angle ABD$ and $\angle DBC$ are complementary. **Check drawings.**

COMMON CORE Standards for Mathematical Content—High School

Mathematical Practices	Lesson
SMP 3	Construct viable arguments and critique the reasoning of others. 2-1, 2-2, 2-3
Reasoning with Equations and Inequalities	
CC.9-12.A.REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2-5

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

- Find the length of a segment with endpoints $A(1, -3)$ and $B(-2, -7)$. **5**
- If $M(4, -3)$ is the midpoint of \overline{RS} , and the coordinates of R are $(8, -2)$, find the coordinates of S . **(0, -4)**
- $\angle A$ and $\angle B$ are supplementary. If the measure of $\angle A$ is three times the measure of $\angle B$, find the measure of $\angle B$. **45°**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you use inductive reasoning in mathematics? **Tell students they will learn how to answer this question by describing patterns suggested by geometric figures and numbers.**

2.1 Use Inductive Reasoning

Before

You classified polygons by the number of sides.

Now

You will describe patterns and use inductive reasoning.

Why?

So you can make predictions about baseball, as in Ex. 32.



Key Vocabulary

- conjecture
- inductive reasoning
- counterexample

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

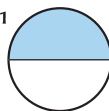


Figure 2

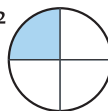
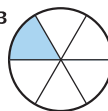


Figure 3



Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

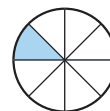


Figure 4



Standard for Mathematical Practice 3 Construct viable arguments and critique the reasoning of others.

READ SYMBOLS

The three dots (...) tell you that the pattern continues.

EXAMPLE 2 Describe a number pattern

Describe the pattern in the numbers $-7, -21, -63, -189, \dots$ and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$\begin{array}{ccccccc} -7, & & -21, & & -63, & & -189, \dots \\ & \nearrow \times 3 & & \nearrow \times 3 & & \nearrow \times 3 & & \nearrow \times 3 \end{array}$$

► Continue the pattern. The next three numbers are $-567, -1701$, and -5103 .



at my.hrw.com



GUIDED PRACTICE for Examples 1 and 2

- Sketch the fifth figure in the pattern in Example 1. **See margin.**
- Describe the pattern in the numbers $5.01, 5.03, 5.05, 5.07, \dots$. Write the next three numbers in the pattern. **The numbers are increasing by 0.02; 5.09, 5.11, 5.13.**

CC.9-12.G.CO.9 Prove theorems about lines and angles.

(preparation for)

CC.9-12.G.CO.10 Prove theorems about triangles. (preparation for)

CC.9-12.G.CO.11 Prove theorems about parallelograms. (preparation for)






INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?

$\xrightarrow{+1}$ $\xrightarrow{+2}$ $\xrightarrow{+3}$ $\xrightarrow{+?}$

► **Conjecture** You can connect five collinear points $6 + 4$, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive integers*. Make and test a conjecture about the sum of any three consecutive integers.

Solution

STEP 1 Find a pattern using a few groups of small numbers.

$$\begin{aligned}
 3 + 4 + 5 &= 12 = 4 \cdot 3 & 7 + 8 + 9 &= 24 = 8 \cdot 3 \\
 10 + 11 + 12 &= 33 = 11 \cdot 3 & 16 + 17 + 18 &= 51 = 17 \cdot 3
 \end{aligned}$$

► **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 Test your conjecture using other numbers. For example, test that it works with the groups $-1, 0, 1$ and $100, 101, 102$.

$$\begin{aligned}
 -1 + 0 + 1 &= 0 = 0 \cdot 3 \quad \checkmark & 100 + 101 + 102 &= 303 = 101 \cdot 3 \quad \checkmark
 \end{aligned}$$

✓ GUIDED PRACTICE for Examples 3 and 4

4. The product of three negative integers is a negative integer.
Sample answer:
 $-2 \cdot (-5) \cdot (-4) = -40$.

- Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points. **You can connect seven collinear points 15 + 6 or 21 different ways.**
- Make and test a conjecture about the sign of the product of any three negative integers.

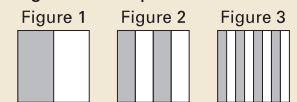
Motivating the Lesson

Your favorite sports team has won 6 consecutive games. Do you think the team will win or lose the 7th game? If you answered, "win", you used inductive reasoning.

3 TEACH

Extra Example 1

Describe how to sketch the fourth figure in the pattern.



Each large region has twice as many equal regions as the previous figure. Sketch the fourth figure by dividing a square into 16 equal-sized vertical rectangles. Shade alternate regions.

Extra Example 2

Describe the pattern in the numbers 1000, 500, 250, 125, ... and write the next three numbers in the pattern. **Each number in the pattern is one-half of the previous number; 62.5, 31.25, 15.625**



An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

Extra Example 3

Given the pattern of triangles below, make a conjecture about the number of segments in a similar diagram with 5 triangles.



$$7 + 2 + 2 = 11 \text{ segments}$$

1. See Additional Answers.

Differentiated Instruction

Below Level If students have difficulty with **Example 2**, encourage them to concentrate on successive pairs of numbers. Ask students whether they can add the same number each time to get from one number to the next or whether they can multiply by the same number each time to get from one number to the next. See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 4

Numbers such as 3, 11, 15, and 29 are odd numbers. Make and test a conjecture about the product of any two odd numbers. **The product of any two odd numbers is odd.**

Extra Example 5

Find a counterexample to disprove the conjecture:

Supplementary angles are always adjacent.

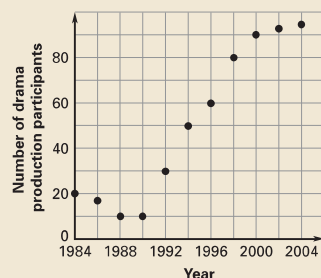
Sample:



Extra Example 6

Which conjecture could you make based on the graph? **D**

Drama Production Participation



- (A) Participants have remained steady since 1984.
- (B) The number of girls participating in a drama production exceeds the number of boys.
- (C) The number of participants for *West Side Story* is over 50.
- (D) Participants have increased steadily since 1990.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you use inductive reasoning in mathematics?

- A conjecture is an unproven statement based on observations.
- You use inductive reasoning to find patterns and make conjectures.
- A counterexample disproves a conjecture.

You use inductive reasoning to discover patterns. You conjecture that the pattern will continue to hold unless you find a counterexample.

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one **counterexample**. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$$\begin{aligned} -2 + -3 &= -5 \\ -5 &\not> -2 \end{aligned}$$

► Because a counterexample exists, the conjecture is false.



EXAMPLE 6 Standardized Test Practice

Which conjecture could a high school athletic director make based on the graph at the right?

- (A) More boys play soccer than girls.
- (B) More girls are playing soccer today than in 2001.
- (C) More people are playing soccer today than in the past because the 2000 World Cup games were held in the United States.
- (D) The number of girls playing soccer was more in 2001 than in 2007.

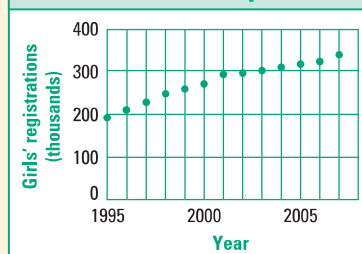
Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase over time. Choice D is a statement that the graph shows is false.

► The correct answer is B. (A) (B) (C) (D)



Girls' Soccer Participation



ELIMINATE CHOICES
Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.



GUIDED PRACTICE for Examples 5 and 6

6. The number of girls playing soccer will increase; the number of girls playing soccer has increased every year for more than 10 years.

- 5. Find a counterexample to show that the following conjecture is false.
Conjecture The value of x^2 is always greater than the value of x . **Sample answer:** $x = \frac{1}{2}$, $x^2 = \frac{1}{4}$
- 6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

Differentiated Instruction

Below Level In Example 4, students can systematically examine the possible sums $0 + 1 + 2$, $1 + 2 + 3$, $2 + 3 + 4$, and so on. The sums are the successive multiples of 3. Students can factor 3 out of each sum and note that each time, the other factor is the middle number in the expression.

Advanced Ask students to use algebra to examine the conjecture for **Example 4**. Ask them to describe what they observe when they simplify $n + (n + 1) + (n + 2)$ and factor the result.

See also the *Differentiated Instruction Resources* for more strategies.

2.1 EXERCISES

HOMWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 7, 15, and 33
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 5, 19, 22, and 36
- ◈ = **MULTIPLE REPRESENTATIONS**
Ex. 35

SKILL PRACTICE

A

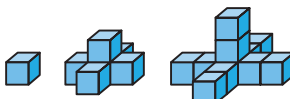
- VOCABULARY** Write a definition of *conjecture* in your own words.
Sample answer: A guess based on observation
- ★ **WRITING** The word *counter* has several meanings. Look up the word in a dictionary. Identify which meaning helps you understand the definition of *counterexample*. *Sample answer: Contrary, opposite, opposing*

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern. 3, 4. See margin.

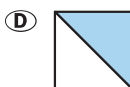
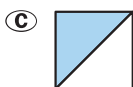
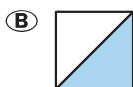
3.



4.



- ★ **MULTIPLE CHOICE** What is the next figure in the pattern? **C**



DESCRIBING NUMBER PATTERNS Describe the pattern in the numbers. Write the next number in the pattern.

- 1, 5, 9, 13, ... The numbers are increasing by 4; 17.
- 3, 12, 48, 192, ...
- 10, 5, 2.5, 1.25, ...
- 4, 3, 1, -2, ... The rate of decrease is increasing by 1; -6.
- 1, $\frac{2}{3}$, $\frac{1}{3}$, 0, ... The numbers are decreasing by $\frac{1}{3}$; $-\frac{1}{3}$.
- 5, -2, 4, 13, ...

MAKING CONJECTURES In Exercises 12 and 13, copy and complete the conjecture based on the pattern you observe in the specific cases.

- Given seven noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Number of points	3	4	5	6	7
Picture					?
Number of connections	3	6	10	15	?

Conjecture You can connect seven noncollinear points 21 different ways. **21**

- Use these sums of odd integers: $3 + 7 = 10$, $1 + 7 = 8$, $17 + 21 = 38$

Conjecture The sum of any two odd integers is even. **even**

EXAMPLE 1
for Exs. 3–5

7. The numbers are 4 times the previous number; 768.

8. The numbers are $\frac{1}{2}$ of the previous number; 0.625.

EXAMPLE 2
for Exs. 6–11

11. The numbers are increasing by successive multiples of 3; 25.

EXAMPLE 3
for Ex. 12

EXAMPLE 4
for Ex. 13

Differentiated Instruction

Visual Learners Before assigning Exercises 6–12, remind students that they should always write a list of the differences or ratios of consecutive numbers in each list of numbers in order to see if there is a pattern.

See also the *Differentiated Instruction Resources* for more strategies.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–8, 12–22, 32–34

Average:

Day 1:

Exs. 1–5, 8–20, 22–28 even, 32–36

Advanced:

Day 1:

Exs. 1, 2, 4, 5, 10–13, 16, 17–21 odd, 22, 23–27 odd, 29–38*

Block:

Exs. 1–5, 8–20, 22–28 even, 32–36 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 12, 14, 33

Average: 4, 8, 13, 16, 33

Advanced: 5, 10, 13, 17, 33

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

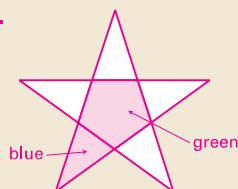
Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

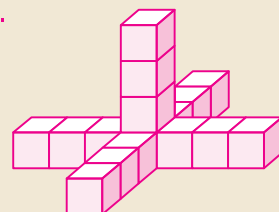
2.1 Use Inductive Reasoning

67

3.



4.



Teaching Strategy

Exercises 6–11 If students have difficulty describing the pattern in the numbers, ask them to write a separate phrase for each number and the next and then generalize from the phrases. For example, the phrases for Exercise 9 might be: subtract 1, subtract 2, subtract 3, and so on.

Mathematical Reasoning

Exercise 12 Ask students if it is possible to write a rule for the number of connections by using n for the number of number of points. They may discover that the number of connections is $\frac{n(n-1)}{2}$.

Avoiding Common Errors

Exercise 16 Students may overlook the one exception to the statement. Suggest that they list the first 12 counting numbers and put a check mark above those that have exactly two divisors.

Mathematical Reasoning

Exercises 20–21 Remind students that a function rule will express the value of y , the dependent variable, in terms of the value of x , the independent variable. To discover the function rule, students can find the differences in successive y -values and then use *guess-and-check* to write y in terms of x . Stress that it is wise to check that the rule works for every pair of values in the table.

EXAMPLE 5

for Exs. 14–17

23. Previous numerator becomes the next denominator while the numerator is one more than the denominator; $\frac{6}{5}$.

24. Successive natural numbers are cubed; 216.

25. 0.25 is being added to each number; 1.45.

26. The rate of increase is increasing by 1; 21.

27. Multiply the first number by 10 to get the second number, take half of the second number to get the third number, and repeat the pattern; 500.

28. The numbers are 6 times the previous number; $0.4(6)^4$.

29. $r > 1$; $0 < r < 1$; raising numbers greater than one by successive natural number powers increases the result while raising a number between 0 and 1 by successive natural number powers decreases the result.

31a. Successive powers of $\frac{1}{2}$ are being added to each number; $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$.

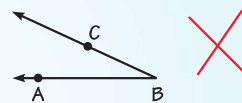
FINDING COUNTEREXAMPLES In Exercises 14–17, show the conjecture is false by finding a counterexample. 14–17. Sample answers are given.

14. If the product of two numbers is positive, then the two numbers must both be positive. $-4 \cdot -7 = 28$
15. The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$. $(3 + 4)^2 = 7^2 = 49 \neq 3^2 + 4^2 = 9 + 16 = 25$
16. All prime numbers are odd. 2
17. If the product of two numbers is even, then the two numbers must both be even. $3 \cdot 6 = 18$

18. **ERROR ANALYSIS** Describe and correct the error in the student's reasoning. All angles are not acute; some angles are obtuse angles, some angles are acute, and some angles are right.

True conjecture: All angles are acute.

Example:



19. **★ SHORT RESPONSE** Explain why only one counterexample is necessary to show that a conjecture is false. To be true, a conjecture must be true for all cases.

xy ALGEBRA In Exercises 20 and 21, write a function rule relating x and y .

20.

x	1	2	3
y	-3	-2	-1

$$y = x - 4$$

21.

x	1	2	3
y	2	4	6

$$y = 2x$$

22. **★ MULTIPLE CHOICE** What is the first number in the pattern? B

$\frac{?}{?}, \frac{?}{?}, \frac{?}{?}, 81, 243, 729$

(A) 1

(B) 3

(C) 9

(D) 27

MAKING PREDICTIONS Describe a pattern in the numbers. Write the next number in the pattern. Graph the pattern on a number line. 23–28. See margin for art.

23. $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \dots$
24. 1, 8, 27, 64, 125, \dots
25. 0.45, 0.7, 0.95, 1.2, \dots
26. 1, 3, 6, 10, 15, \dots
27. 2, 20, 10, 100, 50, \dots
28. $0.4(6), 0.4(6)^2, 0.4(6)^3, \dots$
29. **xy ALGEBRA** Consider the pattern $5, 5r, 5r^2, 5r^3, \dots$. For what values of r will the values of the numbers in the pattern be increasing? For what values of r will the values of the numbers be decreasing? Explain.
30. **REASONING** A student claims that the next number in the pattern 1, 2, 4, \dots is 8, because each number shown is two times the previous number. Is there another description of the pattern that will give the same first three numbers but will lead to a different pattern? Explain. Yes; the rate of increase is increasing by 1, which would make the next number 7.
31. **CHALLENGE** Consider the pattern $1, 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{7}{8}, \dots$

- a. Describe the pattern. Write the next three numbers in the pattern.
- b. What is happening to the values of the numbers? They are getting larger.
- c. Make a conjecture about later numbers. Explain your reasoning. They are getting closer and closer to the number 2; the difference between 2 and each new term is getting smaller.

= See **WORKED-OUT SOLUTIONS** in Student Resources

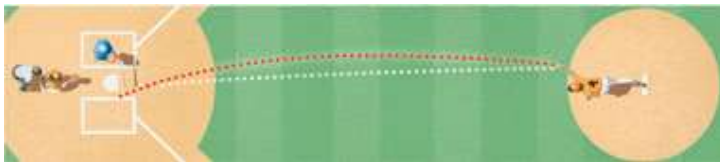
= **STANDARDIZED TEST PRACTICE**

= **MULTIPLE REPRESENTATIONS**



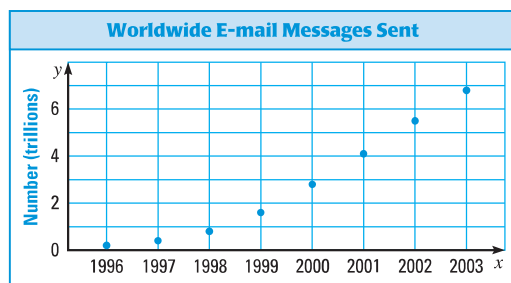
PROBLEM SOLVING

- A** 32. **BASEBALL** You are watching a pitcher who throws two types of pitches, a fastball (F, in white below) and a curveball (C, in red below). You notice that the order of pitches was F, C, F, C, F, C, C, F, F, F. Assuming that this pattern continues, predict the next five pitches. **C, C, C, F, F**

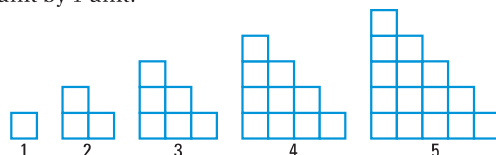


EXAMPLE 6
for Ex. 33

- B** 33. **STATISTICS** The scatter plot shows the number of person-to-person e-mail messages sent each year. Make a conjecture that *could* be true. Give an explanation that supports your reasoning. **Sample answer: The number of e-mail messages will increase in 2004; the number of e-mail messages has increased for the past 7 years.**



- B** 34. **VISUAL REASONING** Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.



- Find the distance around each figure. Organize your results in a table. **See margin.**
- Use your table to *describe* a pattern in the distances. **The distances are increasing each time by 4 units.**
- Predict the distance around the 20th figure in this pattern. **80 units**

35. **MULTIPLE REPRESENTATIONS** Use the given function table relating x and y .

- Making a Table** Copy and complete the table.
- Drawing a Graph** Graph the table of values. **See margin.**
- Writing an Equation** Describe the pattern in words and then write an equation relating x and y . **Double the value of x and add 1 to the result, $y = 2x + 1$.**

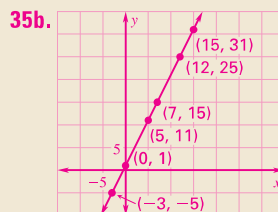
x	y
-3	-5
? 0	1
5	11
? 7	15
12	? 25
15	31

2.1 Use Inductive Reasoning

69

34a.

Figure number	1	2	3	4	5
Distance around (units)	4	8	12	16	20



2.2 Analyze Conditional Statements



Before

You used definitions.

Now

You will write definitions as conditional statements.

Why?

So you can verify statements, as in Example 2.

Key Vocabulary

- conditional statement
- converse, inverse, contrapositive
- if-then form
- hypothesis, conclusion
- negation
- equivalent statements
- perpendicular lines
- biconditional statement



Standard for Mathematical Practice 3 Construct viable arguments and critique the reasoning of others.

A **conditional statement** is a logical statement that has two parts, a **hypothesis** and a **conclusion**. When a conditional statement is written in **if-then form**, the “if” part contains the **hypothesis** and the “then” part contains the **conclusion**. Here is an example:

If **it is raining**, then **there are clouds in the sky**.

Hypothesis
Conclusion

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

- All birds have feathers.
- Two angles are supplementary if they are a linear pair.

Solution

First, identify the **hypothesis** and the **conclusion**. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

- All birds** have **feathers**.
If **an animal is a bird**, then **it has feathers**.
- Two angles are supplementary** if **they are a linear pair**.
If **two angles are a linear pair**, then **they are supplementary**.



GUIDED PRACTICE for Example 1

Rewrite the conditional statement in if-then form.

- All 90° angles are right angles.
- $2x + 7 = 1$, because $x = -3$.
If $x = -3$, then $2x + 7 = 1$.
- When $n = 9$, $n^2 = 81$.
If $n = 9$, then $n^2 = 81$.
- Tourists at the Alamo are in Texas.
If tourists are at the Alamo, then they are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.

Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.

Negation 2 The cat is black.

2.2 Analyze Conditional Statements

71

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Classify each of the following angles as acute, right, or obtuse.

- 102° **obtuse**
- 2.37° **acute**

- Find the complement and supplement of $\angle XYZ$ if $m\angle XYZ = 80^\circ$.

10° ; 100°

- If $XY = YZ$, is Y the midpoint of \overline{XZ} ? If not, give a counterexample.

No; X , Y , and Z need not be collinear.

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with previous lesson
0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you rewrite a biconditional statement? **Tell students they will learn how to answer this question by rewriting a statement and its converse.**



Standards for Mathematical Content High School

CC.9-12.G.CO.9 Prove theorems about lines and angles. (preparation for)

CC.9-12.G.CO.10 Prove theorems about triangles. (preparation for)

CC.9-12.G.CO.11 Prove theorems about parallelograms. (preparation for)

Motivating the Lesson

Students have encountered many situations in which one event leads to another. For example, if they do their homework, then they can watch TV. Ask them about other real-world situations that can be described in if-then form.

3 TEACH

Extra Example 1

Rewrite the conditional statement in if-then form.

- All whales are mammals. **If an animal is a whale, then it is a mammal.**
- Three points are collinear if there is a line containing them. **If there is a line containing three points, then the points are collinear.**

Key Question Example 1

- How can you tell which statement is the hypothesis in part (b). **The hypothesis follows the word "if."**

Extra Example 2

Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Soccer players are athletes." Decide whether each statement is true or false.

If-then form: If you are a soccer player, then you are an athlete; true.

Converse: If you are an athlete, then you are a soccer player; false.

Inverse: If you are not a soccer player, then you are not an athlete; false.

Contrapositive: If you are not an athlete, then you are not a soccer player; true.

READ VOCABULARY

To *negate* part of a conditional statement, you write its negation.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

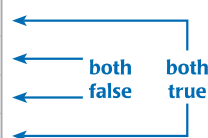
To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99^\circ$, then $\angle A$ is obtuse.

Converse If $\angle A$ is obtuse, then $m\angle A = 99^\circ$.

Inverse If $m\angle A \neq 99^\circ$, then $\angle A$ is not obtuse.

Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99^\circ$.



EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician. *True*, guitar players are musicians.

Converse If you are a musician, then you are a guitar player. *False*, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician. *False*, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.



GUIDED PRACTICE for Example 2

5. If a dog is large, then it is a Great Dane, *false*; if a dog is not a Great Dane, then it is not large, *false*; if a dog is not large, then it is not a Great Dane, *true*.

6. If a polygon is regular, then it is equilateral, *true*; if a polygon is not equilateral, then it is not regular, *true*; if a polygon is not regular, then it is not equilateral, *false*.

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.

5. If a dog is a Great Dane, then it is large.

6. If a polygon is equilateral, then the polygon is regular.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.



Differentiated Instruction

Inclusion One way for students to remember the four types of statements discussed in this lesson is to study the following:

Conditional (If x , then y) *Contrapositive* (If not y , then not x)
Converse (If y , then x) *Inverse* (If not x , then not y)

Stress that the form of each of the last three types is based on the conditional statement.

See also the *Differentiated Instruction Resources* for more strategies.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

KEY CONCEPT

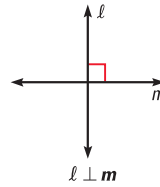
For Your Notebook

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write “line ℓ is perpendicular to line m ” as $\ell \perp m$.



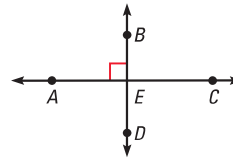
READ DIAGRAMS

In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

- $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$
- $\angle AEB$ and $\angle CEB$ are a linear pair.
- \overrightarrow{EA} and \overrightarrow{EB} are opposite rays.



Solution

- This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.
- This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because \overrightarrow{EA} and \overrightarrow{EC} are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.
- This statement is *false*. Point E does not lie on the same line as A and B , so the rays are not opposite rays.

