



Algebra 1 Unit 4: Functions

Unit Focus

In this unit, students expand and deepen their understanding of functions. They develop new knowledge and skills for communicating about functions clearly and precisely, investigate different kinds of functions, and hone their ability to interpret functions. Students also use functions to model a wider variety of mathematical and real-world situations.

The unit opens with a refresher on what functions are and what they are not. Students use descriptions, tables, and graphs to reason about the idea of “exactly one output for each input.” Then, students learn that function notation is an efficient way to communicate succinctly about functions and devote some focused time to interpret this new notation and use it. They continue this work throughout the unit, employing the notation to perform increasingly sophisticated mathematical work: to analyze and compare functions, to write rules of functions (primarily linear functions), to solve for an input, to graph functions, and more.

Next, students focus their attention on graphs of functions and on how they help to tell stories about the relationships between the quantities in the functions. Students interpret features of graphs and relate them to features of situations, using terms such as “maximum,” “minimum,” and “intercepts” to describe their observations. From a graph, students can see intervals where the values of a function increase or decrease. They learn to use average rates of change to more precisely describe how quickly these values rise or fall. Students also sketch graphs to depict qualitative behavior of functions.

Students then go on to take a closer look at the input and output of a function. They think about possible and reasonable input and output values and learn to identify the domain and range of a function based on contextual and graphical information. This new awareness of input and output in turn helps students make sense of piecewise-defined functions, in which different rules apply to different intervals of the domain, producing different sets of output values.

Two variations of piecewise functions are studied here: step functions and absolute value functions. The latter are introduced with the idea of absolute errors as an entry point. Thinking about “how far away from a value” primes students to regard the absolute value function as a distance function. The graph of such a function is a distinct V shape, which is convenient for noticing the graphical effects of changing an expression that defines a function.

Students close the unit by applying their insights about functions to model real-world situations and solve problems.

Stage 1: Desired Results - Key Understandings

Standard(s)	Transfer	
Common Core Mathematics: 9-12 - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (CCSS.MATH.CONTENT.HSA.REI.A.1) - Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where	<i>Students will be able to independently use their learning to...</i> T1 Apply models to solve problems.	
	Meaning	
	Understanding(s)	Essential Question(s)
	<i>Students will understand that...</i> U1 Mathematicians create or use models to generalize, represent, and solve problems.	<i>Students will keep considering...</i> Q1 What model best represents this problem? Q2 How can we apply knowledge of a pattern to similar problems?

Stage 1: Desired Results - Key Understandings

$f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* (CCSS.MATH.CONTENT.HSA.REI.D.11) - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (CCSS.MATH.CONTENT.HSF.IF.A.1) - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS.MATH.CONTENT.HSF.IF.A.2) - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSS.MATH.CONTENT.HSF.IF.B.4) - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (CCSS.MATH.CONTENT.HSF.IF.B.5) - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (CCSS.MATH.CONTENT.HSF.IF.B.6) - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS.MATH.CONTENT.HSF.IF.C.7.B) - Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS.MATH.CONTENT.HSF.BF.A.1.A) - Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS.MATH.CONTENT.HSF.BF.B.3) - Model with mathematics. (CCSS.MATH.MP.4) Madison Public Schools Profile of a Graduate - Idea Generation: Studying a problem, need or model (mentor text, political piece, documents, art work, etc.) to consider limitations and imagine new solutions/transformations. (POG. 2.1)	Acquisition of Knowledge and Skill	
	Knowledge	Skill(s)
	<i>Students will know...</i> K1 Vocabulary: dependent variable, function, independent variable, function notation, linear function, decreasing (function), horizontal intercept, increasing (function), maximum, minimum, vertical intercept, average rate of change, domain, range, piecewise function, absolute value, inverse (function) K2 When a relationship between two quantities is a function. K3 Different ways to find the value of a function and to solve equations written in function notation. K4 What makes a function a linear function. K5 What makes a function a piecewise function.	<i>Students will be skilled at...</i> S1 Identifying dependent and independent variables in a function. S2 Using function notation to express functions that have specific inputs and outputs and explain what it means in context. S3 Using statements in function notation to sketch a graph of a function. S4 Writing equations that represent the rules of functions. S5 Using technology to graph a function given in function notation, and use the graph to find the values of the function. S6 Identifying important features of graphs of functions and explain what they mean in the situation represented. S7 Estimating or calculating the average rate of change between two points when given the graph of a function. S8 Making sense of statements about two or more functions written in function notation. S9 Determining a reasonable domain and range for a function. S10 Calculating absolute value errors and creating a scatter plot of the data. S11 Describing the effects of adding a number to the expression that defines an absolute value function.