Grade 7 Pre-Algebra - Unit A - Proportional Drawings and Relationships

Unit Focus

In this unit, students study scaled copies of pictures and plane figures, then apply what they have learned to scale drawings (maps and floor plans). Students begin by looking at copies of pictures and use their own words to describe what differentiates scaled and non-scaled copies of a picture. As the unit progresses, students learn that all lengths in a scaled copy are multiplied by a scale factor and all angles stay the same. They draw scaled copies of figures. They learn that if the scale factor is greater than 1, the copy will be larger, and if the scale factor is less than 1, the copy will be smaller. Next, students see that the principles and strategies that they used to reason about scaled copies of figures can be used with scale drawings. They work with scales that involve units (e.g., "1 cm represents 10 km"), and scales that do not include units (e.g., "the scale is 1 to 100"). They understand that actual lengths are products of a scale factor and corresponding lengths in the scale drawing, thus lengths in the drawing are the product of the actual lengths and the reciprocal of that scale factor.

The second portion of this unit focuses on understanding what a proportional relationship is, how it is represented, and what types of contexts give rise to proportional relationships. In a table of equivalent ratios, a multiplicative relationship between the pair of rows is given by a scale factor. By contrast, the multiplicative relationship between the columns is given by a unit rate. The relationship between pairs of values in the two columns is called a proportional relationship, the unit rate that describes this relationship is called a constant of proportionality, and the quantity represented by the right column is said to be proportional to the quantity represented by the left. Students learn that any proportional relationship can be represented by an equation of the form y=kx where k is the constant of proportionality, that its graph lies on a line through the origin and that the constant of proportionality indicates the steepness of the line. By the end of the unit, students should be able to easily work with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether a relationship is proportional or not.

Throughout the unit, they discuss their mathematical ideas and respond to the ideas of others. In the culminating PBA of this unit, students make a floor plan of a room of their choice. This is an opportunity for them to apply what they have learned in the unit to everyday life.

Stage 1: Desired Results - Key Understandings

Standard(s)

Common Core
Mathematics: 7

- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour. (CCSS.MATH.CONTENT.7.RP.A.1)
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS.MATH.CONTENT.7.RP.A.2A)

Transfer

Students will be able to independently use their learning to...
T1 Apply models to solve problems.
T2 Construct viable arguments using clear and appropriate mathematical language and critique the reasoning of others.

Meaning

Understanding(s)

Students will understand that...
U1 Mathematicians construct viable arguments to explain problems, solutions, and mathematical representations.

Essential Question(s)

Students will keep considering...
Q1 Have I sufficiently supported my answer and shown my work?
Q2 How can the model created be tested and improved?

Stage 1: Desired Results - Key Understandings

- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS.MATH.CONTENT.7.RP.A.2B)
- Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. (CCSS.MATH.CONTENT.7.RP.A.2C)
- Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. (CCSS.MATH.CONTENT.7.RP.A.2D)
- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS.MATH.CONTENT.7.G.A.1)

Mathematics: 8

- Construct viable arguments and critique the reasoning of others. (CCSS.MATH.MP.3)
- Model with mathematics. (CCSS.MATH.MP.4)

Madison Public Schools Profile of a Graduate

- Collective Intelligence: Working respectfully and responsibly with others, exchanging and evaluating ideas to achieve a common objective. (POG.3.1)
- Product Creation: Effectively use a medium to communicate important information. (POG.3.2)

U2 Mathematicians can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
U3 Mathematicians create or use models to generalize, represent, and solve problems.

Q3 How can I represent this relationship?

Acquisition of Knowledge and Skill

Knowledge

Students will know...
K1 Vocabulary: area, corresponding, reciprocal, scale, scale drawing, scale factor, scaled copy, equivalent ratios, origin, proportional, relationship, constant of proportionality
K2 that corresponding sides of similar polygons are multiples of the scale factor.
K3 that corresponding angles of similar polygons are equal.
K4 how the magniture of the scale (less than 1, equal to 1, greater than 1) effects the size of the new scaled drawing compared to the original.
K5 that some scales involve units (1cm to 1 foot) while others do not (1 to 20)
K6 that graphs of proportional relationships are straight lines that cross the origin.
K7 the steepness of a proportional line gives information about the rate of change in the scenario.
K8 that proportional relationships can be modeled using an equation in the form y=kx

Skill(s)

Students will be skilled at...
S1 using all four operations on positive, rational numbers.
S2 identifying corresponding parts of similar polygons (with no rotations or reflections).
S3 drawing a scaled copy of a diagram given a scale factor.
S4 determining if a pair of polygons are similar given their dimensions.
S5 finding a missing side of a scaled drawing/ diagram given one side and a similar diagram.
S6 using a scale to convert between actual and diagram dimensions.
S7 using a ratio to complete the table, identify the constant of proportionality and write an equation to represent the proportional relationship.
S8 identifying whether two ratios are equivalent or not equivalent.
S9 determining what the constant of proportionality is and explaining its meaning in context.
S10 explaining what a point on a proportional graph represents in context.
S11 finding other points that are or are not on a graph of a proportional relationship given one point that is on the line.
S12 writing equations for proportional situations in the form y=kx.
S13 using an equation in the form y=kx to create a table or graph