Animated Geometry at my.hrw.com

7. True; linear pairs of angles are supplementary.

8. False; it is not known that $\overline{FM} \cong \overline{MH}$.

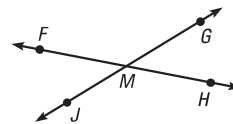
9. True; two intersecting lines form 2 pairs of vertical angles.

10. False; it is not known that the lines intersect to form right angles.

GUIDED PRACTICE for Example 3

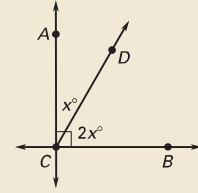
Use the diagram shown. Decide whether each statement is true. Explain your answer using the definitions you have learned.

- $\angle JMF$ and $\angle FMG$ are supplementary.
- Point M is the midpoint of \overline{FH} .
- $\angle JMF$ and $\angle HMG$ are vertical angles.
- $\overline{FH} \perp \overline{JG}$



Extra Example 3

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



- $\overleftrightarrow{AC} \perp \overleftrightarrow{BC}$ **True; the right angle symbol indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.**
- $\angle ACD$ and $\angle DCB$ are complementary. **True; by the Angle Addition Postulate, $m\angle ACD + m\angle DCB = 90^\circ$, so the angles are complementary by definition.**
- \overrightarrow{CD} bisects $\angle ACB$. **False; since x is positive, x° and $2x^\circ$ cannot be equal.**

Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Extra Example 4

Write the definition of supplementary angles as a biconditional.

Definition: If the sum of the measures of two angles is 180° , then the angles are supplementary.

Converse: If two angles are supplementary, then the sum of their measures is 180° . **Biconditional:** The sum of the measures of two angles is 180° if and only if the angles are supplementary.



Key Question Example 4

- What are the key words that let you identify a statement as a biconditional? **if and only if**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question How do you rewrite a biconditional statement?

- A conditional statement can be written in if-then form. The if part is the hypothesis. The then part is the conclusion. The statement can be true or false.
- You can write the converse, inverse, and contrapositive of a conditional.
- A biconditional statement can be written when a conditional statement and its converse are both true.

Write or rewrite the statement in if-then form. Write its converse in if-then form. Determine if both are true. If so, rewrite them together using if and only if.

READ DEFINITIONS

All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase “if and only if.” Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition If **two lines intersect to form a right angle**, then **they are perpendicular**.

Converse If **two lines are perpendicular**, then **they intersect to form a right angle**.

Biconditional **Two lines are perpendicular** if and only if **they intersect to form a right angle**.

11. An angle is a right angle if and only if the measure of the angle is 90° .

12. Mary is in the theater class if and only if she will be in the fall play.



GUIDED PRACTICE for Example 4

- Rewrite the definition of *right angle* as a biconditional statement.
- Rewrite the statements as a biconditional.
If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

2.2 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS** Exs. 11, 17, and 33
- = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

A

- VOCABULARY** Copy and complete: The ? of a conditional statement is found by switching the hypothesis and the conclusion. **converse**
- WRITING** Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional. **Points are collinear if one line contains them; points are collinear if and only if one line contains the points.**

REWRITING STATEMENTS Rewrite the conditional statement in if-then form.

- When $x = 6$, $x^2 = 36$. **If $x = 6$, then $x^2 = 36$.**
- The measure of a straight angle is 180° . **If an angle is a straight angle, then its measure is 180° .**
- Only people who are registered are allowed to vote. **If a person is allowed to vote, then the person is registered to vote.**
- ERROR ANALYSIS** Describe and correct the error in writing the if-then statement.

Given statement: All high school students take four English courses.

If-then statement: If a high school student takes four courses, then all four are English courses.



EXAMPLE 1 for Exs. 3–6

6. The hypothesis and conclusion are not written correctly; if a student is a high school student, then the student takes four English courses.

7. If two angles are complementary, then they add to 90° ; if two angles add to 90° , then they are complementary; if two angles are not complementary, then they do not add to 90° ; if two angles do not add to 90° , then they are not complementary.

8. If it is an ant, then it is an insect; if it is an insect, then it is an ant; if it is not an ant, then it is not an insect; if it is not an insect, then it is not an ant.

9. If $x = 2$, then $3x + 10 = 16$; if $3x + 10 = 16$, then $x = 2$; if $x \neq 2$, then $3x + 10 \neq 16$; if $3x + 10 \neq 16$, then $x \neq 2$.

EXAMPLE 2
for Exs. 7–15

WRITING RELATED STATEMENTS For the given statement, write the if-then form, the converse, the inverse, and the contrapositive. 7–10. See margin.

7. The complementary angles add to 90° .
8. Ants are insects.
9. $3x + 10 = 16$, because $x = 2$.
10. A midpoint bisects a segment.

ANALYZING STATEMENTS Decide whether the statement is *true* or *false*. If false, provide a counterexample.

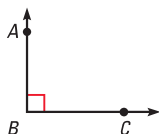
11. If a polygon has five sides, then it is a regular pentagon. **False; see margin for art.**
12. If $m\angle A$ is 85° , then the measure of the complement of $\angle A$ is 5° . **true**
13. Supplementary angles are always linear pairs. **False. Sample answer: $m\angle ABC = 60^\circ$, $m\angle GEF = 120^\circ$**
14. If a number is an integer, then it is rational. **true**
15. If a number is a real number, then it is irrational. **False. Sample answer: 2**

EXAMPLE 3
for Exs. 16–18

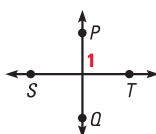
16. True; the diagram indicates that $\angle ABC$ is a right angle and the measure of a right angle is 90° .

USING DEFINITIONS Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

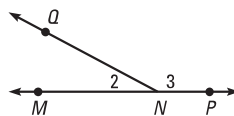
16. $m\angle ABC = 90^\circ$



17. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



18. $m\angle 2 + m\angle 3 = 180^\circ$



EXAMPLE 4
for Exs. 19–21

17. False; there is no indication of a right angle in the diagram.

18. True; by the Angle Addition Postulate, $m\angle 2 + m\angle 3 = m\angle MNP$, $\angle MNP$ is a straight angle and the measure of a straight angle is 180° .

REWRITING STATEMENTS In Exercises 19–21, rewrite the definition as a biconditional statement.

19. An angle with a measure between 90° and 180° is called *obtuse*. **An angle is obtuse if and only if its measure is between 90° and 180° .**
20. Two angles are a *linear pair* if they are adjacent angles whose noncommon sides are opposite rays. **Two angles are a linear pair if and only if they are adjacent angles whose noncommon sides are opposite rays.**
21. *Coplanar points* are points that lie in the same plane. **Points are coplanar if and only if they lie in the same plane.**

DEFINITIONS Determine whether the statement is a valid definition.

22. If two rays are *opposite rays*, then they have a common endpoint. **not a good definition**
23. If the sides of a triangle are all the same length, then the triangle is *equilateral*. **good definition**
24. If an angle is a *right angle*, then its measure is greater than that of an acute angle. **not a good definition**
25. **★ MULTIPLE CHOICE** Which statement has the same meaning as the given statement? **A**

GIVEN ► You can go to the movie after you do your homework.

- (A) If you do your homework, then you can go to the movie afterwards.
- (B) If you do not do your homework, then you can go to the movie afterwards.
- (C) If you cannot go to the movie afterwards, then do your homework.
- (D) If you are going to the movie afterwards, then do not do your homework.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:
Exs. 1–15, 31, 32
Day 2:
Exs. 16–25, 33, 34

Average:

Day 1:
Exs. 1, 2, 4–10, 13–15, 26–29, 31, 32
Day 2:
Exs. 16–25, 33–36

Advanced:

Day 1:
Exs. 1, 2, 4, 5, 9, 10, 14, 15, 26–32*
Day 2:
Exs. 16–25, 33–39*

Block:

Exs. 1, 2, 4–10, 13–15, 26–29, 31, 32 (with previous lesson)
Exs. 16–25, 33–36 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 8, 16, 19, 31

Average: 4, 13, 17, 20, 31

Advanced: 5, 14, 18, 21, 32

Extra Practice

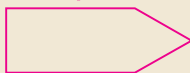
- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

10. If a point is the midpoint of a segment, then the point bisects the segment; if a point bisects a segment, then it is the midpoint of the segment; if a point is not the midpoint of a segment, then the point does not bisect the segment; if a point does not bisect a segment, then it is not the midpoint of the segment.

11. Sample:



Avoiding Common Errors

Exercises 6–10 Students often make errors in deciding which part of the statement is the hypothesis and which part is the conclusion. Suggest that they think of the phrase before the verb as corresponding to the hypothesis and the phrase after the verb as corresponding to the conclusion.

Study Strategy

Exercises 22–24 Ask students to think of counterexamples when trying to decide if a definition is a good definition.

Internet Reference

Exercises 31–32 For more information about volcanoes, visit www.geology.sdsu.edu/how_volcanoes_work/Home.html

34b. If the mean of the data is between x and y , then x and y are the least and greatest values in the data set; false; the mean can lie between two numbers in the data set neither of which is the smallest and/or largest.

34c. Mode. *Sample answer:* The mode is the number that occurs most often and has to be an element of the set. The mean and median can both be a number that is not in the set.

36a. If igneous rock is formed, then molten rock was cooled; if sedimentary rock is formed, then the rock was formed from pieces of other rocks; if metamorphic rock is formed, then there were changes in temperature, pressure, or chemistry.

26. If $x > 0$, then $x > 4$; false.
Sample answer: 2 is greater than zero but not greater than 4. **C**

C

32. False;
 $d = 1$ mm (ash).

34a. True, the mean is the average which lies between the smallest value and the largest value unless all heights are the same.

35. *Sample answer:* If a student is a member of the jazz band, then the student is a member of the band but not the chorus.

xy ALGEBRA Write the converse of each true statement. Tell whether the converse is true. If false, explain why.

26. If $x > 4$, then $x > 0$.

27. If $x < 6$, then $-x > -6$.

28. If $x \leq -x$, then $x \leq 0$.

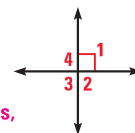
If $-x > -6$, then $x < 6$; true.

If $x \leq 0$, then $x \leq -x$; true.

29. **★ OPEN-ENDED MATH** Write a statement that is true but whose converse is false. *Sample answer:* If the dog sits, she gets a treat.

30. **CHALLENGE** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90^\circ$. Use the definition of linear pairs.

If $\angle 1$ and $\angle 2$ are linear pairs, then $m\angle 2$ is 90° ; if $\angle 1$ and $\angle 4$ are linear pairs, then the $m\angle 4$ is 90° ; if $\angle 4$ and $\angle 3$ are linear pairs, then the $m\angle 3$ is 90° .



PROBLEM SOLVING

EXAMPLE 4 **A**
for Exs. 31–32

In Exercises 31 and 32, use the information about volcanoes to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

VOLCANOES Solid fragments are sometimes ejected from volcanoes during an eruption. The fragments are classified by size, as shown in the table.

31. A fragment is called a *block* or *bomb* if and only if its diameter is greater than 64 millimeters. **true**

32. A fragment is called a *lapilli* if and only if its diameter is less than 64 millimeters.

Type of fragment	Diameter d (millimeters)
Ash	$d < 2$
Lapilli	$2 \leq d \leq 64$
Block or bomb	$d > 64$



33. **★ SHORT RESPONSE** How can you show that the statement, “If you play a sport, then you wear a helmet,” is false? *Explain.*

Find a counterexample. Sample answer: Tennis is a sport, but the participants do not wear helmets.

34. **★ EXTENDED RESPONSE** You measure the heights of your classmates to get a data set.

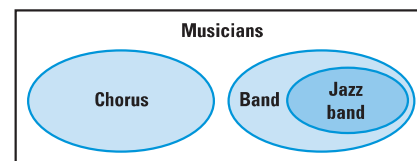
a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y . *Explain* your reasoning.

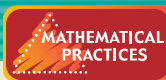
b. Write the converse of the statement in part (a). Is the converse true? *Explain.* **See margin.**

c. Copy and complete the statement using *mean*, *median*, or *mode* to make a conditional that is true for any data set. *Explain* your reasoning. **See margin.**

Statement If a data set has a mean, a median, and a mode, then the ? of the data set will always be one of the measurements.

35. **★ OPEN-ENDED MATH** The Venn diagram at the right represents all of the musicians at a high school. Write an if-then statement that describes a relationship between the various groups of musicians.





Make sense of problems and persevere in solving them.

Investigating Geometry **ACTIVITY** Use before Apply Deductive Reasoning

1 PLAN AND PREPARE

Explore the Concept

- Students will use deductive reasoning to solve a logic problem.
- This activity leads into the study of the Law of Detachment and the Law of Syllogism in Examples 1 and 2 in this lesson.

Materials

Each student will need:

- graph paper
- Activity Support Master (Chapter Resource Book)

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Ask students to rewrite the clues as conditional statements before they begin to draw conclusions from them and mark the grid.

Key Discovery

If-then statements, deductive reasoning, and a chart can be used to solve a logic puzzle.

3 ASSESS AND RETEACH

If statement *A* is true, then statement *B* is true. If statement *B* is true, then statement *C* is true. Suppose statement *A* is true, can you conclude that statement *C* is true? **yes**



CC.9-12.A.SSE.3 Choose and produce an equivalent form

of an expression to reveal and explain properties of the quantity represented by the expression.

Logic Puzzles

MATERIALS • graph paper • pencils

QUESTION How can reasoning be used to solve a logic puzzle?

EXPLORE Solve a logic puzzle

Using the clues below, you can determine an important mathematical contribution and interesting fact about each of five mathematicians.

Copy the chart onto your graph paper. Use the chart to keep track of the information given in Clues 1–7. Place an **X** in a box to indicate a definite “no.” Place an **O** in a box to indicate a definite “yes.”

Clue 1 Pythagoras had his contribution named after him. He was known to avoid eating beans.

Clue 2 Albert Einstein considered Emmy Noether to be one of the greatest mathematicians and used her work to show the theory of relativity.

Clue 3 Anaxagoras was the first to theorize that the moon’s light is actually the sun’s light being reflected.

Clue 4 Julio Rey Pastor wrote a book at age 17.

Clue 5 The mathematician who is fluent in Latin contributed to the study of differential calculus.

Clue 6 The mathematician who did work with *n*-dimensional geometry was not the piano player.

Clue 7 The person who first used perspective drawing to make scenery for plays was not Maria Agnesi or Julio Rey Pastor.

	<i>n</i> -dimensional geometry	Differential calculus	Math for theory of relativity	Perspective drawing	Pythagorean Theorem	Did not eat beans	Studied moonlight	Wrote a math book at 17	Fluent in Latin	Played piano
Maria Agnesi	X	O	X	X	X	X	X	O	X	
Anaxagoras	X	X	O	X	X	X	O	X	X	
Emmy Noether	X	X	O	X	X	X	X	X	O	
Julio Rey Pastor	O	X	X	X	X	X	O	X	X	
Pythagoras	X	X	X	X	O	O	X	X	X	
Did not eat beans	X	X	X	X	O					
Studied moonlight	X	X	X	O	X					
Wrote a math book at 17	O	X	X	X	X					
Fluent in Latin	X	O	X	X	X					
Played piano	X	X	O	X	X					

DRAW CONCLUSIONS Use your observations to complete these exercises 1–3. See margin.

1. Write Clue 4 as a conditional statement in if-then form. Then write the contrapositive of the statement. *Explain* why the contrapositive of this statement is a helpful clue.
2. *Explain* how you can use Clue 6 to figure out who played the piano.
3. *Explain* how you can use Clue 7 to figure out who worked with perspective drawing.

1. If it is Julio Rey Pastor, then he wrote a book at age 17; if he did not write a book at age 17, then it is not Julio Rey Pastor; no other mathematician in the list wrote a book at age 17.
2. *Sample answer:* Find the individual who worked with *n*-dimensional geometry and eliminate him/her from the possible piano players.
3. *Sample answer:* Eliminate both Maria Agnesi and Julio Rey Pastor from the possibilities for perspective drawing.

2.3 Apply Deductive Reasoning



Before

You used inductive reasoning to form a conjecture.

Now

You will use deductive reasoning to form a logical argument.

Why

So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary

• **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

COMMON
CORE

Standard for Mathematical Practice 3 Construct viable arguments and critique the reasoning of others.

READ VOCABULARY

The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

KEY CONCEPT

For Your Notebook

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If **hypothesis p** , then **conclusion q** . \Rightarrow If these statements are true,
If **hypothesis q** , then **conclusion r** .
If **hypothesis p** , then **conclusion r** . \leftarrow then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

- If two segments have the same length, then they are congruent. You know that $BC = XY$.
- Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

- Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.
- First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."
"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

- Write the converse, inverse, and contrapositive of "If the measure of an angle is less than 90° , then the angle is acute." **Converse:** If an angle is acute, then the measure is less than 90° . **Inverse:** If the measure of an angle is not less than 90° , then the angle is not acute. **Contrapositive:** If an angle is not acute, then its measure is not less than 90° .

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with previous lesson
0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you construct a logical argument? **Tell students they will learn how to answer this question by using the Laws of Detachment and Syllogism.**

COMMON
CORE

Standards for Mathematical Content High School

CC.9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

CC.9-12.G.CO.9 Prove theorems about lines and angles. (preparation for)

CC.9-12.G.CO.10 Prove theorems about triangles. (preparation for)

CC.9-12.G.CO.11 Prove theorems about parallelograms. (preparation for)

Motivating the Lesson

Your coach has told you that if you miss more than one practice during the week, you will not be allowed to play on Saturday. You missed practice yesterday and today. What do you think will happen?

3 TEACH

Extra Example 1

Use the Law of Detachment to make a valid conclusion in the true situation.

- If two angles are right angles, then they are congruent. $\angle C$ and $\angle D$ are right angles. **$\angle C \cong \angle D$**
- If Nathan is enrolled at Metro High School, then Nathan has an ID number. Nathan is enrolled at Metro High School. **Nathan has an ID number.**

Extra Example 2

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

- If the radius of a circle is 4 ft, then the diameter is 8 ft. If the radius of a circle is 4 ft, then its area is $16\pi \text{ ft}^2$. **not possible**
- If Joe takes Geometry this year, then he will take Algebra 2 next year. If Joe takes Algebra 2 next year, then he will graduate. **If Joe takes Geometry this year, then he will graduate.**
- If $y^3 = 8$, then $y = 2$. If $y = 2$, then $3y + 4 = 10$. **If $y^3 = 8$, then $3y + 4 = 10$.**



An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

- If Rick takes chemistry this year, then Jesse will be Rick's lab partner.
If Jesse is Rick's lab partner, then Rick will get an A in chemistry.
- If $x^2 > 25$, then $x^2 > 20$.
If $x > 5$, then $x^2 > 25$.
- If a polygon is regular, then all angles in the interior of the polygon are congruent.
If a polygon is regular, then all of its sides are congruent.

Solution

- The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.
If Rick takes chemistry this year, then Rick will get an A in chemistry.
- Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.
If $x > 5$, then $x^2 > 20$.
- Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

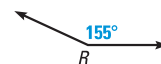
at my.hrw.com

AVOID ERRORS

The order in which the statements are given does not affect whether you can use the Law of Syllogism.

GUIDED PRACTICE for Examples 1 and 2

- If $90^\circ < m\angle R < 180^\circ$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155° . Using the Law of Detachment, what statement can you make? **$\angle R$ is obtuse.**
- If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make? **If Jenelle gets a job, then she will drive to school.**



State the law of logic that is illustrated.

- If you get an A or better on your math test, then you can go to the movies.
If you go to the movies, then you can watch your favorite actor.
If you get an A or better on your math test, then you can watch your favorite actor. **Law of Syllogism**
- If $x > 12$, then $x + 9 > 20$. The value of x is 14.
Therefore, $x + 9 > 20$. **Law of Detachment**

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

xy ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$(-2)(2) = -4, (-1)(2) = -2, 2(2) = 4, 3(2) = 6,$$

$$(-2)(-4) = 8, (-1)(-4) = 4, 2(-4) = -8, 3(-4) = -12$$

Conjecture Even integer \cdot Any integer = Even integer

STEP 2 Let n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m .

$2nm$ is the product of 2 and an integer nm . So, $2nm$ is an even integer.

► The product of an even integer and any integer is an even integer.

EXAMPLE 4 Compare inductive and deductive reasoning

Decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. Explain your reasoning.

- Each time Monica kicks a ball up in the air, it returns to the ground. So the next time Monica kicks a ball up in the air, it will return to the ground.
- All reptiles are cold-blooded. Parrots are not cold-blooded. Sue's pet parrot is not a reptile.



Solution

- Inductive reasoning, because a pattern is used to reach the conclusion.
- Deductive reasoning, because facts about animals and the laws of logic are used to reach the conclusion.



GUIDED PRACTICE for Examples 3 and 4

5. The sum of a number and itself is twice the number; $n + n = 2n$.

- Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true.
- Give an example of when you used deductive reasoning in everyday life.
Check examples.

Extra Example 3

What conclusion can you make about the product of two odd numbers? **odd number \cdot odd number = odd number**



Key Question Example 3

- How does this example use both inductive and deductive reasoning? **Inductive reasoning is used to discover a pattern, and deductive reasoning is used to prove that the pattern always holds.**

Extra Example 4

Tell whether the statement is a result of inductive reasoning or deductive reasoning. Explain your choice.

- Whenever it rains in the morning, afternoon baseball games are canceled. The baseball game this afternoon was not canceled. So, it did not rain this morning. **Deductive reasoning, because it uses the laws of logic.**
- Every time Tom has eaten strawberries, he has had a mild allergic reaction. The next time he eats strawberries, he will have a mild allergic reaction. **Inductive reasoning, because it is based on a pattern.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you construct a logical argument?

- The Law of Detachment allows you to make a valid conclusion in a true situation.**
- The Law of Syllogism allows you to write a new conditional statement that follows from a pair of true statements.**

You construct a logical argument by applying the Law of Detachment and the Law of Syllogism along with known facts and definitions.

Differentiated Instruction

English Learners Inductive reasoning uses patterns in specific examples to lead into the general case. Deductive reasoning leads from the general case to a specific instance. See also the *Differentiated Instruction Resources* for more strategies.

2.3 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 7, 17, and 21
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 3, 12, 20, and 23

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:
Exs. 1–10, 16, 17
Day 2:
Exs. 11–13, 18–24

Average:

Day 1:
Exs. 1–6, 8–10, 14, 16, 17
Day 2:
Exs. 11–13, 18–28

Advanced:

Day 1:
Exs. 1–6, 9, 10, 14–17*
Day 2:
Exs. 11–13, 18–29*

Block:

Exs. 1–6, 8–10, 14, 16, 17, 34–38
(with previous lesson)
Exs. 11–13, 18–28 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 11, 16, 20

Average: 5, 9, 11, 16, 20

Advanced: 6, 10, 11, 17, 20

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

- A** 1. **VOCABULARY** Copy and complete: If the hypothesis of a true if-then statement is true, then the conclusion is also true by the Law of ? . **Detachment**

★ **WRITING** Use deductive reasoning to make a statement about the picture.

2.



3.



2. **Sample answer:** The person in the center of the photo is wearing jeans.
3. **Sample answer:** The door to this room is closed.

EXAMPLE 1
for Exs. 4–6

LAW OF DETACHMENT Make a valid conclusion in the situation.

- If the measure of an angle is 90° , then it is a right angle. The measure of $\angle A$ is 90° . **$\angle A$ is a right angle.**
- If $x > 12$, then $-x < -12$. The value of x is 15. **$-15 < -12$**
- If a book is a biography, then it is nonfiction. You are reading a biography. **It is nonfiction.**

EXAMPLE 2
for Exs. 7–10

LAW OF SYLLOGISM In Exercises 7–10, write the statement that follows from the pair of statements that are given.

- If a rectangle has four equal side lengths, then it is a square. If a polygon is a square, then it is a regular polygon.
- If $y > 0$, then $2y > 0$. If $2y > 0$, then $2y - 5 \neq -5$. **If $y > 0$, then $2y - 5 \neq -5$.**
- If you play the clarinet, then you play a woodwind instrument. If you play a woodwind instrument, then you are a musician. **If you play the clarinet, then you are a musician.**
- If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$. **If $\frac{1}{2}a = 1\frac{1}{2}$, then $5a = 15$.**

EXAMPLE 3
for Ex. 11

- REASONING** What can you say about the sum of an even integer and an even integer? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.

