



Unit Plan

6.9 Putting it All Together

Gateway Regional Middle School / Grade 6 / Mathematics

[Week 33 - Week 35](#) | 6 Curriculum Developers | Last Updated: Apr 8, 2024 by LeBlanc, Deanna[Style Guide](#)

What is the purpose of the unit? What are the major take-aways?

Standards

MA: Mathematics (2017)**MA: Grade 6****Mathematical Practice**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- 7. Look for and make use of structure. [Show Details](#)
- 8. Look for and express regularity in repeated reasoning. [Show Details](#)
- 4. Model with mathematics. [Show Details](#)
- 5. Use appropriate tools strategically. [Show Details](#)
- 6. Attend to precision. [Show Details](#)
- 1. Make sense of problems and persevere in solving them. [Show Details](#)
- 2. Reason abstractly and quantitatively. [Show Details](#)
- 3. Construct viable arguments and critique the reasoning of others. [Show Details](#)

Ratios & Proportional Relationships

6.RP Understand ratio and rate concepts and use ratio and rate reasoning to solve problems.

- 3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
- 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 1. Understand the concept of a ratio including the distinctions between part:part and part:whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. [Show Details](#)
- 2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship, including the use of units. [Show Details](#)

The Number System

6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. [Show Details](#)

6.NS Compute fluently with multi-digit numbers and find common factors and multiples.

- Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime numbers. [Show Details](#)
- 2. Fluently divide multi-digit numbers using the standard algorithm.
- 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Geometry

6.G Solve real-world and mathematical problems involving area, surface area, and volume.

- 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

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Enduring Understandings

Enduring Understandings for Unit 6.9 "Putting it All Together":

- Mathematical problems can be understood and solved by applying a variety of strategies, enabling perseverance and flexibility in finding solutions.
- Utilizing mathematical models helps to conceptualize and solve real-world problems, deepening the understanding of their implications.
- The strategic selection and application of appropriate tools, including digital resources, manipulatives, and mathematical practices, enhance the accuracy and efficiency of problem-solving.
- Mathematical reasoning involves making sense of quantities and their relationships in problem situations, allowing for abstract and quantitative analysis.
- Constructing arguments and critiquing the reasoning of others fosters a mathematical community where ideas are respectfully challenged and refined, leading to deeper comprehension and innovation in mathematics.

Essential Questions

- How can we apply mathematical concepts learned this year to solve real-world problems?
- How do different areas of mathematics connect and influence each other in problem-solving situations?
- In what ways can we use mathematics to make predictions or informed decisions in daily life?
- How do effective problem solvers choose and apply appropriate mathematical strategies?
- How can understanding patterns, relationships, and functions help us in making sense of the world around us?

Content

In this optional unit, students use concepts and skills from previous units. In solving Fermi problems, they use measurement conversions together with their knowledge of volumes or surface areas of right rectangular prisms or the relationship of distance, rate, and time. In answering questions about ratios of two populations, they work with percentages that include numbers expressed in the form a/b or as decimals. In answering questions about diagrams of rectangles with whole-number dimensions, they connect arithmetic features of the dimensions such as remainder or greatest common

Skills

factor with geometric features of the diagrams. In answering questions about votes, voting methods, and equitable distribution, they use their knowledge of equivalent ratios, part-part ratios, percentages, and unit rates.

This optional unit consists of six lessons. Each of the first three lessons is independent of the others, requiring only the mathematics of the previous units. The last three lessons build on each other.

The first lesson concerns Fermi problems—problems that require making rough estimates for quantities that are difficult or impossible to measure directly (MP4). The three problems in this lesson involve measurement conversion and calculation of volumes and surface areas of three-dimensional figures or the relationship of distance, rate, and time.

