Algebra 1 Unit 7: Quadratic Equations

Unit Focus

In this unit, students interpret, write, and solve quadratic equations. They see that writing and solving quadratic equations enables them to find input values that produce certain output values.

Students begin solving quadratic equations by reasoning. Next, students learn that equations of the form $(x - m)(x - n) = 0$ can be easily solved by applying the zero product property, which says that when two factors have a product of 0, one of the factors must be 0. When the equations are not in factored form, students rearrange them so that one side is 0, and rewrite the expressions from standard form to factored form. Students soon recognize that not all quadratic expressions in standard form can be rewritten into factored form. Even when it is possible, finding the right two numbers may be tedious, so another strategy is needed.

Students encounter perfect squares and notice that solving a quadratic equation is pretty straightforward when the equation contains a perfect square on one side and a number on the other. They learn that we can put equations into this helpful format by completing the square, that is, by rewriting the equation such that one side is a perfect square. Although this method can be used to solve any quadratic equation, it is not practical for solving all equations. This challenge motivates the quadratic formula.

Once introduced to the formula, students apply it to solve contextual and abstract problems, including those that they couldn't previously solve. After gaining some experience using the formula, students investigate how it is derived. They find that the formula essentially encapsulates all the steps of completing the square into a single expression. Just like completing the square, the quadratic formula can be used to solve any equation, but it may not always be the quickest method. Students consider how to use the different methods strategically.

Throughout the unit, students see that solutions to quadratic equations are often irrational numbers. Students reason about whether such sums and products are rational or irrational.

Toward the end of the unit, students revisit the vertex form and recall that it can be used to identify the maximum or minimum of a quadratic function. Previously students learned to rewrite expressions from vertex form to standard form. Now they can go in reverse—by completing the square. Being able to rewrite expressions in vertex form allows students to effectively solve problems about maximum and minimum values of quadratic functions.

In the final lesson, students integrate their insights and choose appropriate strategies to solve an applied problem and a mathematical problem (a system of linear and quadratic equations).

Stage 1: Desired Results - Key Understandings

Standard(s)

Common Core
Mathematics: 9-12

- Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS.MATH.CONTENT.HSA.SSE.A.1.A)
- Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (CCSS.MATH.CONTENT.HSA.SSE.A.2)
- Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS.MATH.CONTENT.HSA.SSE.B.3.A)

Transfer

Students will be able to independently use their learning to...
T1 Represent situations using mathematical reasoning and symbols

Meaning

Understanding(s)	Essential Question(s)
Students will understand that... **U1** Mathematicians make sense of quantities to represent situations mathematically.	*Students will keep considering...* **Q1** What do the quantities mean?

Stage 1: Desired Results - Key Understandings

- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS.MATH.CONTENT.HSA.SSE.B.3.B)
- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (CCSS.MATH.CONTENT.HSA.CED.A.1)
- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (CCSS.MATH.CONTENT.HSA.REI.A.1)
- Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. (CCSS.MATH.CONTENT.HSA.REI.B.4.A)
- Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. (CCSS.MATH.CONTENT.HSA.REI.B.4.B)
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. (CCSS.MATH.CONTENT.HSA.REI.C.7)
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (CCSS.MATH.CONTENT.HSA.REI.D.10)
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS.MATH.CONTENT.HSF.IF.B.4)
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS.MATH.CONTENT.HSF.IF.B.5)
- Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS.MATH.CONTENT.HSF.IF.C.7.A)

U2 Mathematicians attend to the underlying meaning of quantities and symbols.

Q2 How can the relationship between quantities be represented?

Acquisition of Knowledge and Skill

Knowledge

Students will know...

K1 Vocabulary: quadratic expression, factored form (of a quadratic expression), quadratic equation, standard form (of a quadratic expression), zero (of a function), zero product property, coefficient, constant term, linear term, perfect square, rational number, completing the square, irrational number, quadratic formula, vertex form (of a quadratic expression), maximum, minimum

K2 Quadratic equations may have one, two, or no solutions.

K3 Dividing by a variable to solve a quadratic equation is not an efficient strategy.

K4 How the numbers and signs in a quadratic expression in factored form relate to the numbers and signs in an equivalent expression in standard form.

K5 That the plus-minus symbol is used to represent both square roots of a number and that the square root notation expresses only the positive square root.

K6 Some methods for solving quadratic equations can be more efficient than others.

Skill(s)

Students will be skilled at...

S1 Finding the solutions to quadratic equations by reasoning about the values that make the equation true, using zero product property, and using the quadratic formula.

S2 Writing equations in standard form, when given in factored form and vice versa.

S3 Factoring conjugate binomials.

S4 Factoring equations in quadratic form, when a is 1 and when a is not 1.

S5 Recognizing perfect square expressions written in different forms.

S6 Completing the square and using it to solve quadratic equations.

S7 Rewriting equations in vertex form when given a quadratic expression in standard form.

S8 Finding the maximum or minimum of a function by writing the quadratic expression that defines it in vertex form.

Stage 1: Desired Results - Key Understandings		
• Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS.MATH.CONTENT.HSF.IF.C.8.A) • Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS.MATH.CONTENT.HSF.IF.C. 9) • Reason abstractly and quantitatively. (CCSS.MATH.MP.2) **Madison Public Schools Profile of a Graduate** • Analyzing: Examining information/data/ evidence from multiple sources to identify possible underlying assumptions, patterns, and relationships in order to make inferences. (POG.1.2)		