- B** 12. ★ **MULTIPLE CHOICE** If two angles are vertical angles, then they have the same measure. You know that $\angle 1$ and $\angle 2$ are vertical angles. Using the Law of Detachment, which conclusion could you make? **B**

- (A) $m\angle 1 > m\angle 2$ (B) $m\angle 1 = m\angle 2$
(C) $m\angle 1 + m\angle 2 = 90^\circ$ (D) $m\angle 1 + m\angle 2 = 180^\circ$

- ERROR ANALYSIS** Describe and correct the error in the argument: “If two angles are a linear pair, then they are supplementary. Angles C and D are supplementary, so the angles are a linear pair.” **See margin.**

82 Chapter 2 Reasoning and Proof

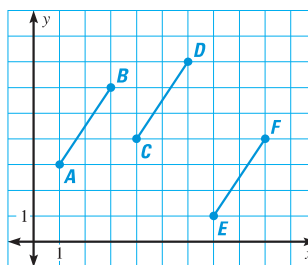
13. Linear pairs are not the only pairs of angles that are supplementary. **Sample answer:** Angles C and D are supplementary, so the sum of their measures is 180° .

14b. If one endpoint is 2 units to the right and 3 units above the other endpoint, then the segment is congruent to the given segments; let $J(1, 1)$ and $K(3, 4)$ then $JK = \sqrt{13}$, use the Distance Formula.

14c. One endpoint is (x, y) and the second is $(x + 2, y + 3)$. Using the Distance Formula the distance between the two points is $\sqrt{13}$.

14. **ALGEBRA** Use the segments in the coordinate plane.

- Use the distance formula to show that the segments are congruent. $AB = CD = EF = \sqrt{13}$.
- Make a conjecture about some segments in the coordinate plane that are congruent to the given segments. Test your conjecture, and *explain* your reasoning.
- Let one endpoint of a segment be (x, y) . Use algebra to show that segments drawn using your conjecture will always be congruent.
- A student states that the segments described below will each be congruent to the ones shown above. Determine whether the student is correct. *Explain* your reasoning.



\overline{MN} , with endpoints $M(3, 5)$ and $N(5, 2)$

\overline{PQ} , with endpoints $P(1, -1)$ and $Q(4, -3)$

\overline{RS} , with endpoints $R(-2, 2)$ and $S(1, 4)$

\overline{MN} , correct, by the Distance Formula;

\overline{PQ} , correct, by the Distance Formula;

\overline{RS} , correct, by the Distance Formula.

15. **CHALLENGE** Make a conjecture about whether the Law of Syllogism works when used with the contrapositives of a pair of statements. Use this pair of statements to *justify* your conjecture. **See margin.**

If a creature is a wombat, then it is a marsupial.

If a creature is a marsupial, then it has a pouch.

PROBLEM SOLVING

EXAMPLES **A**
1 and 2
for Exs. 16–17

USING THE LAWS OF LOGIC In Exercises 16 and 17, what conclusions can you make using the true statement?

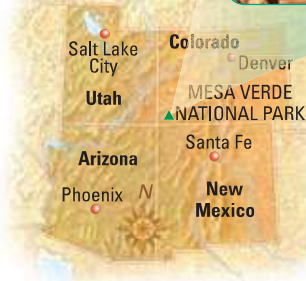
16. **CAR COSTS** If you save at least \$2000, then you can buy a used car. You have saved \$2400. **You can buy a used car.**

17. **PROFIT** The bakery makes a profit if its revenue is greater than its costs. You will get a raise if the bakery makes a profit.

17. You will get a raise if the revenue is greater than its costs.

USING DEDUCTIVE REASONING Select the word(s) that make(s) the conclusion true.

- Mesa Verde National Park is in Colorado. Simone vacationed in Colorado. So, Simone (*must have, may have, or never*) visited Mesa Verde National Park. **may have**
- The cliff dwellings in Mesa Verde National Park are accessible to visitors only when accompanied by a park ranger. Billy is at a cliff dwelling in Mesa Verde National Park. So, Billy (*is, may be, is not*) with a park ranger. **is**



Mathematical Reasoning

Exercise 14 Students should notice that each segment can be thought of as a diagonal of a 2 by 3 rectangle. Using the distance formula gives a length of $\sqrt{13}$. The segments do not have to be parallel.

Internet Reference

Exercises 18–19 For more information on Mesa Verde National Park, go to the National Park Service's website at www.nps.gov/meve

15. Law of Syllogism does apply; the contrapositive of the statements are: if the creature is not a marsupial, then it is not a wombat; if the creature does not have a pouch, then it is not a marsupial; using the Law of Syllogism it can be stated that if the creature does not have a pouch, then it is not a wombat.

Internet Reference

Exercise 20 More information about the Mohs scale of hardness, including a complete listing of the scale, can be found at www.kgs.ku.edu/Extension/KGSrocks/hardness.html

Mathematical Reasoning

Exercises 25–28 Ask students to apply the laws of logic very carefully in these exercises. Point out that the given information implies that Mia will buy popcorn. But we cannot be certain that she will not also buy a hot dog.

20a. Sample answer: If Gypsum is scratched against Calcite, then a scratch mark is left on the Gypsum; if Talc is scratched against Calcite, then a scratch mark is left on the Talc; if Calcite is scratched against Fluorite, then a scratch mark is left on the Calcite.

23. Conjecture: The sum of an even integer and an odd integer is odd; **Reasoning:** $2m + (2n + 1) = (2m + 2n) + 1 = 2(m + n) + 1$, which is odd.

EXAMPLE 4 B

20b. Mineral A cannot be Fluorite since Mineral B can scratch it; Mineral C must be Talc since it is the softest of the minerals.

20c. Sample answer: If Mineral D scratches Mineral B then D is the Fluorite otherwise B is the Fluorite; if Mineral B is Fluorite, take Mineral A and Mineral D and do one scratch test to identify them.

21. Deductive; laws of logic were used to reach the conclusion.

24. For want of a nail the rider is lost; the hypothesis and conclusion are directly linked by the Law of Syllogism.

25. True; since the game is not sold out, Arlo goes and buys a hot dog.

26. True; Arlo and Mia go to the game since it is not sold out.

- 20. ★ EXTENDED RESPONSE** Geologists use the Mohs scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Geologists use scratch tests to help identify an unknown mineral.

Mineral				
	Talc	Gypsum	Calcite	Fluorite
Mohs rating	1	2	3	4

- Use the table to write three if-then statements such as "If talc is scratched against gypsum, then a scratch mark is left on the talc." **See margin.**
- The four minerals in the table are randomly labeled A, B, C, and D. You must identify them. The results of your scratch tests are shown below. What can you conclude? *Explain* your reasoning.
Mineral A is scratched by Mineral B.
Mineral C is scratched by all three of the other minerals.
- What additional test(s) can you use to identify *all* the minerals in part (b)?

REASONING In Exercises 21 and 22, decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. *Explain* your reasoning.

- The rule at your school is that you must attend all of your classes in order to participate in sports after school. You played in a soccer game after school on Monday. Therefore, you went to all of your classes on Monday.
- For the past 5 years, your neighbor goes on vacation every July 4th and asks you to feed her hamster. You conclude that you will be asked to feed her hamster on the next July 4th. **Inductive; a pattern leads to the conclusion.**
- ★ SHORT RESPONSE** Use inductive reasoning to form a conjecture about whether the sum of an even integer and an odd integer is even or odd. Then use deductive reasoning to show that the conjecture is true. (*Hint:* Let the even integer be $2m$ and the odd integer be $2n + 1$.) **See margin.**

- 24. LITERATURE** George Herbert wrote a poem, *Jacula Prudentum*, that includes the statements shown. Use the Law of Syllogism to write a new conditional statement. *Explain* your reasoning.

For want of a nail the shoe is lost,
for want of a shoe the horse is lost,
for want of a horse the rider is lost.

REASONING In Exercises 25–28, use the true statements below to determine whether you know the conclusion is *true* or *false*. *Explain* your reasoning.

If Arlo goes to the baseball game, then he will buy a hot dog.
If the baseball game is not sold out, then Arlo and Mia will go to the game.
If Mia goes to the baseball game, then she will buy popcorn.
The baseball game is not sold out.

- Arlo bought a hot dog.
- Arlo and Mia went to the game.
- Mia bought a hot dog. **False; Mia will buy popcorn.**
- Arlo had some of Mia's popcorn. **False; Arlo buys a hotdog.**

Extension

1 PLAN AND PREPARE

Warm-Up Exercises

- Name the hypothesis and conclusion of the statement, "If an animal flies, then it is a bird."
hypothesis: an animal flies
conclusion: it is a bird
- Write the converse, inverse, and contrapositive of the conditional, "If a polygon is regular, then it has congruent sides."
Converse: If a polygon has congruent sides, then it is regular. Inverse: If a polygon is not regular, then it does not have congruent sides. Contrapositive: If a polygon does not have congruent sides, then it is not regular.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How can you represent a logical statement symbolically? **Tell students they will learn how to answer this question by using variables to represent statements in conditionals.**

COMMON CORE **CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Key Vocabulary

- truth value
- truth table



Standard for Mathematical Practice 2 Reason abstractly and quantitatively.

Symbolic Notation and Truth Tables

GOAL Use symbolic notation to represent logical statements.

Conditional statements can be written using *symbolic notation*, where letters are used to represent statements. An arrow (\rightarrow), read "implies," connects the hypothesis and conclusion. To write the negation of a statement p you write the symbol for negation (\sim) before the letter. So, "not p " is written $\sim p$.

KEY CONCEPT

For Your Notebook

Symbolic Notation

Let p be "the angle is a right angle" and let q be "the measure of the angle is 90° ."

Conditional If p , then q . $p \rightarrow q$

Example: If an angle is a right angle, then its measure is 90° .

Converse If q , then p . $q \rightarrow p$

Example: If the measure of an angle is 90° , then the angle is a right angle.

Inverse If not p , then not q . $\sim p \rightarrow \sim q$

Example: If an angle is not a right angle, then its measure is not 90° .

Contrapositive If not q , then not p . $\sim q \rightarrow \sim p$

If the measure of an angle is not 90° , then the angle is not a right angle.

Biconditional p if and only if q $p \leftrightarrow q$

Example: An angle is a right angle if and only if its measure is 90° .

EXAMPLE 1 Use symbolic notation

Let p be "the car is running" and let q be "the key is in the ignition."

- Write the conditional statement $p \rightarrow q$ in words.
- Write the converse $q \rightarrow p$ in words.
- Write the inverse $\sim p \rightarrow \sim q$ in words.
- Write the contrapositive $\sim q \rightarrow \sim p$ in words.

Solution

- Conditional: If the car is running, then the key is in the ignition.
- Converse: If the key is in the ignition, then the car is running.
- Inverse: If the car is not running, then the key is not in the ignition.
- Contrapositive: If the key is not in the ignition, then the car is not running.

6.

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

7.

p	q	$\sim p$	$\sim p \rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

8.

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

9.

p	q	$q \rightarrow p$	$\sim(q \rightarrow p)$
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	F

2. If polygon $ABCDE$ is equiangular and equilateral, then it is a regular polygon.

3. Polygon $ABCDE$ is not both equiangular and equilateral.

4. If polygon $ABCDE$ is not a regular polygon, then it is not both equiangular and equilateral.

5. Polygon $ABCDE$ is equiangular and equilateral if and only if it is a regular polygon.

READ TRUTH TABLES

A conditional statement and its contrapositive are *equivalent statements* because they have the same truth table. The same is true of the converse and the inverse.

TRUTH TABLES The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table at the right shows the truth values for hypothesis p and conclusion q . The conditional $p \rightarrow q$ is only false when a true hypothesis produces a false conclusion.

Two statements are *logically equivalent* if they have the same truth table.

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE 2 Make a truth table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \rightarrow q$.

Solution

Converse			Inverse					Contrapositive				
p	q	$q \rightarrow p$	p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	T	T	T	F	F	T	T	T	F	F	T
T	F	T	T	F	F	T	T	T	F	T	F	F
F	T	F	F	T	T	F	F	F	T	F	T	T
F	F	T	F	F	T	T	T	F	F	T	T	T

PRACTICE

EXAMPLE 1
for Exs. 1–5

1. **WRITING** Describe how to use symbolic notation to represent the contrapositive of a conditional statement. $\sim q \rightarrow \sim p$

WRITING STATEMENTS Use p and q to write the symbolic statement in words. 2–5. See margin.

p : Polygon $ABCDE$ is equiangular and equilateral.

q : Polygon $ABCDE$ is a regular polygon.

2. $p \rightarrow q$ 3. $\sim p$ 4. $\sim q \rightarrow \sim p$ 5. $p \leftrightarrow q$

MAKING TRUTH TABLES Make a truth table for the logical statement. 6–9. See margin.

6. $p \rightarrow \sim q$ 7. $\sim p \rightarrow q$ 8. $\sim(p \rightarrow q)$ 9. $\sim(q \rightarrow p)$

10. **LOGICAL EQUIVALENCE** The truth table shows that the *conjunction* “ p and q ” is true only when p and q are both true. It also shows that the *disjunction* “ p or q ” is false only when p and q are both false.

- a. Make a truth table for $\sim(p \text{ or } q)$. See margin.
b. Make a truth table for $\sim p$ and $\sim q$. See margin.
c. Show that $\sim(p \text{ or } q)$ and $(\sim p \text{ and } \sim q)$ are logically equivalent. The truth tables are the same.

p	q	Conjunction p and q	Disjunction p or q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Extension: Symbolic Notation and Truth Tables

87

3 TEACH

Extra Example 1

Let p be “snow is falling” and let q be “it is winter”.

- a. Write the conditional statement $p \rightarrow q$ in words. **If snow is falling, then it is winter.**
b. Write the converse $q \rightarrow p$ in words. **If it is winter, then snow is falling.**
c. Write the inverse $\sim p \rightarrow \sim q$ in words. **If snow is not falling, then it is not winter.**
d. Write the contrapositive $\sim q \rightarrow \sim p$ in words. **If it is not winter, then snow is not falling.**

Extra Example 2

Use the truth tables to make a truth table for the biconditional $p \leftrightarrow q$.

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you represent a logical statement symbolically?

- Use different variables to represent the hypothesis and conclusion of a statement. Use \sim to represent “not”.

- Rewrite the conditional using variables, \sim , and an arrow to represent “if-then”.

You can represent a logical statement symbolically by using variables to represent phrases and arrows to represent “if-then”.

4 PRACTICE AND APPLY

Mathematical Reasoning

Exercise 10 The equivalence in this exercise is known as one of De Morgan’s Laws.

10a.

p	q	$p \text{ or } q$	$\sim(p \text{ or } q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

10b.

p	q	$\sim p$	$\sim q$	$\sim p \text{ and } \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2.4 Use Postulates and Diagrams



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

State which postulate justifies each statement.

- If D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$. **Angle Addition Postulate**
- If M is between X and Y , then $XM + MY = XY$. **Segment Addition Postulate**
- Find the measure of MN if N is between M and P , $MP = 6x - 2$, $MN = 4x$, and $MP = 16$. **12**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with previous lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you identify postulates illustrated by a diagram? **Tell students they will learn how to answer this question by interpreting diagrams.**

Before

You used postulates involving angle and segment measures.

Now

You will use postulates involving points, lines, and planes.

Why?

So you can draw the layout of a neighborhood, as in Ex. 39.

Key Vocabulary

- line perpendicular to a plane
- postulate



CC.9-12.G.CO.9 Prove theorems about lines and angles.

In geometry, rules that are accepted without proof are called *postulates* or *axioms*. Rules that are proved are called *theorems*. Postulates and theorems are often written in conditional form. Unlike the converse of a definition, the converse of a postulate or theorem cannot be assumed to be true.

You have already learned four postulates.

- POSTULATE 1 Ruler Postulate
- POSTULATE 2 Segment Addition Postulate
- POSTULATE 3 Protractor Postulate
- POSTULATE 4 Angle Addition Postulate

Here are seven new postulates involving points, lines, and planes.

POSTULATES

For Your Notebook

Point, Line, and Plane Postulates

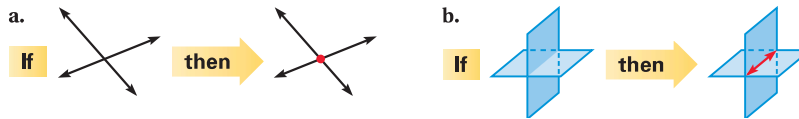
- POSTULATE 5** Through any two points there exists exactly one line.
- POSTULATE 6** A line contains at least two points.
- POSTULATE 7** If two lines intersect, then their intersection is exactly one point.
- POSTULATE 8** Through any three noncollinear points there exists exactly one plane.
- POSTULATE 9** A plane contains at least three noncollinear points.
- POSTULATE 10** If two points lie in a plane, then the line containing them lies in the plane.
- POSTULATE 11** If two planes intersect, then their intersection is a line.

ALGEBRA CONNECTION You have been using many of Postulates 5–11 in previous courses.

One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Postulate 5 guarantees that there is exactly one such line. A familiar way to find a common solution of two linear equations is to graph the lines and find the coordinates of their intersection. This process is guaranteed to work by Postulate 7.

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.



Solution

- a. **Postulate 7** If two lines intersect, then their intersection is exactly one point.
- b. **Postulate 11** If two planes intersect, then their intersection is a line.

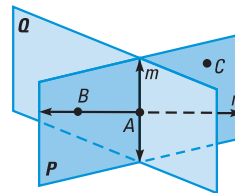
EXAMPLE 2 Identify postulates from a diagram

Use the diagram to write examples of Postulates 9 and 10.

Postulate 9 Plane P contains at least three noncollinear points, A , B , and C .

Postulate 10 Point A and point B lie in plane P , so line n containing A and B also lies in plane P .

at my.hrw.com



GUIDED PRACTICE for Examples 1 and 2

- Use the diagram in Example 2. Which postulate allows you to say that the intersection of plane P and plane Q is a line? **Postulate 11**
- Use the diagram in Example 2 to write examples of Postulates 5, 6, and 7.
Line n passes through points A and B , line n contains points A and B , line m and line n intersect at point A .

CONCEPT SUMMARY

Interpreting a Diagram

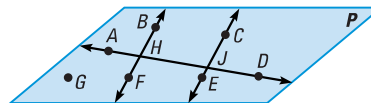
When you interpret a diagram, you can assume information about size or measure only if it is marked.

YOU CAN ASSUME

- All points shown are coplanar.
- $\angle AHB$ and $\angle BHD$ are a linear pair.
- $\angle AHF$ and $\angle BHD$ are vertical angles.
- A , H , J , and D are collinear.
- \overleftrightarrow{AD} and \overleftrightarrow{BF} intersect at H .

YOU CANNOT ASSUME

- G , F , and E are collinear.
- \overleftrightarrow{BF} and \overleftrightarrow{CE} intersect.
- \overleftrightarrow{BF} and \overleftrightarrow{CE} do not intersect.
- $\angle BHA \cong \angle CJA$
- $\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90^\circ$



For Your Notebook

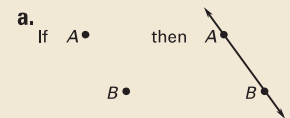
Motivating the Lesson

There are four buildings on the school campus. How many paths need to be constructed to connect all four buildings? To answer this question, it helps to use a postulate of geometry that tells you how many lines pass through two points.

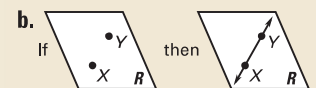
3 TEACH

Extra Example 1

State the postulate illustrated by the diagram.



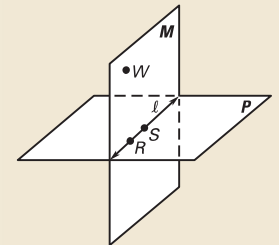
Postulate 5: Through any two points there exists exactly one line.



Postulate 10: If two points lie in a plane, then the line containing them lies in the plane.

Extra Example 2

Use the diagram to write examples of Postulates 6 and 8.



Postulate 6: Line l contains at least two points R and S .

Postulate 8: Through noncollinear points R , S , and W , there exists exactly one plane M .

Differentiated Instruction

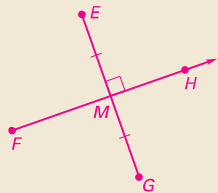
Kinesthetic Learners Some students find it difficult to visualize three-dimensional situations in a sketch. Students may need to build models of some of the figures in this book, or you can create them and bring them to class.

See also the *Differentiated Instruction Resources* for more strategies.

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

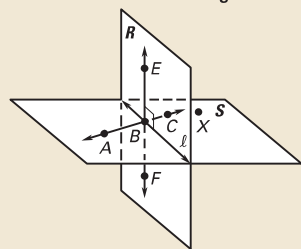
Extra Example 3

Sketch a diagram showing $\overleftrightarrow{FH} \perp \overleftrightarrow{EG}$ at its midpoint M .



Extra Example 4

Which of the following cannot be assumed from the diagram?



$A, B,$ and C are collinear.

$\overleftrightarrow{EF} \perp$ line l

$\overleftrightarrow{BC} \perp$ plane R

\overleftrightarrow{EF} intersects \overleftrightarrow{AC} at B .

line $l \perp \overleftrightarrow{AB}$

Points $B, C,$ and X are collinear.

$\overleftrightarrow{BC} \perp$ plane R , line $l \perp \overleftrightarrow{AB}$.

Points $B, C,$ and X are collinear.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you identify postulates from a diagram?

- Analyze a diagram to determine what you can assume.
- Verify what you can assume by determining which postulates apply.

You can identify postulates from a diagram by determining which information you can assume. Do not assume congruence, perpendicularity, collinearity, midpoints, and bisectors. Determine which postulate applies.

AVOID ERRORS

Notice that the picture was drawn so that W does not look like a midpoint of \overleftrightarrow{PQ} . Also, it was drawn so that \overleftrightarrow{PQ} is not perpendicular to \overleftrightarrow{TV} .

EXAMPLE 3 Use given information to sketch a diagram

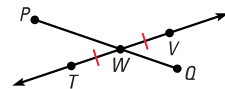
Sketch a diagram showing \overleftrightarrow{TV} intersecting \overleftrightarrow{PQ} at point W , so that $\overline{TW} \cong \overline{WV}$.

Solution

STEP 1 Draw \overleftrightarrow{TV} and label points T and V .

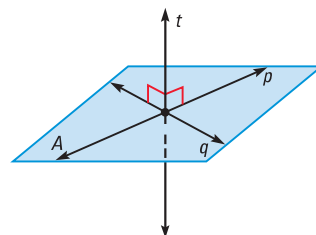
STEP 2 Draw point W at the midpoint of \overleftrightarrow{TV} . Mark the congruent segments.

STEP 3 Draw \overleftrightarrow{PQ} through W .



PERPENDICULAR FIGURES A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right angle symbol.



EXAMPLE 4 Interpret a diagram in three dimensions

Which of the following statements *cannot* be assumed from the diagram?

$A, B,$ and F are collinear.

$E, B,$ and D are collinear.

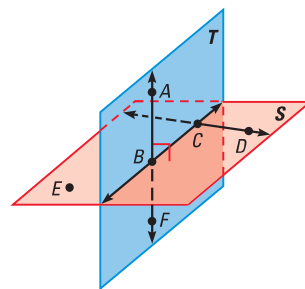
$\overleftrightarrow{AB} \perp$ plane S

$\overleftrightarrow{CD} \perp$ plane T

\overleftrightarrow{AF} intersects \overleftrightarrow{BC} at point B .

Solution

No drawn line connects $E, B,$ and D , so you cannot assume they are collinear. With no right angle marked, you cannot assume $\overleftrightarrow{CD} \perp$ plane T .

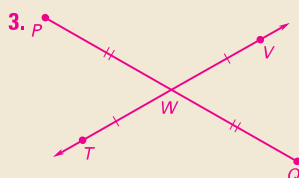


GUIDED PRACTICE for Examples 3 and 4

In Exercises 3 and 4, refer back to Example 3.

- If the given information stated \overline{PW} and \overline{QW} are congruent, how would you indicate that in the diagram? **See margin.**
- Name a pair of supplementary angles in the diagram. *Explain.*
Sample answer: $\angle TWP, \angle VWP$; they form a linear pair.
- In the diagram for Example 4, can you assume plane S intersects plane T at \overleftrightarrow{BC} ? **yes**
- Explain* how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ in Example 4.

6. It is given that line AB is perpendicular to plane S , therefore line AB is perpendicular to every line in the plane that intersects it at point B .



Differentiated Instruction

Below Level Before beginning a discussion of Example 4, have students make a two-column list of statements that can and cannot be assumed from the diagram.