The second lesson involves finding approximately equivalent ratios for groups from two populations, one very large (the population of the world) and one comparatively small (a 30-student class). Students work with percent rates that describe subgroups of the world population, e.g., about 59% of the world population lives in Asia. Using these rates, which include numbers expressed in the form a/b or as decimals, they determine, for example, the number of students who would live in Asia—“if our class were the world” (MP2). Because students choose their own methods to determine these numbers, possibly making strategic use of benchmark percentages or spreadsheets (MP5), there is an opportunity for them to see correspondences between approaches (MP1). Because the size of the world population and its subgroups are estimates, and because pairs of values in ratios may both be whole numbers, considerations of accuracy may arise (MP6).

The third lesson is an exploration of the relationship between the greatest common factor of two numbers, continued fractions, and decomposition of rectangles with whole-number side lengths, providing students an opportunity to perceive this relationship through repeated reasoning (MP8) and to see correspondences between two kinds of numerical relationships, and between numerical and geometric relationships (MP1).

The remaining three lessons explore the mathematics of voting (MP2, MP4). In some activities, students chose how to assign votes and justify their choices (MP3). The first of these lessons focuses on proportions of voters and votes cast in elections in which there are two choices. It requires only the mathematics of the previous units, in particular, equivalent ratios, part-part ratios, percentages, unit rates, and, in the final activity, the concept of area. The second of these lessons focuses on methods for voting when there are more than two choices: plurality, runoff, and instant runoff. They compute percentages, finding that different voting methods have different outcomes. The third of these lessons focuses on representation in the case when voters have two choices. It's not always possible to have the same number of constituents per representative. How can we fairly share a small number of representatives? Students again compute percentages to find outcomes.

How will you gauge student learning?

Assessments

How will students learn?

Learning Activities

Making Connections	<p>Lesson 1 - Optional</p> <ul style="list-style-type: none"> Estimate quantities in a real-world situation and explain (orally and in writing) the estimation strategy. Justify (orally) why it is unreasonable to have an exact answer for a situation that involves estimation, and critique (orally) different estimates. Make simplifying assumptions and determine what information is needed to solve a Fermi problem about distance, volume, or surface area. <p>Lesson 2 - Optional</p> <ul style="list-style-type: none"> Apply reasoning about percentages and equivalent ratios to analyze and approximate characteristics of the world's population. Generate (orally and in writing) mathematical questions about the world's population, e.g., "How many people . . . ?" Present (using words and other representations) a comparison that uses the number of students in the class to represent the proportion of the world's population with a particular characteristic. <p>Lesson 3 - Optional</p> <ul style="list-style-type: none"> Coordinate diagrams and expressions involving equivalent fractions. Interpret and create diagrams involving a rectangle decomposed into squares. Recognize that decomposing rectangles into squares is a geometric way to determine the greatest common factor of two numbers.
Voting	<p>Lesson 4 - Optional</p> <ul style="list-style-type: none"> Apply reasoning about ratios and percentages to analyze (orally and in writing) voting situations involving two choices. Comprehend the terms "majority" and "supermajority" (in spoken and written language). Critique (using words and other representations) a statement reporting the results of a vote. <p>Lesson 5 - Optional</p> <ul style="list-style-type: none"> Apply reasoning about ratios and percentages to analyze (orally and in writing) voting situations involving more than two choices. Choose and justify (orally) which voting system seems the fairest for dealing with more than two choices. Compare and contrast (orally and in writing) different voting systems for dealing with more than two choices, i.e., plurality, runoff, and instant runoff. <p>Lesson 6 - Optional</p> <ul style="list-style-type: none"> Compare and contrast different ways to distribute representatives, and recognize that changing the way the votes are grouped can affect the outcome. Critique (orally and in writing) whether a method for distributing representatives is fair. Suggest a method for distributing representatives and justify (orally) why it is fair.
Section	Teacher-facing learning goals

Differentiated Instruction

Technology Integration

21st Century Skills

Positive Behavior

CASEL

Collaborative for Academic, Social, and Emotional Learning

Resources

Teacher Notes and Reflections