See also the *Differentiated Instruction Resources* for more strategies.

2.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 7, 13, and 31

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 10, 24, 25, 33, 39, and 41

SKILL PRACTICE

- A** 1. **VOCABULARY** Copy and complete: A ? is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it. **line perpendicular to a plane**
2. ★ **WRITING** Explain why you cannot assume $\angle BHA \cong \angle CJA$ in the Concept Summary in this lesson. **We don't know that they have the same measure.**

EXAMPLE 1
for Exs. 3–5

IDENTIFYING POSTULATES State the postulate illustrated by the diagram.



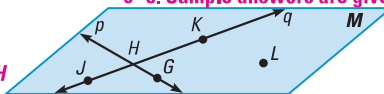
5a. If three points are not collinear, then there exists exactly one plane that contains all three points.

EXAMPLE 2
for Exs. 6–8

5. **CONDITIONAL STATEMENTS** Postulate 8 states that through any three noncollinear points there exists exactly one plane.
- Rewrite Postulate 8 in if-then form.
 - Write the converse, inverse, and contrapositive of Postulate 8. **See margin.**
 - Which statements in part (b) are true? **all of them**

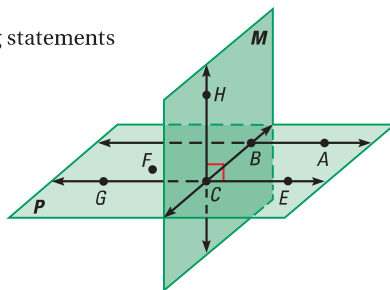
USING A DIAGRAM Use the diagram to write an example of each postulate. **6–8. Sample answers are given.**

6. Postulate 6
line g containing points K and H
7. Postulate 7
lines p and q intersecting in point H
8. Postulate 8
points G, K, L contained in plane M
9. **SKETCHING** Sketch a diagram showing \overleftrightarrow{XY} intersecting \overleftrightarrow{WV} at point T , so $\overleftrightarrow{XY} \perp \overleftrightarrow{WV}$. In your diagram, does \overline{WT} have to be congruent to \overline{TV} ? Explain your reasoning. **See margin for art; no; \overleftrightarrow{XY} does not necessarily bisect \overleftrightarrow{WV} .**



10. ★ **MULTIPLE CHOICE** Which of the following statements cannot be assumed from the diagram? **B**

- Points A, B, C , and E are coplanar.
- Points F, B , and G are collinear.
- $\overline{HC} \perp \overline{GE}$
- \overline{EC} intersects plane M at point C .



- B** **ANALYZING STATEMENTS** Decide whether the statement is true or false. If it is false, give a real-world counterexample.

- Through any three points, there exists exactly one line.
- A point can be in more than one plane. **true**
- Any two planes intersect. **False. Sample answer: Consider any pair of opposite sides of a rectangular prism.**

11. False. Sample answer: Consider a highway with two houses on the right side and one house on the left.

5b. If there exists exactly one plane that contains three points, then the three points are noncollinear; if three points are collinear, then there does not exist exactly one plane that contains all three; if there is not exactly one plane containing three points, then the three points are collinear.

2.4 Use Postulates and Diagrams

91

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–18, 30–41

Average:

Day 1:

Exs. 1, 2, 4, 5, 8–10, 15–23 odd, 24–28, 30–44

Advanced:

Day 1:

Exs. 1, 2, 5, 8–12 even, 19–29*, 32–45*

Block:

Exs. 1, 2, 4, 5, 8–10, 15–23 odd, 24–28, 30–44 (with previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 9, 14, 35

Average: 4, 7, 10, 19, 36

Advanced: 5, 8, 10, 22, 37

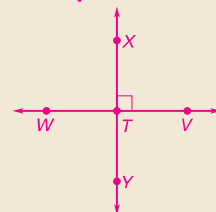
Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

9. Sample:



Avoiding Common Errors

Exercise 10 Some students may say that choice B can be assumed. Point out that while the points do appear to line up, the diagram does not show a line that passes through them.

Teaching Strategy

Exercises 14–23 If students chose “false”, ask them to explain why. You may want to have students approach this group of problems by drawing the intersecting planes first, then individually trying to add/justify each of the statements as they compose the figure.

Mathematical Reasoning

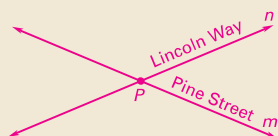
Exercise 27 Have students think about the order in which Postulates 5 and 9 must be applied. Also, ask students to show that a plane must contain at least three lines.

25. **Sample:**



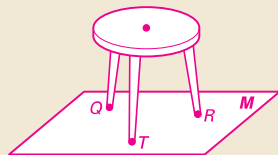
Postulate 5

Sample:



Postulate 7

Sample:



Postulate 8

26. **Sample answer:** A line contains at least two points; three points are sometimes contained in a line.

27. **Sample answer:** Postulate 9 guarantees three noncollinear points in a plane while Postulate 5 guarantees that through any two there exists exactly one line; therefore there exists at least one line in the plane.

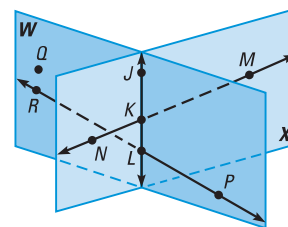
28. **Sample answer:** Postulate 9 guarantees three noncollinear points in the plane, one of them being X. If A and B are the other two then Postulate 5 guarantees \overleftrightarrow{XA} and \overleftrightarrow{XB} exist on plane M.

29. See margin for art; 1 plane. **Sample answer:** Postulate 6 guarantees the existence of two points on line m and Postulate 8 guarantees the existence of one plane containing those two points and point C.

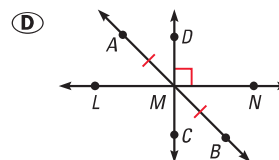
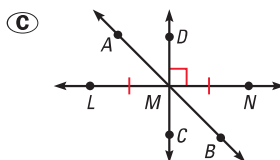
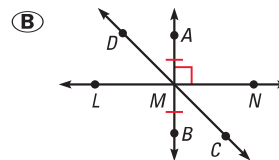
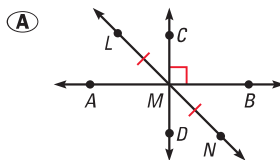
C

USING A DIAGRAM Use the diagram to determine if the statement is true or false.

14. Planes W and X intersect at \overleftrightarrow{KL} . **true**
15. Points Q, J, and M are collinear. **false**
16. Points K, L, M, and R are coplanar. **false**
17. \overleftrightarrow{MN} and \overleftrightarrow{RP} intersect. **false**
18. $\overleftrightarrow{RP} \perp$ plane W **false**
19. \overleftrightarrow{JK} lies in plane X. **true**
20. $\angle PLK$ is a right angle. **false**
21. $\angle NKL$ and $\angle JKM$ are vertical angles. **true**
22. $\angle NKJ$ and $\angle JKM$ are supplementary angles. **true**
23. $\angle JKM$ and $\angle KLP$ are congruent angles. **false**

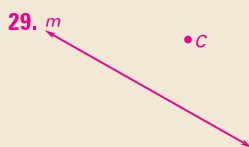


24. **★ MULTIPLE CHOICE** Choose the diagram showing \overleftrightarrow{LN} , \overleftrightarrow{AB} , and \overleftrightarrow{DC} intersecting at point M, \overleftrightarrow{AB} bisecting \overleftrightarrow{LN} , and $\overleftrightarrow{DC} \perp \overleftrightarrow{LN}$. **C**



25. **★ OPEN-ENDED MATH** Sketch a diagram of a real-world object illustrating three of the postulates about points, lines, and planes. List the postulates used. **See margin.**
26. **ERROR ANALYSIS** A student made the false statement shown. Change the statement in two different ways to make it true. **See margin.**
27. **REASONING** Use Postulates 5 and 9 to *explain* why every plane contains at least one line. **See margin.**
28. **REASONING** Point X lies in plane M. Use Postulates 5 and 9 to *explain* why there are at least two lines in plane M that contain point X. **See margin.**
29. **CHALLENGE** Sketch a line m and a point C not on line m. Make a conjecture about how many planes can be drawn so that line m and point C lie in the plane. Use postulates to justify your conjecture.

Three points are always contained in a line. **X**



PROBLEM SOLVING

A REAL-WORLD SITUATIONS Which postulate is suggested by the photo?



Postulate 5



Postulate 7



Postulate 11

33. **Sample answer:** A stoplight with a red, yellow, and green light.

33. ★ **SHORT RESPONSE** Give a real-world example of Postulate 6, which states that a line contains at least two points.

34. **DRAW A DIAGRAM** Sketch two lines that intersect, and another line that does not intersect either one. **See margin.**

35. The line ZU exists through points Z and U .

36. \vec{SZ} and \vec{TU} intersect at point Z .

37. The floor is a plane containing points W , X , and Y .

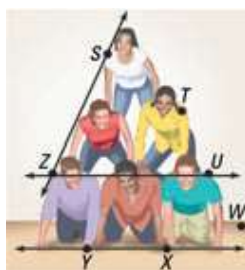
USING A DIAGRAM Use the pyramid to write examples of the postulate indicated.

35. Postulate 5 **35–38. Sample answers are given.**

36. Postulate 7

37. Postulate 9

38. Postulate 10 **Points X and Y lie in the plane that is the floor, so \vec{XY} also lies in the plane of the floor.**



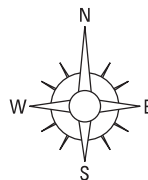
B 39. ★ **EXTENDED RESPONSE** A friend e-mailed you the following statements about a neighborhood. Use the statements to complete parts (a)–(e).

Subject Neighborhood

Building B is due west of Building A.
Buildings A and B are on Street 1.
Building D is due north of Building A.
Buildings A and D are on Street 2.
Building C is southwest of Building A.
Buildings A and C are on Street 3.
Building E is due east of Building B.
 $\angle CAE$ formed by Streets 1 and 3 is obtuse.

39d. No; since $\angle CAE$ is obtuse, Building E must be on the east side of Building A.

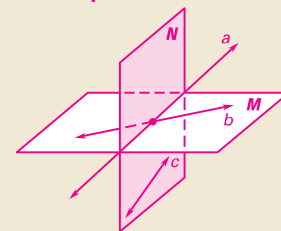
- Draw a diagram of the neighborhood. **See margin.**
- Where do Streets 1 and 2 intersect? **Building A**
- Classify the angle formed by Streets 1 and 2. **right angle**
- Is Building E between Buildings A and B? **Explain.**
- What street is Building E on? **Street 1**



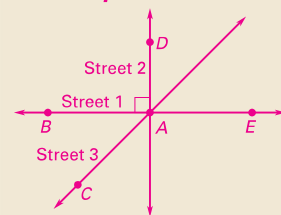
Teaching Strategy

Exercises 40–41 You may want to pair advanced students with other students who are struggling with these exercises, which have multiple steps. This will give the struggling students assistance as well as help the advanced students review what they know.

34. **Sample:**



39a. **Sample:**



MIXED REVIEW of Problem Solving



Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** The table below shows the time of the sunrise on different days in Galveston, Texas.

Date in 2006	Time of sunrise (Central Standard Time)
Jan. 1	7:14 A.M.
Feb. 1	7:08 A.M.
Mar. 1	6:45 A.M.
Apr. 1	6:09 A.M.
May 1	5:37 A.M.
June 1	5:20 A.M.
July 1	5:23 A.M.
Aug. 1	5:40 A.M.

- a. Describe the pattern, if any, in the times shown in the table. **See margin.**
- b. Use the times in the table to make a reasonable prediction about the time of the sunrise on September 1, 2006.
Sample answer: 6:12 A.M.
2. **SHORT RESPONSE** As shown in the table below, hurricanes are categorized by the speed of the wind in the storm. Use the table to determine whether the statement is *true* or *false*. If false, provide a counterexample.

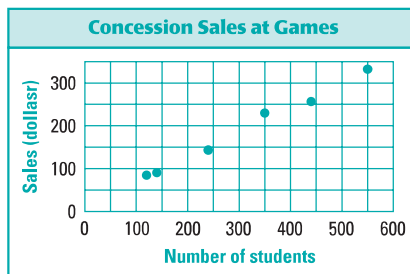
Hurricane category	Wind speed w (mi/h)
1	$74 \leq w \leq 95$
2	$96 \leq w \leq 110$
3	$111 \leq w \leq 130$
4	$131 \leq w \leq 155$
5	$w > 155$

- a. A hurricane is a category 5 hurricane if and only if its wind speed is greater than 155 miles per hour. **true**
- b. A hurricane is a category 3 hurricane if and only if its wind speed is less than 130 miles per hour. **See margin.**

3. **GRIDDED ANSWER** Write the next number in the pattern.

1, 2, 5, 10, 17, 26, ... **37**

4. **EXTENDED RESPONSE** The graph shows concession sales at six high school football games. Tell whether each statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your thinking.



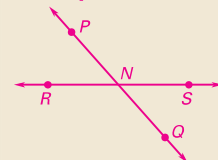
- a. If 500 students attend a football game, the high school can expect concession sales to reach \$300. **Inductive reasoning; conclusion is based on an observation.**
- b. Concession sales were highest at the game attended by 550 students. **Deductive reasoning; it's a fact.**
- c. The average number of students who come to a game is about 300. **Inductive reasoning; conclusion is based on an observation.**
5. **SHORT RESPONSE** Select the phrase that makes the conclusion true. Explain your reasoning.
- a. A person needs a library card to check out books at the public library. You checked out a book at the public library. You (*must have, may have, or do not have*) a library card. **Must have; you can't check out a book unless you have a library card.**
- b. The islands of Hawaii are volcanoes. Bob has never been to the Hawaiian Islands. Bob (*has visited, may have visited, or has never visited*) volcanoes. **See margin.**
6. **SHORT RESPONSE** Sketch a diagram showing \overleftrightarrow{PQ} intersecting \overleftrightarrow{RS} at point N . In your diagram, $\angle PNS$ should be an obtuse angle. Identify two acute angles in your diagram. Explain how you know that these angles are acute. **See margin.**

1a. **Sample answer:** The sun rises earlier each month until July.

2b. **False. Sample answer:** 100 miles per hour is less than 130 miles per hour but this hurricane is a category 2.

5b. **May have visited; just because Bob has not been to the Hawaiian Islands doesn't mean he has not visited a volcano elsewhere.**

6. **Sample:**



$\angle RNP, \angle SNQ$; they each form a linear pair with an obtuse angle.



Construct viable arguments and critique the reasoning of others.

1 PLAN AND PREPARE

Explore the Concept

- Students will use algebra to verify the steps of a number trick.
- This activity leads into the study of solving equations by using algebraic properties in Example 1 in this lesson.

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Students may want to use a calculator for this activity.

Alternative Strategy

Ask students to write their own number trick, describe the pattern, and verify that the number trick works. Then have them switch with a classmate.

Key Discovery

Number tricks work because algebra justifies the steps.

3 ASSESS AND RETEACH

Let x represent the number for a number trick. If you want the answer to be 12 more than the original number, express the answer in terms of x . $x + 12$

1. $x, 2x, 2x + 4, 5(2x + 4), 5(2x + 4) + 12, 10[5(2x + 4) + 12], 10[5(2x + 4) + 12] - 320, [10[5(2x + 4) + 12] - 320] \div 100 = x$

2. x is chosen, $2x$ doubles x , $2x + 4$ is four more than $2x$, $5(2x + 4)$ is



CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Justify a Number Trick

MATERIALS • paper • pencil



Construct viable arguments and critique the reasoning of others.

QUESTION How can you use algebra to justify a number trick?

Number tricks can allow you to guess the result of a series of calculations.

EXPLORE Play the number trick

STEP 1 *Pick a number* Follow the directions below.

- Pick any number between 11 and 98 that does not end in a zero.
- Double the number.
- Add 4 to your answer.
- Multiply your answer by 5.
- Add 12 to your answer.
- Multiply your answer by 10.
- Subtract 320 from your answer.
- Cross out the zeros in your answer.

23
 $23 \cdot 2$
 $46 + 4$
 $50 \cdot 5$
 $250 + 12$
 $262 \cdot 10$
 $2620 - 320$
~~2300~~

STEP 2 *Repeat the trick* Repeat the trick three times using three different numbers. What do you notice? **The result is the same as the number you start with.**

DRAW CONCLUSIONS Use your observations to complete these exercises 1–4. See margin.

- Let x represent the number you chose in the Explore. Write algebraic expressions for each step. Remember to use the Order of Operations.
- Justify* each expression you wrote in Exercise 1.
- Another number trick is as follows:
 Pick any number.
 Multiply your number by 2.
 Add 18 to your answer.
 Divide your answer by 2.
 Subtract your original number from your answer.

What is your answer? Does your answer depend on the number you chose? How can you change the trick so your answer is always 15? *Explain.*

- REASONING** Write your own number trick.

five times the previous number, $5(2x + 4) + 12$ is 12 more than the previous number, $10[5(2x + 4) + 12]$ multiplies the previous number by 10, $10[5(2x + 4) + 12] - 320$ reduces the previous number by 320, crossing out the zeros (dividing by 100) leaves x .

3. 9; no; change the third step to add 30 to your answer, instead of 18, the algebraic expression would then be $\frac{2x + 30}{2} - x$, which simplifies to $x + 15 - x$, which will always be 15.

4. **Sample answer:** Pick any number, triple your number, add 10 to your answer, then subtract twice the original number from your answer.

2.5 Reason Using Properties from Algebra



Before

You used deductive reasoning to form logical arguments.

Now

You will use algebraic properties in logical arguments too.

Why

So you can apply a heart rate formula, as in Example 3.

Key Vocabulary

- equation
- solve an equation



CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT

For Your Notebook

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property If $a = b$, then $a + c = b + c$.

Subtraction Property If $a = b$, then $a - c = b - c$.

Multiplication Property If $a = b$, then $ac = bc$.

Division Property If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Substitution Property If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	Addition Property of Equality
$5x + 5 = 20$	Combine like terms.	Simplify.
$5x = 15$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 3$	Divide each side by 5.	Division Property of Equality

► The value of x is 3.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Solve each equation.

1. $6x + 2 = -3x - 16$ **-2**

2. $-4(x - 8) = -16$ **12**

3. Solve $P = 2\ell + 2w$ for w .

$w = \frac{P - 2\ell}{2}$

4. $\angle S$ is the complement of $\angle R$. If $m\angle R = 38^\circ$, find $m\angle S$. If $\angle T$ is also the complement of $\angle R$, what is true about $m\angle S$ and $m\angle T$? **52° ; $m\angle S = m\angle T$**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you solve an equation?

Tell students they will learn how to answer this question by using algebraic properties to justify the steps in finding the solution.



Standards for Mathematical Content High School

CC.9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

CC.9-12.G.CO.9 Prove theorems about lines and angles. (preparation for)

CC.9-12.G.CO.10 Prove theorems about triangles. (preparation for)

CC.9-12.G.CO.11 Prove theorems about parallelograms. (preparation for)

Motivating the Lesson

The performance of athletes has improved over the years as formulas for optimum heart rate and other components of sports have been developed. Tell students that they will learn how to use algebra to work with formulas in this lesson.

3 TEACH

Extra Example 1

Solve $3x + 8 = -4x - 34$. Write a reason for each step.

Equation (Reason)

$$3x + 8 = -4x - 34 \text{ (Given)}$$

$$3x + 8 + 4x = -4x - 34 + 4x \text{ (Addition Property of Equality)}$$

$$7x + 8 = -34 \text{ (Simplify.)}$$

$$7x + 8 - 8 = -34 - 8$$

(Subtraction Property of Equality)

$$7x = -42 \text{ (Simplify.)}$$

$$x = -6 \text{ (Division Property of Equality)}$$

The value of x is -6 .

Extra Example 2

Solve $60 = -3(8x - 4)$. Write a reason for each step.

Equation (Reason)

$$60 = -3(8x - 4) \text{ (Given)}$$

$$60 = -24x + 12 \text{ (Distributive Property)}$$

$$48 = -24x \text{ (Subtraction Property of Equality)}$$

$$-2 = x \text{ (Division Property of Equality)}$$

The value of x is -2 .



An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

KEY CONCEPT

For Your Notebook

Distributive Property

$a(b + c) = ab + ac$, where a , b , and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44 .	Division Property of Equality

at my.hrw.com

EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a .

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70 .	Division Property of Equality



GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1. $4x + 9 = -3x + 2$

2. $14x + 3(7 - x) = -1$

3. Solve the formula $A = \frac{1}{2}bh$ for b . $b = \frac{2A}{h}$

1, 2. See margin.

Differentiated Instruction

Below Level While discussing **Example 1**, remind students that the goal is to isolate the variable on one side of the equation and to make the coefficient equal to 1. Have students consider whether the steps in the solution could be done in a different order.

See also the *Differentiated Instruction Resources* for more strategies.

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT

For Your Notebook

Reflexive Property of Equality

Real Numbers	For any real number a , $a = a$.
Segment Length	For any segment AB , $AB = AB$.
Angle Measure	For any angle A , $m\angle A = m\angle A$.

Symmetric Property of Equality

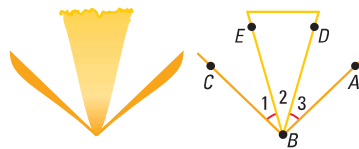
Real Numbers	For any real numbers a and b , if $a = b$, then $b = a$.
Segment Length	For any segments AB and CD , if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles A and B , if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments AB , CD , and EF , if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles A , B , and C , if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.



Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	Transitive Property of Equality

2.5 Reason Using Properties from Algebra

99

Extra Example 3

The cost C of using a certain cell phone can be modeled by the plan rate formula $C = 0.30(m - 300) + 39.99$, where m represents the number of minutes over 300. Solve the formula for m .

Equation (Reason)

$$C = 0.30(m - 300) + 39.99 \text{ (Given)}$$

$$C = 0.30m - 90 + 39.99$$

(Distributive Property)

$$C = 0.3m - 50.01 \text{ (Simplify.)}$$

$$C + 50.01 = 0.3m \text{ (Addition Property of Equality)}$$

$$\frac{C + 50.01}{0.3} = m \text{ (Division Property of Equality)}$$



Key Question Example 3

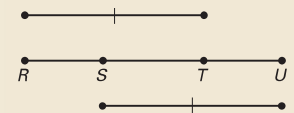
- When would you use the new form of the formula? **when you want to find the age that corresponds to a known heart rate**

Avoiding Common Errors

Students may have difficulty interpreting the properties of equality for geometric quantities. Remind them that the properties of equality can be used for the measures of segments and angles.

Extra Example 4

The city is planning to add two stations between the beginning and end of a commuter train line. Use the information given. Determine whether $RS = TU$.



Equation (Reason)

$$RT = SU \text{ (Given)}$$

$$ST = ST \text{ (Reflexive Property)}$$

$$RT - ST = SU - ST \text{ (Subtraction Property of Equality)}$$

$$RT - ST = RS \text{ (Segment Addition Postulate)}$$

$$SU - ST = TU \text{ (Segment Addition Postulate)}$$

$$RS = TU \text{ (Substitution Property of Equality)}$$

1. Equation (Reason)

$$4x + 9 = -3x + 2 \text{ (Given)}$$

$$7x + 9 = 2 \text{ (Addition Property of Equality)}$$

$$7x = -7 \text{ (Subtraction Property of Equality)}$$

$$x = -1 \text{ (Division Property of Equality)}$$

2. Equation (Reason)

$$14x + 3(7 - x) = -1 \text{ (Given)}$$

$$14x + 21 - 3x = -1 \text{ (Distributive Property)}$$

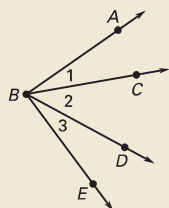
$$11x + 21 = -1 \text{ (Simplify.)}$$

$$11x = -22 \text{ (Subtraction Property of Equality)}$$

$$x = -2 \text{ (Division Property of Equality)}$$

Extra Example 5

In the diagram $m\angle ABD = m\angle CBE$. Show that $m\angle 1 = m\angle 3$.



Equation (Reason)

$$m\angle ABD = m\angle CBE \text{ (Given)}$$

$$m\angle ABD - m\angle 2 = m\angle 1 \text{ (Angle Addition Postulate)}$$

$$m\angle CBE - m\angle 2 = m\angle 3 \text{ (Angle Addition Postulate)}$$

$$m\angle ABD - m\angle 2 = m\angle CBE - m\angle 2 \text{ (Substitution Property of Equality)}$$

$$m\angle 1 = m\angle 3 \text{ (Substitution Property of Equality)}$$

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you solve an equation?

- The algebraic properties of equality can be used to justify each step of solving an equation.
- The algebraic properties of equality and the reflexive, symmetric, and transitive properties of equality can be used to justify steps in solving geometric problems.

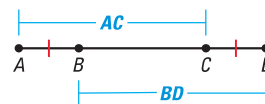
You solve an equation by applying the properties of algebra to justify the reason for each step.

3. Subtraction Property of Equality, Addition Property of Equality, Division Property of Equality

4. Distributive Property, Subtraction Property of Equality, Addition Property of Equality 6–20. See Additional Answers.

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	Given
$AC = AB + BC$	Add lengths of adjacent segments.	Segment Addition Postulate
$BD = BC + CD$	Add lengths of adjacent segments.	Segment Addition Postulate
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	Addition Property of Equality
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	Substitution Property of Equality



GUIDED PRACTICE for Examples 4 and 5

Name the property of equality the statement illustrates.

- If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$. **Symmetric Property of Equality**
- If $JK = KL$ and $KL = 12$, then $JK = 12$. **Transitive Property of Equality**
- $m\angle W = m\angle W$ **Reflexive Property of Equality**

2.5 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS** Exs. 9, 21, and 31
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 27, and 35
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

- VOCABULARY** The following statement is true because of what property? The measure of an angle is equal to itself. **Reflexive Property of Equality for Angle Measure**
 - ★ **WRITING** Explain how to check the answer to Example 3. **Substitute the value of a into the original equation to see if it is a solution.**
- WRITING REASONS** Copy the logical argument. Write a reason for each step.
- | | | | | | |
|----|--------------------|-------|----|----------------------|-------|
| 3. | $3x - 12 = 7x + 8$ | Given | 4. | $5(x - 1) = 4x + 13$ | Given |
| | $-4x - 12 = 8$ | ? | | $5x - 5 = 4x + 13$ | ? |
| | $-4x = 20$ | ? | | $x - 5 = 13$ | ? |
| | $x = -5$ | ? | | $x = 18$ | ? |

EXAMPLES 1 and 2
for Exs. 3–14

26. x should be subtracted from each side, not added.

Equation (Reason)

$$7x = x + 24 \text{ (Given)}$$

$$6x = 24 \text{ (Subtraction Property of Equality)}$$

$$x = 4 \text{ (Division Property of Equality)}$$

Differentiated Instruction

Below Level Students may have difficulty with Example 5 because they find the notation confusing. Suggest that they use specific numbers for the lengths of the segments to help them follow the reasoning in the argument.

See also the *Differentiated Instruction Resources* for more strategies.

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$. **D**
- (A) Substitution (B) Reflexive (C) Symmetric (D) Transitive

WRITING REASONS Solve the equation. Write a reason for each step. 6–14. See margin.

6. $5x - 10 = -40$ 7. $4x + 9 = 16 - 3x$ 8. $5(3x - 20) = -10$
 9. $3(2x + 11) = 9$ 10. $2(-x - 5) = 12$ 11. $44 - 2(3x + 4) = -18x$
 12. $4(5x - 9) = -2(x + 7)$ 13. $2x - 15 - x = 21 + 10x$ 14. $3(7x - 9) - 19x = -15$

★ **ALGEBRA** Solve the equation for y . Write a reason for each step. 15–20. See margin.

15. $5x + y = 18$ 16. $-4x + 2y = 8$ 17. $12 - 3y = 30x$
 18. $3x + 9y = -7$ 19. $2y + 0.5x = 16$ 20. $\frac{1}{2}x - \frac{3}{4}y = -2$

COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD = \underline{\hspace{1cm}}$. **20 + CD**
 22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $\underline{\hspace{1cm}}$. **$m\angle 2 = m\angle 1$**
 23. Addition Property of Equality: If $AB = CD$, then $\underline{\hspace{1cm}} + EF = \underline{\hspace{1cm}} + EF$. **AB, CD**
 24. Distributive Property: If $5(x + 8) = 2$, then $\underline{\hspace{1cm}}x + \underline{\hspace{1cm}} = 2$. **5, 40**
 25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\hspace{1cm}}$. **$m\angle 1 = m\angle 3$**
 26. **ERROR ANALYSIS** Describe and correct the error in solving the equation for x . See margin.

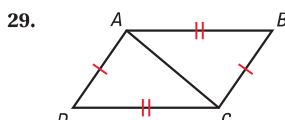
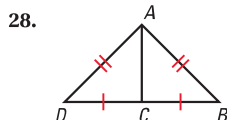
$7x = x + 24$	Given
$6x = 24$	Addition Property of Equality
$x = 3$	Division Property of Equality



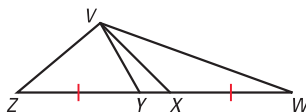
27. **Sample answer:** Look in the mirror and see your reflection; 12 in. = 1 ft, so 1 ft = 12 in.; 10 pennies = 1 dime and 1 dime = 2 nickels, so 10 pennies = 2 nickels.

27. ★ **OPEN-ENDED MATH** Write examples from your everyday life that could help you remember the *Reflexive*, *Symmetric*, and *Transitive* Properties of Equality.

PERIMETER In Exercises 28 and 29, show that the perimeter of triangle ABC is equal to the perimeter of triangle ADC . 28–29. See margin.



30. **CHALLENGE** In the figure at the right, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW . **9, 9**



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1: SRH SR4 Exs. 1–6
Exs. 1–20

Day 2:

Exs. 21–27, 31–35

Average:

Day 1:

Exs. 1–20

Day 2:

Exs. 21–29, 31–36

Advanced:

Day 1:

Exs. 1–20

Day 2:

Exs. 21–30*, 32–38*

Block:

Exs. 1–29, 31–36

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 16, 22, 31, 35

Average: 4, 18, 24, 31, 35

Advanced: 4, 20, 25, 32, 35

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

28. Equation (Reason)

$AD = AB$, $DC = BC$ (Given)

$AC = AC$ (Reflexive Property of Equality)

$AD + DC = AB + DC$ (Addition Property of Equality)

$AD + DC = AB + BC$ (Substitution)

$AD + DC + AC = AB + BC + AC$ (Addition Property of Equality)

29. Equation (Reason)

$AD = CB$, $DC = BA$ (Given)

$AC = AC$ (Reflexive Property of Equality)

$AD + DC = CB + DC$ (Addition Property of Equality)

$AD + DC = CB + BA$ (Substitution)

$AD + DC + AC = CB + BA + AC$ (Addition Property of Equality)

PROBLEM SOLVING

Avoiding Common Errors

Exercises 6–14 Students who are proficient in algebra may have errors in the reasons they give. If this seems due to their combining steps to arrive at the solution quickly, urge them not to combine steps and explain why this is important for these exercises.

Reading Strategy

Exercises 21–25 If students have difficulty getting started with these exercises, suggest that they compare each statement they are to complete with the appropriate algebraic property in this lesson.

Avoiding Common Errors

Exercises 28–29 Some students may fail to include the shared side of triangle ABC and triangle ADC . Help students see that all three sides of each triangle must be included.

31. Equation (Reason)

$$P = 2\ell + 2w \text{ (Given)}$$

$$P - 2w = 2\ell \text{ (Subtraction Property of Equality)}$$

$$\frac{P - 2w}{2} = \ell \text{ (Division Property of Equality)}$$

$$16.5 \text{ m}$$

32. Equation (Reason)

$$A = \frac{1}{2}bh \text{ (Given)}$$

$$2A = bh \text{ (Multiplication Property of Equality)}$$

$$\frac{2A}{b} = h \text{ (Division Property of Equality)}$$

$$68 \text{ in.}$$

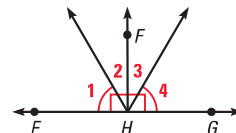
33. Row 1: Marked in diagram;
Row 2: Substitute $m\angle GHF$ for 90° ;
Row 3: Angle Addition Postulate;
Row 4: Substitution Property of Equality;
Row 5: $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$; Substitution Property of Equality;
Row 6: Subtract $m\angle 1$ from both sides.

EXAMPLE 3 A
for Exs. 31–32

31. PERIMETER The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$ where ℓ is the length and w is the width. Solve the formula for ℓ and write a reason for each step. Then find the length of a rectangular lawn whose perimeter is 55 meters and whose width is 11 meters. **See margin.**

32. AREA The formula for the area A of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. Solve the formula for h and write a reason for each step. Then find the height of a triangle whose area is 1768 square inches and whose base is 52 inches. **See margin.**

33. PROPERTIES OF EQUALITY Copy and complete the table to show $m\angle 2 = m\angle 3$. **See margin.**



Equation	Explanation	Reason
$m\angle 1 = m\angle 4, m\angle EHF = 90^\circ, m\angle GHF = 90^\circ$?	Given
$m\angle EHF = m\angle GHF$?	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	?
?	Substitute $m\angle 1$ for $m\angle 4$.	?
$m\angle 2 = m\angle 3$?	Subtraction Property of Equality

34. MULTI-STEP PROBLEM Points A, B, C , and D represent stops, in order, along a subway route. The distance between Stops A and C is the same as the distance between Stops B and D . **a–c. See margin.**

- Draw a diagram to represent the situation.
- Use the Segment Addition Postulate to show that the distance between Stops A and B is the same as the distance between Stops C and D .
- Justify part (b) using the Properties of Equality.

EXAMPLE 4
for Ex. 35

35. ★ SHORT RESPONSE A flashlight beam is reflected off a mirror lying flat on the ground. Use the information given below to find $m\angle 2$. **116°**

$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

$$m\angle 1 + m\angle 2 = 148^\circ$$

$$m\angle 1 = m\angle 3$$


34a. Sample:



34b. $AC = BD$

$$AB + BC = AC, BC + CD = BD$$

$$AB + BC = BC + CD$$

$$AB = CD$$

34c. Equation (Reason)

$$AC = BD \text{ (Given)}$$

$$AB + BC = AC, BC + CD = BD \text{ (Segment Addition Postulate)}$$

$$AB + BC = BC + CD \text{ (Substitution)}$$

$$AB = CD \text{ (Subtraction Property of Equality)}$$

- B** 36. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$) is $C = \frac{5}{9}(F - 32)$. **a–c. See margin.**

- Writing an Equation** Solve the formula for F . Write a reason for each step.
- Making a Table** Make a table that shows the conversion to Fahrenheit for each temperature: 0°C , 20°C , 32°C , and 41°C .
- Drawing a Graph** Use your table to graph the temperature in degrees Fahrenheit ($^{\circ}\text{F}$) as a function of the temperature in degrees Celsius ($^{\circ}\text{C}$). Is this a linear function?

- C** **CHALLENGE** In Exercises 37 and 38, decide whether the relationship is *reflexive*, *symmetric*, or *transitive*.

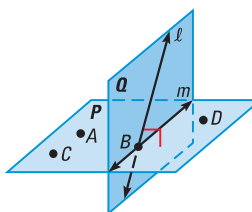
37. **Group:** two employees in a grocery store
Relationship: “worked the same hours as”
Example: Yen worked the same hours as Jim. **symmetric**

38. **Group:** negative numbers on a number line
Relationship: “is less than”
Example: -4 is less than -1 . **transitive**

QUIZ

Use the diagram to determine if the statement is *true* or *false*.

- Points B , C , and D are coplanar. **true**
- Point A is on line ℓ . **false**
- Plane P and plane Q are perpendicular. **true**



Solve the equation. Write a reason for each step. **4, 5. See margin.**

- $x + 20 = 35$
- $5x - 14 = 16 + 3x$

Use the property to copy and complete the statement.

- Subtraction Property of Equality: If $AB = CD$, then $\underline{\quad} - EF = \underline{\quad} - EF$. **AB, CD**
- Transitive Property of Equality: If $a = b$ and $b = c$, then $\underline{\quad} = \underline{\quad}$. **a, c**

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

103

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Solve. Give a reason for each step.

1. $-5x + 18 = 3x - 38$

Equation (Reason)

$-5x + 18 = 3x - 38$ (Given)

$-8x + 18 = -38$ (Subtr. Prop. of Eq.)

$-8x = -56$ (Subtr. Prop. of Eq.)

$x = 7$ (Division Prop. of Eq.)

2. $-3(x - 5) = 2(x + 10)$

Equation (Reason)

$-3(x - 5) = 2(x + 10)$ (Given)

$-3x + 15 = 2x + 20$

(Distributive Prop.)

$15 = 5x + 20$ (Addition Prop. of Eq.)

$-5 = 5x$ (Subtr. Prop. of Eq.)

$-1 = x$ (Division Prop. of Eq.)

Identify the property of equality.

3. If $m\angle 3 = m\angle 5$ and $m\angle 5 = m\angle 8$, then $m\angle 3 = m\angle 8$

Transitive Prop. of Eq.

4. If $CD = EF$, then $EF = CD$.

Symmetric Prop. of Eq.

Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

36a–c, Quiz 4, 5. See Additional Answers.

2.6 Prove Statements about Segments and Angles



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Use a property of equality to complete the statement.

- If $m\angle 1 = m\angle 3$, then $m\angle 3 = \underline{\quad}$.
 $m\angle 1$
- If $AB = CD$ and $CD = TU$, then $\underline{\quad}$. $AB = TU$
- If $RS = WX$, then $\underline{\quad} + AB = \underline{\quad} + AB$. $RS; WX$
- If $m\angle EFG = 28^\circ$ and $m\angle GFH = 62^\circ$, then $\underline{\quad} + 62^\circ = m\angle EFG + m\angle GFH$. 28°

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How do you write a geometric proof? **Tell students they will learn how to answer this question using geometric theorems and algebraic properties to do proofs.**

Before

You used deductive reasoning.

Now

You will write proofs using geometric theorems.

Why?

So you can prove angles are congruent, as in Ex. 21.

Key Vocabulary

- proof
- two-column proof
- theorem



CC.9-12.G.CO.9 Prove theorems about lines and angles.

A **proof** is a logical argument that shows a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

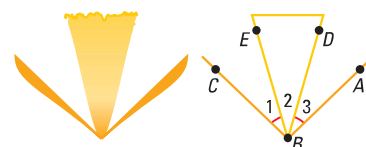
In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

EXAMPLE 1 Write a two-column proof

Write a two-column proof for the situation in Example 4 from the previous lesson.

GIVEN $\triangleright m\angle 1 = m\angle 3$

PROVE $\triangleright m\angle EBA = m\angle DBC$



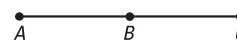
STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle EBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate
3. $m\angle EBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle DBC$	4. Angle Addition Postulate
5. $m\angle EBA = m\angle DBC$	5. Transitive Property of Equality

GUIDED PRACTICE for Example 1

- Four steps of a proof are shown. Give the reasons for the last two steps.

GIVEN $\triangleright AC = AB + AB$

PROVE $\triangleright AB = BC$



STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. Segment Addition Postulate
3. $AB + AB = AB + BC$	3. $\underline{\quad}$ Transitive Property of Equality
4. $AB = BC$	4. $\underline{\quad}$ Subtraction Property of Equality

THEOREMS The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

TAKE NOTES

Be sure to copy all new theorems in your notebook.

THEOREMS

For Your Notebook

THEOREM 2.1 Congruence of Segments

Segment congruence is reflexive, symmetric, and transitive.

- Reflexive** For any segment AB , $\overline{AB} \cong \overline{AB}$.
Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

THEOREM 2.2 Congruence of Angles

Angle congruence is reflexive, symmetric, and transitive.

- Reflexive** For any angle A , $\angle A \cong \angle A$.
Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

EXAMPLE 2 Name the property shown

Name the property illustrated by the statement.

- If $\angle R \cong \angle T$ and $\angle T \cong \angle P$, then $\angle R \cong \angle P$.
- If $\overline{NK} \cong \overline{BD}$, then $\overline{BD} \cong \overline{NK}$.

Solution

- Transitive Property of Angle Congruence
- Symmetric Property of Segment Congruence



GUIDED PRACTICE for Example 2

Name the property illustrated by the statement.

- $\overline{CD} \cong \overline{CD}$ **Reflexive Property of Congruence**
- If $\angle Q \cong \angle V$, then $\angle V \cong \angle Q$. **Symmetric Property of Congruence**

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs so that you will be prepared to write more complicated proofs in later chapters.

Motivating the Lesson

Much of science was developed because scientists had a “theory” about how things work, and they tried to prove the theory. Have students give examples from science that they know.

3 TEACH

Extra Example 1

Write a two-column proof for Extra Example 4 in this lesson.

Given: $RT = SU$

Prove: $RS = TU$

Statements (Reasons)

- $RT = SU$ (Given)
- $ST = ST$ (Reflexive Prop.)
- $RT - ST = SU - ST$ (Subtraction Prop. of Eq.)
- $RT - ST = RS$ (Segment Addition Post.)
- $SU - ST = TU$ (Segment Addition Post.)
- $RS = TU$ (Substitution Prop. of Eq.)

Extra Example 2

Name the property illustrated by each statement.

- If $\angle RST \cong \angle MNP$, then $\angle MNP \cong \angle RST$. **Symmetric Prop. of Angle Congruence**
- If $\overline{AB} \cong \overline{FG}$ and $\overline{FG} \cong \overline{MN}$, then $\overline{AB} \cong \overline{MN}$. **Transitive Prop. of Segment Congruence**



Key Question Example 2

- Why is the reason for part a the Transitive Property of Angle Congruence and not the Transitive Property of Equality? **Congruence is a relation between geometric figures rather than numbers.**

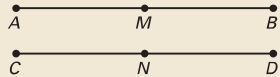
Differentiated Instruction

Below Level In the discussion of **Example 2**, remind students of the Reflexive, Symmetric, and Transitive Properties of Equality from algebra. Point out similarities to the corresponding properties of congruence for segments and for angles.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

If you know that M is the midpoint of \overline{AB} , N is the midpoint of \overline{CD} , and $AB = CD$, prove that $AM = CN$.



Given: M is the midpoint of \overline{AB} ;

N is the midpoint of \overline{CD} ; $AB = CD$.

Prove: $AM = CN$

Statements (Reasons)

1. M is the midpoint of \overline{AB} ; N is the midpoint of \overline{CD} ; $AB = CD$. (Given)
2. $\overline{AM} \cong \overline{MB}$; $\overline{CN} \cong \overline{ND}$ (Definition of midpoint)
3. $AM = MB$; $CN = ND$ (Definition of congruent segments)
4. $AM + MB = AB$; $CN + ND = CD$ (Segment Addition Post.)
5. $AM + AM = AB$; $CN + CN = CD$ (Substitution Prop. of Eq.)
6. $2AM = AB$; $2CN = CD$ (Distributive Prop.)
7. $2AM = 2CN$ (Substitution Prop. of Eq.)
8. $AM = CN$ (Division Prop. of Eq.)



Key Question Example 3

- Why can't you go from step 1 to step 3 in the proof? **The definition of midpoint is given in terms of congruence, not equality.**

Teaching Strategy

Have students analyze the Concept Summary Box carefully. If possible, photocopy the box onto a transparency, then cut out each section of the summary. Mix up the sections, then rearrange them correctly as a demonstration as you discuss them.

EXAMPLE 3 Use properties of equality

Prove this property of midpoints: If you know that M is the midpoint of \overline{AB} , prove that AB is two times AM and AM is one half of AB .



GIVEN \blacktriangleright M is the midpoint of \overline{AB} .

PROVE \blacktriangleright a. $AB = 2 \cdot AM$

b. $AM = \frac{1}{2} AB$

WRITE PROOFS

Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the GIVEN and PROVE statements.

STATEMENTS

1. M is the midpoint of \overline{AB} .
2. $\overline{AM} \cong \overline{MB}$
3. $AM = MB$
4. $AM + MB = AB$
5. $AM + AM = AB$
- a. 6. $2AM = AB$
- b. 7. $AM = \frac{1}{2} AB$

REASONS

1. Given
2. Definition of midpoint
3. Definition of congruent segments
4. Segment Addition Postulate
5. Substitution Property of Equality
6. Distributive Property
7. Division Property of Equality



GUIDED PRACTICE for Example 3

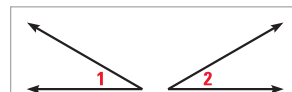
4. **WHAT IF?** Look back at Example 3. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2} AB$ instead? **In steps 5, 6, and 7, AM would be replaced by MB .**

CONCEPT SUMMARY

For Your Notebook

Writing a Two-Column Proof

In a proof, you make one statement at a time, until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs.

Proof of the Symmetric Property of Angle Congruence

GIVEN $\blacktriangleright \angle 1 \cong \angle 2$

PROVE $\blacktriangleright \angle 2 \cong \angle 1$

STATEMENTS

1. $\angle 1 \cong \angle 2$
2. $m\angle 1 = m\angle 2$
3. $m\angle 2 = m\angle 1$
4. $\angle 2 \cong \angle 1$

Statements based on facts that you know or on conclusions from deductive reasoning

The number of statements will vary.

REASONS

1. **Given**
2. Definition of congruent angles
3. Symmetric Property of Equality
4. Definition of congruent angles

Remember to give a reason for the last statement.

Definitions, postulates, or proven theorems that allow you to state the corresponding statement

Differentiated Instruction

Inclusion For many students, proofs are the most difficult part of geometry. Suggest that students write all the postulates and theorems on index cards as well as in a notebook. When they are given a problem, they can try to arrange some of the index cards as a method of completing the proof.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 4 Solve a multi-step problem

SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.



Solution

STEP 1 Draw and label a diagram.



STEP 2 Draw separate diagrams to show mathematical relationships.



STEP 3 State what is given and what is to be proved for the situation. Then write a proof.

GIVEN ▶ B is the midpoint of \overline{AC} .
 C is the midpoint of \overline{BD} .

PROVE ▶ $AB = CD$

STATEMENTS

1. B is the midpoint of \overline{AC} .
 C is the midpoint of \overline{BD} .
2. $\overline{AB} \cong \overline{BC}$
3. $\overline{BC} \cong \overline{CD}$
4. $\overline{AB} \cong \overline{CD}$
5. $AB = CD$

REASONS

1. Given
2. Definition of midpoint
3. Definition of midpoint
4. Transitive Property of Congruence
5. Definition of congruent segments



GUIDED PRACTICE for Example 4

5. In Example 4, does it matter what the actual distances are in order to prove the relationship between AB and CD ? *Explain.*
No; the critical factor is the midpoint.
6. In Example 4, there is a clothing store halfway between the music store and the shoe store. What other two store entrances are the same distance from the entrance of the clothing store? **food court, bookstore**

Extra Example 4

The distance from the park to the pool is the same as the distance from your house to the school. The school is between the pool and the house. Prove that the distance from the park to the school is the same as the distance from the pool to your house.

Step 1:

Step 2:

Given: $AB = CD$

Prove: $AC = BD$

Statements (Reasons)

1. $AB = CD$ (Given)
2. $BC = BC$ (Reflexive Prop. of Eq.)
3. $AB + BC = BC + CD$ (Addition Prop. of Eq.)
4. $AB + BC = AC$; $BC + CD = BD$ (Segment Addition Post.)
5. $AC = BD$ (Substitution Prop. of Eq.)

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you write a geometric proof?

- A two-column proof has numbered statements and corresponding reasons in a logical order.
- A theorem is a statement that can be proved. Proved theorems can be used as reasons in proofs.

You write a geometric proof by organizing the steps as described below.

- Draw and label a diagram.
- Show mathematical relationships in the diagram.
- State what is given and what is to be proved for the situation.
- Write the proof using given information, definitions, theorems, and postulates.

2.6 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 7, 15, and 21
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 4, 12, 19, 27, and 28

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:
Exs. 1–12
Day 2:
Exs. 13–17, 21–28

Average:

Day 1:
Exs. 1–12
Day 2:
Exs. 13–19, 21–29

Advanced:

Day 1:
Exs. 1–12
Day 2:
Exs. 15–20*, 22–30*

Block:

Exs. 1–19, 21–29

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 14, 22, 23

Average: 3, 6, 14, 22, 24

Advanced: 4, 10, 15, 22, 25

Extra Practice

- Student Edition
- Chapter Resource Book: Practice Levels A, B, C

Practice Worksheet

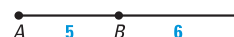
An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

- A** 1. **VOCABULARY** What is a *theorem*? How is it different from a *postulate*? **A theorem is a statement that can be proven; a postulate is a rule that is accepted without proof.**
2. ★ **WRITING** You can use theorems as reasons in a two-column proof. What other types of statements can you use as reasons in a two-column proof? Give examples. **Sample answer: Definitions, properties, postulates; Definition of a Right Angle, Transitive Property, Angle Addition Postulate**
3. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN ▶ $AB = 5$, $BC = 6$

PROVE ▶ $AC = 11$



STATEMENTS	REASONS
1. $AB = 5$, $BC = 6$	1. Given
2. $AC = AB + BC$	2. Segment Addition Postulate
3. $AC = 5 + 6$	3. ? Substitution Property of Equality
4. ? $AC = 11$	4. Simplify.

4. ★ **MULTIPLE CHOICE** Which property listed is the reason for the last step in the proof? **A**

GIVEN ▶ $m\angle 1 = 59^\circ$, $m\angle 2 = 59^\circ$

PROVE ▶ $m\angle 1 = m\angle 2$

STATEMENTS	REASONS
1. $m\angle 1 = 59^\circ$, $m\angle 2 = 59^\circ$	1. Given
2. $59^\circ = m\angle 2$	2. Symmetric Property of Equality
3. $m\angle 1 = m\angle 2$	3. ?
(A) Transitive Property of Equality	(B) Reflexive Property of Equality
(C) Symmetric Property of Equality	(D) Distributive Property

USING PROPERTIES Use the property to copy and complete the statement.

5. Reflexive Property of Congruence: $\underline{\quad} \cong \overline{SE}$ **\overline{SE}**
6. Symmetric Property of Congruence: If $\underline{\quad} \cong \underline{\quad}$, then $\angle RST \cong \angle JKL$. **$\angle JKL$, $\angle RST$**
7. Transitive Property of Congruence: If $\angle F \cong \angle J$ and $\underline{\quad} \cong \underline{\quad}$, then $\angle F \cong \angle L$. **$\angle J$, $\angle L$**

NAMING PROPERTIES Name the property illustrated by the statement.

8. If $\overline{DG} \cong \overline{CT}$, then $\overline{CT} \cong \overline{DG}$. **9. $\angle VWX \cong \angle VWX$**
10. If $\overline{JK} \cong \overline{MN}$ and $\overline{MN} \cong \overline{XY}$, then $\overline{JK} \cong \overline{XY}$. **11. $YZ = ZY$ Reflexive Property of Equality**
12. ★ **MULTIPLE CHOICE** Name the property illustrated by the statement "If $\overline{CD} \cong \overline{MN}$, then $\overline{MN} \cong \overline{CD}$." **C**
- (A) Reflexive Property of Equality (B) Symmetric Property of Equality
- (C) Symmetric Property of Congruence (D) Transitive Property of Congruence

EXAMPLE 1
for Exs. 3–4

EXAMPLES 2 and 3
for Exs. 5–13

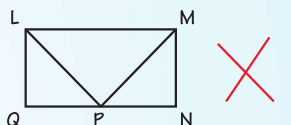
8. Symmetric Property of Congruence

9. Reflexive Property of Congruence

17. Equation	Explanation	Reason
$\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$	Write original statement.	Given
$\overline{QR} \cong \overline{RS}$	Both segments are congruent to \overline{PQ} .	Transitive Property of Congruent Segments
$2x + 5 = 10 - 3x$	Write an equation using the equal segment lengths.	Definition of congruent segments
$5x + 5 = 10$	Add $3x$ to each side.	Addition Property of Equality
$5x = 5$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 1$	Divide each side by 5.	Division Property of Equality

13. **ERROR ANALYSIS** In the diagram below, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. Describe and correct the error in the reasoning. **The reason is the Transitive Property of Congruence, not the Reflexive Property of Congruence.**

Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$,
then $\overline{MN} \cong \overline{PN}$ by the Reflexive
Property of Segment Congruence.



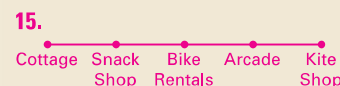
EXAMPLE 4
for Exs. 14–15

MAKING A SKETCH In Exercises 14 and 15, sketch a diagram that represents the given information. **14, 15. See margin.**

14. **CRYSTALS** The shape of a crystal can be represented by intersecting lines and planes. Suppose a crystal is *cubic*, which means it can be represented by six planes that intersect at right angles.



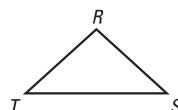
15. **BEACH VACATION** You are on vacation at the beach. Along the boardwalk, the bike rentals are halfway between your cottage and the kite shop. The snack shop is halfway between your cottage and the bike rentals. The arcade is halfway between the bike rentals and the kite shop.



16. **DEVELOPING PROOF** Copy and complete the proof.

GIVEN $\triangleright RT = 5, RS = 5, \overline{RT} \cong \overline{TS}$

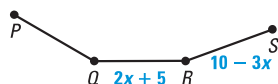
PROVE $\triangleright \overline{RS} \cong \overline{TS}$



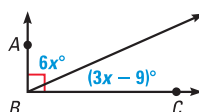
STATEMENTS	REASONS
1. $RT = 5, RS = 5, \overline{RT} \cong \overline{TS}$	1. ? Given
2. $RS = RT$	2. Transitive Property of Equality
3. $RT = TS$	3. Definition of congruent segments
4. $RS = TS$	4. Transitive Property of Equality
5. $\overline{RS} \cong \overline{TS}$	5. ? Definition of congruent segments

- xy ALGEBRA** Solve for x using the given information. **Explain your steps.** **17, 18. See margin.**

17. **GIVEN** $\triangleright \overline{QR} \cong \overline{PQ}, \overline{RS} \cong \overline{PQ}$



18. **GIVEN** $\triangleright m\angle ABC = 90^\circ$



19. **★ SHORT RESPONSE** Explain why writing a proof is an example of deductive reasoning, not inductive reasoning. **A proof is deductive reasoning because it uses facts, definitions, accepted properties, and laws of logic.**

20. **CHALLENGE** Point P is the midpoint of \overline{MN} and point Q is the midpoint of \overline{MP} . Suppose \overline{AB} is congruent to \overline{MP} , and \overline{PN} has length x . Write the length of the segments in terms of x . **Explain.**

- a. \overline{AB} b. \overline{MN} c. \overline{MQ} d. \overline{NQ}

20a. x . **Sample answer:** $AB = MP$, and $MP = PN$, so $AB = PN$, by the Transitive Property of Equality. $PN = x$, therefore $AB = x$.

20b. $2x$. **Sample answer:** $MP = PN = x$, $MP + PN = MN$, therefore $MN = 2x$.

20c. $\frac{x}{2}$. **Sample answer:** $MQ = QP$, $MQ + QP = x$, $MQ + MQ = x$, $2MQ = x$, therefore $MQ = \frac{x}{2}$.

20d. $\frac{3x}{2}$. **Sample answer:** $NP = x$, $PQ = \frac{x}{2}$, $NP + PQ = NQ$, $x + \frac{x}{2} = NQ$, therefore $\frac{3x}{2} = NQ$.

Avoiding Common Errors

Exercises 8–12 Naming the property illustrated by the statement may be difficult for some students because they confuse the properties. Encourage them to use note cards with each property listed, then check the note cards as they do these exercises. They can also add these exercises as examples to their note cards.

14.

15.

17. See previous page.

18. See below.

18. Equation	Explanation	Reason
$m\angle ABC = 90^\circ$	Write original statement.	Given
$m\angle ABC = 6x^\circ + (3x - 9)^\circ$	Marked in diagram.	Angle Addition Postulate
$6x^\circ + (3x - 9)^\circ = 90^\circ$	Replace $m\angle ABC$ with $6x^\circ + (3x - 9)^\circ$.	Substitution
$9x - 9 = 90$	Add $6x$ to $3x$.	Combine like terms.
$9x = 99$	Add 9 to each side.	Addition Property of Equality
$x = 11$	Divide each side by 9.	Division Property of Equality

PROBLEM SOLVING

Teaching Strategy

Exercises 22–26 Have students work with a partner to write the proofs and verify the correctness of each reason. Encourage students to think of alternate ways to prove the statements.

Mathematical Reasoning

Exercise 23 Students may want to include the definition of midpoint to do this proof. Remind them that a midpoint divides a segment into two congruent segments.



An **Animated Geometry** activity is available online for **Exercise 29**. This activity is also part of **Power Presentations**.

23. Statements (Reasons)

- $2AB = AC$ (Given)
- $AC = AB + BC$ (Segment Addition Postulate)
- $2AB = AB + BC$ (Transitive Property of Segment Equality)
- $AB = BC$ (Subtraction Property of Equality)

24. Statements (Reasons)

- $m\angle 1 + m\angle 2 = 180^\circ$,
 $m\angle 1 = 62^\circ$ (Given)
- $62^\circ + m\angle 2 = 180^\circ$
(Substitution)
- $m\angle 2 = 118^\circ$ (Subtraction Property of Equality)

25. Statements (Reasons)

- A is an angle. (Given)
- $m\angle A = m\angle A$ (Reflexive Property of Equality)
- $\angle A \cong \angle A$ (Definition of congruent angles)

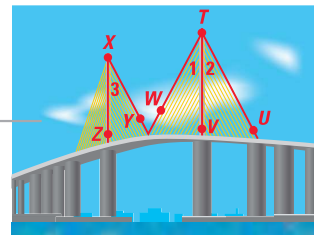
26. Statements (Reasons)

- $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$ (Given)
- $WX = XY$ and $XY = YZ$ (Definition of congruent segments)
- $WX = YZ$ (Transitive Property of Equality)
- $\overline{WX} \cong \overline{YZ}$ (Definition of congruent segments)

21. Definition of angle bisector; Transitive Property of Congruence

- 21. BRIDGE** In the bridge in the illustration, it is known that $\angle 2 \cong \angle 3$ and \overline{TV} bisects $\angle UTW$. Copy and complete the proof to show that $\angle 1 \cong \angle 3$.

STATEMENTS	REASONS
1. \overline{TV} bisects $\angle UTW$.	1. Given
2. $\angle 1 \cong \angle 2$	2. ?
3. $\angle 2 \cong \angle 3$	3. Given
4. $\angle 1 \cong \angle 3$	4. ?



EXAMPLE 3...
for Ex. 22

- 22. DEVELOPING PROOF** Write a complete proof by matching each statement with its corresponding reason.

GIVEN \overline{QS} is an angle bisector of $\angle PQR$.

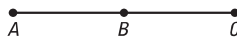
PROVE $m\angle PQS = \frac{1}{2} m\angle PQR$

STATEMENTS	REASONS
1. \overline{QS} is an angle bisector of $\angle PQR$. D	A. Definition of angle bisector
2. $\angle PQS \cong \angle SQR$ A	B. Distributive Property
3. $m\angle PQS = m\angle SQR$ F	C. Angle Addition Postulate
4. $m\angle PQS + m\angle SQR = m\angle PQR$ C	D. Given
5. $m\angle PQS + m\angle PQS = m\angle PQR$ G	E. Division Property of Equality
6. $2 \cdot m\angle PQS = m\angle PQR$ B	F. Definition of congruent angles
7. $m\angle PQS = \frac{1}{2} m\angle PQR$ E	G. Substitution Property of Equality

PROOF Use the given information and the diagram to prove the statement.

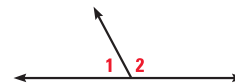
- 23. GIVEN** $2AB = AC$

PROVE $AB = BC$



- 23, 24. See margin.**
24. GIVEN $m\angle 1 + m\angle 2 = 180^\circ$
 $m\angle 1 = 62^\circ$

PROVE $m\angle 2 = 118^\circ$

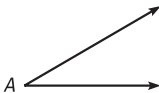


PROVING PROPERTIES Prove the indicated property of congruence. **25, 26. See margin.**

- 25. Reflexive Property of Angle Congruence**

GIVEN A is an angle.

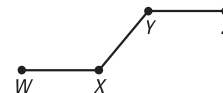
PROVE $\angle A \cong \angle A$



- 26. Transitive Property of Segment Congruence**

GIVEN $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$

PROVE $\overline{WX} \cong \overline{YZ}$



110

= See **WORKED-OUT SOLUTIONS** in Student Resources

= **STANDARDIZED TEST PRACTICE**



29c. Statements (Reasons)

- $RS = CF$, $SM = MC = FD$ (Given)
- $RS + SM = RM$ (Segment Addition Postulate)
- $CF + FD = CD$ (Segment Addition Postulate)
- $CF + FD = RM$ (Substitution)
- $RM = CD$ (Transitive Property of Equality)

27. **Equiangular; Transitive Property of Congruent Angles implies $\angle 1 \cong \angle 3$, so all angle measures are the same.**

EXAMPLE 4
for Ex. 29

28. **Sample answer:** The length of each segment is the same, therefore the segments are congruent.

29b. **Given:**
 $RS = CF$,
 $SM = MC = FD$;
Prove: $RM = CD$

27. **★ SHORT RESPONSE** In the sculpture shown, $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. Classify the triangle and *justify* your reasoning.

28. **★ SHORT RESPONSE** You use a computer drawing program to create a line segment. You copy the segment and paste it. You copy the pasted segment and then paste it, and so on. How do you know all the line segments are congruent?
See margin.

29. **MULTI-STEP PROBLEM** The distance from the restaurant to the shoe store is the same as the distance from the cafe to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the cafe, and from the florist to the dry cleaners.



Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the cafe to the dry cleaners.

- Draw and label a diagram to show the mathematical relationships. **See margin.**
- State what is given and what is to be proved for the situation.
- Write a two-column proof. **See margin.**

Animated Geometry at my.hrw.com

- C** 30. **CHALLENGE** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville is. Moon Valley is 50 miles farther from Springfield than Lakewood City is.
- Assume all five cities lie in a straight line. Draw a diagram that represents this situation. **See margin.**
 - Suppose you do not know that all five cities lie in a straight line. Draw a diagram that is different from the one in part (a) to represent the situation. **See margin.**
 - Explain the differences in the two diagrams. **Sample answer:** In the second diagram Bettsville and Lakewood City are not in opposite directions from Springfield.



5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

1. Copy and complete the proof.

$M \quad A \quad T \quad H$

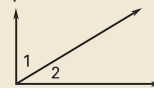
Given: $MA = TH$

Prove: $MT = AH$

Statements (Reasons)

- $MA = TH$ (?) **Given**
- ? (Reflexive Prop. of Eq.)
 $AT = AT$
- $MA + AT = AT + TH$ (?)
Addition Prop. of Eq.
- $MA + AT = MT$; $AT + TH = AH$ (?) **Segment Add. Post.**
- ? (Substitution Prop. of Eq.)
 $MT = AH$

2. Use the given information to prove the statement.



Given: $m\angle 1 + m\angle 2 = 90^\circ$;
 $m\angle 1 = 59^\circ$

Prove: $m\angle 2 = 31^\circ$

Statements (Reasons)

- $m\angle 1 + m\angle 2 = 90^\circ$;
 $m\angle 1 = 59^\circ$ (Given)
- $m\angle 2 = 90^\circ - m\angle 1$
(Subtraction Prop. of Eq.)
- $m\angle 2 = 90^\circ - 59^\circ$
(Substitution Prop. of Eq.)
- $m\angle 2 = 31^\circ$ (Simplify.)

Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

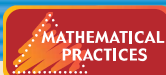
- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

30a–b. **See Additional Answers.**

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com



Make sense of problems and persevere in solving them.

Alternative Strategy

Example 4 in this lesson was solved by writing a two-column proof. The method shown on this page allows students to visualize a plan for the proof using a visual organizer or diagram, then write a proof using the verbal statements from the organizer.

Teaching Strategy

Emphasize that the deductions flow from the given information as sentences, and then the sentences are translated into mathematical language. The proofs may not appear the same as previous two-column proofs.

Mathematical Reasoning

Multiple Representations

Encourage students to use the method in this workshop to help them organize and write their proofs. They could also write statements and reasons on separate slips of paper and organize them like a flowchart with arrows connecting them.

PROBLEM SOLVING WORKSHOP

LESSON 2.6



Make sense of problems and persevere in solving them.

PROBLEM

Using ALTERNATIVE METHODS

Another Way to Solve Example 4

MULTIPLE REPRESENTATIONS The first step in writing any proof is to make a plan. A diagram or *visual organizer* can help you plan your proof. The steps of a proof must be in a logical order, but there may be more than one correct order.

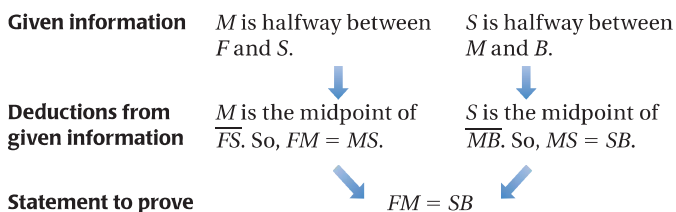
SHOPPING MALL Walking down a hallway at the mall, you notice the music store is halfway between the food court and the shoe store. The shoe store is halfway between the music store and the bookstore. Prove that the distance between the entrances of the food court and music store is the same as the distance between the entrances of the shoe store and bookstore.

METHOD

Using a Visual Organizer

STEP 1 Use a visual organizer to map out your proof.

The music store is halfway between the food court and the shoe store.
The shoe store is halfway between the music store and the bookstore.



STEP 2 Write a proof using the lengths of the segments.

GIVEN ► M is halfway between F and S .
 S is halfway between M and B .

PROVE ► $FM = SB$

STATEMENTS	REASONS
1. M is halfway between F and S .	1. Given
2. S is halfway between M and B .	2. Given
3. M is the midpoint of \overline{FS} .	3. Definition of midpoint
4. S is the midpoint of \overline{MB} .	4. Definition of midpoint
5. $FM = MS$ and $MS = SB$	5. Definition of midpoint
6. $MS = MS$	6. Reflexive Property of Equality
7. $FM = SB$	7. Substitution Property of Equality

PRACTICE

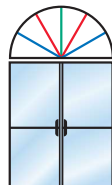
- COMPARE PROOFS** Compare the proof on the previous page and the proof in Example 4 in the preceding lesson.
 - How are the proofs the same? How are they different? **See margin.**
 - Which proof is easier for you to understand? *Explain.* **Sample answer: Both the same; the logic is similar.**
- REASONING** Below is a proof of the Transitive Property of Angle Congruence. What is another reason you could give for Statement 3? *Explain.* **substitution; replace $m\angle B$ with $m\angle C$.**

GIVEN $\angle A \cong \angle B$ and $\angle B \cong \angle C$

PROVE $\angle A \cong \angle C$

STATEMENTS	REASONS
1. $\angle A \cong \angle B, \angle B \cong \angle C$	1. Given
2. $m\angle A = m\angle B, m\angle B = m\angle C$	2. Definition of congruent angles
3. $m\angle A = m\angle C$	3. Transitive Property of Equality
4. $\angle A \cong \angle C$	4. Definition of congruent angles

- SHOPPING MALL** You are at the same mall as in the example on the previous page and you notice that the bookstore is halfway between the shoe store and the toy store. Draw a diagram or make a visual organizer, then write a proof to show that the distance from the entrances of the food court and music store is the same as the distance from the entrances of the book store and toy store. **See margin.**
- WINDOW DESIGN** The entrance to the mall has a decorative window above the main doors as shown. The colored dividers form congruent angles. Draw a diagram or make a visual organizer, then write a proof to show that the angle measure between the red dividers is half the measure of the angle between the blue dividers. **See margin.**



- COMPARE PROOFS** Below is a proof of the Symmetric Property of Segment Congruence.

GIVEN $\overline{DE} \cong \overline{FG}$



PROVE $\overline{FG} \cong \overline{DE}$



STATEMENTS	REASONS
1. $\overline{DE} \cong \overline{FG}$	1. Given
2. $DE = FG$	2. Definition of congruent segments
3. $FG = DE$	3. Symmetric Property of Equality
4. $\overline{FG} \cong \overline{DE}$	4. Definition of congruent segments

- Compare this proof to the proof of the Symmetric Property of Angle Congruence in the Concept Summary of this lesson. What makes the proofs different? *Explain.* **See margin.**
- Explain why Statement 2 above cannot be $\overline{FG} \cong \overline{DE}$. **Sample answer: If $\overline{FG} \cong \overline{DE}$ is the second statement, the reason would have to be Symmetric Property of Segment Congruence and you cannot use a property that you are proving as a reason in the proof.**

Using Alternative Methods

113

Avoiding Common Errors

Doing Exercise 1 carefully can help students understand the proof process better and help avoid some common errors in how one step flows from the next.

Reading Strategy

For Exercise 3, have students who draw diagrams or make visual organizers point to corresponding parts of verbal information in the exercise and in the diagram or organizer.

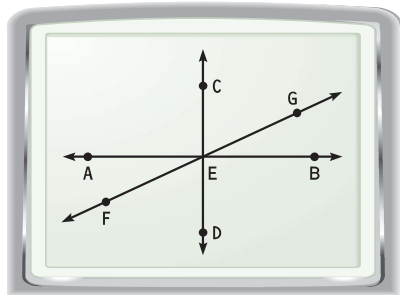
1a. Sample answer: The logic used is similar; one uses segment length and the other uses segment congruence.

3, 4. See Additional Answers.

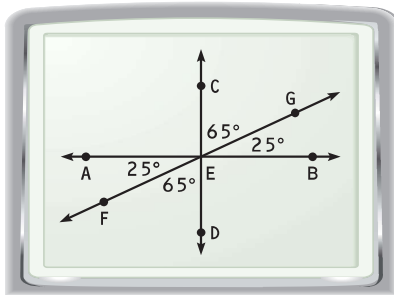
5a. Sample answer: The proof in this lesson involves angle congruence while this one involves segment congruence.

EXPLORE 2 Measure complementary angles

STEP 1 *Draw two perpendicular lines* Draw and label \overleftrightarrow{AB} . Draw point E on \overleftrightarrow{AB} . Draw and label $\overleftrightarrow{EC} \perp \overleftrightarrow{AB}$. Draw and label point D on \overleftrightarrow{EC} so that E is between C and D as shown in Step 2.

STEP 2

Draw another line Draw and label \overleftrightarrow{EG} so that G is in the interior of $\angle CEB$. Draw point F on \overleftrightarrow{EG} as shown.

STEP 3

Measure angles Measure $\angle AEF$, $\angle FED$, $\angle CEG$, and $\angle GEB$. Save as "EXPLORE2". Move point G to change the angles.

EXPLORE 3 Measure vertical angles formed by intersecting lines

STEP 1 *Draw two intersecting lines* Draw and label \overleftrightarrow{AB} . Draw and label \overleftrightarrow{CD} so that it intersects \overleftrightarrow{AB} . Draw and label the point of intersection E .

STEP 2 *Measure angles* Measure $\angle AEC$, $\angle AED$, $\angle BEC$, and $\angle DEB$. Move point C to change the angles. Save as "EXPLORE3".

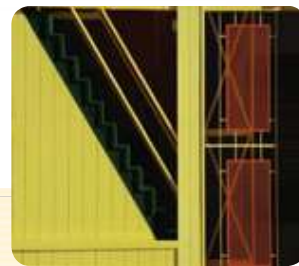
DRAW CONCLUSIONS Use your observations to complete these exercises

- In Explore 2, does the angle relationship stay the same as you move G ? **yes**
- In Explore 2, make a conjecture about the relationship between $\angle CEG$ and $\angle GEB$. Write your conjecture in if-then form. **They are complementary; if $\angle CEB$ is a right angle, then $\angle CEG$ and $\angle GEB$ are complementary.**
- In Explore 3, the intersecting lines form two pairs of vertical angles. Make a conjecture about the relationship between any two vertical angles. Write your conjecture in if-then form. **The vertical angles are congruent; if two lines intersect, then the vertical angles formed are congruent.**
- Name the pairs of vertical angles in Explore 2. Use this drawing to test your conjecture from Exercise 8. **$\angle AEC$ and $\angle BED$, $\angle AEF$ and $\angle BEG$, $\angle DEF$ and $\angle CEG$, $\angle CEB$ and $\angle DEA$, $\angle AEG$ and $\angle FEB$, $\angle GED$ and $\angle CEF$**

3 ASSESS AND RETEACH

- Given that lines m and n intersect to form an angle of 82° , find the measures of the other three angles formed by the lines.
 82° , 98° , 98°
- If all four angles formed by two intersecting lines are congruent, what can be said about the lines?
They are perpendicular.

2.7 Prove Angle Pair Relationships



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Give a reason for each statement.

- If $m\angle 1 = 90^\circ$ and $m\angle 2 = 90^\circ$, then $m\angle 1 = m\angle 2$ (Transitive Prop. of Eq.)
- If $\overline{AB} \perp \overline{BC}$, then $\angle ABC$ is a right angle. (Def. of perpendicular)
- If $\overline{FG} \cong \overline{RS}$, then $FG = RS$. (Def. of segment congruence)

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See Teaching Guide/Lesson Plan.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

What is the relationship between vertical angles, between two angles that are supplementary to the same angle, and between two angles that are complementary to the same angle? **Tell students they will learn how to answer this question by proving theorems about special pairs of angles.**

Before

You identified relationships between pairs of angles.

Now

You will use properties of special pairs of angles.

Why?

So you can describe angles found in a home, as in Ex. 44.

Key Vocabulary

- complementary angles
- supplementary angles
- linear pair
- vertical angles



CC.9-12.G.CO.9 Prove theorems about lines and angles.

THEOREM

For Your Notebook

THEOREM 2.3 Right Angles Congruence Theorem

All right angles are congruent.

PROOF

Right Angles Congruence Theorem

WRITE PROOFS

When you prove a theorem, write the hypothesis of the theorem as the GIVEN statement. The conclusion is what you must PROVE.

GIVEN $\angle 1$ and $\angle 2$ are right angles.

PROVE $\angle 1 \cong \angle 2$



STATEMENTS

- $\angle 1$ and $\angle 2$ are right angles.
- $m\angle 1 = 90^\circ$, $m\angle 2 = 90^\circ$
- $m\angle 1 = m\angle 2$
- $\angle 1 \cong \angle 2$

REASONS

- Given
- Definition of right angle
- Transitive Property of Equality
- Definition of congruent angles

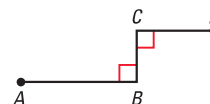
EXAMPLE 1

Use right angle congruence

Write a proof.

GIVEN $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

PROVE $\angle B \cong \angle C$



AVOID ERRORS

The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

STATEMENTS

- $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$
- $\angle B$ and $\angle C$ are right angles.
- $\angle B \cong \angle C$

REASONS

- Given
- Definition of perpendicular lines
- Right Angles Congruence Theorem

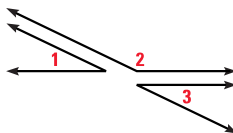
THEOREMS

For Your Notebook

THEOREM 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

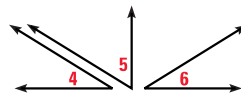
If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.



THEOREM 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.



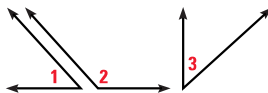
To prove Theorem 2.4, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of Theorem 2.5 also requires two cases.

EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 2$ are supplements.

PROVE $\angle 1 \cong \angle 3$



STATEMENTS

- $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 2$ are supplements.
- $m\angle 1 + m\angle 2 = 180^\circ$
 $m\angle 3 + m\angle 2 = 180^\circ$
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
- $m\angle 1 = m\angle 3$
- $\angle 1 \cong \angle 3$

REASONS

- Given
- Definition of supplementary angles
- Transitive Property of Equality
- Subtraction Property of Equality
- Definition of congruent angles

at my.hrw.com



GUIDED PRACTICE for Examples 1 and 2

- How many steps do you save in the proof in Example 1 by using the *Right Angles Congruence Theorem*? **2 steps**
- Draw a diagram and write GIVEN and PROVE statements for a proof of each case of the *Congruent Complements Theorem*. **See margin.**

Differentiated Instruction

Below Level Use cardboard models of angles to illustrate the Congruent Supplements Theorem. Do the same for the Congruent Complements Theorem.

See also the *Differentiated Instruction Resources* for more strategies.

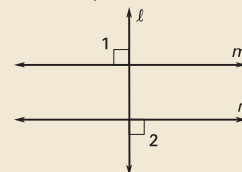
Motivating the Lesson

Are there pairs of angles in architectural structures that you can be sure are congruent without making measurements? The theorems in this lesson will help you answer this question.

3 TEACH

Extra Example 1

Write a proof.



Given: $l \perp m$, $l \perp n$

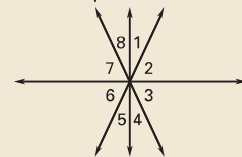
Prove: $\angle 1 \cong \angle 2$

Statements (Reasons)

- $l \perp m$; $l \perp n$ (Given)
- $\angle 1$ and $\angle 2$ are right angles.
(Def. of perpendicular)
- $\angle 1 \cong \angle 2$ (Right Angles Congruence Thm.)

Extra Example 2

Write a proof.



Given: $\angle 1$ and $\angle 3$ are complements;
 $\angle 3$ and $\angle 5$ are complements.

Prove: $\angle 1 \cong \angle 5$

Statements (Reasons)

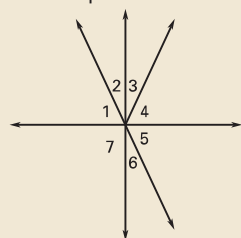
- $\angle 1$ and $\angle 3$ are complements;
 $\angle 3$ and $\angle 5$ are complements.
(Given)
- $\angle 1 \cong \angle 5$ (Congruent Complements Thm.)

2. See Additional Answers.

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

Extra Example 3

Write a proof.



Given: $\angle 2 \cong \angle 3$

Prove: $\angle 3 \cong \angle 6$

Statements (Reasons)

1. $\angle 2 \cong \angle 3$ (Given)
2. $\angle 2 \cong \angle 6$ (Vertical Angles Congruence Thm.)
3. $\angle 3 \cong \angle 6$ (Transitive Prop. of Congruence)



Key Question Example 3

- What law of logic is used to proceed from step 2 to step 3?

Law of Detachment

Avoiding Common Errors

The converse of the Linear Pair Postulate is not true, although sometimes students make this error. Ask students to demonstrate this fact.

Mathematical Reasoning

Some students may think that the proof of the Vertical Angles Congruence Theorem is incomplete because we did not go on to prove that the other set of vertical angles are congruent. Discuss why it is not necessary to consider this as a separate case.

INTERSECTING LINES When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you have used for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used in the proof of the *Vertical Angles Congruence Theorem*.

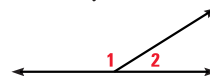
POSTULATE

For Your Notebook

POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.

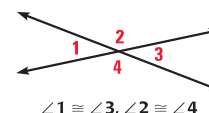


THEOREM

For Your Notebook

THEOREM 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



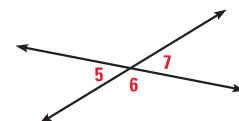
$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN $\angle 5$ and $\angle 7$ are vertical angles.

PROVE $\angle 5 \cong \angle 7$



STATEMENTS

1. $\angle 5$ and $\angle 7$ are vertical angles.
2. $\angle 5$ and $\angle 6$ are a linear pair.
 $\angle 6$ and $\angle 7$ are a linear pair.
3. $\angle 5$ and $\angle 6$ are supplementary.
 $\angle 6$ and $\angle 7$ are supplementary.
4. $\angle 5 \cong \angle 7$

REASONS

1. Given
2. Definition of linear pair, as shown in the diagram
3. Linear Pair Postulate
4. Congruent Supplements Theorem

USE A DIAGRAM

You can use information labeled in a diagram in your proof.



GUIDED PRACTICE for Example 3

In Exercises 3–5, use the diagram.

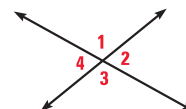
3. If $m\angle 1 = 112^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

4. If $m\angle 2 = 67^\circ$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.

5. If $m\angle 4 = 71^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

6. Which previously proven theorem is used in Example 3 as a reason?

Congruent Supplements Theorem



Differentiated Instruction

Auditory Learners Have students work with a partner to answer **Guided Practice Exercises 3–6**. In addition to finding the angle measures, have students tell their partner which theorem or postulate they applied when finding each angle measure. See also the *Differentiated Instruction Resources* for more strategies.



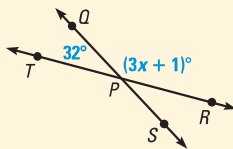
EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

Which equation can be used to find x ?

- (A) $32 + (3x + 1) = 90$
- (B) $32 + (3x + 1) = 180$
- (C) $32 = 3x + 1$
- (D) $3x + 1 = 212$



Solution

Because $\angle TPQ$ and $\angle QPR$ form a linear pair, the sum of their measures is 180° .

► The correct answer is B. (A) (B) (C) (D)



GUIDED PRACTICE for Example 4

Use the diagram in Example 4.

7. Solve for x . 49

8. Find $m\angle TPS$. 148°

2.7 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 5, 13, and 39

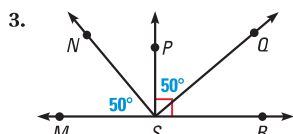
★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 7, 16, 30, and 45

SKILL PRACTICE

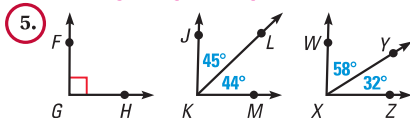
[A]

- VOCABULARY** Copy and complete: If two lines intersect at a point, then the ? angles formed by the intersecting lines are congruent. **vertical**
- ★ WRITING** Describe the relationship between the angle measures of complementary angles, supplementary angles, vertical angles, and linear pairs. **The sum is 90° , the sum is 180° , same, the sum is 180° .**

IDENTIFY ANGLES Identify the pair(s) of congruent angles in the figures below. *Explain how you know they are congruent.*

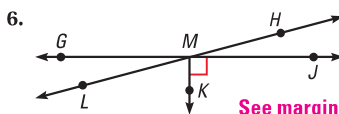
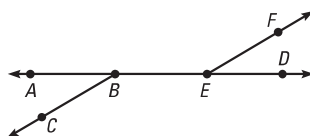


$\angle MSN$ and $\angle PSQ$, $\angle NSP$ and $\angle QSR$, $\angle MSP$ and $\angle PSR$; indicated in diagram, Congruent Complements Theorem, Right Angles Congruence Theorem



$\angle FGH$ and $\angle WXZ$; Right Angles Congruence Theorem

4. $\angle ABC$ is supplementary to $\angle CBD$.
 $\angle CBD$ is supplementary to $\angle DEF$.



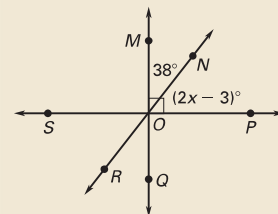
See margin.

2.7 Prove Angle Pair Relationships

119

Extra Example 4

Which equation can be used to find x ? **A**



- (A) $38 + (2x - 3) = 90$
- (B) $38 + (2x - 3) = 180$
- (C) $38 = (2x - 3)$
- (D) $(2x - 3) = 128$



Key Question Example 4

- What is the relationship between $\angle TPQ$ and $\angle QPR$? **They are supplementary.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What is the relationship between vertical angles, between two angles that are supplementary to the same angle, and between two angles that are complementary to the same angle?

- All right angles are congruent.**
 - Supplements of congruent angles or the same angle are congruent.**
 - Complements of congruent angles or the same angle are congruent.**
 - Vertical angles are congruent.**
- Vertical angles are congruent. Supplements of the same angle or of congruent angles are congruent, and complements of the same angle or of congruent angles are congruent.

6. $\angle GML$ and $\angle HMJ$, $\angle GMH$ and $\angle LMJ$, $\angle JMK$ and $\angle GMK$; Vertical Angles Congruence Theorem, Vertical Angles Congruence Theorem, Right Angles Congruence Theorem

EXAMPLES 1 and 2
for Exs. 3–7

4. $\angle ABC$ and $\angle DEF$, $\angle CBD$ and $\angle FEB$; Congruent Supplements Theorem

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:

Day 1:

Exs. 1–7, 36–39

Day 2:

Exs. 8–26, 40

Average:

Day 1:

Exs. 1–7, 36–39

Day 2:

Exs. 9–15 odd, 16, 17–29 odd, 30, 40–45

Advanced:

Day 1:

Exs. 1–7, 36–39, 43, 44

Day 2:

Exs. 10, 11, 14–30 even, 31–35*, 40–42, 45–48*

Block:

Exs. 1–7, 9–15 odd, 16, 17–29 odd, 30, 36–45

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 8, 12, 18, 36

Average: 4, 10, 13, 20, 38

Advanced: 6, 11, 14, 22, 40

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

EXAMPLE 3 for Exs. 8–11

EXAMPLE 4 for Exs. 12–14

7. ★ **SHORT RESPONSE** The x -axis and y -axis in a coordinate plane are perpendicular to each other. The axes form four angles. Are the four angles congruent right angles? *Explain.* **Yes; perpendicular lines form right angles, and all right angles are congruent.**

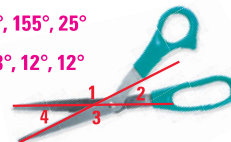
FINDING ANGLE MEASURES In Exercises 8–11, use the diagram at the right.

8. If $m\angle 1 = 155^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$. **$25^\circ, 155^\circ, 25^\circ$**

9. If $m\angle 3 = 168^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$. **$168^\circ, 12^\circ, 12^\circ$**

10. If $m\angle 4 = 27^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. **$153^\circ, 27^\circ, 153^\circ$**

11. If $m\angle 2 = 32^\circ$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$. **$148^\circ, 148^\circ, 32^\circ$**



xy ALGEBRA Find the values of x and y .

12. **$x = 11, y = 17$**

13. **$x = 13, y = 20$**

14. **$x = 4, y = 9$**

15. **ERROR ANALYSIS** Describe the error in stating that $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$.

Sample answer: It was assumed that $\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$ are linear pairs, but they are not; $\angle 1$ and $\angle 4$, and $\angle 2$ and $\angle 3$ are not vertical angles and are not congruent.



$$\angle 1 \cong \angle 4$$

$$\angle 2 \cong \angle 3$$



16. ★ **MULTIPLE CHOICE** In a figure, $\angle A$ and $\angle D$ are complementary angles and $m\angle A = 4x^\circ$. Which expression can be used to find $m\angle D$? **D**

- (A) $(4x + 90)^\circ$ (B) $(180 - 4x)^\circ$ (C) $(180 + 4x)^\circ$ (D) $(90 - 4x)^\circ$

FINDING ANGLE MEASURES In Exercises 17–21, copy and complete the statement given that $m\angle FHE = m\angle BHG = m\angle AHF = 90^\circ$.

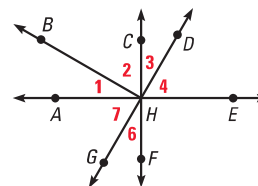
17. If $m\angle 3 = 30^\circ$, then $m\angle 6 = ?$. **30°**

18. If $m\angle BHF = 115^\circ$, then $m\angle 3 = ?$. **25°**

19. If $m\angle 6 = 27^\circ$, then $m\angle 1 = ?$. **27°**

20. If $m\angle DHF = 133^\circ$, then $m\angle CHG = ?$. **133°**

21. If $m\angle 3 = 32^\circ$, then $m\angle 2 = ?$. **58°**



B ANALYZING STATEMENTS Two lines that are not perpendicular intersect such that $\angle 1$ and $\angle 2$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 3$ are vertical angles. Tell whether the statement is true or false.

22. $\angle 1 \cong \angle 2$ **false** 23. $\angle 1 \cong \angle 3$ **true** 24. $\angle 1 \cong \angle 4$ **false**
25. $\angle 3 \cong \angle 2$ **false** 26. $\angle 2 \cong \angle 4$ **true** 27. $m\angle 3 + m\angle 4 = 180^\circ$ **true**

xy ALGEBRA Find the measure of each angle in the diagram.

28. **$130^\circ, 50^\circ, 130^\circ, 50^\circ$**

29. **$140^\circ, 40^\circ, 140^\circ, 40^\circ$**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

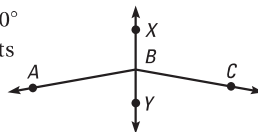
32. $\angle 1$ and $\angle 9$; Congruent Supplements Theorem

33. Sample answer: $\angle CEB$ and $\angle DEB$; Right Angles Congruence Theorem

34. $\angle 5$ and $\angle 1$; Congruent Complements Theorem

30. ★ **OPEN-ENDED MATH** In the diagram, $m\angle CBY = 80^\circ$ and \overleftrightarrow{XY} bisects $\angle ABC$. Give two more true statements about the diagram.

Sample answer: $m\angle CBX = 100^\circ$, $m\angle ABX = 100^\circ$



DRAWING CONCLUSIONS In Exercises 31–34, use the given statement to name two congruent angles. Then give a reason that justifies your conclusion.

31. In triangle GFE , \overleftrightarrow{GH} bisects $\angle EGF$. $\angle FGH$ and $\angle EGH$; Definition of angle bisector
32. $\angle 1$ is a supplement of $\angle 6$, and $\angle 9$ is a supplement of $\angle 6$.
33. \overleftrightarrow{AB} is perpendicular to \overleftrightarrow{CD} , and \overleftrightarrow{AB} and \overleftrightarrow{CD} intersect at E .
34. $\angle 5$ is complementary to $\angle 12$, and $\angle 1$ is complementary to $\angle 12$.
35. **CHALLENGE** Sketch two intersecting lines j and k . Sketch another pair of lines ℓ and m that intersect at the same point as j and k and that bisect the angles formed by j and k . Line ℓ is perpendicular to line m . Explain why this is true.

See margin for art. Sample answer: ℓ and m bisect supplementary angles.

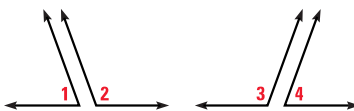
PROBLEM SOLVING

EXAMPLE 2 [A] for Ex. 36

36. **PROVING THEOREM 2.4** Prove the second case of the Congruent Supplements Theorem where two angles are supplementary to congruent angles. See margin.

GIVEN $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 4$ are supplements.
 $\angle 1 \cong \angle 4$

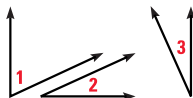
PROVE $\angle 2 \cong \angle 3$



37. **PROVING THEOREM 2.5** Copy and complete the proof of the first case of the Congruent Complements Theorem where two angles are complementary to the same angle.

GIVEN $\angle 1$ and $\angle 2$ are complements.
 $\angle 1$ and $\angle 3$ are complements.

PROVE $\angle 2 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complements. $\angle 1$ and $\angle 3$ are complements.	1. ? Given
2. $m\angle 1 + m\angle 2 = 90^\circ$ $m\angle 1 + m\angle 3 = 90^\circ$	2. ? Definition of complementary angles
3. ? $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	3. Transitive Property of Equality
4. ? $m\angle 2 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	5. ? Definition of congruent angles

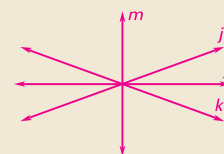
Teaching Strategy

Exercises 12–14 These exercises can be done by solving two equations in one variable or by solving a system of two equations in two variables. Discuss both approaches and ask which theorem is being applied to justify each approach.

Mathematical Reasoning

Exercises 22–27 Ask students to draw a diagram that reflects the information given in the directions for the exercises. Discuss whether there is more than one way to draw the diagram.

35. Sample:



36. **Statements (Reasons)**

- $\angle 1$ and $\angle 2$ are supplements;
 $\angle 3$ and $\angle 4$ are supplements;
 $\angle 1 \cong \angle 4$. (Given)
- $m\angle 1 + m\angle 2 = 180^\circ$; $m\angle 3 + m\angle 4 = 180^\circ$ (Definition of supplementary angles)
- $m\angle 1 = m\angle 4$ (Definition of congruent angles)
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ (Transitive Property of Equality)
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$ (Substitution)
- $m\angle 2 = m\angle 3$ (Subtraction Property of Equality)
- $\angle 2 \cong \angle 3$ (Definition of congruent angles)

38. Statements (Reasons)

- $\angle ABD$ is a right angle;
 $\angle CBE$ is a right angle. (Given)
- $\angle ABC$ and $\angle CBD$ are complementary. (Definition of complementary angles)
- $\angle DBE$ and $\angle CBD$ are complementary. (Definition of complementary angles)
- $\angle ABC \cong \angle DBE$ (Congruent Complements Theorem)

39. Statements (Reasons)

- $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$, $\angle J \cong \angle M$,
 $\angle K \cong \angle L$ (Given)
- $\angle J$ and $\angle L$ are right angles.
(Definition of perpendicular lines)
- $m\angle J = 90^\circ$ and $m\angle L = 90^\circ$
(Definition of right angles)
- $m\angle J = m\angle M$ and $m\angle L = m\angle K$
(Definition of congruent angles)
- $m\angle M = 90^\circ$ and $m\angle K = 90^\circ$
(Transitive Property of Equality)
- $\angle M$ and $\angle K$ are right angles.
(Definition of right angles)
- $\overline{JM} \perp \overline{ML}$, $\overline{JK} \perp \overline{KL}$ (Definition of perpendicular lines)

41. Statements (Reasons)

- $\angle 1$ and $\angle 2$ are complementary;
 $\angle 3$ and $\angle 4$ are complementary;
 $\angle 1 \cong \angle 4$. (Given)
- $m\angle 1 + m\angle 2 = 90^\circ$; $m\angle 3 + m\angle 4 = 90^\circ$ (Definition of complementary)
- $m\angle 1 = m\angle 4$ (Definition of congruent angles)
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$
(Transitive Property of Equality)
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$
(Substitution)
- $m\angle 2 = m\angle 3$ (Subtraction Property of Equality)
- $\angle 2 \cong \angle 3$ (Definition of congruent angles)

42. Statements (Reasons)

- $\angle 1 \cong \angle 3$ (Given)
- $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$ (Vertical Angles Theorem)
- $\angle 2 \cong \angle 3$ (Transitive Property of Angle Congruence)
- $\angle 2 \cong \angle 4$ (Transitive Property of Angle Congruence)

40a. $m\angle 2 = (180 - x)^\circ$,
 $m\angle 3 = x^\circ$,
 $m\angle 4 = (180 - x)^\circ$

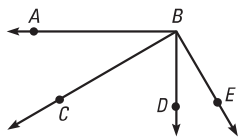
40b. Sample answer:
 $x \approx 110$, $m\angle 2 \approx 70^\circ$, $m\angle 3 \approx 110^\circ$,
 $m\angle 4 \approx 70^\circ$

40c. $\angle 2$ gets smaller, while $\angle 1$ and $\angle 3$ will get larger. Sample answer: $\angle 3$ and $\angle 4$, and $\angle 1$ and $\angle 2$ form linear pairs. Since the sum of the angles in a linear pair is always 180° , as one angle in a linear pair gets smaller the other angle of the linear pair will get larger.

B PROOF Use the given information and the diagram to prove the statement. **38, 39. See margin.**

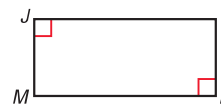
- 38. GIVEN** $\angle ABD$ is a right angle.
 $\angle CBE$ is a right angle.

PROVE $\angle ABC \cong \angle DBE$



- 39. GIVEN** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
 $\angle J \cong \angle M$, $\angle K \cong \angle L$

PROVE $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



- 40. MULTI-STEP PROBLEM** Use the photo of the folding table.

- If $m\angle 1 = x^\circ$, write expressions for the other three angle measures.
- Estimate the value of x . What are the measures of the other angles?
- As the table is folded up, $\angle 4$ gets smaller. What happens to the other three angles? Explain your reasoning.

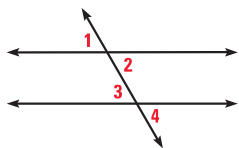


- 41. PROVING THEOREM 2.5** Write a two-column proof for the second case of Theorem 2.5 where two angles are complementary to congruent angles. **See margin.**

WRITING PROOFS Write a two-column proof. **42, 43. See margin.**

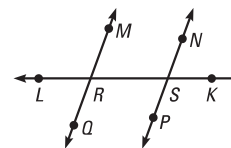
- 42. GIVEN** $\angle 1 \cong \angle 3$

PROVE $\angle 2 \cong \angle 4$



- 43. GIVEN** $\angle QRS$ and $\angle PSR$ are supplementary.

PROVE $\angle QRL \cong \angle PSR$



- 44. STAIRCASE** Use the photo and the given information to prove the statement. **See margin.**

- GIVEN** $\angle 1$ is complementary to $\angle 3$.
 $\angle 2$ is complementary to $\angle 4$.

PROVE $\angle 1 \cong \angle 4$



- 45. ★ EXTENDED RESPONSE** $\angle STV$ is bisected by \overleftrightarrow{TW} , and \overleftrightarrow{TX} and \overleftrightarrow{TW} are opposite rays. You want to show $\angle STX \cong \angle VTX$. **a-c. See margin.**

- Draw a diagram.
- Identify the GIVEN and PROVE statements for the situation.
- Write a two-column proof.

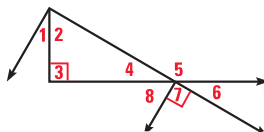
43. Statements (Reasons)

- $\angle QRS$ and $\angle PSR$ are supplementary. (Given)
- $\angle QRS$ and $\angle QRL$ are a linear pair. (Definition of linear pair)
- $\angle QRS$ and $\angle QRL$ are supplementary. (Linear Pair Postulate)
- $\angle QRL \cong \angle PSR$ (Congruent Supplements Theorem)

44, 45a-c. See Additional Answers.

- C** 46. **USING DIAGRAMS** Copy and complete the statement with $<$, $>$, or $=$.

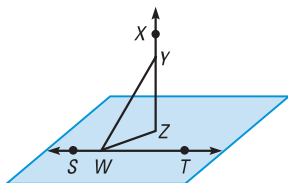
- $m\angle 3$ $\underline{\quad ? \quad}$ $m\angle 7$ $\underline{\quad = \quad}$
- $m\angle 4$ $\underline{\quad ? \quad}$ $m\angle 6$ $\underline{\quad = \quad}$
- $m\angle 8 + m\angle 6$ $\underline{\quad ? \quad}$ 150° $\underline{\quad < \quad}$
- If $m\angle 4 = 30^\circ$, then $m\angle 5$ $\underline{\quad ? \quad}$ $m\angle 4$ $\underline{\quad > \quad}$



CHALLENGE In Exercises 47 and 48, write a two-column proof. 47, 48. See margin.

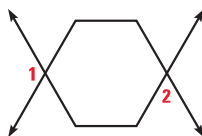
47. **GIVEN** $\triangleright m\angle WYZ = m\angle TWZ = 45^\circ$

PROVE $\triangleright \angle SWZ \cong \angle XYW$



48. **GIVEN** \triangleright The hexagon is regular.

PROVE $\triangleright m\angle 1 + m\angle 2 = 180^\circ$



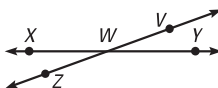
QUIZ

Match the statement with the property that it illustrates.

- If $\overline{HJ} \cong \overline{LM}$, then $\overline{LM} \cong \overline{HJ}$. **B**
- If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 4$, then $\angle 1 \cong \angle 4$. **C**
- $\angle XYZ \cong \angle XYZ$ **A**
- Write a two-column proof. See margin.

GIVEN $\triangleright \angle XWY$ is a straight angle.
 $\angle ZWV$ is a straight angle.

PROVE $\triangleright \angle XWV \cong \angle ZWY$



A. Reflexive Property of Congruence

B. Symmetric Property of Congruence

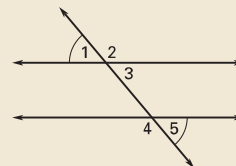
C. Transitive Property of Congruence

5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

1. Give the reason for each step.



Given: $\angle 1 \cong \angle 5$

Prove: $\angle 1$ is supplementary to $\angle 4$.

Statements (Reasons)

- $\angle 1 \cong \angle 5$ (**Given**)
- $m\angle 1 = m\angle 5$ (**Def. of \cong**)
- $\angle 4$ and $\angle 5$ are a linear pair. (**Def. of linear pair**)
- $\angle 4$ and $\angle 5$ are supplementary. (**Linear Pair Post.**)
- $m\angle 4 + m\angle 5 = 180$ (**Def. of supplementary**)
- $m\angle 4 + m\angle 1 = 180$ (**Substitution Prop. of Eq.**)
- $\angle 1$ is supplementary to $\angle 4$. (**Def. of supplementary**)

Online Quiz

Available at my.hrw.com

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

47, 48, Quiz 4. See Additional Answers.

MIXED REVIEW of Problem Solving



Make sense of problems and persevere in solving them.

1a. Statements (Reasons)

- \overrightarrow{BD} bisects $\angle ABC$;
 \overrightarrow{BC} bisects $\angle DBE$. (Given)
- $m\angle ABD = m\angle DBC$;
 $m\angle DBC = m\angle CBE$ (Definition of angle bisector)
- $m\angle ABD = m\angle CBE$ (Transitive Property of Equality)

1b. 33° ; $m\angle DBC = \frac{1}{3} m\angle ABE$

2. Yes; each piece is $\frac{1}{4}$ the original width.

4. **Sample answer:** If $\angle 1$ is a right angle, then $m\angle 1 = 90^\circ$ by the definition of right angle. If $\angle 1$ and $\angle 2$ are supplementary, then $m\angle 1 + m\angle 2 = 180^\circ$ by the definition of supplementary angles. Use the Subtraction and Substitution Properties of Equality to show that $m\angle 2 = 180^\circ - m\angle 1 = 180^\circ - 90^\circ = 90^\circ$. If $m\angle 2 = 90^\circ$, then $m\angle 2$ must be a right angle by the definition of right angles.

5a. Equation (Reason)

$$T = c(1 + s) \text{ (Given)}$$

$$\frac{T}{c} = 1 + s \text{ (Division Property of Equality)}$$

$$\frac{T}{c} - 1 = s \text{ (Subtraction Property of Equality)}$$

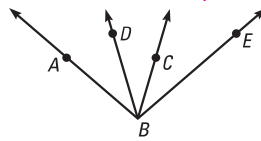
5c. Yes; distribute the c , then subtract c from both sides, followed by dividing both sides by c .

6. **Sample answer:** Either $m\angle BAC$ or $m\angle CAD$; since $\angle GAD$ is a straight angle, if two of the three angles are known, the third angle can be found.

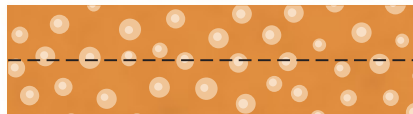
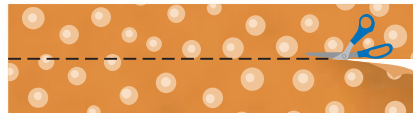
7. $m\angle 1 = m\angle 2 = 45^\circ$, $m\angle 3 = m\angle 4 = 135^\circ$; $m\angle 1 + m\angle 3 = m\angle 1 + 3m\angle 1 = 4m\angle 1 = 180^\circ$

8. $\angle EAF$ and $\angle BAC$ are complementary. **Sample answer:** $m\angle BAC + m\angle CAD + m\angle DAE + m\angle EAF = m\angle BAF$ by the Angle Addition Postulate; $m\angle BAF = 180^\circ$ and $m\angle CAD + m\angle DAE = 90^\circ$, so $m\angle BAC + m\angle EAF = 90^\circ$.

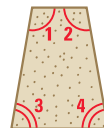
1. **MULTI-STEP PROBLEM** In the diagram below, \overrightarrow{BD} bisects $\angle ABC$ and \overrightarrow{BC} bisects $\angle DBE$.
1a, b. **See margin.**



- Prove $m\angle ABD = m\angle CBE$.
 - If $m\angle ABE = 99^\circ$, what is $m\angle DBC$? **Explain.**
2. **SHORT RESPONSE** You are cutting a rectangular piece of fabric into strips that you will weave together to make a placemat. As shown, you cut the fabric in half lengthwise to create two congruent pieces. You then cut each of these pieces in half lengthwise. Do all of the strips have the same width? **Explain** your reasoning. **See margin.**



3. **GRIDDED ANSWER** The cross section of a concrete retaining wall is shown below. Use the given information to find the measure of $\angle 1$ in degrees. **100°**



$$m\angle 1 = m\angle 2$$

$$m\angle 3 = m\angle 4$$

$$m\angle 3 = 80^\circ$$

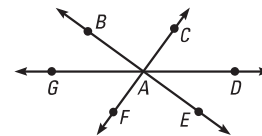
$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$$

4. **EXTENDED RESPONSE** Suppose you know that $\angle 1$ is a right angle, and $\angle 1$ and $\angle 2$ are supplementary. **Explain** how to use definitions and properties of equality to prove that $\angle 2$ is a right angle. **See margin.**

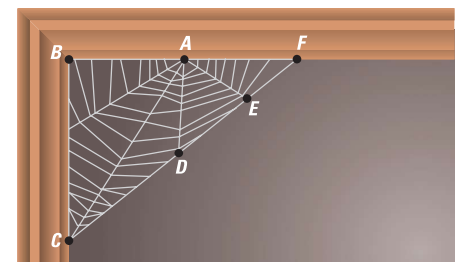
5. **EXTENDED RESPONSE** A formula you can use to calculate the total cost of an item including sales tax is $T = c(1 + s)$, where T is the total cost including sales tax, c is the cost not including sales tax, and s is the sales tax rate written as a decimal.

- Solve the formula for s . Give a reason for each step. **See margin.**
- Use your formula to find the sales tax rate on a purchase that was \$26.75 with tax and \$25 without tax. **0.07, or 7%**
- Look back at the steps you used to solve the formula for s . Could you have solved for s in a different way? **Explain.** **See margin.**

6. **OPEN-ENDED** In the diagram below, $m\angle GAB = 36^\circ$. What additional information do you need to find $m\angle BAC$ and $m\angle CAD$? **Explain** your reasoning. **See margin.**



7. **SHORT RESPONSE** Two lines intersect to form $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$. The measure of $\angle 3$ is three times the measure of $\angle 1$ and $m\angle 1 = m\angle 2$. Find all four angle measures. **Explain** your reasoning. **See margin.**
8. **SHORT RESPONSE** Part of a spider web is shown below. If you know that $\angle CAD$ and $\angle DAE$ are complements and that \overrightarrow{AB} and \overrightarrow{AF} are opposite rays, what can you conclude about $\angle BAC$ and $\angle EAF$? **Explain** your reasoning. **See margin.**



2 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Using Inductive and Deductive Reasoning

When you make a conjecture based on a pattern, you use inductive reasoning. You use deductive reasoning to show whether the conjecture is true or false by using facts, definitions, postulates, properties, or proven theorems. If you can find one counterexample to the conjecture, then you know the conjecture is false.

Big Idea 2

Understanding Geometric Relationships in Diagrams

The following can be assumed from the diagram:

A , B , and C are coplanar.

$\angle ABH$ and $\angle HBF$ are a linear pair.

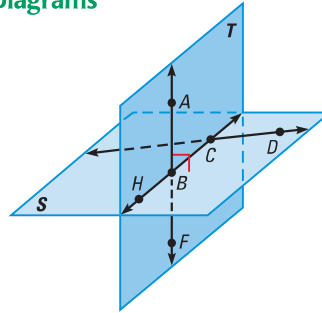
Plane T and plane S intersect in \overleftrightarrow{BC} .

\overleftrightarrow{CD} lies in plane S .

$\angle ABC$ and $\angle HBF$ are vertical angles.

$\overleftrightarrow{AB} \perp$ plane S .

Diagram assumptions are reviewed on page 89.



Big Idea 3

Writing Proofs of Geometric Relationships

You can write a logical argument to show a geometric relationship is true. In a two-column proof, you use deductive reasoning to work from GIVEN information to reach a conjecture you want to PROVE.

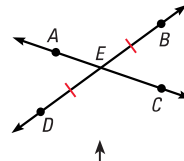


Diagram of geometric relationship with given information labeled to help you write the proof

GIVEN ► The hypothesis of an if-then statement

PROVE ► The conclusion of an if-then statement

STATEMENTS

1. Hypothesis

n . Conclusion

Statements based on facts that you know or conclusions from deductive reasoning

REASONS

1. Given

n .

Use postulates, proven theorems, definitions, and properties of numbers and congruence as reasons.

Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish

@HomeTutor

Vocabulary Practice

Vocabulary practice is available at my.hrw.com

Extra Example 1

Describe the pattern in the numbers 6, 24, 96, 384, ..., and write the next three numbers in the pattern. **Each number is 4 times the previous number; 1536, 6144, 24,576**

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. PT2.

- conjecture
- inductive reasoning
- counterexample
- conditional statement
- converse, inverse, contrapositive
- if-then form
- hypothesis, conclusion
- negation
- equivalent statements
- perpendicular lines
- biconditional statement
- deductive reasoning
- line perpendicular to a plane
- proof
- two-column proof
- theorem

2. In the inverse the hypothesis and conclusion are negated while in the converse the hypothesis and conclusion are switched.

VOCABULARY EXERCISES

1. Copy and complete: A statement that can be proven is called a(n) ? . **theorem**
2. **WRITING** Compare the inverse of a conditional statement to the converse of the conditional statement.
3. You know $m\angle A = m\angle B$ and $m\angle B = m\angle C$. What does the Transitive Property of Equality tell you about the measures of the angles? **$m\angle A = m\angle C$**

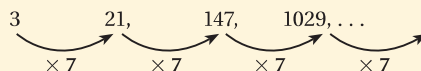
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

2.1 Use Inductive Reasoning**EXAMPLE**

Describe the pattern in the numbers 3, 21, 147, 1029, ..., and write the next three numbers in the pattern.

Each number is seven times the previous number.



So, the next three numbers are 7203, 50,421, and 352,947.

4. Each number is $\frac{1}{4}$ of the previous number; -80, -20, -5.

EXAMPLES 2 and 5 for Exs. 4-5

EXERCISES

4. Describe the pattern in the numbers -20,480, -5120, -1280, -320, ... Write the next three numbers.
5. Find a counterexample to disprove the conjecture:
If the quotient of two numbers is positive, then the two numbers must both be positive. **Sample answer: $\frac{-10}{-2} = 5$**

2.2 Analyze Conditional Statements

EXAMPLE

Write the if-then form, the converse, the inverse, and the contrapositive of the statement “Black bears live in North America.”

- If-then form: If a bear is a black bear, then it lives in North America.
- Converse: If a bear lives in North America, then it is a black bear.
- Inverse: If a bear is not a black bear, then it does not live in North America.
- Contrapositive: If a bear does not live in North America, then it is not a black bear.

EXERCISES

- Write the if-then form, the converse, the inverse, and the contrapositive of the statement “An angle whose measure is 34° is an acute angle.” **See margin.**
- Is this a valid definition? *Explain* why or why not.
“If the sum of the measures of two angles is 90° , then the angles are complementary.”
Yes. Sample answer: This is the definition for complementary angles.
- Write the definition of an *equiangular polygon* as a biconditional statement.
The interior angles of a polygon are congruent if and only if the polygon is equiangular.

EXAMPLES
2, 3, and 4
for Exs. 6–8

2.3 Apply Deductive Reasoning

EXAMPLE

Use the Law of Detachment to make a valid conclusion in the true situation.

If two angles have the same measure, then they are congruent. You know that $m\angle A = m\angle B$.

- Because $m\angle A = m\angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

EXERCISES

- Use the Law of Detachment to make a valid conclusion.
If an angle is a right angle, then the angle measures 90° . $\angle B$ is a right angle.
 $\angle B$ measures 90° .
- Use the Law of Syllogism to write the statement that follows from the pair of true statements. **If $4x = 12$, then $2x = 6$.**
If $x = 3$, then $2x = 6$.
If $4x = 12$, then $x = 3$.
- What can you say about the sum of any two odd integers? Use inductive reasoning to form a conjecture. Then use deductive reasoning to show that the conjecture is true.
The sum of two odd integers is even. Sample answer: $7 + 1 = 8$; $2n + 1$ and $2m + 1$ are odd, but their sum $(2n + 1) + (2m + 1) = 2m + 2n + 2 = 2(m + n + 1)$ is even.

EXAMPLES
1, 2, and 4
for Exs. 9–11

Extra Example 2

Write the if-then form, the converse, the inverse, and the contrapositive of the statement “The heart of a mouse beats at least 600 times a minute.” **If-then: If an animal is a mouse, then its heart beats at least 600 times a minute. Converse: If an animal’s heart beats at least 600 times a minute, then the animal is a mouse. Inverse: If an animal is not a mouse, then its heart does not beat at least 600 times a minute. Contrapositive: If an animal’s heart does not beat at least 600 times a minute, then the animal is not a mouse.**

Extra Example 3

Use the Law of Detachment to make a valid conclusion. If two segments have the same length, then they are congruent. The length of \overline{MP} is the same as the length of \overline{WU} . **$\overline{MP} \cong \overline{WU}$**

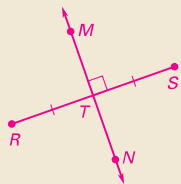
6. If an angle measures 34° , then the angle is an acute angle; if an angle is an acute angle, then it measures 34° ; if an angle does not measure 34° , then the angle is not an acute angle; if an angle is not an acute angle, then it does not measure 34° .

2

CHAPTER REVIEW

Extra Example 4

\overleftrightarrow{MN} intersects \overleftrightarrow{RS} at its midpoint T so that $\overleftrightarrow{MN} \perp \overleftrightarrow{RS}$. Sketch a diagram that represents the given information.



Extra Example 5

Solve $-4x + 2(3x + 8) = -(x + 8)$ and write a reason for each step.

Equation (Reason)

$$-4x + 2(3x + 8) = -(x + 8)$$

(Write the original equation.)

$$-4x + 6x + 16 = -x - 8$$

(Distributive Prop.)

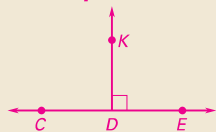
$$2x + 16 = -x - 8$$

$$3x = -24$$

$$x = -8$$

(Division Prop. of Eq.)

12. Sample:



14. Equation (Reason)

$$-9x - 21 = -20x - 87$$

(Given)

$$11x - 21 = -87$$

(Addition Property of Equality)

$$11x = -66$$

(Addition Property of Equality)

$$x = -6$$

(Division Property of Equality)

$$x = -6$$

(Division Property of Equality)

$$15x + 22 = 7x + 62$$

(Given)

$$8x + 22 = 62$$

(Subtraction Property of Equality)

$$8x = 40$$

(Subtraction Property of Equality)

$$x = 5$$

(Division Property of Equality)

$$3(2x + 9) = 30$$

(Given)

$$2x + 9 = 10$$

(Division Property of Equality)

$$2x = 1$$

(Subtraction Property of Equality)

$$x = \frac{1}{2}$$

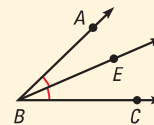
(Division Property of Equality)

2.4 Use Postulates and Diagrams

EXAMPLE

$\angle ABC$, an acute angle, is bisected by \overleftrightarrow{BE} . Sketch a diagram that represents the given information.

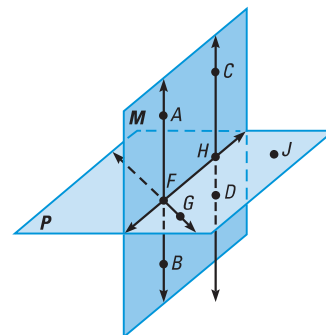
1. Draw $\angle ABC$, an acute angle, and label points A , B , and C .
2. Draw angle bisector \overleftrightarrow{BE} . Mark congruent angles.



EXERCISES

12. Straight angle CDE is bisected by \overleftrightarrow{DK} . Sketch a diagram that represents the given information. See margin.
13. Which of the following statements *cannot* be assumed from the diagram? B

- (A) A , B , and C are coplanar.
- (B) $\overleftrightarrow{CD} \perp$ plane P
- (C) A , F , and B are collinear.
- (D) Plane M intersects plane P in \overleftrightarrow{FH} .



2.5 Reason Using Properties from Algebra

EXAMPLE

Solve $3x + 2(2x + 9) = -10$. Write a reason for each step.

$$3x + 2(2x + 9) = -10$$

Write original equation.

$$3x + 4x + 18 = -10$$

Distributive Property

$$7x + 18 = -10$$

Simplify.

$$7x = -28$$

Subtraction Property of Equality

$$x = -4$$

Division Property of Equality

EXERCISES

Solve the equation. Write a reason for each step. 14–17. See margin.

$$14. -9x - 21 = -20x - 87$$

$$15. 15x + 22 = 7x + 62$$

$$16. 3(2x + 9) = 30$$

$$17. 5x + 2(2x - 23) = -154$$

EXAMPLES 3 and 4

for Exs. 12–13

EXAMPLES 1 and 2

for Exs. 14–17

2.6 Prove Statements about Segments and Angles

EXAMPLE

Prove the Reflexive Property of Segment Congruence.

GIVEN \overline{AB} is a line segment.

PROVE $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. \overline{AB} is a line segment.	1. Given
2. AB is the length of \overline{AB} .	2. Ruler Postulate
3. $AB = AB$	3. Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	4. Definition of congruent segments

EXERCISES

Name the property illustrated by the statement.

18. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$. **Reflexive Property of Congruence**
 19. $\angle C \cong \angle C$ **Reflexive Property of Congruence**
 20. If $MN = PQ$ and $PQ = RS$, then $MN = RS$. **Transitive Property of Equality**
 21. Prove the Transitive Property of Angle Congruence. **See margin.**

EXAMPLES 2 and 3
for Exs. 18–21

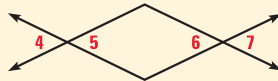
18. Symmetric Property of Congruence

2.7 Prove Angle Pair Relationships

EXAMPLE

GIVEN $\angle 5 \cong \angle 6$

PROVE $\angle 4 \cong \angle 7$

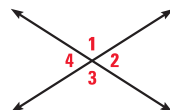


STATEMENTS	REASONS
1. $\angle 5 \cong \angle 6$	1. Given
2. $\angle 4 \cong \angle 5$	2. Vertical Angles Congruence Theorem
3. $\angle 4 \cong \angle 6$	3. Transitive Property of Congruence
4. $\angle 6 \cong \angle 7$	4. Vertical Angles Congruence Theorem
5. $\angle 4 \cong \angle 7$	5. Transitive Property of Congruence

EXERCISES

In Exercises 22 and 23, use the diagram at the right.

22. If $m\angle 1 = 114^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$. **$66^\circ, 114^\circ, 66^\circ$**
 23. If $m\angle 4 = 57^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. **$123^\circ, 57^\circ, 123^\circ$**
 24. Write a two-column proof. **See margin.**



GIVEN $\angle 3$ and $\angle 2$ are complementary.
 $m\angle 1 + m\angle 2 = 90^\circ$

PROVE $\angle 3 \cong \angle 1$

EXAMPLES 2 and 3
for Exs. 22–24

Extra Example 6

Prove the Transitive Property of Angle Congruence.

Given: $\angle A \cong \angle B$, $\angle B \cong \angle C$

Prove: $\angle A \cong \angle C$

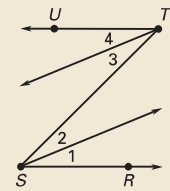
Statements (Reasons)

- $\angle A \cong \angle B$, $\angle B \cong \angle C$ (Given)
- $m\angle A = m\angle B$, $m\angle B = m\angle C$ (Def. of congruent angles)
- $m\angle A = m\angle C$ (Transitive Prop. of Eq.)
- $\angle A \cong \angle C$ (Def. of congruent angles)

Extra Example 7

Given: $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

Prove: $\angle RST \cong \angle STU$



Statements (Reasons)

- $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$ (Given)
- $m\angle 1 = m\angle 3$, $m\angle 2 = m\angle 4$ (Def. of congruent angles)
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ (Addition Prop. of Eq.)
- $m\angle 1 + m\angle 2 = m\angle RST$, $m\angle 3 + m\angle 4 = m\angle STU$ (Angle Addition Post.)
- $m\angle RST = m\angle STU$ (Substitution Prop. of Eq.)
- $\angle RST \cong \angle STU$ (Def. of congruent angles)

24. Statements (Reasons)

- $\angle 3$ and $\angle 2$ are complementary angles; $m\angle 1 + m\angle 2 = 90^\circ$. (Given)
- $m\angle 2 + m\angle 3 = 90^\circ$ (Definition of complementary angles)
- $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 2$ (Transitive Property of Equality)
- $m\angle 3 = m\angle 1$ (Subtraction Property of Equality)
- $\angle 1 \cong \angle 3$ (Definition of angle congruence)

21. Statements (Reasons)

- $\angle A \cong \angle B$, $\angle B \cong \angle C$ (Given)
- $m\angle A = m\angle B$, $m\angle B = m\angle C$ (Definition of angle congruence)
- $m\angle A = m\angle C$ (Transitive Property of Equality)
- $\angle A \cong \angle C$ (Definition of angle congruence)